A treatment of the twin paradox based on the assumption of an instantaneous acceleration

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Abstract

We investigate the twin paradox assuming the acceleration acts instantaneously in one of the twins and whose effect is just to revert the relative movement of the twins keeping the same relative speed. The relative motion of the twins is then split in two stages: one where they move away and another when they approach each other. Each stage is described by specific Lorentz transformations that obey certain boundary conditions related to the reversion of motion. We then show how the paradox arises from the particular form of the Lorentz transformation describing the approaching movement of the twins.

1 Introduction

The validity of the theory of Special Relativity (SR) is strongly enforced by many experiments in high-energy physics which, so far, has shown no signs of violation of any of the SR laws. However, this constitutes no more than an indirect proof of the validity of the theory as there still remains issues concerning the foundations of SR that calls for a convenient treatment. Among those, the so-called twin paradox, with its implications on the nature of time, occupies a prominent role. The paradox has stirred much debate in the past as it is seen in letters published mainly through the 50's and 60's [1] and that, as a whole, constitute a sequence of argument-counterarguments that are far to settle the paradox on a common accepted basis. Since then, the paradox is still under investigation where new forms of thinking have been presented [2].

The main difficulty on the paradox lies on the role of the acceleration acting on one of the twins and, consequently, in the recurrent use of arguments that leave the domain of special relativity and go far beyond the mere use of the standard Lorentz transformation [1] which, after all, continue to be valid during most of the motion of the twins. In fact, one would expect a departure from it only insofar as there is acceleration. Therefore, in the limit case when

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[1]It is important to notice that by "standard" we mean the Lorentz transformation describing non-accelerated frames.

the acceleration happens *instantaneously* we should perform the analysis solely on the basis of the standard Lorentz transformation since there is no possibility for considering additional assumptions associated to a *finite* time interval [2] during which acceleration takes place. A necessity for a treatment that uses concepts from SR alone has been envisaged long ago by McCrea and Crawford [3] though they have not provided one. It is the purpose of our work to fill in this gap. The main motivation is to avoid the introduction of more elements than necessary to solve the paradox. Therefore, keeping the paradox in its most simple form we are able to contemplate its aspects in a more precise way.

In our work we analyze the problem by identifying three stages as follows: (I) consisting on the movement of the twins getting apart from each other with relative speed v, (II) consisting on the reversion of the movement of the twins [3], and (III) consisting on the movement of the twins approaching each other with speed v. Here, we consider the stage (II) as happening instantaneously (otherwise we would have to consider accelerated frames in special relativity [4]). This assumption brings to the problem "boundary" conditions that must be obeyed by the Lorentz transformations when one shifts from stages (I) to (III) and, in particular, they play an important role on the form of the Lorentz transformation describing stage (III) which, as we will see, is the source of the paradox.

Our work is organized as follows. In Section **2.1** we analyze stage (I) assuming the twins start their movement at the same position and with their clocks marking $t = t' = 0$. Then by using the Lorentz transformation in its standard form we determine the events associated to the reversion of the movement of the twins which then become boundary conditions to be imposed on the transformation of stage (III). In Sections **2.2.** and **2.3** we analyze stage (III) considering two forms for the Lorentz transformation that, in principle, would serve to describe the movement, e.g. $x'^{\mu} = A^{\mu}_{\nu}x^{\nu} + B^{\mu}$ or $x^{\mu} = A'^{\mu}_{\nu}x'^{\nu} + B'^{\mu}$ with $A^{\mu}_{\nu}, A'^{\mu}_{\nu}, B^{\mu}, B'^{\mu}$ parameters that will be determined by imposing the boundary conditions of Section 2.1. We then show the two transformations are not inverse of each other, thus giving different descriptions to the movement of stage (III). Finally, in Section **2.4** and **2.5** we analyze the paradox giving a solution in terms of physical arguments.

2 Deriving the transformations

2.1 Stage I: The twins depart from each other

Initially, we assume the twins are fixed respectively at the origins of two inertial frames S, S', with S' moving relative to S with velocity $\vec{v}$. Both frames have coordinates axis, x^1, x^2, x^3 and x'^1, x'^2, x'^3 that are respectively parallel to each other and have their origins coinciding when

[2] i.e. a non-degenerate interval $[a, b]$ with $a \neq b$.

[3] In fact, each twin sees the other revert his movement, even though we assume only one of them suffer acceleration.

$t = t' = 0$. Under this initial condition the standard Lorentz transformation is written as [5], [6]

$$\begin{cases} \vec{x}\,' = \vec{x} - (1-\gamma)\frac{\vec{x}\cdot\vec{v}}{v^2}\,\vec{v} - \gamma\vec{v}t \\ t' = \gamma\Big(t - \frac{\vec{x}\cdot\vec{v}}{c^2}\Big) \end{cases} \tag{1}$$

with $\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$. We also assume the twin that will suffer acceleration is the one moving with the frame $S\,'$. For simplicity of notation we identify the twins with the frames S, $S\,'$, more specifically with the origins of the frames, then instead of saying "the twin that moves together with the frame S (resp. $S\,'$)" we simply say the twin S (resp. $S\,'$).

Setting the boundary conditions

From the perspective of the twin S, he sees $S\,'$ moving with velocity $\vec{v}$. Suppose that S sees $S\,'$ revert its movement at an instant

$$t^{(I)}_{O'} = \eta\ .$$

From (1) we obtain that the reversion of the movement of $S\,'$ corresponds to the event

$$\begin{cases} \big(t^{(I)}_{O'}, \vec{x}^{(I)}_{O'}\big) = (\eta, \eta\vec{v}) \\ \big(t'^{(I)}_{O'}, \vec{x}\,'^{(I)}_{O'}\big) = \Big(\frac{\eta}{\gamma}, 0\Big)\ . \end{cases} \tag{2}$$

From the perspective of the twin $S\,'$, he sees S moving with velocity $\vec{u} = -\vec{v}$. Suppose that S sees $S\,'$ revert its movement at an instant

$$t'^{(I)}_{O} = \xi\ ,$$

then from (1) we now obtain that the reversion of the movement of S corresponds to the event

$$\begin{cases} \big(t^{(I)}_{O}, \vec{x}^{(I)}_{O}\big) = \Big(\frac{\xi}{\gamma}, 0\Big) \\ \big(t'^{(I)}_{O}, \vec{x}\,'^{(I)}_{O}\big) = (\xi, -\xi\vec{v})\ . \end{cases} \tag{3}$$

In this first stage each twin measure the following time intervals

$$\begin{cases} \Delta t'^{(I)}_{O'} := t'^{(I)}_{O'} - 0 = \frac{\eta}{\gamma} \\ \Delta t^{(I)}_{O'} := t^{(I)}_{O'} - 0 = \eta \end{cases} , \qquad \begin{cases} \Delta t^{(I)}_{O} := t^{(I)} - 0 = \frac{\xi}{\gamma} \\ \Delta t'^{(I)}_{O} := t'^{(I)}_{O} - 0 = \xi \end{cases} \tag{4}$$

Remarks:

(i) In stage (I) our analysis focus on the time interval corresponding to two situations: one where initially the twins occupy the same position and the other when it happens the reversion of their relative movement. Therefore there are two perspectives to be considered, one from what the twin S sees and the other from what the twin $S\,'$ sees. As we have identified each twin with the origin of the frames S, $S\,'$ the analysis we intend to carry out is then equivalent to the analysis of the movement of the origins of each frame. Therefore, with respect to the origin $O\,'$ we have the intervals $\Delta t'^{(I)}_{O'}$, $\Delta t^{(I)}_{O'}$, while with respect to the origin O we have $\Delta t^{(I)}_{O}, \Delta t'^{(I)}_{O}$. In this way, one should not try to relate time intervals associated to the movement of O and $O\,'$ since what Lorentz transformation gives is solely $\Delta t^{(I)}_{O'} = \gamma\Delta t'^{(I)}_{O'}$ and $\Delta t'^{(I)}_{O} = \gamma\Delta t^{(I)}_{O}$.

(ii) At this point we don't make any inference on the relation between η and ξ, rather we will look for this relation from the analysis of the consistency condition that has to be fulfilled by the Lorentz transformations describing the movement of departure and then returning of the twins.

2.2 Stage III: the twins approach each other

Now we must obtain the Lorentz transformation that describes the movement of returning of both twins. Let us assume the transformation has the form

$$\begin{aligned} x'^0 &= A^0_{\ 0}x^0 + A^0_{\ i}x^i + B^0 \\ x'^i &= A^i_{\ 0}x^0 + A^i_{\ j}x^j + B^i \end{aligned} \tag{5}$$

with B^0, B^i constants to be determined by imposing that conditions (2, 3) are fulfilled by transformation (5).

Fulfillment of condition (3) gives

$$B^0 = c\xi - A^0_{\ 0}\frac{c\xi}{\gamma} \tag{6}$$

$$B^i = -\xi v^i - A^i_{\ 0}\frac{c\xi}{\gamma} \tag{7}$$

while fulfillment of condition (2) gives

$$B^0 = \frac{c\eta}{\gamma}\xi - A^0_{\ 0}c\eta - A^0_{\ i}\eta v^i \tag{8}$$

$$B^i = -A^i_{\ 0}c\eta - A^i_{\ j}\eta v^j \ . \tag{9}$$

From (6,8) and (5) we obtain that

$$x'^0 = A^0_{\ 0}x^0 + A^0_{\ i}x^i + B^0 \Rightarrow \begin{cases} x'^0 - c\xi = A^0_{\ 0}\big(x^0 - \frac{c\xi}{\gamma}\big) + A^0_{\ i}x^i \\ x'^0 - c\xi = A^0_{\ 0}\big(x^0 - c\eta\big) + A^0_{\ i}x^i - A^0_{\ i}\eta v^i + \frac{c\eta}{\gamma} - \xi c \end{cases}$$

which gives

$$\eta = \frac{\xi}{\gamma} \tag{10}$$

$$A^0_{\ i}v^i = c\frac{(1-\gamma^2)}{\gamma} \ . \tag{11}$$

From (7, 9) and (5) we obtain that

$$x'^i = A^i_{\ 0}x^0 + A^i_{\ j}x^j + B^i \Rightarrow \begin{cases} x'^i + \xi v^i = A^i_{\ 0}\big(x^0 - \frac{c\xi}{\gamma}\big) + A^i_{\ j}x^j \\ x'^i + \xi v^i = A^i_{\ 0}\big(x^0 - c\eta\big) + A^i_{\ j}x^j - A^i_{\ j}\eta v^i + \xi v^i \end{cases}$$

which gives

$$A^i_{\ j}v^j = \gamma v^i \ . \tag{12}$$

Let us denote

$$\begin{aligned} \widetilde{x}'^0 &:= x'^0 - c\gamma\eta, \quad \widetilde{x}'^i := x'^i + \gamma\eta v^i \\ \widetilde{x}^0 &:= x^0 - c\eta, \qquad \widetilde{x}^i := x^i \end{aligned} \tag{13}$$

then transformation (5) is rewritten as

$$\begin{aligned} \widetilde{x}'^0 &= A^0_{\ 0}\widetilde{x}^0 + A^0_{\ i}\widetilde{x}^i \\ \widetilde{x}'^i &= A^i_{\ 0}\widetilde{x}^0 + A^i_{\ j}\widetilde{x}^j \ . \end{aligned} \tag{14}$$

In order to derive the Lorentz transformation of the returning movement of the twins we use the standard argument for deriving Lorentz transformations as used in [5]. First we decompose $\widetilde{\vec{x}}$, $\widetilde{\vec{x}}\,'$ in components parallel and perpendicular to $\vec{u}$. e.g.

$$\widetilde{\vec{x}} = \widetilde{\vec{x}}_{\parallel} + \widetilde{\vec{x}}_{\perp} \quad \text{with} \quad \begin{cases} \widetilde{\vec{x}}_{\parallel} := \frac{\widetilde{\vec{x}}\cdot\vec{u}}{u^2}\vec{u} \\ \widetilde{\vec{x}}_{\perp} := \widetilde{\vec{x}} - \widetilde{\vec{x}}_{\parallel} \end{cases}$$

with a similar expression for $\widetilde{\vec{x}}\,'$. The Lorentz transformation is expected not to change the perpendicular component, i.e. we impose that

$$\widetilde{\vec{x}}\,'_{\perp} = \widetilde{\vec{x}}_{\perp}$$

from which we obtain the relations

$$A^i_{\ 0} - A^j_{\ 0}\frac{u^i u^j}{u^2} \;=\; 0 \tag{15}$$

$$A^i_{\ j} - A^k_{\ j}\frac{u^i u^k}{u^2} \;=\; \delta^i_j - \frac{u^i u^j}{u^2} \tag{16}$$

that allow us to express the transformation coefficients $A^i_{\ 0}, A^i_{\ j}$ in terms of two parameters λ, α as follows

$$A^i_{\ 0} \;\equiv\; \lambda u^i \tag{17}$$

$$A^i_{\ j} \;\equiv\; \delta^i_j + \alpha\frac{u^i u^j}{u^2} \ . \tag{18}$$

From (16,18) we obtain

$$\alpha = \gamma - 1$$

which momentarily allows us to write transformation (14) as

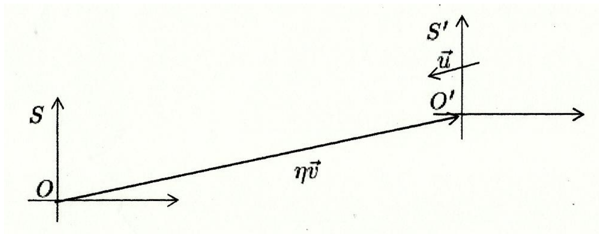
$$\begin{aligned} \widetilde{x}\,'^0 &= A^0_{\ 0}\widetilde{x}^0 + A^0_{\ i}\widetilde{x}^i_{\parallel} + A^0_{\ i}\widetilde{x}^i_{\perp} \\ \widetilde{x}\,'^i_{\parallel} &= \lambda u^i\widetilde{x}^0 + \gamma\widetilde{x}^i_{\parallel} \ . \end{aligned} \tag{19}$$

We consider now the movement of S' relative to S. For $t \geq \eta$ we have

$$\vec{x}_{\parallel O'} = (t_{O'} - 2\eta)\vec{u}$$

then from (19)

$$\widetilde{\vec{x}}\,'_{\parallel O'} = \lambda\,\widetilde{x}^0_{O'}\,\vec{u} + \gamma\widetilde{\vec{x}}_{\parallel O'} \;\Rightarrow\; 0 = t_{O'}(\lambda c + \gamma)\vec{u} + (-\lambda c\eta - \gamma\eta)\vec{u} \;\Rightarrow\; \lambda = -\frac{\gamma}{c} \ . \tag{20}$$



Transformation (19) will have an inverse if we consider it as a transformation relating the pairs $(\widetilde{x}^0, \widetilde{\vec{x}}_{\parallel}) \leftrightarrow (\widetilde{x}'^0, \widetilde{\vec{x}}'_{\parallel})$ which force us to impose that $A^0_{\ i}\widetilde{x}^i_{\perp} = 0$, therefore $A^0_{\ i} = ku^i$ with k a certain parameter that is determined from (11) as

$$k = \frac{\gamma}{c}$$

i.e. $A^0_{\ i} = \frac{\gamma}{c}u^i$. Then, transformation (19) assumes the form

$$\left\{ \begin{array}{l} \widetilde{t}\,' = A^0_{\ 0}\widetilde{t} + \gamma\frac{\widetilde{\vec{x}}_{\parallel}\cdot\vec{u}}{c^2} \\ \widetilde{\vec{x}}\,'_{\parallel} = -\gamma\vec{u}\,\widetilde{t} + \gamma\widetilde{\vec{x}}_{\parallel} \end{array} \right. \quad \text{or} \quad \left\{ \begin{array}{l} \widetilde{t}\,' = A^0_{\ 0}\widetilde{t} + \gamma\frac{\widetilde{\vec{x}}\cdot\vec{u}}{c^2} \\ \widetilde{\vec{x}}\,' = \widetilde{\vec{x}}\,' - (1-\gamma)\frac{\widetilde{\vec{x}}\cdot\vec{u}}{u^2} - \gamma\vec{u}\widetilde{t}\,. \end{array} \right. \tag{21}$$

Finally, we determine the last coefficient $A^0_{\ 0}$ by imposing the transformation (21) is compatible with the requirement of the constancy of the speed of light. We have

$$\frac{d\widetilde{\vec{x}}\,'}{d\widetilde{t}'} = \Big(\frac{d\widetilde{\vec{x}}}{d\widetilde{t}} - (1-\gamma)\frac{\frac{d\widetilde{\vec{x}}}{dt}\cdot\vec{u}}{u^2}\vec{u} - \gamma\vec{u}\Big)\Big/\Big(A^0_{\ 0} + \gamma\frac{\vec{u}\cdot\frac{d\widetilde{\vec{x}}}{dt}}{c^2}\Big) \tag{22}$$

and assuming that

$$\frac{d\widetilde{\vec{x}}\,'}{d\widetilde{t}'} = \frac{c}{u}\,\vec{u} = \frac{d\widetilde{\vec{x}}}{d\widetilde{t}}$$

we obtain

$$A^0_{\ 0} = \gamma\Big(1 - \frac{2u}{c}\Big)$$

that determines the Lorentz transformation for the stage (III) as having the form

$$\begin{array}{l} t\,' = \gamma\Big[\Big(1 - \frac{2u}{c}\Big)t + \frac{\vec{x}\cdot\vec{u}}{c^2}\Big] + \frac{2\gamma u}{c}\eta \\ \vec{x}\,' = \vec{x} - (1-\gamma)\frac{\vec{x}\cdot\vec{u}}{u^2}\vec{u} - \gamma\vec{u}t + 2\gamma\eta\vec{u} \end{array} \tag{23}$$

which satisfies the boundary conditions stated in (2, 3), with $\xi = \gamma\eta$ (10).

The twins meet again when their origins coincide i.e. when $\vec{x}_{O'} = 0 \equiv \vec{x}\,'_{O'}$. Let us denote the respective instants of time by $t^{(III)}_{O'}, t'^{(III)}_{O'}$. From (23) we obtain

$$t^{(III)}_{O'} = 2\eta, \qquad t'^{(III)}_{O'} = 2\gamma\eta\Big(1 - \frac{u}{c}\Big)$$

and the time interval corresponding to the duration of stage (III) is

$$\begin{array}{l} \Delta^{(III)}t_{O'} := t^{(III)}_{O'} - t^{(I)}_{O'} = 2\eta - \eta = \eta \\ \Delta^{(III)}t'_{O'} := t'^{(III)}_{O'} - t'^{(I)}_{O'} = 2\gamma\eta\Big(1 - \frac{u}{c}\Big) - \frac{\eta}{\gamma}\,. \end{array}$$

From (4) we obtain that the total time interval corresponding to the movement of the origin O' departing from O until its returning back to O is

$$\begin{aligned}\Delta t_{O'} &:= \Delta^{(I)} t_{O'} + \Delta^{(III)} t_{O'} = 2\eta \\ \Delta t'_{O'} &:= \Delta^{(I)} t'_{O'} + \Delta^{(III)} t'_{O'} = 2\gamma\eta\Big(1 - \frac{u}{c}\Big) \,.\end{aligned}$$

We can also think the twins meet again when $\vec{x}'_O = 0 \equiv \vec{x}_O$, which corresponds to instants of time we denote by $t_O^{(III)}, t_O^{\prime(III)}$. Again, from (23) we obtain

$$t_O^{(III)} = 2\eta, \qquad t_O^{\prime(III)} = 2\gamma\eta\Big(1 - \frac{u}{c}\Big)$$

and the corresponding intervals

$$\begin{aligned}\Delta^{(III)} t_O &:= t_O^{(III)} - t_O^{(I)} = 2\eta - \eta = \eta \\ \Delta^{(III)} t'_O &:= t_O^{\prime(III)} - t_O^{\prime(I)} = 2\gamma\eta\Big(1 - \tfrac{u}{c}\Big) - \gamma\eta\end{aligned}$$

that together with (4) and (10) gives the total time interval corresponding to the movement of the origin O departing from O' until its returning back to O'

$$\begin{aligned}\Delta t_O &:= \Delta^{(I)} t_O + \Delta^{(III)} t_O = 2\eta \\ \Delta t'_O &:= \Delta^{(I)} t'_O + \Delta^{(III)} t'_O = 2\gamma\eta\Big(1 - \frac{u}{c}\Big) \,.\end{aligned}$$

We then conclude that

$$\Delta t_O = \Delta t_{O'} = 2\eta \tag{24}$$
$$\Delta t'_{O'} = \Delta t'_O = 2\eta(\gamma - \sqrt{\gamma^2 - 1}) \tag{25}$$
$$\Delta t'_{O'} = (\gamma - \sqrt{\gamma^2 - 1})\Delta t_O < \Delta t_O \tag{26}$$

Remarks:
(i) Eq. (24) is an important consistency relation as it tells us that the clocks constituting the frame S measure the same time interval for the movement of the both origins O and O' in their movement of getting far apart and then returning back as it was expected. The same applies to (25) and the clocks of the frame S'.
(ii) Eq. (26) shows that *when the twins meet again, the twin S' is younger that the twin S.*

2.3 Another development for stage (III)

We now show how the paradox arises. Instead of writing (5) let us put the transformation of stage (III) in the form

$$\begin{aligned}x^0 &= A'^0_0 x'^0 + A'^0_i x'^i + B'^0 \\ x^i &= A'^i_0 x'^0 + A'^i_j x'^j + B'^i \,.\end{aligned} \tag{27}$$

The fulfillment of conditions (2, 3) now determines

$$\eta = \gamma\xi \tag{28}$$

and by a similar procedure we obtain the transformation

$$\begin{aligned} t &= \gamma\Big[\Big(1-\tfrac{2v}{c}\Big)t' + \tfrac{\vec{x}\,'\cdot\vec{v}}{c^2}\Big] + 2\gamma\tfrac{v}{c}\xi \\ \vec{x} &= \vec{x}\,' - (1-\gamma)\tfrac{\vec{x}\,'\cdot\vec{v}}{v^2}\,\vec{v} - \gamma\vec{v}t' + 2\xi\gamma\vec{v}\ . \end{aligned} \tag{29}$$

Fixing η as the time S sees $S\,'$ revert its movement, there is an irreconcilable disagreement between (10, 28) which are the instants when $S\,'$ sees S revert its movement. This disagreement is due to the form one has adopted for the analysis of stage (III) given by (5) or (27).

From (29) we obtain the following expressions for the time interval it takes for the twins to meet again as seen by S and $S\,'$

$$\Delta t_O = \Delta t_{O'} = 2\xi(\gamma - \sqrt{\gamma^2-1}) \tag{30}$$

$$\Delta t'_{O'} = \Delta t'_O = 2\xi \tag{31}$$

$$\Delta t_O = (\gamma - \sqrt{\gamma^2-1})\Delta t'_{O'} < \Delta t'_{O'} \tag{32}$$

and from (32) we conclude that *when the twins meet again, the twin S is younger than the twin $S\,'$.*

2.4 The emergence of the paradox

We have then arrived at a paradox as the conclusion of the last section contradicts a previous one where we obtained it was the twin $S\,'$ that was younger upon their returning. This is how the twin paradox appears in our approach. Analytically, its origin is due to the two possible choices for the transformation set for stage (III) and given by (5, 27), and also the different values obtained for ξ and η. These transformations constitutes, in fact, two different transformations that are not inverse of one another (as a straightforward calculation shows this would happen only if $\eta = \xi$, which is not allowed), then using one or another leads to different descriptions of the reality.

2.5 Solving the paradox

In order to solve the paradox we will analyze some physical aspects of the time intervals measured by each twin during stages (I) and (III).

First let us focus on the transformation (23). Here, the twin S sees the same duration for the two stages of the movement of the twin $S\,'$ e.g.

$$\Delta t^{(I)}_{O'} = \eta = \Delta t^{(III)}_{O'}$$

which is a physically reasonable fact since the acceleration acts only to revert the movement of $S\,'$ maintaining the same relative speed during stages (I) and (III). Since the acceleration acts on $S\,'$ we expect it doesn't exert any influence whatsoever on the clocks that constitute the frame S. Still on the basis of the transformation (23), the twin $S\,'$ sees different durations for the two stages of the movement of the twin S e.g.

$$\Delta t'^{(I)}_O = \gamma\eta \neq 2\gamma\eta\Big(1-\frac{u}{c}\Big) - \gamma\eta = \Delta t'^{(III)}_O\ .$$

This asymmetry must be somehow related to the effect of the acceleration undergone by the twin S' that, in this case, affects the clocks that constitute the frame S'.

Now, let us focus on the transformation (29). Here, the twin S sees different durations for the two stages of movement of S' e.g.

$$\Delta t_{O'}^{(I)} = \gamma\xi \neq 2\gamma\xi\Big(1 - \frac{v}{c}\Big) - \gamma\xi = \Delta t_{O'}^{(III)}$$

and from our previous argument this is not an expected outcome as there is no acceleration acting on the clocks of S that could ultimately lead to such a difference on the time readings.

For the sake of completeness, we observe that S' sees the same duration for the movement of S, e.g.

$$\Delta t_{O}'^{(I)} = \xi = \Delta t_{O}'^{(III)} \ .$$

Then, the physical predictions based on transformation (29) are not correct, and the transformation we must use for the analysis of the movement of the twins is the one given by (23). Therefore, the time measured by each twin from the beginning until they meet again is given by (26), i.e

$$\Delta t'_{O'} = (\gamma - \sqrt{\gamma^2 - 1})\Delta t_O < \Delta t_O$$

and we conclude that *the twin S' that suffers acceleration is younger than S upon their returning.* As the limit case of an acceleration that happens instantaneously, the above expression provides a possible numerical check for the difference of the time intervals measured by the clocks.

3 Conclusion

We investigated the twin paradox assuming the acceleration acts instantaneously on one of the twins. This allowed us to focus our attention only on the Lorentz transformation describing the movement of the twins - first when they depart and then when they approach each other. We have obtained that the source of the paradox lies on the two possible forms one can write for the transformation describing the returning of the twins, e.g. $x'^{\mu} = A^{\mu}_{\nu}x^{\nu} + B^{\mu}$ or $x^{\mu} = A'^{\mu}_{\nu}x'^{\nu} + B'^{\mu}$ and the different conditions $\eta = \frac{\xi}{\gamma}$ (10), $\eta = \gamma\xi$ that arise when we impose the boundary conditions (2, 3). The fact the transformations are not inverse of one another indicates that, from a physical point of view, one of them might be somehow privileged over the other, and it was this fact that ultimately provided a solution of the paradox. One tacit assumption we made is that the acceleration doesn't affect the boundary conditions (2, 3). If so, we would not have necessarily the expressions $\eta = \frac{\xi}{\gamma}$ or $\eta = \gamma\xi$ and the analysis of the paradox would be different. Therefore, it may be interesting to analyze the paradox assuming modified boundary conditions caused by the acceleration.

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[4] See also the objection of W. McCrea on pp. 784-785, and the answer of Dingle on pp. 785.
[5] See also the objection of W. McCrea on pp. 681-682.
[6] See also a comment of H. Dingle on pp. 500.