Quickly identifying the presence of the golden ratio in the logistic map

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Abstract

A brief visual demonstration of the presence of the golden ratio in the logistic map is given.

1 Branching in the 2nd iteration of the logistic map

The following is an image of some iterations of the logistic map

$$x' = rx(1-x) \tag{1}$$

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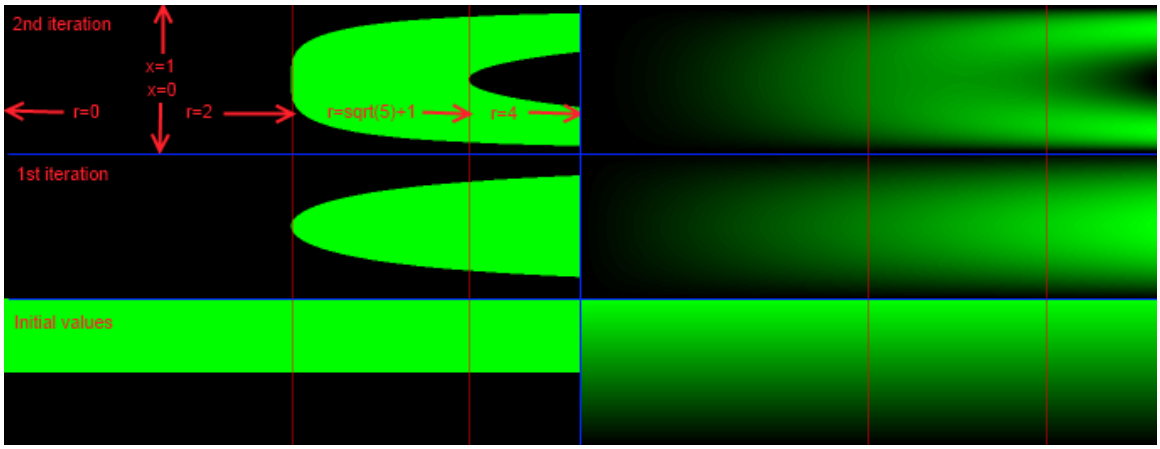

To get a sense of what's going on, notice that each of the six regions represent $x = [0,1]$ and $r = [0,4]$.

The bottom-right region shows the values of x ranging from 0 to 1 as a green gradient. The bottom-right region shows the same set of values, but using only two colours – green for values $>= 0.5$ or black for values < 0.5.

After running each value through the logistic map one time, we arrive at the middle region – the result of the first iteration. After running each value through the logistic map one last time, we arrive at the top region – the result of the second iteration.

You will notice that there is most definitely a branch formed by the values in the top row. It's unmistakable in the two colour version. You will also notice from the two colour version that the trunk starts at $r = 2$, the branches start at $r = \sqrt{5}+1$, and the branches effectively end at $r = 4$.

Doing some arithmetic to get the distances involved:

$$D_1 = (\sqrt{5}+1)-2 = \sqrt{5}-1 \simeq 1.236068, \tag{2}$$

$$D_2 = 4-(\sqrt{5}+1) \simeq 0.763932, \tag{3}$$

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you will notice that D_1/D_2 (ie. trunk length to branch length ratio) is equal to the golden ratio

$$\phi = \frac{\sqrt{5}+1}{2}. \tag{4}$$

Also, you will notice that $2/D_1$ (ie. pretrunk length to trunk length ratio) is also equal to the golden ratio.

Presumably, this relationship pops up in all similar maps.

The paper "Fibonacci order in the period-doubling cascade to chaos" by Linage et al [1] shows from another point of view how the golden ratio is present in the logistic map.

References

[1] Linage G, Montoya F, Sarmiento A, Showalter K, Parmananda P. Fibonacci order in the period-doubling cascade to chaos. (2006) Physics Letters A 359