On Covariant Relation between Relativity of Electric Potential and Maxwell Equations

Yingtao Yang

(Toronto, Canada, former employing institution: Xi'an Institute of China

Coal Technology and Engineering Group Corp)

yangyingtao2012@hotmail.com

Abstract: The paper discussed in detail the covariant relation between relativity of electric potential and Maxwell equations, deduced new Maxwell equations. At the same time, the equation of relation between the velocity of light and the electric potential has been got, and two new physic effects, i.e., lenticular effect and frequency effect of electric potential, have been found. In addition, the paper has deduced the relation between electric potential and electromagnetic physic quantity of capacitance, resistance inductance. It is indicated that the present Maxwell equations are the approximation of the new Maxwell equations and tenable only under conditions of low electric potential. The paper is the fourth one of the serial papers of the author.

Keywords: lunular effect of electric potential; frequency effect of electric potential; new Maxwell's equations; relativity of electric potential; covariant relation; equation of speed of light and electric potential.

According to the relativity of electromotive space-time[1], we know that the relativity of electric potential and general relativity are all special cases of the relativity of complex space-time, and general relativity and Maxwell's equation are covariant. Therefore it is necessary to discuss the relation between the relativity of electric potential and Maxwell's equation. From the relativity of electromotive space-time [1], it can be known that the basic equation of complex relativity is as follows:

$$\mathrm{R}_{\mathbf{w}}{}' = \mathrm{R}_{\mathbf{w}} - \mathrm{V}_{\mathrm{w}}\mathrm{t}_{\mathrm{w}} \tag{1}$$

$$\mathrm{t}' = \gamma\left(\mathrm{t} - \frac{|\mathrm{V}_{\mathrm{w}}|}{C_0{}^2}\mathrm{X}_1\right) \tag{2}$$

Where, $$\gamma = \frac{1}{\sqrt{1-\frac{|\mathrm{V}_{\mathrm{w}}|^2}{C_0{}^2}}} \tag{3}$$

$$\mathrm{t}_{\mathrm{w}} = \frac{(\mathrm{X}_1 - \mathrm{X}_2)}{|\mathrm{V}_{\mathrm{w}}|} \tag{4}$$

$$\mathrm{X}_2 = \gamma(\frac{\mathbf{R}_{\mathbf{p}} \cdot \mathbf{V}_{\theta}}{|\mathrm{V}_{\mathrm{w}}|} - |\mathrm{V}_{\mathrm{w}}|\mathrm{t}) \tag{5}$$

$$\mathrm{X}_1 = \mathrm{X}\cos\theta + \mathrm{F}\sin\theta \tag{6}$$

According to complex relativity, it can be known that when the movement velocity is equal to zero, its special case is the relativity of electric potential.

$$\mathrm{F}'i = \gamma\left(\mathrm{F}i - \frac{C_0}{\Phi_0}\phi i t\right) \tag{7}$$

$$X' = X \tag{8}$$

$$t' = \gamma\left(t - \frac{\phi}{\Phi_0 C_0}F\right) \tag{9}$$

$$\gamma = \frac{1}{\sqrt{1-\frac{\phi}{\Phi_0{}^2}^2}} \tag{10}$$

Because X' and X are taken arbitrarily, equation(8) indicates that the tri-dimensional spatial real length r', r in reference system of electric potential is invariable, so there is:

$$r' = r \tag{11}$$

Formula for time expansion: $\Delta t' = \gamma \Delta t$ (12)

Both sides of equation(7) is divided by imaginary number i, we can obtain Lorentz Transform in scalar form:

$$F' = \gamma\left(F - \frac{C_0}{\Phi_0}\phi t\right) \tag{13}$$

Lorentz transform of electric potential can be expressed by hyperbolic function:

$$F' = F\cosh(\varphi) - C_0 t\sinh(\varphi) \tag{14}$$

$$C_0 t' = -F\sinh(\varphi) + C_0 t\cosh(\varphi) \tag{15}$$

Where, $\cosh(\varphi) = \frac{1}{\sqrt{1-(\frac{\phi}{\Phi_0})^2}}$ (16)

Since, $\cosh^2(\varphi) - \sinh^2(\varphi) = 1$ (17)

From(16) and(17), we can obtain: $\tanh(\varphi) = \frac{\phi}{\Phi_0}$ (18)

Assume: $\varphi = \varphi_1 + \varphi_2$, $\frac{\phi_1}{\Phi_0} = \tanh(\varphi_1)$和$\frac{\phi_2}{\Phi_0} = \tanh(\varphi_2)\frac{\phi_1}{\Phi_0} = \tanh(\varphi_1)$ and $\frac{\phi_2}{\Phi_0} = \tanh(\varphi_2)$

Where, assume ϕ_1 as the electric potential of reference system 2 related to reference system 1, ϕ_2 as the electric potential of reference system 3 related to reference system 2. ϕ is the electric potential of reference system 3 related to reference system 1, that is the total electric potential after superposition of electric potential ϕ_1 and ϕ_2.

$$\frac{\phi}{\Phi_0} = \tanh(\varphi_1 + \varphi_2) \tag{19}$$

The formula for electric potential superposition can be obtained:

$$\phi = \frac{\phi_2 + \phi_1}{1 + \frac{\phi_1 \phi_2}{\Phi_0{}^2}} \tag{20}$$

In(18), what does the physical meaning of φ have? It will be discussed further in following section.

In series circuit of k completely same batteries, assume that the electric potential of the negative pole of the first battery is zero, the electric potential of its positive pole is ϕ_1, the electric potential of the positive pole of battery No. k is ϕ_k relative to the point of zero electric potential. There are two ways to measure the electric potential difference of the positive and negative poles of the battery No. k in the circuit, one way is to measure electric potential difference ΔV_k of the positive and negetive poles of the battery No.K by using a voltmete, it is not related to the amplitude of electric potential where the battery is situated, i.e., not related to the amplitude of K. ΔV_k is a constant. It is equivalent to the voltage the battery measured in the reference system of aero electric potential. The present electromagnetic theory calculates $\Delta\phi_k = \phi_{k+} - \phi_{k-}$， i.e. ，$\Delta\phi_k = \Delta V$. Assume that the total voltage of the circuit is V_k， and when k tends to be infinite,V_k may be infinite. So there is:

$$V_k = k\Delta V_k \tag{21}$$

Another way is to measure the electric potential relative to zero electric potential of the negative and positive pole of the battery No. k, its electric potential value is ϕ_{k-} 和 ϕ_{k+}. If assume $\phi = \phi_{k+}$， $\phi_1 = \phi_{k-}$， $\phi_2 = \Delta\phi_k$， according to the calculation of formula(20) for superposition of the relativity of electric potential, the amplitude of $\Delta\phi_k$ is not a constant, and closely related to the amplitude of ϕ_{k-} 和 ϕ_{k+}. That is, $\Delta\phi_k$ the voltage of the battery measured in different reference systems of electric potential. From the formula for calculation of electric potential of the relativity of electric potential(18) we obtain：

$$\phi = \frac{\Delta\phi_k + \phi_1}{1 + \frac{\phi_1 \Delta\phi_k}{{\Phi_0}^2}} \tag{22}$$

When $\Delta\phi_k$ tends to be zero or $\Phi_0 = \infty$， the above formula becomes $\phi = \Delta\phi_k + \phi_1$， i.e. ,

$$\Delta\phi_k = \Delta V_k \tag{23}$$

According to(18), it can be obtained：

$$\phi = \Phi_0 \tanh(\varphi)$$

Assume $\varphi = k\Delta\varphi_k$ (24)

$$\Delta\phi_k = \Phi_0 \tanh(\Delta\varphi_k) \tag{25}$$

From(21) and(24), the functional relation between $V_k$and $\varphi$$V_k$和 φ can be obtained：

$$\varphi = \frac{V_k}{\Delta V_k} \Delta\varphi_k$$

When ϕ remains invariant， the quantity of batteries k tends to be infinite,

ϕ_{k-} and ϕ_{k+} $\Delta\varphi_k$和$\Delta\phi_k$ tend to be zero. Therefore from equation(23), there is:

$$\Delta V_k = \Phi_0 \tanh(\Delta\varphi_k) \tag{26}$$

Substituting it into the above equation to search for the limit：

$$\varphi = \lim_{\Delta\varphi_k \to 0} \frac{V_k}{\Phi_0 \tanh(\Delta\varphi_k)} \Delta\varphi_k = \frac{V_k}{\Phi_0} \tag{27}$$

Therefore it can be known that the physical meaning of the angle of hyperbolic function in equation(18). V_k is defined as the absolute electric potential, it is the relative electric potential when the limit electric potential Φ_0 is infinite.

$$\frac{\phi}{\Phi_0} = \tanh\left(\frac{V_k}{\Phi_0}\right) \tag{28}$$

By searching the derivative for the above formula, we obtain：

$$\frac{d\phi}{dV_k} = 1 - \frac{\phi^2}{{\Phi_0}^2} \tag{29}$$

In the above equation,dV_k is the micro voltage of the micro battery No. k in the reference system with zero electric potential,$d\phi$ is the micro voltage of the battery in the reference system with electric potential ϕ. If the same parallel polar plate vacuum dielectric capacitor C is connected parallels to both ends of each battery, then，the voltage at two ends of the capacitor and the voltage of the battery are completely equal. So there is：

$$\frac{d\phi}{dV_k} = \frac{d{V_c}'}{dV_c} = 1 - \frac{\phi^2}{{\Phi_0}^2} \tag{30}$$

Where， dV_c and $d{V_c}'$$dV_c$和 dV_c' are respectively the voltage differential of the same capacitor in the reference system with zero electric potential and the reference system with electric potential ϕ， equation(30) have universal significance, therefore if it is assumed that V'和 V $V'$and V are respectively the electric potential difference in the reference system with electric potential ϕ and the electric potential in the reference system with zero electric potential, there is:

$$\frac{V'}{V} = 1 - \frac{\phi^2}{{\Phi_0}^2} \tag{31}$$

According to the principle of relativity, the voltage, the capacitance and electric charge of the same capacitor in the reference system with zero electric potential and reference system with electric potential ϕ must meet satisfy the same relational formula:

$$dV_c = \frac{Q}{C} \text{ and } d{V_c}' = \frac{Q'}{C'} \tag{32}$$

dV_c，Q，C，S，D，ε_0 are respectively the voltage, the electric charge, the capacitance, the area of electric pole, the distance of two parallel polar plates of the capacitor in the reference system with zero electric potential, they are all vacuum dielectric constants.。

$d{V_c}'$，Q'，C'，S'，D'，${\varepsilon_0}'$ are respectively the voltage, the electric charge, the capacitance, the area of electric pole, the distance of two parallel polar plates of the capacitor in the reference system with electric potentialϕ, they are all vacuum dielectric constants.

Therefore there is $d{V_c}' = \frac{CQ'}{C'Q} dV_c$ (33)

By calculating the relation between the capacitance and the geometric size of the capacitor, we can obtain：

$$C = \frac{S\varepsilon_0}{D},$$

$$C' = \frac{S'{\varepsilon_0}'}{D'}$$

From(11) we can obtain：Under any electric potential, the tri-dimensional geometric size is invariable, i.e., $\mathrm{D} = \mathrm{D}'$， $\mathrm{S} = \mathrm{S}'$.So there is:

$$\frac{\mathrm{C}}{\varepsilon_0} = \frac{\mathrm{C}'}{\varepsilon_0{}'} \tag{34}$$

By substituting (34) into(33), we can obtain：

$$\frac{\mathrm{dV_c}'}{\mathrm{dV_c}} = \frac{(\frac{\mathrm{Q}'}{\varepsilon_0{}'})}{(\frac{\mathrm{Q}}{\varepsilon_0})}$$

Therefore， $\frac{(\frac{\mathrm{Q}'}{\varepsilon_0{}'})}{(\frac{\mathrm{Q}}{\varepsilon_0})} = (1 - \frac{\phi^2}{\Phi_0{}^2})$ (35)

Evidently， equation(35) is contradictory to Gauss's law in Maxwell's electromagnetic theory. In Gauss's law, the right side of equation(35) must be equal to 1. Therefore, Maxwell equation must be modified correspondingly.

Maxwell equation in vacuum[2]：

$$\oiint \mathbf{E} \cdot \mathrm{d}\mathbf{s} = \frac{\mathrm{Q}}{\varepsilon_0} \tag{36}$$

$$\oint \mathbf{E} \cdot \mathrm{d}\boldsymbol{l} = -\iint \frac{\partial \mathbf{B}}{\partial \mathrm{t}} \cdot \mathrm{d}\mathbf{s} \tag{37}$$

$$\oiint \mathbf{B} \cdot \mathrm{d}\mathbf{s} = 0 \tag{38}$$

$$\oint \mathbf{B} \cdot \mathrm{d}\boldsymbol{l} = \mu_0 \mathrm{I} + \mu_0 \varepsilon_0 \iint \frac{\partial \mathbf{E}}{\partial \mathrm{t}} \cdot \mathrm{d}\mathbf{s} \tag{39}$$

1. Gauss's law of electric field

According to relativistic principle of electric potential, for the reference system with zero electric potential and the reference system with electric potentialϕ, the equations have the same form：

$$\oiint \mathbf{E} \cdot \mathrm{d}\mathbf{s} = \frac{\mathrm{Q}}{\varepsilon_0} \tag{40}$$

$$\oiint \mathbf{E}' \cdot \mathrm{d}\mathbf{s}' = \frac{\mathrm{Q}'}{\varepsilon_0{}'} \tag{41}$$

For the same reason, for the reference system with zero electric potential and the reference system with electric potential ϕ, the intensity of electric field is respectively：

$$\mathbf{E} = \frac{\mathrm{Q}}{4\pi\varepsilon_0 \mathbf{r}^2} \tag{42}$$

$$\mathbf{E}' = \frac{\mathrm{Q}'}{4\pi'\varepsilon_0' \mathbf{r}'^2} \tag{43}$$

From the relational expression for electric potential transform, it is known that tri-dimensional geometric size is not related to electric potential, so there is: $\mathbf{r} = \mathbf{r}'$, $\pi = \pi'$

Therefore $\frac{\mathbf{E}'}{\mathbf{E}} = \frac{(\frac{Q'}{\varepsilon_0'})}{(\frac{Q}{\varepsilon_0})}$ (44)

From equation(35) we obtain the relation between the intensity of electric field and the electric potential of the reference system:

$$\mathbf{E}' = \mathbf{E}(1 - \frac{\phi^2}{{\Phi_0}^2}) \tag{45}$$

Therefore after integration of both sides of equation (45), Gauss's law is:

$$\oiint \mathbf{E}' \cdot \mathrm{d}\mathbf{s}' = \oiint \mathbf{E}(1 - \frac{\phi^2}{{\Phi_0}^2}) \cdot \mathrm{d}\mathbf{s}' \tag{46}$$

Since electric potential is not related to tri-dimensional geometric size, i.e. $\mathrm{d}\mathbf{s} = \mathrm{d}\mathbf{s}'$，the modified Gauss's law becomes：

$$\oiint \mathbf{E}' \cdot \mathrm{d}\mathbf{s}' = (1 - \frac{\phi^2}{{\Phi_0}^2}) \oiint \mathbf{E} \cdot \mathrm{d}\mathbf{s} \tag{47}$$

So there is： $\oiint \mathbf{E}' \cdot \mathrm{d}\mathbf{s}' = (1 - \frac{\phi^2}{{\Phi_0}^2}) \ \frac{Q}{\varepsilon_0}$ (48)

Thus it can be seen that the amplitude of electric flux is related to the amplitude of electric potential of the reference system, Gauss's law of electric field in Maxwell equation is only a particular case in equation (48) when $\phi = 0$.

2. Gauss's law of magnetic field

According to relativistic principle of electric potential, there is:

$$\oiint \mathbf{B} \cdot \mathrm{d}\mathbf{s} = 0$$

$$\oiint \mathbf{B}' \cdot \mathrm{d}\mathbf{s}' = 0 \tag{49}$$

Therefore, it is not necessary to modify Gauss's law for magnetics. However magnetic induction intensity is related to electric potential. Suppose the magnetic induction intensity in infinitely long spiral coil in the reference system F with zero electric potential and that in the reference system F′ with electric potential ϕ are respectively B 和 B′，there is：

$$\mathrm{B} = \mu_0 \mathrm{nI} \tag{50}$$

$$\mathrm{B}' = \mu_0' \mathrm{n}' \mathrm{I}' \tag{51}$$

$\mu_0, \mathrm{n}, \mathrm{I}$ and $\mu_0', \mathrm{n}', \mathrm{I}'$ are respectively magnetic induction coefficient, circles of coil and current intensity of reference systems F and F′. Then there is:

$$\frac{\mathrm{B}'}{\mathrm{B}} = \frac{\mu_0' \mathrm{n}' \mathrm{I}'}{\mu_0 \mathrm{nI}} \tag{52}$$

From equations(12) and(35) , we can obtain：

$$\frac{\mathrm{I}'}{\mathrm{I}} = \frac{\varepsilon_0'}{\varepsilon_0}(1 - \frac{\phi^2}{{\Phi_0}^2})^{\frac{3}{2}} \tag{53}$$

Because geometric parameters are not related to electric potential, therefore $\mathrm{n}' = \mathrm{n}$，and $\mathbf{B}'$和$\mathbf{B}$ have the same direction.

$$\frac{\mathbf{B}'}{\mathbf{B}} = \frac{\mu_0' I'}{\mu_0 I} \tag{54}$$

$$\frac{\mathbf{B}'}{\mathbf{B}} = \frac{\varepsilon_0' \mu_0'}{\varepsilon_0 \mu_0} (1 - \frac{\phi^2}{{\Phi_0}^2})^{\frac{3}{2}} \tag{55}$$

3. Ampere's loop law of electric field：

$$\oint \mathbf{E} \cdot \mathrm{d}\boldsymbol{l} = -\iint \frac{\partial \mathbf{B}}{\partial \mathrm{t}} \cdot \mathrm{d}\mathbf{s} \tag{56}$$

$$\oint \mathbf{E}' \cdot \mathrm{d}\boldsymbol{l}' = -\iint \frac{\partial \mathbf{B}'}{\partial \mathrm{t}'} \cdot \mathrm{d}\mathbf{s}' \tag{57}$$

Substituting (55) and(12) into(57)，and because $\mathrm{d}\mathbf{s} = \mathrm{d}\mathbf{s}'$ **,** we can get：

$$\oint \mathbf{E}' \cdot \mathrm{d}\boldsymbol{l}' = -\frac{\varepsilon_0' \mu_0'}{\varepsilon_0 \mu_0} (1 - \frac{\phi^2}{{\Phi_0}^2})^2 \iint \frac{\partial \mathbf{B}}{\partial \mathrm{t}} \cdot \mathrm{d}\mathbf{s} \tag{58}$$

Substituting(56) into(58), we obtain：

$$\oint \mathbf{E}' \cdot \mathrm{d}\boldsymbol{l}' = \frac{\varepsilon_0' \mu_0'}{\varepsilon_0 \mu_0} (1 - \frac{\phi^2}{{\Phi_0}^2})^2 \oint \mathbf{E} \cdot \mathrm{d}\boldsymbol{l} \tag{59}$$

From(45), we obtain： $\oint \mathbf{E}' \cdot \mathrm{d}\boldsymbol{l}' = (1 - \frac{\phi^2}{{\Phi_0}^2}) \oint \mathbf{E} \cdot \mathrm{d}\boldsymbol{l}$ (60)

$$\frac{\varepsilon_0' \mu_0'}{\varepsilon_0 \mu_0} = \frac{1}{(1 - \frac{\phi^2}{{\Phi_0}^2})} \tag{61}$$

Substituting the above equation into (55) and (59), we get：

$$\frac{\mathbf{B}'}{\mathbf{B}} = (1 - \frac{\phi^2}{{\Phi_0}^2})^{\frac{1}{2}} \tag{62}$$

Substituting (61) into(58), we obtain the modified Ampere's loop law of electric field:

$$\oint \mathbf{E}' \cdot \mathrm{d}\boldsymbol{l}' = -(1 - \frac{\phi^2}{{\Phi_0}^2}) \iint \frac{\partial \mathbf{B}}{\partial \mathrm{t}} \cdot \mathrm{d}\mathbf{s} \tag{63}$$

Because the speed of light of the reference system with zero electric potential $C_0 = \frac{1}{\sqrt{\varepsilon_0 \mu_0}}$, the optical velocity of the reference system with electric potential ϕ is ${C_0}' = \frac{1}{\sqrt{\varepsilon_0' \mu_0'}}$, From(61) we get speed of light—electric potential equation：

$$ {C_0}' = C_0 \sqrt{(1 - \frac{\phi^2}{{\Phi_0}^2})} \tag{64}$$

That is, the speed of light is related to the electric potential of the reference system（${C_0}' \leq C_0$）. In fact, it reflects the basic assumption that there exists a limit in the module of electromotive velocity. Because the speed of light in Maxwell equation is the speed of light in real space, it is the limit of object movement in real space. When the electric potential of electromotive reference system is not zero, it has virtual velocity $V_\phi i$，the limit of the real velocity $C_0{}'$ must become smallerto ensure that the module of complex velocity is equal to C_0. From(64) it can be known that they satisfy：

$$C_0{}'^2 + V_\phi{}^2 = C_0{}^2$$

Where，$V_\phi = C_0 \frac{\phi}{\Phi_0}$

It indicates that there exists certainly refraction effect of electric potential, i.e., lunular effect of electric potential. When light rays pass nearby an object, because electric potential continues to increase with the decrease of r， inducing the decrease of speed of light with the decrease of r，therefore light path will be a bending line, the same as the lunular effect of gravity.

4. Law of magnetic loop：

According to the principle of relativity of electric potential, there is:

$$\oint \mathbf{B} \cdot d\boldsymbol{l} = \mu_0 I + \varepsilon_0 \mu_0 \iint \frac{\partial \mathbf{E}}{\partial t} \cdot d\mathbf{s} \tag{65}$$

$$\oint \mathbf{B}' \cdot d\boldsymbol{l}' = \mu_0' I' + \varepsilon_0' \mu_0' \iint \frac{\partial \mathbf{E}'}{\partial t'} \cdot d\mathbf{s}' \tag{66}$$

From(54) and (62) we get：

$$\frac{\mu_0' I'}{\mu_0 I} = (1 - \frac{\phi^2}{\Phi_0{}^2})^{\frac{1}{2}} \tag{67}$$

Substituting(67)(45)(12)(61) into(66)，and $d\mathbf{s} = d\mathbf{s}'$， we get the modified law magnetic loop：

$$\oint \mathbf{B}' \cdot d\boldsymbol{l}' = (1 - \frac{\phi^2}{\Phi_0{}^2})^{\frac{1}{2}} (\mu_0 I + \varepsilon_0 \mu_0 \iint \frac{\partial \mathbf{E}}{\partial t} \cdot d\mathbf{s}) \tag{68}$$

$$\oint \mathbf{B}' \cdot d\boldsymbol{l}' = (1 - \frac{\phi^2}{\Phi_0{}^2})^{\frac{1}{2}} \oint \mathbf{B} \cdot d\boldsymbol{l} \tag{69}$$

Equations(47)(49)(63)(68) constitute the form of integration in Maxwell equations. It is easy to get its form of differential by mathematic transform:

$$\nabla \cdot \mathbf{E}' = \frac{\rho_0}{\varepsilon_0} \left(1 - \frac{\phi^2}{\Phi_0{}^2}\right) \tag{70}$$

$$\nabla \times \mathbf{E}' = -(1 - \frac{\phi^2}{\Phi_0{}^2}) \frac{\partial \mathbf{B}}{\partial t} \tag{71}$$

$$\nabla \cdot \mathbf{B}' = 0 \tag{72}$$

$$\nabla \times \mathbf{B}' = \left(1 - \frac{\phi^2}{\Phi_0{}^2}\right)^{\frac{1}{2}} \left(\mu_0 \boldsymbol{j}_0 + \varepsilon_0 \mu_0 \frac{\partial \mathbf{E}}{\partial t}\right) \tag{73}$$

Where，ρ_0 is density of electric charge，$\boldsymbol{j}_0$ is current density.

Application of new Maxwell equations：

(1) Suppose the resistance in the reference system with zero electric potential is R and the resistance in the reference system with electric potential ϕ is R'， there is:

$$R = \frac{V}{I} \tag{74}$$

$$R' = \frac{V'}{I'} \tag{75}$$

Substituting(31)(53)(74) into(75), we get：

$$R' = \frac{\varepsilon_0}{\varepsilon_0'}(1 - \frac{\phi^2}{\Phi_0{}^2})^{-\frac{1}{2}} R \tag{76}$$

(2) Suppose the capacitance in the reference system with zero electric potential is C and the capacitance in the reference system with electric potential ϕ is C'， there is：

$$C' = \frac{\varepsilon_0'}{\varepsilon_0} C \tag{77}$$

(3) Suppose the electric inductance in the reference system with zero electric potential is L and the electric inductance in the reference system with electric potential ϕ is L'， there is:

$$L = -\frac{V}{(\frac{dI}{dt})} \tag{78}$$

$$L' = -\frac{V'}{(\frac{dI'}{dt'})} \tag{79}$$

Substituting (31)(53)(12)(78) into(79), we get：

$$L' = \frac{\varepsilon_0}{\varepsilon_0'} \frac{L}{(1 - \frac{\phi^2}{\Phi_0{}^2})} \tag{80}$$

(4) Suppose the frequency of LC circuit in the reference system with zero electric potential is f and the frequency in the reference system with electric potential ϕ is ν'， there is：

$$\nu = \frac{2\pi}{\sqrt{LC}} \tag{81}$$

$$\nu' = \frac{2\pi}{\sqrt{L'C'}} \tag{82}$$

Substituting (80)(77)(81) into(82), we get：

$$\nu' = \frac{2\pi}{\sqrt{L'C'}} = \nu\sqrt{1 - \frac{\phi^2}{\Phi_0{}^2}} \tag{83}$$

From (83) it can be known that the frequency of electromagnetic wave is related to electric potential. That is, the spectral frequency of an atom in the reference system with high electric potential is lower than that the reference system with zero electric potential, we call it frequency effect of electric potential. At the same time, equation(83) can be transformed into equation (12)， which shows that equations are consistent and harmonious.

Suppose in RC circuit and in the reference system with zero electric potential, time constant is t， resistance R， voltage V， current I， capacitance C, and in the reference system with electric potential ϕ, time constant t'’ resistance R', voltage V', current I', capacitance C',

$$RC = t$$

$$R'C' = t' \tag{84}$$

According to Ohm's law $R' = \frac{V'}{I'}$, substituting (31)(53)(34) into(84) we get：

$$t' = (1 - \frac{\phi^2}{\Phi_0{}^2})^{-\frac{1}{2}}\, t \tag{85}$$

(85) and (12) are equivalent.

References:

[1] Yingtao Yang, The relativity of electromotive space-time, National Science and Technology Library, www.nstl.gov.cn/preprint

[2] Zhao Kaihua, Chen Ximu, Electromagnetics, Publising House of High Education, December 2006