The resurfacing of Rindler's coordinate theory

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ABSTRACT

In the general relativity theory, in the Rindler's coordinate theory, find the present accelerated theory's problem, resurface Rindler's coordinate theory that used the tetrad on the new method.The theory's concept is that save Rindler's coordinate transformation in the other way.

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I.Introduction

This theory is that studying the Rindler's coordinate theory, find the theory's problem and resurface the Rindler's coordinate theory.

Finding the Rindler's coordinate theory , use following the formula about the constant accelerated matter.

$$x+\frac{c^2}{a_0}=\frac{c^2}{a_0}\cosh(\frac{a_0\tau}{c}),t=\frac{c}{a_0}\sinh(\frac{a_0\tau}{c}) \quad (1)$$

x and t is the coordinate and the time in the inertial system about the constant accelerated matter. a_0 is the constant acceleration, τ is invariable time about the constant accelerated matter, c is light speed in the inertial system in the free space-time.

The tetrad $e_a{}^{\mu}$ is the unit vector that is each other orthographic and it used the following formula.

$$e_a{}^{\mu}e_b{}^{\upsilon}g_{\mu\upsilon}=\eta_{ab} \quad (2)$$

$e^a{}_{\mu}$ is

$$e^a{}_{\mu}=\eta^{ab}g_{\mu\upsilon}e_b{}^{\upsilon} \quad (3)$$

and it is $e_a{}^{\mu}$'s inverse-metrix.And it is

$$e^a{}_{\mu}e_b{}^{\mu}=\delta^a{}_b \ , \ e^a{}_{\mu}e_a{}^{\upsilon}=\delta_{\mu}{}^{\upsilon}$$

$$e^a{}_{\mu}e^b{}_{\upsilon}\eta_{ab}=g_{\mu\upsilon} \quad (4)$$

According to the tetrad $e^a{}_{\mu}$,the flat Minkowski space's inertial coordinate system transform the accelerated system ξ

$$d\tau^2=dt^2-\frac{1}{c^2}[dx^2+dy^2+dz^2]$$

$$=-\frac{1}{c^2}\eta_{ab}\frac{\partial x^a}{\partial\xi^{\mu}}\frac{\partial x^b}{\partial\xi^{\upsilon}}d\xi^{\mu}d\xi^{\upsilon} \quad (5)$$

$$=-\frac{1}{c^2}\eta_{ab}e^a{}_{\mu}e^b{}_{\upsilon}d\xi^{\mu}d\xi^{\upsilon}=-\frac{1}{c^2}g_{\mu\upsilon}d\xi^{\mu}d\xi^{\upsilon} \quad (6) \quad e^a{}_{\mu}=\frac{\partial x^a}{\partial\xi^{\mu}} \quad (7)$$

$e^{\alpha}{}_{\mu}(\tau)$ is the tetrad that if $\xi^1=\xi^2=\xi^3=0$, $d\xi^1=d\xi^2=d\xi^3=0$ .It is not the accelerated system and it is the point's the accelerate motion. Therefore $\xi^0=\tau$, in this case, it is in the flat Minkowski space's inertial coordinate system therefore it does $g_{\mu\upsilon}=\eta_{\mu\upsilon}$,According to Eq (1),Eq (6),Eq(7)

$$e^{\alpha}{}_{0}(\tau)=\frac{\partial x^{\alpha}}{c\partial\xi^{0}}=\frac{1}{c}\frac{dx^{\alpha}}{d\tau}=(\cosh(\frac{a_0\tau}{c}),\sinh(\frac{a_0\tau}{c}),0,0) \quad (8)$$

About y -axis's and z -axis's orientation

$e^{\alpha}{}_{2}(\tau)=(0,0,1,0)$ (9), $e^{\alpha}{}_{3}(\tau)=(0,0,0,1)$ (10)

and the other unit vector $e^{\alpha}{}_{1}(\tau)$ is that satisfy tetrad condition, Eq (4)

$$e^{\alpha}{}_{1}(\tau)=(\sinh(\frac{a_0\tau}{c}),\cosh(\frac{a_0\tau}{c}),0,0) \quad (11)$$

According to the accelerated system, $e^{\alpha}{}_{\mu}(\xi^{0})$ is used by Eq (8),Eq (9),Eq (10),Eq(11) that used ξ^{0} instead of τ and it is the accelerated system .By Eq (7)

$$e^{\alpha}{}_{0}(\xi^{0})=\frac{\partial x^{\alpha}}{c\partial\xi^{0}}=(\cosh(\frac{a_0\xi^{0}}{c}),\sinh(\frac{a_0\xi^{0}}{c}),0,0) \quad (12)$$

About y -axis's and z -axis's orientation

$e^{\alpha}{}_{2}(\xi^{0})=\frac{\partial x^{\alpha}}{\partial\xi^{2}}=(0,0,1,0)$ (13), $e^{\alpha}{}_{3}(\xi^{0})=\frac{\partial x^{\alpha}}{\partial\xi^{3}}=(0,0,0,1)$ (14)

and the other unit vector $e^{\alpha}{}_{1}(\xi^{0})$ is

$$e^{\alpha}{}_{1}(\xi^{0})=\frac{\partial x^{\alpha}}{\partial\xi^{1}}=(\sinh(\frac{a_0\xi^{0}}{c}),\cosh(\frac{a_0\xi^{0}}{c}),0,0) \quad (15)$$

In the Rindler's coordinate theory, the inertial system x^{α}'s and the accelerated system ξ^{α}'s the coordinate transformation

$$x^{\alpha}=\xi^{1}e^{\alpha}{}_{1}(\xi^{0})+\xi^{2}e^{\alpha}{}_{2}(\xi^{0})+\xi^{3}e^{\alpha}{}_{3}(\xi^{0})+z^{\alpha}(\xi^{0}) \quad (16)$$

ξ^{1},ξ^{2},ξ^{3} is the accelerated system ξ^{α}'s coordinate, $z^{\alpha}(\xi^{0})$ is the distance that is the origin of the accelerated system ξ^{α} in the inertial system.

Therefore in Eq (16) used Eq (1) and Eq (13),Eq (14),Eq (15),finally the Rindler's coordinate transformation is found.

$ct=(\frac{c^{2}}{a_0}+\xi^{1})\sinh(\frac{a_0\xi^{0}}{c})$ (17)

$x=(\frac{c^{2}}{a_0}+\xi^{1})\cosh(\frac{a_0\xi^{0}}{c})-\frac{c^{2}}{a_0}$ (18)

$y=\xi^{2}, z=\xi^{3}$ (19)

II. Additional chapter-I

In Eq (12), $\frac{c\partial t}{c\partial\xi^{0}}=\cosh(\frac{a_0\xi^{0}}{c})$ (20), $\frac{\partial x}{c\partial\xi^{0}}=\sinh(\frac{a_0\xi^{0}}{c})$ (21),

In Eq (15), $\frac{c\partial t}{\partial\xi^{1}}=\sinh(\frac{a_0\xi^{0}}{c})$ (22), $\frac{\partial x}{\partial\xi^{1}}=\cosh(\frac{a_0\xi^{0}}{c})$ (23)

And in Eq (13),Eq(14), it is $\frac{c\partial t}{\partial\xi^{2}}=\frac{c\partial t}{\partial\xi^{3}}=0$, $\frac{\partial x}{\partial\xi^{2}}=\frac{\partial x}{\partial\xi^{3}}=0$

According to Eq (20) and Eq (21),

$$ct = \frac{c^2}{a_0}\sinh(\frac{a_0\xi^0}{c}) + f(\xi^1),\ \ x = \frac{c^2}{a_0}\cosh(\frac{a_0\xi^0}{c}) + g(\xi^1) \quad (24)$$

$f(\xi^1), g(\xi^1)$ is the function of only ξ^1

According to Eq (22) and Eq (23),

$$ct = (\xi^1 + h(\xi^0))\sinh(\frac{a_0\xi^0}{c}) + C_1, x = (\xi^1 + i(\xi^0))\cosh(\frac{a_0\xi^0}{c}) + C_2 \quad (25)$$

$h(\xi^0), i(\xi^0)$ is the function of only ξ^0, C_1, C_2 is a constant number.

In this time, If Eq(24) is the right formula, for saving the new $e^\alpha{}_1$,

$$\frac{\partial e^\alpha{}_0}{\partial\xi^1} - \frac{\partial e^\alpha{}_1}{c\partial\xi^0} = \frac{\partial}{\partial\xi^1}(\frac{\partial x^\alpha}{c\partial\xi^0}) - \frac{\partial}{c\partial\xi^0}(\frac{\partial x^\alpha}{\partial\xi^1}) = 0, e^\alpha{}_0 = \frac{\partial x^\alpha}{c\partial\xi^0}, e^\alpha{}_1 = \frac{\partial x^\alpha}{\partial\xi^1}$$

$$e^\alpha{}_0(\xi^0) = \frac{\partial x^\alpha}{c\partial\xi^0} = (\cosh(\frac{a_0\xi^0}{c}), \sinh(\frac{a_0\xi^0}{c}), 0, 0)$$

$$\frac{\partial e^\alpha{}_1}{c\partial\xi^0} = \frac{\partial e^\alpha{}_0}{\partial\xi^1} = (0,0,0,0),\ e^\alpha{}_1 = (C_3, C_4, C_5, C_6), C_3, C_4, C_5, C_6 \text{ is a constant number.}$$

Therefore, Eq(24) don't the wrong formula therefore Eq (12) is wrong formula.

In this time, Eq(16) is used by $e^\alpha{}_1(\xi^0), e^\alpha{}_2(\xi^0), e^\alpha{}_3(\xi^0)$, Eq(13),Eq(14),Eq(15) has to the right tetrad.Therefore Eq(25) is the right formula.

Therefore Eq(15) is the right tetrad, If save the new $e^\alpha{}_0$,

$$\frac{\partial e^\alpha{}_0}{\partial\xi^1} - \frac{\partial e^\alpha{}_1}{c\partial\xi^0} = \frac{\partial}{\partial\xi^1}(\frac{\partial x^\alpha}{c\partial\xi^0}) - \frac{\partial}{c\partial\xi^0}(\frac{\partial x^\alpha}{\partial\xi^1}) = 0, e^\alpha{}_0 = \frac{\partial x^\alpha}{c\partial\xi^0}, e^\alpha{}_1 = \frac{\partial x^\alpha}{\partial\xi^1}$$

$$e^\alpha{}_1(\xi^0) = \frac{\partial x^\alpha}{\partial\xi^1} = (\sinh(\frac{a_0\xi^0}{c}), \cosh(\frac{a_0\xi^0}{c}), 0, 0) \quad (25\text{-}1)$$

$$\frac{\partial e^\alpha{}_0}{\partial\xi^1} = \frac{\partial e^\alpha{}_1}{c\partial\xi^0} = (\frac{a_0}{c^2}\cosh(\frac{a_0\xi^0}{c}), \frac{a_0}{c^2}\sinh(\frac{a_0\xi^0}{c}), 0, 0)$$

Therefore, the new $e^\alpha{}_0$ is

$$e^\alpha{}_0 = \frac{\partial x^\alpha}{c\partial\xi^0} = ((C_7 + \xi^1\frac{a_0}{c^2})\cosh(\frac{a_0\xi^0}{c}), (C_8 + \xi^1\frac{a_0}{c^2})\sinh(\frac{a_0\xi^0}{c}), 0, 0)$$

C_7, C_8 is a constant number.

If $\xi^1 = \xi^2 = \xi^3 = 0, d\xi^1 = d\xi^2 = d\xi^3 = 0. \xi^0 = \tau$

$$e^\alpha{}_0(\tau) = \frac{dx^\alpha}{cd\tau} = (C_7\cosh(\frac{a_0\tau}{c}), C_8\sinh(\frac{a_0\tau}{c}), 0, 0) = (\cosh(\frac{a_0\tau}{c}), \sinh(\frac{a_0\tau}{c}), 0, 0)$$

$$C_7 = C_8 = 1$$

Therefore, the new $e^\alpha{}_0$ is

$$e^\alpha{}_0 = \frac{\partial x^\alpha}{c\partial\xi^0} = ((1 + \xi^1\frac{a_0}{c^2})\cosh(\frac{a_0\xi^0}{c}), (1 + \xi^1\frac{a_0}{c^2})\sinh(\frac{a_0\xi^0}{c}), 0, 0) \quad (25\text{-}2)$$

Therefore, new Rindler's coordinate theory is the theory that the tetrad $e^{\alpha}{}_{\mu} = e^{\alpha}{}_{\mu}(\xi^0, \frac{\xi^i}{c})$ is. ($i = 1,2,3$).

$$dx^{\alpha} = \frac{\partial x^{\alpha}}{\partial \xi^{\mu}} d\xi^{\mu}$$

$$= e^{\alpha}{}_{0}(\xi^0, \frac{\xi^1}{c}) c d\xi^0 + e^{\alpha}{}_{1}(\xi^0, \frac{\xi^1}{c}) d\xi^1 + e^{\alpha}{}_{2}(\xi^0, \frac{\xi^2}{c}) d\xi^2 + e^{\alpha}{}_{3}(\xi^0, \frac{\xi^3}{c}) d\xi^3 \quad (25\text{-}3)$$

By Eq(25-2), $e^{\alpha}{}_{0}(\xi^0, \frac{\xi^1}{c})$ is

$$e^{\alpha}{}_{0}(\xi^0, \frac{\xi^1}{c}) = \frac{\partial x^{\alpha}}{c\partial \xi^0} = (\cosh(\frac{a_0 \xi^0}{c})(1 + \frac{a_0 \xi^1}{c^2}), \sinh(\frac{a_0 \xi^0}{c})(1 + \frac{a_0 \xi^1}{c^2}), 0, 0) \quad (26)$$

$$e^{\alpha}{}_{2}(\xi^0, \frac{\xi^2}{c}) = \frac{\partial x^{\alpha}}{\partial \xi^2} = (0,0,1,0) \ (27), \quad e^{\alpha}{}_{3}(\xi^0, \frac{\xi^3}{c}) = \frac{\partial x^{\alpha}}{\partial \xi^3} = (0,0,0,1) \ (28)$$

and by Eq(25-1),the other unit vector $e^{\alpha}{}_{1}(\xi^0, \frac{\xi^1}{c})$

$$e^{\alpha}{}_{1}(\xi^0, \frac{\xi^1}{c}) = \frac{\partial x^{\alpha}}{\partial \xi^1} = (\sinh(\frac{a_0 \xi^0}{c}), \cosh(\frac{a_0 \xi^0}{c}), 0, 0) \quad (29)$$

In Eq(26),Eq(29),

$$\frac{\partial (ct)}{c\partial \xi^0} = \cosh(\frac{a_0 \xi^0}{c})(\frac{a_0 \xi^1}{c^2} + 1), \ \frac{\partial (ct)}{\partial \xi^1} = \sinh(\frac{a_0 \xi^0}{c})$$

Therefore,

$$ct = \frac{c^2}{a_0} \sinh(\frac{a_0 \xi^0}{c})(1 + \frac{a_0 \xi^1}{c^2}) = (\frac{c^2}{a_0} + \xi^1) \sinh(\frac{a_0 \xi^0}{c}) \quad (30)$$

In Eq(26),Eq(29),

$$\frac{\partial x}{c\partial \xi^0} = \sinh(\frac{a_0 \xi^0}{c})(\frac{a_0 \xi^1}{c^2} + 1), \ \frac{\partial x}{\partial \xi^1} = \cosh(\frac{a_0 \xi^0}{c})$$

Therefore,

$$x = \frac{c^2}{a_0} \cosh(\frac{a_0 \xi^0}{c})(1 + \frac{a_0 \xi^1}{c^2}) - \frac{c^2}{a_0} = (\frac{c^2}{a_0} + \xi^1) \cosh(\frac{a_0 \xi^0}{c}) - \frac{c^2}{a_0} \ (31), \ y = \xi^2, \ z = \xi^3 \ (31\text{-}1)$$

Therefore, the Rindler's coordinate transformation is

$$ct = \frac{c^2}{a_0} \sinh(\frac{a_0 \xi^0}{c})(1 + \frac{a_0 \xi^1}{c^2}) = (\frac{c^2}{a_0} + \xi^1) \sinh(\frac{a_0 \xi^0}{c}) \quad (32),$$

$$x = \frac{c^2}{a_0} \cosh(\frac{a_0 \xi^0}{c})(1 + \frac{a_0 \xi^1}{c^2}) - \frac{c^2}{a_0} = (\frac{c^2}{a_0} + \xi^1) \cosh(\frac{a_0 \xi^0}{c}) - \frac{c^2}{a_0} \quad (33),$$

$$y = \xi^2, \ z = \xi^3 \ (34)$$

By this theory, can save Rindler's coordinate transformation in the other way.

Compute coordinate differentiation's transformation,

$$cdt = c\cosh(\frac{a_0\xi^0}{c})d\xi^0(1+\frac{a_0}{c^2}\xi^1) + \sinh(\frac{a_0\xi^0}{c})d\xi^1$$

$$dx = c\sinh(\frac{a_0\xi^0}{c})d\xi^0(1+\frac{a_0}{c^2}\xi^1) + \cosh(\frac{a_0\xi^0}{c})d\xi^1, dy = d\xi^2, \quad dz = d\xi^3$$

$$c^2dt^2 - dx^2 - dy^2 - dz^2$$

$$= [c\cosh(\frac{a_0\xi^0}{c})d\xi^0(1+\frac{a_0}{c^2}\xi^1) + \sinh(\frac{a_0\xi^0}{c})d\xi^1]^2$$

$$-[c\sinh(\frac{a_0\xi^0}{c})d\xi^0(1+\frac{a_0}{c^2}\xi^1) + \cosh(\frac{a_0\xi^0}{c})d\xi^1]^2 - (d\xi^2)^2 - (d\xi^3)^2$$

$$= c^2(1+\frac{a_0\xi^1}{c^2})^2(d\xi^0)^2 - (d\xi^1)^2 - (d\xi^2)^2 - (d\xi^3)^2$$

$$= -g_{\mu\upsilon}d\xi^\mu d\xi^\upsilon \quad (35)$$

III. Additional chapter-II

According to Eq (32),Eq (33),Eq (34),Eq (35)

$$\eta_{\alpha\beta}\frac{\partial x^\alpha}{\partial\xi^\mu}\frac{\partial x^\beta}{\partial\xi^\upsilon} = e^\alpha{}_\mu(\xi^0,\xi^i/c)e^\beta{}_\upsilon(\xi^0,\xi^j/c)\eta_{\alpha\beta} = g_{\mu\upsilon}$$

$$\eta_{\alpha\alpha}e^\alpha{}_0(\xi^0,\frac{\xi^1}{c})\cdot e^\alpha{}_0(\xi^0,\frac{\xi^1}{c}) = [-\cosh^2(\frac{a_0\xi^0}{c}) + \sinh^2(\frac{a_0\xi^0}{c})](1+\frac{a_0\xi^1}{c^2})^2$$

$$= -(1+\frac{a_0\xi^1}{c^2})^2 = g_{00} \quad ,$$

$$\eta_{\alpha\alpha}e^\alpha{}_0(\xi^0,\frac{\xi^1}{c})\cdot e^\alpha{}_1(\xi^0,\frac{\xi^1}{c}) = g_{01} = \eta_{\alpha\alpha}e^\alpha{}_1(\xi^0,\frac{\xi^1}{c})\cdot e^\alpha{}_0(\xi^0,\frac{\xi^1}{c}) = g_{10} = 0,$$

$$\eta_{\alpha\alpha}e^\alpha{}_1(\xi^0,\frac{\xi^1}{c})\cdot e^\alpha{}_1(\xi^0,\frac{\xi^1}{c}) = g_{11} = [-\sinh^2(\frac{a_0\xi^0}{c}) + \cosh^2(\frac{a_0\xi^0}{c})] = 1$$

$$\eta_{\alpha\alpha}e^\alpha{}_2(\xi^0,\xi^2/c)\cdot e^\alpha{}_2(\xi^0,\xi^2/c) = g_{22} = 1, \quad \eta_{\alpha\alpha}e^\alpha{}_3(\xi^0,\xi^3/c)\cdot e^\alpha{}_3(\xi^0,\xi^3/c) = g_{33} = 1,$$

$$\eta_{\alpha\alpha}e^\alpha{}_i(\xi^0,\xi^i/c)\cdot e^\alpha{}_j(\xi^0,\xi^j/c) = 0, (i \neq j, i, j = 1,2,3) \qquad (36)$$

Therefore, Eq (4) is satisfied.

$$e^a{}_\mu e^b{}_\upsilon\eta_{ab} = g_{\mu\upsilon} \qquad (37)$$

According to Eq (30-1),Eq (30-3),Eq(30-4),Eq(30-5)

$$e^{\alpha}{}_{\mu} = \frac{\partial x^{\alpha}}{\partial \xi^{\mu}} = \begin{pmatrix} \frac{c\partial t}{c\partial \xi^0} & \frac{c\partial t}{\partial \xi^1} & \frac{c\partial t}{\partial \xi^2} & \frac{c\partial t}{\partial \xi^3} \\ \frac{\partial x}{c\partial \xi^0} & \frac{\partial x}{\partial \xi^1} & \frac{\partial x}{\partial \xi^2} & \frac{\partial x}{\partial \xi^3} \\ \frac{\partial y}{c\partial \xi^0} & \frac{\partial y}{\partial \xi^1} & \frac{\partial y}{\partial \xi^2} & \frac{\partial y}{\partial \xi^3} \\ \frac{\partial z}{c\partial \xi^0} & \frac{\partial z}{\partial \xi^1} & \frac{\partial z}{\partial \xi^2} & \frac{\partial z}{\partial \xi^3} \end{pmatrix}$$

$$= A = \begin{pmatrix} \cosh(\frac{a_0\xi^0}{c})(1+\frac{a_0\xi^1}{c^2}) & \sinh(\frac{a_0\xi^0}{c}) & 0 & 0 \\ \sinh(\frac{a_0\xi^0}{c})(1+\frac{a_0\xi^1}{c^2}) & \cosh(\frac{a_0\xi^0}{c}) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$dx^{\alpha} = \frac{\partial x^{\alpha}}{\partial \xi^{\mu}} d\xi^{\mu} = e^{\alpha}{}_{\mu} d\xi^{\mu} \tag{38}$$

$$e_{\mu}{}^{\alpha} = \frac{\partial \xi^{\alpha}}{\partial x^{\mu}} = A^{-1} = \begin{pmatrix} \frac{c\partial \xi^0}{c\partial t} & \frac{c\partial \xi^0}{\partial x} & \frac{c\partial \xi^0}{\partial y} & \frac{c\partial \xi^0}{\partial z} \\ \frac{\partial \xi^1}{c\partial t} & \frac{\partial \xi^1}{\partial x} & \frac{\partial \xi^1}{\partial y} & \frac{\partial \xi^1}{\partial z} \\ \frac{\partial \xi^2}{c\partial t} & \frac{\partial \xi^2}{\partial x} & \frac{\partial \xi^2}{\partial y} & \frac{\partial \xi^2}{\partial z} \\ \frac{\partial \xi^3}{c\partial t} & \frac{\partial \xi^3}{\partial x} & \frac{\partial \xi^3}{\partial y} & \frac{\partial \xi^3}{\partial z} \end{pmatrix}$$

$$= \begin{pmatrix} \frac{\cosh(\frac{a_0\xi^0}{c})}{(1+\frac{a_0\xi^1}{c^2})} & -\frac{\sinh(\frac{a_0\xi^0}{c})}{(1+\frac{a_0\xi^1}{c^2})} & 0 & 0 \\ -\sinh(\frac{a_0\xi^0}{c}) & \cosh(\frac{a_0\xi^0}{c}) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$d\xi^{\alpha} = \frac{\partial \xi^{\alpha}}{\partial x^{\mu}} dx^{\mu} = e_{\mu}{}^{\alpha} dx^{\mu} \tag{39}$$

Eq (2) and Eq (3) are satisfied by Eq(36), Eq (38) and Eq (39)

$$e^a{}_\mu e^b{}_\upsilon \eta_{ab} = g_{\mu\upsilon} \quad \to \quad A^T \eta A = g$$

$$e_a{}^\mu e_b{}^\upsilon g_{\mu\upsilon} = \eta_{ab} \to \quad (A^T)^{-1} g A^{-1} = (A^T)^{-1} A^T \eta A A^{-1} = \eta$$

$$e^a{}_\mu = \eta^{ab} g_{\mu\upsilon} e_b{}^\upsilon \to \eta^{-1} (A^T)^{-1} A^T \eta A = A = \eta^{-1} (A^T)^{-1} g \qquad (40)$$

And both the differentiation transformation and the differentiation operator is

$$dx^\mu = \frac{\partial x^\mu}{\partial \xi^\upsilon} d\xi^\upsilon \ (41), \quad \frac{\partial}{\partial x^\mu} = \frac{\partial \xi^\upsilon}{\partial x^\mu} \frac{\partial}{\partial \xi^\upsilon} \ (42)$$

According to the Eq (39), Eq(42) is

$$\frac{1}{c}\frac{\partial}{\partial t} = \frac{c\partial \xi^0}{c\partial t}\frac{1}{c}\frac{\partial}{\partial \xi^0} + \frac{\partial \xi^1}{c\partial t}\frac{\partial}{\partial \xi^1}$$

$$= \frac{\cosh(\frac{a_0 \xi^0}{c})}{(1+\frac{a_0 \xi^1}{c^2})}\frac{\partial}{c\partial \xi^0} - \sinh(\frac{a_0 \xi^0}{c})\frac{\partial}{\partial \xi^1}$$

$$\frac{\partial}{\partial x} = \frac{c\partial \xi^0}{\partial x}\frac{1}{c}\frac{\partial}{\partial \xi^0} + \frac{\partial \xi^1}{\partial x}\frac{\partial}{\partial \xi^1}$$

$$= -\frac{\sinh(\frac{a_0 \xi^0}{c})}{(1+\frac{a_0 \xi^1}{c^2})}\frac{\partial}{c\partial \xi^0} + \cosh(\frac{a_0 \xi^0}{c})\frac{\partial}{\partial \xi^1}$$

$$\frac{\partial}{\partial y} = \frac{\partial}{\partial \xi^2}, \frac{\partial}{\partial z} = \frac{\partial}{\partial \xi^3}$$

Therefore

$$\begin{pmatrix} cdt \\ dx \\ dy \\ dz \end{pmatrix} = \begin{pmatrix} \cosh(\frac{a_0 \xi^0}{c})(1+\frac{a_0 \xi^1}{c^2}) & \sinh(\frac{a_0 \xi^0}{c}) & 0 & 0 \\ \sinh(\frac{a_0 \xi^0}{c})(1+\frac{a_0 \xi^1}{c^2}) & \cosh(\frac{a_0 \xi^0}{c}) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} cd\xi^0 \\ d\xi^1 \\ d\xi^2 \\ dz^3 \end{pmatrix}$$

$$= A \begin{pmatrix} cd\xi^0 \\ d\xi^1 \\ d\xi^2 \\ dz^3 \end{pmatrix}$$

$$\begin{pmatrix} \frac{1}{c}\frac{\partial}{\partial t} \\ \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \\ \frac{\partial}{\partial z} \end{pmatrix} = (A^{-1})^T \begin{pmatrix} \frac{1}{c}\frac{\partial}{\partial \xi^0} \\ \frac{\partial}{\partial \xi^1} \\ \frac{\partial}{\partial \xi^2} \\ \frac{\partial}{\partial \xi^3} \end{pmatrix} = (A^T)^{-1} \begin{pmatrix} \frac{1}{c}\frac{\partial}{\partial \xi^0} \\ \frac{\partial}{\partial \xi^1} \\ \frac{\partial}{\partial \xi^2} \\ \frac{\partial}{\partial \xi^3} \end{pmatrix}$$

$$= \begin{pmatrix} \frac{\cosh(\frac{a_0\xi^0}{c})}{(1+\frac{a_0\xi^1}{c^2})} & -\sinh(\frac{a_0\xi^0}{c}) & 0 & 0 \\ -\frac{\sinh(\frac{a_0\xi^0}{c})}{(1+\frac{a_0\xi^1}{c^2})} & \cosh(\frac{a_0\xi^0}{c}) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \frac{1}{c}\frac{\partial}{\partial \xi^0} \\ \frac{\partial}{\partial \xi^1} \\ \frac{\partial}{\partial \xi^2} \\ \frac{\partial}{\partial \xi^3} \end{pmatrix}$$

(43)

By Eq (36),save metric tensor $g_{\mu\upsilon}$ and $g^{\mu\upsilon}$

$$g_{\mu\upsilon} = \begin{pmatrix} -(1+\frac{a_0\xi^1}{c^2})^2 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad (44)$$

$$g^{\mu\upsilon} = \begin{pmatrix} -\frac{1}{(1+\frac{a_0\xi^1}{c^2})^2} & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad (45)$$

In the constant accelerated system,the differentiation operator's relation is

$$\frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \nabla^2 = -\eta^{\alpha\beta}\frac{\partial}{\partial x^\alpha}\frac{\partial}{\partial x^\beta}$$

$$= -(\eta^{\alpha\beta}\frac{\partial \xi^\mu}{\partial x^\alpha}\frac{\partial \xi^\upsilon}{\partial x^\beta})\frac{\partial}{\partial \xi^\mu}\frac{\partial}{\partial \xi^\upsilon}$$

$$= -g^{\mu\upsilon}\frac{\partial}{\partial\xi^{\mu}}\frac{\partial}{\partial\xi^{\upsilon}} \quad (46)$$

To check that Eq (46) does, Eq (43),Eq (45) use,

$$\eta^{\alpha\beta}\frac{\partial\xi^{\mu}}{\partial x^{\alpha}}\frac{\partial\xi^{\upsilon}}{\partial x^{\beta}} = \eta^{\alpha\beta}e_{\alpha}{}^{\mu}e_{\beta}{}^{\upsilon} = g^{\mu\upsilon} \quad \rightarrow (A^{T}\eta A)^{-1} = A^{-1}\eta^{-1}(A^{T})^{-1} = g^{-1}$$

$$\frac{1}{c^{2}}\frac{\partial^{2}}{\partial t^{2}} - \nabla^{2} = \frac{1}{c^{2}}\frac{\partial^{2}}{\partial t^{2}} - \frac{\partial^{2}}{\partial x^{2}} - \frac{\partial^{2}}{\partial y^{2}} - \frac{\partial^{2}}{\partial z^{2}}$$

$$= [\frac{\cosh(\frac{a_{0}\xi^{0}}{c})}{(1+\frac{a_{0}\xi^{1}}{c^{2}})}\frac{\partial}{c\partial\xi^{0}} - \sinh(\frac{a_{0}\xi^{0}}{c})\frac{\partial}{\partial\xi^{1}}]^{2}$$

$$-[-\frac{\sinh(\frac{a_{0}\xi^{0}}{c})}{(1+\frac{a_{0}\xi^{1}}{c^{2}})}\frac{\partial}{c\partial\xi^{0}} + \cosh(\frac{a_{0}\xi^{0}}{c})\frac{\partial}{\partial\xi^{1}}]^{2} - (\frac{\partial}{\partial\xi^{2}})^{2} - (\frac{\partial}{\partial\xi^{3}})^{2}$$

$$= \frac{1}{(1+\frac{a_{0}\xi^{1}}{c^{2}})^{2}}\frac{1}{c^{2}}(\frac{\partial}{\partial\xi^{0}})^{2} - (\frac{\partial}{\partial\xi^{1}})^{2} - (\frac{\partial}{\partial\xi^{2}})^{2} - (\frac{\partial}{\partial\xi^{3}})^{2} = -g^{\mu\upsilon}\frac{\partial}{\partial\xi^{\mu}}\frac{\partial}{\partial\xi^{\upsilon}} \quad (47)$$

In this time,

$$g_{\mu\upsilon}e_{\alpha}{}^{\mu}e_{\beta}{}^{\upsilon} = \eta_{\alpha\beta} \qquad \rightarrow \quad (A^{T})^{-1}gA^{-1} = (A^{T})^{-1}A^{T}\eta AA^{-1} = \eta$$

$$e^{a}{}_{\mu}e_{b}{}^{\mu} = \delta^{a}{}_{b} \;\rightarrow AA^{-1} = I\,, \qquad e^{a}{}_{\mu}e_{a}{}^{\upsilon} = \delta_{\mu}{}^{\upsilon} \;\rightarrow A^{-1}A = I$$

$$A = \begin{pmatrix} \cosh(\frac{a_{0}\xi^{0}}{c})(1+\frac{a_{0}\xi^{1}}{c^{2}}) & \sinh(\frac{a_{0}\xi^{0}}{c}) & 0 & 0 \\ \sinh(\frac{a_{0}\xi^{0}}{c})(1+\frac{a_{0}\xi^{1}}{c^{2}}) & \cosh(\frac{a_{0}\xi^{0}}{c}) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} e^{0}{}_{0} & e^{0}{}_{1} & e^{0}{}_{2} & e^{0}{}_{3} \\ e^{1}{}_{0} & e^{1}{}_{1} & e^{1}{}_{2} & e^{1}{}_{3} \\ e^{2}{}_{0} & e^{2}{}_{1} & e^{2}{}_{2} & e^{2}{}_{3} \\ e^{3}{}_{0} & e^{3}{}_{1} & e^{3}{}_{2} & e^{3}{}_{3} \end{pmatrix}$$

$$A^{-1}=\begin{pmatrix}\frac{\cosh(\frac{a_0\xi^0}{c})}{(1+\frac{a_0\xi^1}{c^2})} & -\frac{\sinh(\frac{a_0\xi^0}{c})}{(1+\frac{a_0\xi^1}{c^2})} & 0 & 0\\ -\sinh(\frac{a_0\xi^0}{c}) & \cosh(\frac{a_0\xi^0}{c}) & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{pmatrix}=\begin{pmatrix}e_0{}^0 & e_1{}^0 & e_2{}^0 & e_3{}^0\\ e_0{}^1 & e_1{}^1 & e_2{}^1 & e_3{}^1\\ e_0{}^2 & e_1{}^2 & e_2{}^2 & e_3{}^2\\ e_0{}^3 & e_1{}^3 & e_2{}^3 & e_3{}^3\end{pmatrix}$$

(48)

IV.Conclusion

If $\xi^1=\xi^2=\xi^3=0, d\xi^1=d\xi^2=d\xi^3=0, \xi^0=\tau$,the tetrad $e^\alpha{}_\mu=e^\alpha{}_\mu(\xi^0,\xi^i/c)$ is the unit vector $e^\alpha{}_\mu=e^\alpha{}_\mu(\tau)$ that is each other orthographic. Therefore if the tetrad $e^\alpha{}_\mu$ is the unit vector $e^\alpha{}_\mu$ that is each other orthographic, the tetrad $e^\alpha{}_\mu$ has to be the function of a same coordinate τ.

By the theory,can save Rindler's coordinate transformation in the other way.

Therefore, Rindler's coordinate theory resurfaces by the tetrad on the new method

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