Fermat's Marvelous Proofs for Fermat's Last Theorem

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Abstract

Using the complex hyperbolic functions and complex trigonometric functions we reappear the Fermat's marvelous proofs for Fermat's last theorem (FLT) [1-7]. We present three proofs: (1) Jiang's marvelous proofs, (2) Fermat's marvelous proofs and (3) Frey-Ribet-Wiles proofs. Ribenbiom [11] points out that there are some mathematicians who are not satisfied with the method of proof using elliptic curves and modular form, perhaps wrongly? or rightly?

1. Introduction

In 1637, the reading of Diophantus' *Arithmetica*, in particular, the part on the Pythagorean equation, inspired Fermat to write in his copy of Diophantus' monograph:

It is impossible for a cube to be written as a sum of two cubes or a fourth power to be written as the sum of two fourth powers or, in general, for any number which is a power greater than the second to be written as a sum of two like powers. I have a truly marvelous demonstration of this proposition which this margin is too narrow to contain.

Fermat never published a proof and, by the unsuccessful quest for a solution of Fermat's last theorem, mathematicians started to believe the that Fermat actually had no proof. However, no counterexample was found..

In this paper using the complex hyperbolic functions and complex trigonometric functions we reappear Fermat's marvelous proofs for FLT. Fermat proved that there are no integral solutons for the FLT exponent $n=4$, Euler proved FLT exponent $n=3$.

2. Jiang's Marvelous Proofs

Theorem 1. It is sufficient to prove that the FLT exponents n are odd primes. But this proof has great difficulty. We consider that FLT exponents n are the composite numbers. Let $n=4P$, where P is an odd prime. Using the complex hyperbolic functions we have the Fermat's equations [2, 5, 7]

(1) $$S_1^{4P}-S_2^{4P}=1,$$

(2) $$S_1^{P}+S_2^{P}=\left[\exp(t_P+t_{2P}+t_{3P})\right]^P .$$

Fermat proved (1), therefore (2) has no rational soultions for any odd prime P.

Note. Let $n=4\sum_{P>2} P$. Every factor of the FLT exponent n has a Fermat's equation [1-7].

Theorem 2. Let $n=3P$, where $P>3$ is an odd prime. Using the complex hyperbolic functions we have the Fermat's equations [1, 3, 4, 5, 6, 7]

(3) $$S_1^{3P}+S_2^{3P}=1,$$

(4) $$S_1^3 + S_2^3 = \left[\exp \sum_{\alpha=1}^{P-1} t_{3\alpha} \right]^3,$$

(5) $$S_1^P + S_2^P = \left[\exp(t_P + t_{2P}) \right]^P.$$

Euler proved (3) and (4), therefore (5) has no rational solutions for any odd prime $P > 3$.

Note. Let $n = \sum_{P>2} P$. Every factor of the FLT exponent n has a Fermat's equation [1-7].

3. Fermat's Marvelous Proofs

Theorem 3. Fermat's equation

(6) $$x^n \pm y^n = z^n$$

has no integral soultions (x, y, z) with $xyz \neq 0$, if $n \geq 3$. We assume that if x and y are integral numbers, then z is irrational numbers.

Proof 1. Let $n = 4P$, where P is an odd prime. From (6) we have the Fermat's equations

(7) $$x^{4P} - y^{4P} = z^{4P},$$

(8) $$(x^P)^4 - (y^P)^4 = (z^P)^4,$$

(9) $$(x^4)^P - (y^4)^P = (z^4)^P.$$

Since Fermat proved the FLT exponent $n = 4$, we prove that (7) and (8) hae no integral solutions, that is z and z^P are irrational numbers. We prove that (9) has no integral solutions for any odd prime P, that is z^4 is irrational numbers.

We rewrite (8) and (9) as

(10) $$x^4 - y^4 = A^4,$$

(11) $$x^P + y^P = B^P,$$

where $A = \dfrac{z^P}{[x^{4P-4} + x^{4P-8}y^4 + \cdots + y^{4P-4}]^{\frac{1}{4}}}$, $B = \dfrac{z^4}{[(x^{2P} + y^{2P})(x^P - y^P)]^{\frac{1}{P}}}$.

Fermat proved (7) and (10), therefore (11) has no integral solutions for any odd prime P.

Note. Let $n = 4 \prod_{P>2} P$. Every factor of the FLT exponent n has a Fermat's equation [1-7].

Proof 2. Let $n = 3P$, where P is an odd prime. From (6) we have the Fermat's equations

(12) $x^{3P} + y^{3P} = z^{3P}$,

(13) $(x^P)^3 + (y^P)^3 = (z^P)^3$,

(14) $(x^3)^P + (y^3)^P = (z^3)^P$.

Since Euler proved the FLT exponent $n = 3$, therefore (12) and (13) have no integral solutions, that is z and z^P are irrational numbers. We prove (14) has no integral solutions, that is z^3 is irrational number.

We rewrite (13) and (14) as

(15) $x^3 + y^3 = C^3$

(16) $x^P + y^P = D^P$

where $C = \dfrac{z^P}{[x^{3P-3} - x^{3P-6}y^3 + \cdots + y^{3P-3}]^{\frac{1}{3}}}$, $D = \dfrac{z^3}{[(x^{2P} - x^P y^P + y^{2P}]^{\frac{1}{P}}}$.

Euler proved (12) and (15), therefore (16) has no integral solutions for any odd prime $P > 3$, that is C and D are irrational numbers.

Note. Let $n = \prod_{P>2} P$. Every factor of the FLT exponent n has a Fermat's equation [1-7].

4. Frey-Ribet-Wiles Proofs

(Ⅰ) From elliotic curve to Fermat's equation

Using elliptic curve we prove FLT. We discuss Fermat's equation

(17) $$A^n + B^n = C^n, \quad n > 2$$

integral solutions. Frey [8] write (17) Fermat's equation as elliptic curve

(18) $$y^2 = x^3 + (A^n - B^n)x^2 - A^n B^n.$$

He conjectures that (18) elliptic curve would imply (17) Fermat's equation. Ribet [9] prove that (18) elliptic curve should imply (17) Fermat's equation. Wiles [10] prove that (18) elliptic curve over Q is modular. But he does not discuss and prove (17) Fermat's equation.

Note. Fermat's equation is n th power, but elliptic curve is 3th power. This proof is incredible to number theorists. Ribenboim [11] points out that there are some mathematicians who are not satisfied with the method of proof using elliptic curves and modular form, perhaps wrongly ? or rightly ?

(Ⅱ) From Fermat's equation to elliptic curve

Using Fermat's equation we prove elliptic curve. We discuss elliptic curve

(19) $$a^2 = b^3 + b$$

integral solutions. Fabs write (19) elliptic curve as Fermat's equation

(20) $$Z^n = (x + a^2)^n + (y + b^3 + b)^n, \quad n > 2.$$

He conjectures that (20) Fermat's equation would imply (19) elliptic curve. Rabs prove that (20) Fermat's equation should imply (19) elliptic curve. Jiang [1-7] prove that (20) Fermat's equation no integral solutions. But he does not discuss and prove (19) elliptic curve.

Note. (I) and (II) cases are the same. Both proofs are incredible to mathematicians.

References

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