

Unruh Radiation Temperature Gradient in a Rotating Global Non-Inertial Frame
in Minkowski Space-Time

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Abstract

There should be a small classical gravity frequency shift between two transceivers one on the axis of rotation and the other clamped to a fixed point on the rim of a rotating rigid disc. Light arriving at the axis of rotation from the edge of the disk will be redshifted. Light moving the opposite way will be blue shifted. However, there will not be any shift for a light signal connecting two points on the rim. In addition the quantum gravity Unruh effect implies a small radial temperature gradient from the increasing radial centripetal acceleration. Furthermore, in general there should be a gravimagnetic inertial force doublet splitting of the vacuum speed of light locally measured at the rim of the rotating disk for a ray of light in the plane of the rotating disk.

The Minkowski metric for a Global Inertial Frame (GIF) in flat space-time is

$$c^2 d\tau^2 = c^2 dT^2 - dX^2 - dY^2 - dZ^2 \quad (1.1)$$

The Global Inertial Frame (GIF) coordinates are in Roman CAPS. The rotating Global Non-Inertial Frame (GNIF) coordinates are in small Roman letters. Actually we should use Local Non-Inertial Frame (LNIF) for a single observer clamped to a fixed point out on the rim of the rotating rigid disk in the X-Y plane with axis of rotation Z. That observer will feel physical quantum electro-dynamic g-forces of constraint from the clamp.ⁱ

Neglecting special relativity time dilation, i.e. $r\omega \ll c$

$$\begin{aligned} T &\equiv t \\ X &\equiv x \cos \omega t - y \sin \omega t \\ Y &\equiv x \sin \omega t + y \cos \omega t \\ Z &\equiv z \end{aligned} \quad (1.2)$$

Therefore, the physical metric field seen by the rotating non-inertial (GNIF) detector clamped to the rigid rotating disk has an inertial fictitious force gravimagnetic field in the sense of Ray Chiao:

$$\begin{aligned} d\tau^2 &= \left(1 - \frac{r^2 \omega^2}{c^2}\right) dt^2 + \frac{2\omega y}{c^2} dx dt - \frac{2\omega x}{c^2} dy dt - \frac{dx^2 - dy^2 - dz^2}{c^2} \\ &= \left(1 - \frac{r^2 \omega^2}{c^2}\right) dt^2 + \frac{2\omega r \sin \omega t}{c^2} dx dt - \frac{2\omega r \cos \omega t}{c^2} dy dt - \frac{dx^2 - dy^2 - dz^2}{c^2} \\ c &\equiv c_{LIF(\text{vacuum})} \\ r^2 &\equiv x^2 + y^2 \\ A_{(g)x} &= \frac{2\omega y}{c} \\ A_{(g)y} &= -\frac{2\omega x}{c} \\ A_{(g)z} &= 0 \\ \left(\vec{\nabla} \times \vec{A}_{(g)}\right)_z &= \frac{\partial A_{(g)x}}{\partial y} - \frac{\partial A_{(g)y}}{\partial x} = \frac{4\omega}{c} \end{aligned} \quad (1.3)$$

Quantum field theory also dictates a very tiny Unruh acceleration black body radiation temperature gradient in the material of the rotating disk. The physical covariant acceleration is radial

$$g(r) = \frac{\omega^2 r}{\sqrt{1 - \frac{\omega^2 r^2}{c^2}}}$$

$$\frac{\omega r}{c} \ll 1 \tag{1.4}$$

$$T_{Unruh} = \frac{\hbar g}{ck_B}$$

To go all the way to the horizon requires special relativity and is a much more complicated calculation.

Now what about the gravity redshift? Obviously for the clamped observer-detector out on the rim $dx = dy = dz = 0$, therefore we can use the usual gravity redshift formula

Gravitational redshift	any stationary spacetime (e.g. the Schwarzschild geometry)	$1 + z = \sqrt{\frac{g_{tt}(\text{receiver})}{g_{tt}(\text{source})}}$ (for the Schwarzschild geometry, $1 + z = \sqrt{\frac{1 - \frac{2GM}{c^2 r_{\text{receiver}}}}{1 - \frac{2GM}{c^2 r_{\text{source}}}}$)
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<http://en.wikipedia.org/wiki/Redshift>

Consider a light signal sent from the edge of the rotating disk to the receiver at the stationary axis of rotation.

$$1 + z = \frac{f_{\text{source}}}{f_{\text{receiver}}} = \sqrt{\frac{g_{00}(\text{receiver})}{g_{00}(\text{source})}}$$

$$\rightarrow \sqrt{\frac{1}{1 - \frac{\omega^2 r^2}{c^2}}} \approx 1 + \frac{\omega^2 r^2}{2c^2} > 1 \tag{1.5}$$

This is a small gravity redshift. Similarly, a signal going the opposite way from the axis of rotation to the outer rim will be blue shifted. However, by cylindrical symmetry, it should be obvious that there is not frequency shift of any kind, i.e. $z = 0$, in the rim-to-rim measurement. This is consistent with Misner, Thorne and Wheeler’s calculation by a different method on p. 63 of “Gravitation.”

What about local measurements of the speed of light between two closely spaced events in the LNIF of the observer out on the rim of the rigid rotating disk? The light ray obeys

$$d\tau = 0$$

Therefore,

$$\begin{aligned}
 0 &= \left(1 - \frac{r^2 \omega^2}{c^2}\right) dt^2 + \frac{2\omega y}{c^2} dx dt - \frac{2\omega x}{c^2} dy dt - \frac{dx^2 - dy^2 - dz^2}{c^2} \\
 r^2 &\equiv x^2 + y^2 \\
 A_{(g)x} &= \frac{2\omega y}{c} \\
 A_{(g)y} &= -\frac{2\omega x}{c} \\
 0 &= \left(1 - \frac{r^2 \omega^2}{c^2}\right) + \frac{2\omega y}{c^2} \frac{dx}{dt} - \frac{2\omega x}{c^2} \frac{dy}{dt} - \frac{1}{c^2} \left(\frac{dL}{dt}\right)^2 \\
 dt_{\text{measured}} &= \sqrt{g_{00}} dt \\
 0 &= 1 + \bar{A}_{(g)} \cdot \frac{\vec{c}_{LNIF}}{c} - \frac{c_{LNIF}^2}{c^2}
 \end{aligned} \tag{1.6}$$

This is a quadratic equation with two possible roots for the speed of light in vacuum as measured locally by a detector clamped to a fixed point on a rotating rigid disk. This shift will be seen for a light ray in the plane of the rotating disk, e.g. Sagnac effect.

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ⁱ Clamped accelerometer pointer will move.