Proca Equations and the Photon Imaginary Mass

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It has been recently proposed that the photon has *imaginary mass* and null real mass. Proca equations are the unique simplest relativistic generalization of Maxwell equations. They are the theoretical expressions of possible nonzero photon rest mass. The fact that the photon has imaginary mass introduces relevant modifications in Proca equations which point to a deviation from the Coulomb's inverse square law.

For quite a long time it has been known that the effects of a *nonzero photon rest mass* can be incorporated into electromagnetism through the *Proca equations* [1-2]. It is also known that particles with imaginary mass can be described by a real Proca field with a negative mass square [3-5]. They could be generated in storage rings, jovian magnetosphere, and supernova remnants. The existence of imaginary mass associated to the neutrino is already well-known. It has been reported by different groups of experimentalists that the mass square of the neutrino is *negative* [6]. Although the imaginary mass is not a measurable amount, its square is [7]. Recently, it was shown that an imaginary mass exist associated to the *electron* and the *photon* too [8]. The photon *imaginary mass* is given by

$$m_\gamma = \tfrac{2}{\sqrt{3}}\left(hf/c^2\right)i \qquad (1)$$

This means that the *photon* has null *real* mass and an *imaginary mass*, m_γ, expressed by the previous equation.

Proca equations may be found in many textbooks [9-11]. They provide a complete and self-consistent description of electromagnetic phenomena [12]. In the presence of sources ρ and $\vec{j}$, these equations may be written as (in SI units)

$$\nabla \cdot \vec{E} = \frac{\rho}{\varepsilon_0} - \mu_\gamma^2 \phi \qquad (2)$$

$$\nabla \cdot \vec{B} = 0 \qquad (3)$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \qquad (4)$$

$$\nabla \times \vec{B} = \mu_0 \vec{j} + \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t} - \mu_\gamma^2 \vec{A} \qquad (5)$$

where $\mu_\gamma = m_\gamma c/\hbar$, with the *real* variables μ_γ and m_γ. However, according to Eq. (1) m_γ is an *imaginary mass*. Then, μ_γ must be also an imaginary variable. Thus, μ_γ^2 is a *negative real* number similarly to m_γ^2. Consequently, we can write that

$$\mu_\gamma^2 = \frac{m_\gamma^2 c^2}{\hbar^2} = \frac{4}{3}\left(\frac{2\pi}{\lambda}\right)^2 = \frac{4}{3}k_r^2 \qquad (6)$$

whence we recognize $k_r = 2\pi/\lambda$ as the real part of the *propagation vector* $\vec{k}$;

$$k = \left|\vec{k}\right| = \left|k_r + ik_i\right| = \sqrt{k_r^2 + k_i^2} \qquad (7)$$

Substitution of Eq. (6) into Proca equations, gives

$$\nabla \cdot \vec{E} = \frac{\rho}{\varepsilon_0} - \frac{4}{3}k_r^2 \phi \qquad (8)$$

$$\nabla \cdot \vec{B} = 0 \qquad (9)$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \qquad (10)$$

$$\nabla \times \vec{B} = \mu_0 \vec{j} + \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t} - \frac{4}{3}k_r^2 \vec{A} \qquad (11)$$

In four-dimensional space these equations can be rewritten as

$$\left(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \frac{4}{3}k_r^2\right)A_\mu = -\mu_0 \vec{j}_\mu \qquad (12)$$

where A_μ and $\vec{j}_\mu$ are the 4-vector of potential $(A, i\phi/c)$ and the current density $(\vec{j}, ic\rho)$, respectively. In free space the above equation reduces to

$$\left(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \frac{4}{3}k_r^2\right)A_\mu = 0 \qquad (13)$$

which is essentially the Klein-Gordon equation for the photon.

Therefore, the presence of a photon in a static electric field modifies the wave equation for all potentials (including the Coulomb potential) in the form

$$\left(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \frac{4}{3}k_r^2\right)\phi = -\frac{\rho}{\varepsilon_0} \quad (14)$$

For a point charge, we obtain

$$\phi(r) = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}e^{-\frac{2}{\sqrt{3}}(k_r r)} \quad (15)$$

and the electric field

$$E(r) = \frac{q}{4\pi\varepsilon_0 r^2}\left[1 + \frac{2}{\sqrt{3}}(k_r r)\right]e^{-\frac{2}{\sqrt{3}}(k_r r)} \quad (16)$$

Note that only in the absence of the photon $(k_r = 0)$ the expression of $E(r)$ reduces to the well-known expression: $E(r) = q/4\pi\varepsilon_0 r^2$. Thus, these results point to an exponential deviation from Coulomb's inverse square law, which, as we know, is expressed by the following equation (in SI units):

$$\vec{F}_{12} = -\vec{F}_{21} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2 \vec{r}_{12}}{|\vec{r}_{12}|^3} \quad (17)$$

As seen in Eq. (16), the term

$$\frac{2}{\sqrt{3}}(k_r r)$$

only becomes significant if

$$r > \sim 10^{-4}\lambda \quad (18)$$

This means that the Coulomb's law is a good approximation when $r < \sim 10^{-4}\lambda$. However, if $r > \sim 10^{-4}\lambda$, the expression of the force departs from the prediction of Maxwell's equations.

The lowest-frequency photons of the primordial radiation of 2.7K is about $10^8\,Hz$ [13]. Therefore, the wavelength of these photons is $\lambda \approx 1m$. Consider the presence of these photons in a terrestrial experiment designed to measure the force between two electric charges separated by a distance r. According to Eq. (18), the deviation from the Coulomb's law only

becomes relevant if $r > 10^{-4}m$. Then, if we take $r = 0.1m$, the result is

$$\frac{2}{\sqrt{3}}(k_r r) = \frac{4\pi}{\sqrt{3}}\left(\frac{r}{\lambda}\right) = 0.73$$

and

$$\left[1 + \frac{2}{\sqrt{3}}(k_r r)\right]e^{-\frac{2}{\sqrt{3}}(k_r r)} = 0.83$$

Therefore, a deviation of 17% in respect to the value predicted by the Coulomb's law.

Then, why the above deviation is not experimentally observed? Theoretically because of the presence of *Schumann radiation* $\left(f_1 = 7.83\,Hz, \lambda_1 = 3.8 \times 10^7\,m\right)$ [14-15]. According to Eq. (18), for $\lambda_1 = 3.8 \times 10^7\,m$, the deviation only becomes significant if

$$r > \sim 10^{-4}\lambda_1 = 3.8Km$$

Since the values of r in usual experiments are much smaller than $3.8Km$ the result is that the deviation is negligible. In fact, this is easy to verify. For example, if $r = 0.1m$, we get

$$\frac{2}{\sqrt{3}}(k_r r) = \frac{4\pi}{\sqrt{3}}\left(\frac{r}{\lambda_1}\right) = \frac{4\pi}{\sqrt{3}}\left(\frac{0.1}{3.8 \times 10^7}\right) = 1.9 \times 10^{-8}$$

and

$$\left[1 + \frac{2}{\sqrt{3}}(k_r r)\right]e^{-\frac{2}{\sqrt{3}}(k_r r)} = 0.999999999$$

Now, if we put the experiment inside an aluminum box whose thickness of the walls are equal to 21cm [*] the experiment will be shielded for the Schumann radiation. By putting inside the box a photons source of $\lambda \approx 1m$, and making $r = 0.1m$, then it will be possible to observe the deviation previously computed of 17% in respect to the value predicted by the Coulomb's law.

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[*] The thickness δ necessary to shield the experiment for Schumann radiation can be calculated by means of the well-known expression [16]: $\delta = 5z = 10/\sqrt{2\pi\mu\sigma f}$ where μ and σ are, respectively, the permeability and the electric conductivity of the material; f is the frequency of the radiation to be shielded.

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