

# Discussion on the distribution regularities and energy of electric field of electric charges

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**Abstract:** The paper, based on the relativity of electric potential and the new Gauss's law of electric field [1], has obtained the equation for the distribution of electric field of basic electric charges and the relational expression for the energy of electric field, i. e. the equation for energy of electric charges. It is indicated that net residual electric charges have energy much higher than atomic energy. At the same time, it has resolved the problem of the divergence of electrons and electric field induced by current Coulomb's law, and given new explanation for the concept of negative energy. It has enriched our understanding on energy. The equation for the relation between Planck's constant and electric potential has been gotten. The paper is the fifth one of the author's serial papers.

**Keywords:** relativity of electromotive space-time; relativity of electric potential; new Gauss's law of electric field; energy of electric charges, net residual energy of electric charges; equation for energy of electric charges; negative energy; negative kinetic energy; limit electric potential; limit of electric potential; equation for the relation between Planck's constant and electric potential

Suppose electron is a spherical symmetric elementary particle where no structure exists, and there is only one electron in the whole space. Suppose the electric potential on the sphere with the center of the electron as the center of circle and  $r$  as radius is  $\phi$ , and  $r = \infty$ ,  $\phi = 0$ . According to Gauss's law of electric field in a reference system with electric potential  $\phi$ [1]:

$$\oiint \mathbf{E}' \cdot d\mathbf{s}' = \frac{Q}{\epsilon_0} \left(1 - \frac{\phi^2}{\Phi_0^2}\right) \quad (1)$$

$$\frac{d\phi}{dr} = \frac{Q}{4\pi\epsilon_0 r^2} \left(1 - \frac{\phi^2}{\Phi_0^2}\right) \quad (2)$$

Because electric potential  $\phi$  is the function of  $r$ , suppose:

$$\frac{\phi}{\Phi_0} = f(r) \quad (3)$$

$$f'(r) = \frac{Q}{4\pi\epsilon_0\Phi_0} \frac{(1-f(r)^2)}{r^2} \quad (4)$$

$$\text{Solving this differential equation, we get: } \frac{\phi}{\Phi_0} = \tanh\left(\frac{Q}{4\pi\epsilon_0\Phi_0 r} + c\right) \quad (5)$$

When  $r = \infty$ ,  $\phi = 0$ , so,  $c = 0$ .

$$\text{Suppose } r_0 = \frac{Q}{4\pi\epsilon_0\Phi_0} \quad (6)$$

Therefore, the equation for electric potential distribution of the electron is:

$$\phi = \Phi_0 \tanh\left(\frac{r_0}{r}\right) \quad (7)$$

When  $r = 0$ ,  $\phi = \Phi_0$ . The electric potential at the center of the electron is the limit electric potential.

The equation for electric field distribution of the electron is:

$$E' = \frac{Q}{4\pi\epsilon_0 r^2} (1 - \tanh^2\left(\frac{r_0}{r}\right)) \quad (8)$$

When  $r \gg r_0$ ,  $\tanh^2\left(\frac{r_0}{r}\right) \approx 0$ , the above equation becomes the equation of Coulomb's electric field. When  $r \rightarrow 0$ , the intensity of electric field in the surrounding area of the electron center is zero, so, the equation has resolved the problem of electric field divergence of electron induced by Coulomb's law. We can calculate the total energy of electric field of an electron.

According to electromagnetics, we can get the formula to calculate the energy density of unit volume of electric field in the reference system with electric potential  $\phi$ :

$$w' = \frac{1}{2} \epsilon'_0 E'^2 \quad (9)$$

$w'$  is energy density of unit volume of electric field.

$E'$  is the intensity of electric field.

$\epsilon'_0$  is the vacuum dielectric constant.

Therefore, the energy of electric field of the electron is:

$$E_e = \int_0^\infty \frac{1}{2} \epsilon'_0 E'^2 dv \quad (10)$$

$V$  is the spatial volume of electric field distribution. That is,  $dv = 4\pi r^2 dr$

$E_e$  is the total energy of electric field.

Provided the relational formula for  $\epsilon'_0$  and  $\epsilon_0$ , we can calculate the total energy  $E_e$  of electric field of the electron. Suppose the electric potential of the observed reference system is  $\phi$  relative to the observing reference system, if there is a ideal plate capacitor with capacitance volume  $C'$  in the observed reference system, and its media is ideal vacuum,  $\epsilon'_0$  is the vacuum dielectric constant, when it is connected in series with a battery of voltage  $V'$ , the voltage of the two poles of the capacitor after recharge is also equal to  $V'$ , the electric quantity in the capacitor is  $Q'$ , its stored energy is  $E'$ .

$$E' = \frac{1}{2} Q' V'$$

Suppose in the reference system with zero electric potential,  $C$ ,  $V$ ,  $Q$ ,  $\epsilon_0$ ,  $E_e$  are respectively the capacitance, electric potential, electric charge, vacuum dielectric constant and energy of the capacitor, there is:

$$E_e = \frac{1}{2} QV$$

According to the relativity of electric potential [2] and Maxwell equations[1], it can be known:

$$Q' = \frac{\varepsilon'_0}{\varepsilon_0} Q \left(1 - \frac{\phi^2}{\Phi_0^2}\right) \quad (11)$$

$$V' = V \left(1 - \frac{\phi^2}{\Phi_0^2}\right)$$

$$\text{Therefore there is: } E_e' = E_e \frac{\varepsilon'_0}{\varepsilon_0} \left(1 - \frac{\phi^2}{\Phi_0^2}\right)^2 \quad (12)$$

At the same time when the recharged capacitor is disconnected with the battery, an ideal electric induction coil is connected in series, constituting an ideal LC electromagnetic oscillation circuit and transmitting electromagnetic radiation. From electromagnetics, it can be known that the frequency of electromagnetic wave is:

$$v = \frac{2\pi}{\sqrt{LC}}$$

$$v' = \frac{2\pi}{\sqrt{L'C'}}$$

Where, in the reference system with electric potential  $\phi$  and in the reference system with zero electric potential, the electric induction quantity of the coil is respectively  $L'$  and  $L$ , the frequency in LC electromagnetic oscillation circuit (that is the frequency of quantum of electromagnetic radiation) is respectively  $v'$  and  $v$ .

According to quantum mechanics and the law of conservation of energy, in the reference system with electric potential  $\phi$  and the reference system  $E_e$  with the radiation energy  $E_e'$  and zero electric potential, that is:

$$E_e' = n'v'h' \quad (13)$$

$$E_e = nvh \quad (14)$$

$h'$  and  $n'$  are respectively Planck's constant and the total quantum quantity of the reference system with electric potential  $\phi$ ,  $h$  and  $n$  are respectively Planck's constant of the reference system with zero electric potential the total quantum quantity of electromagnetic radiation. Because, the quantum quantity will not vary with the reference system, that is,  $n' = n$ , so there is:

$$E_e' = E_e \frac{v'h'}{vh} \quad (15)$$

From (12) and (15), we can obtain:

$$\frac{v'h'}{vh} = \frac{\varepsilon'_0}{\varepsilon_0} \left(1 - \frac{\phi^2}{\Phi_0^2}\right)^2 \quad (16)$$

And suppose Planck's constant will not vary with the reference system of electric potential, according to the relativity of electric potential and the new Maxwell equation[1], the following relational equation for the effect of electric potential and frequency can be known:

$$v' = v \sqrt{1 - \frac{\phi^2}{\Phi_0^2}}$$

Therefore, there is:

$$\frac{h'}{h} = \frac{\varepsilon'_0}{\varepsilon_0} \left(1 - \frac{\phi^2}{\Phi_0^2}\right)^{\frac{3}{2}} \quad (17)$$

If suppose in any inertial reference system of electric potential, electric charges are invariable, that is,  $Q' = Q$ , from(11), we get:

$$\varepsilon'_0 = \frac{\varepsilon_0}{1 - \frac{\phi^2}{\Phi_0^2}} \quad (18)$$

At the same time, from(17) and (18), we can obtain:

$$\frac{h'}{h} = \left(1 - \frac{\phi^2}{\Phi_0^2}\right)^{\frac{1}{2}} \quad (19)$$

It is can be seen that Planck's constant is related to electric potential, and Planck's constant is an important parameter of quantum mechanics. That is, the relativity of electric potential would make some modification for quantum mechanics. The details will be discussed in other papers.

Substituting(18) and (8) into (10), we get:

$$E_e = \frac{Q^2}{8\pi\varepsilon_0} \int_0^\infty \frac{1}{r^2} \left(1 - \tanh^2\left(\frac{r_0}{r}\right)\right) dr$$

$$E_e = \frac{\pi}{4} Q\Phi_0 \quad (20)$$

That is, the energy contained in electric field of electron is  $\frac{\pi}{4} Q\Phi_0$ , and whether the electron is positive or negative, its electric field is always positive. It is called net electric charge or energy of electric charges.

The value of limit electric potential should be rather big, if it is approximate to Planck's electric potential, then, the electric charge of electron is much higher than its mass energy. The electric field energy and the mass energy are completely different.

From the above calculation, in a cosmos, when a electron or a positive electron occurs as net residual electric charge, its energy corresponding to the system is positive  $\frac{\pi}{4} Q\Phi_0$ . While in past according to calculation of electromagnetics, this energy is infinitely big for electron or positive electron. The new Maxwell equations eliminate the problem of infinitely big energy of the electric field of electron, which has being boring the physics for many years. In a confined system, the magnitude of the electric field of electric charge is proportional to the magnitude of net residual electric charge in the system. That is, the more general equation for electric field energy of electric charge is:

$$E_e = k\Phi_0 Q \quad (21)$$

Where  $Q$  is the net residual basic electric charge,  $k$  is the proportional coefficient of which the magnitude depends on the relation between  $\varepsilon'_0$  and  $\varepsilon_0$ . In the above section, we have gotten  $k = \frac{\pi}{4}$ , it contains an assumption that the magnitude of electric charge is not related to the electric potential, which needs to be verified by experiment. If the condition is not tenable,  $k$  would change.

However, the annihilation experiment of positive and negative electrons pair indicates that the annihilation of positive and negative electron pair releases only the net mass energy of positive and negative electron pair. While the electric field of electric charge is not released. This runs counter to the law of conservation of energy. In classic electromagnetics and new electromagnetics, the electric field energy of electron exists objectively. Therefore to satisfy the experimental fact, theoretic calculation and the law of conservation of energy, the negative energy must exist between positive and negative electrons, as well the following relational formula:

$$2E_e + E_i = 2m_0c^2 \quad (22)$$

Where  $E_i$  is the negative energy,  $m_0$  is the net mass of electron,  $c$  is the velocity of light. It is the concept similar to “negative energy sea” in quantum mechanics. The difference is that here the negative energy is of finite magnitude:

$$E_i = -2k\Phi_0Q + 2m_0c^2 \quad (23)$$

We can analyze further this negative energy. When the positive and negative electron pair is relative to the distance  $r$ , their relative electric potential is  $\phi$ . According to the electromotive relativity, there is a relative virtual velocity  $V_i i = i \frac{\phi C_0}{\Phi_0}$  between them, the kinetic energy is:

$$\frac{1}{2} m_0 (V_i i)^2 = -\frac{1}{2} m_0 \left(\frac{\phi C_0}{\Phi_0}\right)^2 \quad (24)$$

That is, the essence of the negative energy is the negative kinetic energy when the mass has virtual velocity. Whether a electron is positive or negative, its kinetic energy is certainly negative. The concrete calculation depends on the process of annihilation of positive and negative electron. There is electric potential difference between the electron and the proton of hydrogen atom. Therefore there exists also negative energy.

However, for the positive electric charges ( or negative electric charges) with the identical sign, there is no electric potential difference between them, that is,  $\phi = 0$ , therefore, the negative energy is zero. The electric field of electrified particle can be manifested. Suppose the total energy of the particle is  $E_t$ , that is:

$$E_t = E_e + E_m = k\Phi_0Q + m_0c^2 \quad (25)$$

Where, the energy  $E_e$  of the net residual electric charge is the new energy higher than the mass energy  $E_m$ . The net residual electric charges would be a new inexhaustible energy for human being. At the same time, The net residual electric charges are of great cosmological significance, which will be discussed in other paper.

#### References:

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