Non Associative Algebraic Structures Using Finite Complex Numbers

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PREFACE

Authors in this book for the first time have constructed nonassociative structures like groupoids, quasi loops, non associative semirings and rings using finite complex modulo integers. The Smarandache analogue is also carried out. We see the nonassociative complex modulo integers groupoids satisfy several special identities like Moufang identity, Bol identity, right alternative and left alternative identities. P-complex modulo integer groupoids and idempotent complex modulo integer groupoids are introduced and characterized.

This book has six chapters. The first one is introductory in nature. Second chapter introduces complex modulo integer groupoids and complex modulo integer loops using $C(Z_n)$. This chapter gives 77 examples and forty theorems. Chapter three introduces the notion of nonassociative complex rings both finite and infinite using complex groupoids and complex loops. This chapter gives over 120 examples and thirty theorems.

Forth chapter introduces nonassociative structures using complex modulo integer groupoids and quasi loops. This new notion is well illustrated by 140 examples.

These can find applications only in due course of time, when these new concepts become familiar. The final chapter suggests over 300 problems some of which are research problems.

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W.B.VASANTHA KANDASAMY
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Chapter One

INTRODUCTION

In this chapter we for just recall some definitions and give the notations to make this book a self contained one. Z_n, Z, Q, R and C denote the modulo integers, integers, rationals, reals and complex numbers respectively.

$C(Z_n) = \{a + bi_F \mid a, b \in Z_n,\ i_F^2 = n - 1\}$ denotes the ring of complex modulo integers.

$C(Z) = \{a + bi \mid a, b \in Z,\ i^2 = -1\}$ denotes the ring of complex integers.

$C(Q) = \{a + bi \mid a, b \in Q,\ i^2 = -1\}$ is the field of rational complex numbers.

$C(R) = C = \{a + bi \mid a, b \in R,\ i^2 = -1\}$ is the field of complex numbers.

$R^+ \cup \{0\}$, $Q^+ \cup \{0\}$, $Z^+ \cup \{0\}$ denote the positive numbers and they form the semifield.

However, since $i^2 = -1$ we cannot have complex semifield.

Now we proceed onto define groupoids and loops and for more about these concepts refer [14-5].

DEFINITION 1.1: *Let*

$$G = \{Z_n, *, (t, u); t, u \in Z_n \setminus \{0, 1\}, (t, u) = 1;$$

t and u are primes} be a groupoid of type I.

If in this definition (t, u) = 1; t and u are not primes but relatively prime we get a groupoid of type II. If in this definition 1.1 if we replace (t, u) = d ≠ 0; (t, u ∈ $Z_n \setminus \{0\}$) we get groupoid of type III. (Also (t, t) gives a groupoid of type V), if t or u = 0 then also (t, 0) or (0, t) will give a groupoid of type IV.

We give examples of them.

Example 1.1: Let G = {Z_{45}, (7, 13), *} be groupoid of type I.

Example 1.2: Let G = {Z_{36}, (15, 8), *} be groupoid of type II.

Example 1.3: Let G = {Z_{40}, (9, 24), *} be groupoid of type III.

Example 1.4: Let G = {Z_{29}, (12, 12), *} be groupoid of type IV.

Example 1.5: Let G = {Z_{20}, (13, 0), *} be groupoid of type V.

Now we have several associated properties with them. For more please refer [14].

DEFINITION 1.2: *Let (G, *) be a groupoid. If H ⊆ G and (H, *) is a groupoid then we call (H, *) to be a subgroupoid of G. If (H, *) is a semigroup we define (G, *) to be a Smarandache groupoid.*

All groupoids in general are not Smarandache groupoids. We can define special identities on groupoids [14].

Now we proceed onto recall the definition of the new class of loops.

DEFINITION 1.3: *Let $L_n(m) = \{e, 1, 2, \ldots, n\}$ be the set where $n > 3$, n is odd and m is a positive integer such that $(m, n) = 1$ and $(m-1, n) = 1$ with $m < n$.*

Define on $L_n(m)$ a binary operation 'o' as follows.

(i) $e \circ i = i \circ e = i$ for all $i \in L_n(m)$

(ii) $i^2 = i \circ i = e$ for all $i \in L_n(m)$

(iii) $i \circ j = t$ where $t = (mj - (m - 1)i)$

for all $i, j \in L_n(m)$, $i \neq j$, $i \neq e$ and $j \neq e$; then $L_n(m)$ is a loop under the binary operation.

We just give one or two examples.

Example 1.6: Let $L_5(2) = \{e, 1, 2, 3, 4, 5\}$. The table for $L_5(2)$ is as follows:

0	e	1	2	3	4	5
e	e	1	2	3	4	5
1	1	e	3	5	2	4
2	2	5	e	4	1	3
3	3	4	1	e	5	2
4	4	3	5	2	e	1
5	5	2	4	1	3	e

$L_5(2)$ is a loop of order six. Clearly $L_5(2)$ is non associative and non commutative.

Example 1.7: $L_9(8)$ be the loop given by the following table.

0	e	1	2	3	4	5	6	7	8	9
e	e	1	2	3	4	5	6	7	8	9
1	1	e	9	8	7	6	5	4	3	2
2	2	3	e	1	9	8	7	6	5	4
3	3	5	4	e	2	1	9	8	7	6
4	4	7	6	5	e	3	2	1	9	8
5	5	9	8	7	6	e	4	3	2	1
6	6	2	1	9	8	7	e	5	4	3
7	7	4	3	2	1	9	8	e	6	5
8	8	6	5	4	3	2	1	9	e	7
9	9	8	7	6	5	4	3	2	1	e

$L_9(8)$ is a Smarandache loop [15].

We can study the special identities satisfied by them.

$L_n = \{L_n(m) \mid 1 < m , n, (m, n) (m - 1, n) = 1, n \text{ odd } n > 3\}$ denotes the class of all loops of order n + 1.

We just recall the definition of groupoid rings.

DEFINITION 1.4: *Let G be a groupoid. R be a commutative ring with unit or a field. The groupoid ring RG consists of all finite formal sums of the form $\sum r_i g_i$ (i; running over a finite number) where $r_i \in R$ and $g_i \in G$ satisfying the following conditions.*

(i) $$\sum_{i=1}^{n} r_i g_i = \sum_{i=1}^{n} s_i g_i \text{ if and only}$$

$$\text{if } r_i = s_i \text{ for } i=1, 2, \ldots, n$$

(ii) $$\sum_{i=1}^{n} a_i g_i + \sum_{i=1}^{n} b_i g_i = \sum_{i=1}^{n} (a_i + b_i) g_i$$

(iii) $$\left(\sum_{i=1}^{n} a_i g_i\right)\left(\sum_{i=1}^{n} b_i g_i\right) = \sum_{k=1}^{n} c_k g_k$$

$$\text{where } g_i\, g_j = m_k \;\; c_k = \sum a_i\, b_j$$

(iv) $r_i\, g_i = g_i\, r_i$ *for all* $r_i \in R$ *and* $g_i \in G$.

(v) $r\sum_{i=1}^{n} r_i g_i = \sum_{i=1}^{n} (r r_i) g_i$ *for all* $r, r_i \in R$ *and* $g_i \in G$.

Since $1 \in R$ and $g_i \in G$ we have $G = 1.G \subseteq R\,G$ and $R \subseteq RG$ if and only if G has identity, otherwise $R \not\subseteq RG$.

Clearly RG is a non associative ring with $0 \in R$ as the additive identity. The groupoid ring RG is an alternative ring if $(x\,x)\,y = x\,(xy)$ and $x\,(yy) = (xy)\,y$ for all $x, y \in RG$.

We just give some examples of groupoid rings.

Example 1.8: Let G = {Z_9, *, (3, 8)} be a groupoid F = Z_2 = {0, 1} be the finite field. FG is the groupoid ring of G over F. Clearly FG is non commutative and non associative ring but of finite order.

Example 1.9: Let G = {Z_{40}, *, (3, 3)} be a groupoid Z = F be the ring of integers, FG is the groupoid ring of infinite order.

FG is commutative but a non associative ring.

Example 1.10: Let G = {Z_{120}, *, (23, 0)} be a groupoid. F = Z_{12} be the ring of modulo integers FG be the groupoid ring. FG has two sided ideals given by

$$I = KG = \left\{ \sum a_i g_i \,\middle|\, a_i \in \{0, 2, 4, 6, 8, 10\} \subseteq Z_{12},\ g_i \in G \right\} \subseteq FG$$

is an ideal of FG. Further FG has right ideals which are not left ideals.

DEFINITION 1.5: *Let R be a commutative ring with unit or a field. L be any loop. The loop ring of the loop L over the ring R denoted by RL consists of all finite formal sums of the form $\sum a_i m_i$ (i-runs over a finite number) where $a_i \in R$ and $m_i \in L$ satisfying the following conditions.*

(i) $\sum_{i=1}^{n} a_i m_i = \sum_{i=1}^{n} b_i m_i$ *if and only if* $a_i = b_i$ *for* $i = 1, 2, \ldots, n$

(ii) $\sum_{i=1}^{n} a_i m_i + \sum_{i=1}^{n} b_i m_i = \sum_{i=1}^{n} (a_i + b_i) m_i$

(iii) $\left(\sum_{i=1}^{n} a_i m_i\right)\left(\sum_{j=1}^{n} b_j m_j\right) = \sum_{m_k = m_i m_j} c_k m_k$ *where* $c_k = \sum a_i b_j$

(iv) $r_i m_i = m_i r_i$ *for all* $r_i \in R$ *and* $m_i \in L$.

(v) $r\left(\sum_i r_i m_i\right) = \sum_i (rr_i)m_i$ *for all* $r \in R$ *and* $\sum r_i m_i \in RL$.

RL is a non associative ring with $0 \in R$ *as the additive identity. Since* $1 \in R$ *we have* $L = 1 . L \subseteq RL$ *and* $R.e = R \subseteq RL$ *where e is the identity element of L.*

We will illustrate this situation by some examples.

Example 1.11: Let F = Z_2 = {0, 1} be the finite field L = $L_{11}(5)$ be the loop of order 12. FL be the loop ring. Clearly FL is non commutative and a non associative ring of finite order.

Example 1.12: Let F = Z be the ring of integers. L = $L_{19}(10)$ be a loop of order 20. FL be the loop ring. FL is a commutative but non associative ring of infinite order.

Chapter Two

COMPLEX MODULO INTEGER GROUPOIDS

In this chapter we for the first time introduce the notion of complex modulo integer groupoids built using complex modulo integers $C(Z_n) = \{a + bi_F \mid a, b \in Z_n;\ i_F^2 = n-1\}$.

Clearly $o(C(Z_n)) = n^2$.

DEFINITION 2.1: *Let $G = \{a + bi_F \mid a, b \in Z_n;\ i_F^2 = n-1\}$ be the collection of complex modulo integers. Define $*$ on G as follows for every x, y in G where $x = a + bi_F$ and $y = c+di_F$*

$$
\begin{aligned}
x*y &= (a+bi_F) * (c + di_F) \\
&= (ta + sc) \bmod n + (tb + sd)i_F \pmod n
\end{aligned}
$$

with $t, s \in Z_n \setminus \{0\}$ $(t, s) = 1$, t and s are primes; $(G, *, (t, s))$ is defined as the complex modulo integer groupoid of type I.

If $(t, u) = 1$ but t and u are not primes then we define them as type II groupoids.

We will illustrate this situation by some examples.

Example 2.1: Let (G, *, (7, 3)) = {a + i_Fb | a, b ∈ Z_{10}, *, (7, 3)} be a complex modulo integer groupoid of type I. Let x = 5 + 2i_F and y = 1 + 9i_F be in G. Now

$$\begin{aligned} x+y &= (5 + 2i_F) * (1+9i_F) \\ &= (5*1) \pmod{10} + (2*9)\, i_F \pmod{10} \\ &= (35 + 3) \pmod{10} + (14 + 27)\, i_F \pmod{10} \\ &= 8 + i_F. \text{ Clearly } x*y \in G. \end{aligned}$$

Example 2.2: Let G = {C(Z_{12}), *, (7, 8)} be a complex modulo integer groupoid of order 12^2.

Example 2.3: Let K = {C(Z_3), *, (1, 2)} be a complex modulo integer groupoid of order 3^2.

Example 2.4: Let G = {C(Z_{14}), *, (9, 5)} be a complex modulo integer groupoid of order 14^2.

Example 2.5: Let K = {C(Z_{19}), *, (12, 11)} be a complex modulo integer groupoid of order 19^2.

All these groupoids are of type I.

Now we can define type III complex modulo integer groupoids.

DEFINITION 2.2: *Let*

$G = \{C(Z_n), *, (t, s) = d \neq 1, n \geq 5, t, s \in Z_n \setminus \{0,1\}\}$. G is a complex modulo integer groupoid of type III.

We will give examples of them.

Example 2.6: Let M = {C(Z_{12}), *, (3, 9)} be a complex modulo integer groupoid of order 12^2.

It is clear from the very definition if (t, s) = d ≠ 0 then n must naturally be greater than or equal to 5 for type II groupoids.

Example 2.7: Let $G = \{C(Z_5), *, (2, 4)\}$ be a complex modulo integer groupoid of order 5.

Suppose $x = 3 + 2i_F$ and $y = 1+i_F$ are in G then

$$\begin{aligned} x*y &= (3+2i_F) * (1+i_F) = 3*1 + (2*1)i_F \\ &= [(6+4)+ (4+4)i_F) \pmod 5 \\ &= 3i_F \in G. \end{aligned}$$

It is interesting to note that the product can be a complex number with real part equal to zero.

Now if $x = (2+3i_F)$ and $y = (4+i_F)$ are in G then

$$\begin{aligned} x*y &= (2+3i_F) * (4+i_F) = (2*4)\ (3i_F * i_F) \pmod 5 \\ &= (4+16) \bmod 5 + (6 + 4)\ i_F \pmod 5 \\ &= 0 + 0i_F = 0 \in G. \end{aligned}$$

Thus we can have zero divisors in G. Just like a semigroup we can in case of groupoids also have the notion of zero divisors.

Take $x = 2+3i_F$ and $y = 1+i_F$ in G, then

$$\begin{aligned} x*y &= (2.2 + 1.4 + (3.2 + 4)\ i_F) \pmod 5 \\ &= 8+10i_F \pmod 5 = 3 \in G. \end{aligned}$$

We see the product is just a real value.

Thus we have seen a groupoid can have modulo integers or imaginary modulo integers as elements. It is also interesting to note that these groupoids can have zero divisors.

The concept of the definition of subgroupoid is a matter of routine, hence left as an exercise to the reader.

Example 2.8: Let $G = \{C(Z_5), *, (2, 4)\}$ be a complex modulo integer groupoid.

Clearly $H_1 = \{1, i_F, 1 + i_F) \subseteq G$ is a complex modulo integer subgroupoid of G.

$H_2 = \{2, 2i_F, 2+2i_F\} \subseteq G$ is also a complex modulo integer subgroupoid of G.

$H_3 = \{3, 3i_F, 3+3i_F\} \subseteq G$ is again a complex modulo integer subgroupoid and $H_4 = \{4, 4i_F, 4+4i_F\} \subseteq G$ is again a complex modulo integer subgroupoid of G.

Example 2.9: Let $G = \{C(Z_6), *, (2, 4)\}$ be a complex modulo integer groupoid. Consider

$H = \{0, 2, 4, 2i_F, 4i_F, 2+2i_F, 2+4i_F, 4i_F + 2, 4i_F + 4\} \subseteq G$ is a complex modulo integer subgroupoid.

Example 2.10: Let $G = \{C(Z_8), *, (2, 6)\}$ be a complex modulo integer groupoid. Consider

$H = \{0, 2, 4, 6, 2i_F, 4i_F, 6i_F, 2+2i_F, 2+4i_F, 2+6i_F, 6+6i_F\} \subseteq G$; H is a complex modulo integer subgroupoid of G.

Example 2.11: Let $G = \{C(Z_{12}), *, (2, 10)\}$ be a complex modulo integer groupoid. Consider

$H = \{0, 2, 4, 6, 8, 10, 2+2i_F, 2+4i_F, 2+6i_F, 2+8i_F, 2+10i_F, 4+2i_F, 4+4i_F, 4+6i_F, 4+8i_F, 4+10i_F, 6+2i_F, 6+4i_F, 6+6i_F, 6+8i_F, 6+10i_F, 8+2i_F, 8+4i_F, 8+6i_F, 8+8i_F, 8+10i_F, 10+2i_F, 10+4i_F, 10+6i_F, 10+8i_F, 10+10i_F, 2i_F, 4i_F, 6i_F, 8i_F, 10i_F\} \subseteq G$, is a complex modulo integer subgroupod of G. $|H| = 36$ and $|G| = 12 \times 12$. Clearly $|H| / |G|$.

Example 2.12: Let $G = \{C(Z_{12}), *, (10, 8)\}$ be a complex modulo integer groupoid. Consider

$H_1 = \{0, 4, 8, 4i_F, 8i_F, 4+4i_F, 4+8i_F, 8+4i_F, 8+8i_F\} \subseteq G$ is a complex modulo integer subgroupoid of G. $H_2 = \{2, 6, 10, 2i_F, 6i_F, 10i_F, 2+2i_F, 2+6i_F, 2+10i_F, 6+2i_F, 6+6i_F, 6+10i_F, 10+2i_F, 10+6i_F, 10+10i_F\} \subseteq G$;

H_2 is a complex modulo integer subgroupoid of G.

Example 2.13: Let $G = \{C(Z_{10}), *, (8, 4)\}$ be a complex modulo integer groupoid. Take $H = \{0, 2, 4, 8, 6, 2i_F, 6i_F, 4i_F,$

$8i_F$, $2+2i_F$, $2+4i_F$, $2+6i_F$, $2+8i_F$, $4+2i_F$, $4 + 4i_F$, $4+6i_F$, $4+8i_F$, $6+2i_F$, $6+4i_F$, $6+6i_F$, $6+8i_F$, $8+2i_F$, $8+4i_F$, $8+6i_F$, $8+8i_F\} \subseteq G$; is a complex modulo integer subgroupod of G.

Next we proceed onto define type IV complex modulo integer groupoid.

DEFINITION 2.3: *Let $G = \{C(Z_n), *, (t, 0); t \in Z_n\}$ be a complex modulo integer groupoid of type IV.*

Example 2.14: Let $G = \{C(Z_6), *, (0, 2)\}$ be a complex modulo integer groupoid of type IV.

Example 2.15: Let $G = \{C(Z_{42}), *, (0, 9)\}$ be a complex modulo integer groupoid of type IV.

Example 2.16: Let $G = \{C(Z_{40}), *, (11, 0)\}$ be a complex modulo integer groupoid of type IV.

Now we give one example of a subgroupoid.

Example 2.17: Let $G = \{C(Z_6), *, (0, 2)\}$ be a complex modulo integer groupoid. Take

$H = \{0, 2, 4, 2i_F, 4i_F, 2+2i_F, 2+4i_F, 4+2i_F, 4+4i_F\} \subseteq G$ is a complex modulo integer subgroupoid of G of type IV.

Example 2.18: Let $G = \{C(Z_{23}), *, (0, 11)\}$ be a complex modulo integer groupoid of order 23^2.

Now we have the fifth type of groupoid using modulo complex integers.

DEFINITION 2.4: *Let $G = \{C(Z_n), *, (t, t); t \in Z_n \setminus \{0, 1\}\} = G$ be the collection of elements. For $a = x+yi_F$ and $b = m+ni_F$ in G we define*

$a*b$ = $(x+yi_F) * (m+ni_F) = x*m + (n*y)i_F = (tx + tm) + (tn + ty)\, i_F \in G$. (addition modulo n). Thus G is a groupoid defined as the groupoid of modulo complex integers of type V.

We give examples of them.

Example 2.19: Let G = $\{C(Z_5), *, (2, 2)\}$ be a complex modulo integer groupoid of type V of order 5.

The only subgroupoids of G are H = $\{Z_5, *, (2, 2)\} \subseteq G$, and P = $\{Z_5\, i_F, *, (2, 2)\} \subseteq G$.

Example 2.20: Let G = $\{C(Z_7), *, (3, 3)\}$ be a complex modulo integer groupoid of type V of order 7^2. Clearly G has only two subgroupoids given by P = $\{Z_7, *, (3, 3)\} \subseteq G$ and T = $\{Z_7\, i_F, *, (3, 3)\} \subseteq G$.

Example 2.21: Let G = $\{C(Z_6), *, (2, 2)\}$ be a complex modulo integer groupoid. Take

H = $\{0, 2, 4, 2i_F, 4i_F, 2+2i_F, 2+4i_F, 4+2i_F, 4+4i_F\} \subseteq G$, H is a complex modulo integer subgroupoid of G.

Example 2.22: Let G = $\{C(Z_6), *, (5, 5)\}$ be a complex modulo integer groupoid. Take $H_1 = \{4, 4i_F, 4+4i_F\} \subseteq G$ is a subgroupoid of G. Also $H_2 = \{2, 2i_F, 2+2i_F\} \subseteq G$ is a subgroupoid of G.

These groupoids of type V are all commutative complex modulo integer groupoids. Now we proceed onto define Smarandache complex modulo integer groupoids and groupoids that satisfy special class of identity.

Let G = $\{C(Z_n), *, (u, v); u, v \in Z_n\}$ be a complex modulo integer groupoid. If $H \subseteq G$ (be a proper subset of G) is such that, H under the operations of G, is a semigroup then we define G to be a Smarandache complex modulo integer groupoid.

We provide some examples from the three types of groupoids.

Example 2.23: Let G = $\{C(Z_{10}), *, (1, 5)\}$ be a Smarandache complex modulo integer groupoid.

Clearly S = $\{0, 5, 5i_F, 5+5i_F\}$ is a semigroup of complex modulo integer groupoid, G.

Example 2.24: Let G = $\{C(Z_6), *, (4, 5)\}$ be a complex modulo integer groupoid. Take H = $\{3, 3i_F\} \subseteq$ G. H is a complex modulo integer semigroup. So G is a Smarandache complex modulo integer groupoid.

Example 2.25: Let G = $\{C(Z_6), *, (2, 4)\}$ be a complex modulo integer groupoid. H = $\{0, 3, 3i_F, 3 +3i_F\} \subseteq$ G is a complex modulo integer semigroup. Hence G is a Smarandache complex modulo integer groupoid.

We see if G is Smarandache complex modulo integer groupoid then every subgroupoid of G need not be a Smarandache complex modulo integer subgroupoid of G.

Example 2.26: Let G = $\{C(Z_6), *, (2, 4)\}$ be a Smarandache complex modulo integer groupoid. Consider

H = $\{0, 2, 4, 2i_F, 4i_F, 2 +2i_F, 4+2i_F, 4+4i_F, 2+4i_F\} \subseteq$ G. H is only a complex modulo integer subgroupoid of G but H is not a Smarandache complex modulo integer subgroupoid of G. However G is a Smarandache complex modulo integer groupoid as

S = $\{0, 3, 3i_F, 3+3i_F\} \subseteq$ G is a complex modulo integer semigroup.

Example 2.27: Let G = $\{C(Z_6), *, (4, 5)\}$ be a complex modulo integer groupoid. Take

H = $\{1, i_F, 3, 3i_F, 5, 5i_F, 1+i_F, 1+3i_F, 1+5i_F, 3+i_F, 3+3i_F, 3+5i_F, 5+i_F, 5+3i_F, 5 + 5i_F\} \subseteq G$, is a complex modulo integer subgroupoid of G. Clearly H is a Smarandache complex modulo integer subgroupoid of G as S = $\{3, 3i_F, 3+3i_F\} \subseteq H$ is a complex modulo integer semigroup. Hence the claim. Since S $\subseteq$ G, G is also a Smarandache complex modulo integer groupoid.

However

P = $\{0, 2, 4, 2i_F, 4i_F, 2+2i_F, 2+4i_F, 4+2i_F, 4+4i_F\} \subseteq G$ is only a complex modulo integer subgroupoid which is not Smarandache.

Here on wards we do not mention the type of the groupoid by very inspection it is clear.

Inview of this we have the following interesting theorem.

THEOREM 2.1: *Let G = $\{C(Z_n), *, (t, u); t, u \in Z_n\}$ be a complex modulo integer groupoid. If H $\subseteq$ G is such that H is a Smarandache modulo integer subgroupoid, then G is a Smarandache complex modulo integer groupoid. But every subgroupoid of G need not be a Smarandache complex modulo interger subgroupoid even if G is a Smarandache groupoid.*

Proof is direct and hence is left as an exercise to the reader.

Example 2.28: Consider G = $\{C(Z_8), *, (2, 4)\}$, a complex modulo integer groupoid. Take

P = $\{0, 3, 2, 4, 6, 2i_F, 3i_F, 4i_F, 6i_F, 3+2i_F, 3+3i_F, 3+4i_F, 3+6i_F, 2+2i_F, 2+3i_F, 2+4i_F, 2+6i_F, 4+2i_F, 4+3i_F, 4+6i_F, 4 + 4i_F, 6+3i_F, 6+2i_F, 6+6i_F, 6+4i_F\}$ to be a Smarandache complex modulo integer subgroupoid of G. Hence G itself is a Smarandache complex modulo integer groupoid.

We now just recall the definition of Smarandache ideal of a complex modulo integer groupoid G. Let G = $\{C(Z_n), *, (t, u)\}$ be a Smarandache complex modulo integer groupoid. A $\subseteq$ G

($A \neq \phi$ or $\{0\}$) is said to be a Smarandache complex modulo integer left ideal of G if the following conditions are true.

(i) A is a Smarandache complex modulo integer subgroupoid of G.
(ii) For $x \in G$ and $a \in A$, $x * a \in A$.

Similarly we can define Smarandache right ideal. If A is both a S-left ideal and S-right ideal of G then we define A to be a Smarandache ideal of G.

We give examples of them.

Example 2.29: Let $G = \{C(Z_6), *, (4, 5)\}$ be a Smarandache complex modulo integer groupoid. Let

$A = \{1, 3, 5, i_F, 3i_F, 5i_F, 1+ i_F, i_F+3, 1+ 3_F, 1+5i_F, 3+3i_F, 3+5i_F, 5+5i_F, 5+3i_F\} \subseteq V$ be a Smarandache left ideal of G. Clearly A is not a Smarandache right ideal of G.

Example 2.30: Let $G = \{C(Z_6), *, (4, 5)\}$ be a complex modulo integer groupoid. Consider

$A = \{1, 3, 5, i_F, 3i_F, 5i_F, 1+i_F, 1+3i_F, 1+5i_F, 3+i_F, 3+3i_F, 3+5i_F, 5+i_F, 5+3i_F, 5+5i_F\} \subseteq G$; A is a Smarandache left ideal of G and is not a Smarandache right ideal of G.

Example 2.31: Let $G = \{C(Z_6), *, (2, 4)\}$ be a complex modulo integer groupoid. Consider

$P = \{0, 2, 4, 2i_F, 4i_F, 2+2i_F, 2+4i_F, 4+2i_F, 4+4i_F\} \subseteq G$;

Clearly P is an ideal of G but clearly P is not a Smarandache ideal of G; infact P is not even a Smarandache subgroupoid of G.

Inview of this we have the following theorem.

THEOREM 2.2: *Let $G = \{C(Z_n), *, (t, s), t, s \in Z_n\}$ be a complex modulo integer groupoid. If I is a Smarandache complex modulo integer ideal of G then I is a complex modulo integer ideal of G. Conversely if I is a complex modulo integer ideal of*

G, then I is general need not be a Smarandache complex modulo integer ideal of G.

We will just define the notion of Smarandache seminormal groupoid.

Let G = {C(Z_n), *, (t, s), t, s $\in$ Z_n} be a Smarandache complex modulo integer groupoid. Suppose S $\subseteq$ G be a Smarandache complex modulo integer subgroupoid of G; we say S is a Smarandache complex modulo integer seminormal subgroupoid of G if

(i) aS = X for all a $\in$ G,
(ii) Sa = Y for all a $\in$ G,

where either X or Y is a Smarandache subgroupoid of G; but both X and Y are subgroupoids of G.

We will give one example of this definition.

Example 2.32: Let G = {C(Z_6), *, (4, 5)} be a Smarandache complex modulo integer groupoid.

A = {1, 3, 5, i_F, $3i_F$, $5i_F$, $1+i_F$, $3i_F+1$, $1+5i_F$, $3+i_F$, $3+3i_F$, $3+5i_F$, $5+i_F$, $5+3i_F$, $5+5i_F$} $\subseteq$ G, A is also a Smaradache subgroupoid of G. Clearly aA = A for all a $\in$ G but

Aa = {0, 2, 4, $2i_F$, $4i_F$, $2+2i_F$, $2+4i_F$, $4+2i_F$, $4+4i_F$} is not a Smarandache subgroupoid of G. Thus A is a Smarandache seminormal subgroupoid of G.

Inview of this we have the following theorem.

THEOREM 2.3: *Every Smarandache complex modulo integer normal groupoid is a Smarandache complex modulo integer seminormal groupoid and not conversely.*

The proof is direct and hence is left as an exercise to the reader.

Example 2.33: Let $G = \{C(Z_8), *, (2, 8)\}$ be a complex modulo integer groupoid. $S = \{0, 4, 4i_F, 4+4i_F\} \subseteq G$ is a complex modulo integer semigroup under $*$. Consider

$A = \{0, 2, 4, 6, 2i_F, 4i_F, 6i_F, 2+2i_F, 2+4i_F, 2+6i_F, 4+2i_F, 4+4i_F, 4+6i_F, 6+2i_F, 6+4i_F, 6+6i_F\} \subseteq G$ is a Smarandache complex modulo integer subgroupoid. It is easily verified for every $x \in G$, $xA = A$ and $Ax = A$.

Thus A is a Smarandache complex modulo integer normal subgroupoid of G.

We now just recall the notion of Smarandache semiconjugate subgroupoid of complex modulo integers.

Let $G = \{C(Z_n), *, (t, u), t, u \in Z_n\}$ be a Smarandache complex modulo integer groupoid.

Let I and J be any two complex modulo integer subgroupoids of G. We say I and J are Smarandache semiconjugate subgroupoids of G if

(i) I and J are Smarandache complex modulo integer subgroupoids of G.
(ii) I = xJ or Jx or
(iii) J = xI or Ix for some $x \in G$.

We give examples of Smarandache complex modulo integer subgroupoids of G which are semiconjugate.

Example 2.34: Let $G = \{C(Z_8), *, (2, 4)$ be a complex modulo integer groupoid. Consider

$J = \{0, 2, 3, 4, 6, 2i_F, 3i_F, 4i_F, 6i_F, 2+2i_F, 2+3i_F, 2+4i_F, 2+6i_F, 4+2i_F, 4+3i_F, 4+4i_F, 4 + 6i_F, 3+2i_F, 3+3i_F, 3+4i_F, 3+6i_F, 6+2i_F, 6+3i_F, 6+4i_F, 6+6i_F\} \subseteq G$, J is a Smarandache complex integer subgroupoid of G. Take

$I = \{0, 2, 4, 6, 2i_F, 4i_F, 6i_F, 2+2i_F, 2+4i_F, 2+6i_F, 4+2i_F, 4+4i_F, 4+6i_F, 6+2i_F, 6+4i_F, 6+6i_F\} \subseteq G$; I is also a Smarandache complex modulo integer subgroupoid of G.

Now TJ = I (or 7, $7i_F$, J = I or $7+7i_F$ J = I). Hence J and I are Smarandache complex modulo integer semiconjugate subgroupoids of G.

Now we proceed onto define Smarandache complex modulo integer conjugate subgroupoids of a complex modulo integer groupoid G.

Let $G = \{C(Z_n), *, (t, u); t, u \in Z_n\}$ be a Smarandache complex modulo integer groupoid. H and P be complex modulo integer subgroupoids of G. We say H and P are Smarandache conjugate complex modulo integer subgroupoids of G if

(i) H and P are Smarandache subgroupoids of complex modulo integers of G.
(ii) H = xP or Px and
(iii) P = xH or Hx.

We will first illustrate this situation by an example.

Example 2.35: Let $G = \{C(Z_{12}), *, (1, 3)\}$ be a complex modulo integer groupoid.

$S = \{0, 6, 6i_F, 6+6i_F\} \subseteq G$ is a complex modulo integer semigroup of G. So G is a Smarandache complex modulo integer groupoid. Consider

$H_1 = \{0, 3, 6, 9, 3i_F, 6i_F, 9i_F, 3+3i_F, 3+6i_F, 3+9i_F, 6+3i_F, 6+6i_F, 6+9i_F, 9+3i_F, 9+6i_F, 9+9i_F\} \subseteq G$ and

$H_2 = \{2, 5, 8, 11, 2i_F, 5i_F, 8i_F, 11i_F, 2+2i_F, 2+5i_F, 2+8i_F, 2+11i_F, 5+2i_F, 5+5i_F, 5+8i_F, 5+11i_F, 8+2i_F, 8+5i_F, 8+8i_F, 8+11i_F, 11+2i_F, 11+5i_F, 11+8i_F, 11+11i_F\} \subseteq G$; are two Smarandache complex modulo integer subgroupoids of G.

Now $H_1 = 3H_2$ ($3i_F H_2$, $3+3i_F H_2$)

Further $H_2 = 2H_1$ ($2i_F H_1$ or $2+2i_F H_1$).

So H_1 and H_2 are Smarandache conjugate complex modulo integer subgroupoids of G.

In view of this we have the following theorem.

THEOREM 2.4: *Let G be a Smarandache complex modulo integer groupoid. If H_1 and H_2 are two Smarandache complex modulo integer subgroupoids of G which are Smarandache conjugate then they are Smarandache semiconjugate. But if we have two subgroupoids to be Smarandache semiconjugate then they need not in general be Smarandache conjugate.*

We give the following theorems proofs of which are left to be reader [ ].

THEOREM 2.5: *The complex modulo integer groupoids $G = \{C(Z_n), *, (t, t), t < n\}$ are commutative.*

THEOREM 2.6: *The complex modulo integer groupoids $G = \{C(Z_p), *, (t, t), t , p, p$ a prime$\}$ are normal.*

THEOREM 2.7: *The complex modulo integer groupoids $G = \{C(Z_n), *, (t, t); t < n\}$ are P-groupoids.*

THEOREM 2.8: *The complex modulo integer groupoids $G = \{C(Z_n\}, *, (t, t), 1 < t < n\}$ are not alternative groupoids if n is a prime.*

THEOREM 2.9: *The complex modulo integer groupoids $G = \{C(Z_n\}, *, (t, t)\}$, n not a prime are alternative groupoids if and only if $t^2 \equiv t \pmod n$.*

THEOREM 2.10: *The complex modulo integer groupoids $G = \{C(Z_n\}, *, (t, u)\}$ are simple if $t + u = n$ and t and u are primes.*

We will give examples of them.

Example 2.36: Let G = {C(Z_2), *, (7, 13)} be complex modulo integer groupoid. G is simple.

Example 2.37: Let G = {C(Z_{13}), *, (7, 6)} be a complex modulo integer groupoid. G is simple.

Inview of this example we can easily prove the following theorem.

THEOREM 2.11: *Let*

*G = {C(Z_p), *, (t, u) such that t + u = p; p a prime and (t, u) = 1} be a complex modulo integer groupoid. G is simple.*

THEOREM 2.12: *Let*

*G = {C(Z_n), *, (t, u); (t, u) = 1 and t, u ∈ Z_n \ {0}} be a complex modulo integer groupoid. In G, {0} is not an ideal.*

THEOREM 2.13: *P is a left ideal of G = {C(Z_n), *, (t, u) = 1} the complex modulo integer groupoid if and only if P is a right ideal of G′ = {C(Z_n), *, (u, t) = 1}, the complex modulo integer groupoid.*

We will just give an example of this situation.

Example 2.38: Let G = {C(Z_4), *, (2,3)} be a complex modulo integer groupoid. Let P = {0, 2, 2i_F, 2+2i_F} and

Q = {1, 3, i_F, 3i_F, i_F+1, i_F+3, 3i_F+1, 3i_F+3} be complex modulo integer left ideals of G. Clearly P and Q are not right ideals of G. Now consider the complex modulo integer groupoid

H = {C(Z_4), *, (3, 2)}. Take T = {0, 2, 2i_F, 2+2i_F} ⊆ H and

R = {1, 3, i_F, 3i_F, 1+i_F, 1+3i_F, i_F+3, 3i_F+3} ⊆ H. Clearly T and R are complex modulo integer right ideals of H and not left ideals of H.

$G(Z_n) = \{C(Z_n), *, (0, t)\}$, is a complex modulo integer P-groupoid and alternative groupoid if and only if $t^2 \equiv t \mod n$.

Proof is left as an exercise to the reader.

Example 2.39: Let $G = \{C(Z_6), *, (0, 3)\}$ be a complex modulo integer P-groupoid and alternative groupoid.

Example 2.40: Let $G = \{C(Z_{12}), *, (0, 4)\}$ be a complex modulo integer P-groupoid and alternative groupoid.

Example 2.41: Let $G = \{C(Z_{10}), *, (0, 5)\}$ be a complex modulo integer P-groupoid and alternative groupoid.

We can as in case of usual groupoids define Smarandache left ideals and Smarandache right ideals in case of complex modulo integer groupoids.

Example 2.42: Let $G = \{C(Z_6), *, (4, 5)\}$ be a Smarandache complex modulo integer groupoid.

$A = \{1, 3, 5, i_F, 3i_F, 5i_F, 1+1i_F, 1+3i_F, 1+5i_F, 3+i_F, 3+3i_F, 3+5i_F, 5+i_F, 5+3i_F, 5+5i_F\} \subseteq G$ is a Smarandache left ideal of G and is not a Smarandache right ideal of G. Thus in general a Smarandache left ideal of G need not be a Smarandache right ideal of G. Also every ideal (right or left) need not be Smarandache (right or left) ideal of G.

We can define all identities as in case of usual groupoids. Also the Smarandache analogue is done as in case of usual groupoids.

Example 2.43: Let $G = \{C(Z_{10}), *, (5, 6)\}$ be a complex modulo integer groupoid. G is a Smarandache strong Moufang complex modulo integer groupoid.

Example 2.44: Let $G = \{C(Z_{12}), *, (3, 9)\}$ be a complex modulo integer groupoid, G is only a Smarandache complex modulo integer Moufang groupoid and is not a Smarandache strong Moufang groupoid.

However this theorem can be easily proved.

THEOREM 2.14: *Every Smarandache strong Moufang complex modulo integer groupoid is a Smarandache Moufang complex modulo integer groupoid and not conversely.*

Example 2.45: Let G = {C(Z_{12}), *, (3, 4)} be a complex modulo integer Smarandache strong Bol groupoid. For if we take x, y,z $\in$ G; then

(x*y)*z) *y = [(3x + 4y) * z] * y = 3x + 4y
(if x = 7+3i_F . y = 2+5i_F and z = 1+i_F).

Then (x*y)*z) *y = 3 (7 + 3i_F) + 4 (2+5i_F)
= 21 + 9i_F + 8 + 20i_F = 5 + 5i_F.

Also x * [(y*z) * y] = 5 + 5i_F.

Example 2.46: Let G = {C(Z_4), *, (2, 3)} be a complex modulo integer groupoid. G is a Smarandache Bol groupoid but is not a Smarandache strong Bol groupoid.

It is left for the reader to prove that every Smarandache strong Bol groupoid of complex modulo integers is a Smarandache Bol groupoid and not conversely.

Example 2.47: Let G = {C(Z_6), *, (4, 3)} be a complex modulo integer groupoid. G is a Smarandache strong P-groupoid.

Example 2.48: Let G = {C(Z_6), *, (3, 5)} be a complex modulo integer groupoid. G is not a Smarandache strong P-groupoid.

Example 2.49: Let G = {C(Z_{14}), *, (7, 8)} be a complex modulo integer groupoid which is a Smarandache strong alternative groupoid.

Now we proceed onto find conditions on Smarandache complex modulo integer groupoids.

Example 2.50: Let $G = \{C(Z_9), *, (5, 3)\}$ be a complex modulo integer groupoid. G is not a Smarandache groupoid.

Example 2.51: Let $G = \{C(Z_8), *, (1, 6)\}$ be a complex modulo integer groupoid. G is a Smarandache groupoid, as

$H = \{4, 4i_F, 4+4i_F\} \subseteq G$ is a semigroup.

The following theorems are simple and hence left as exercises to the reader.

THEOREM 2.15: *Let $G = \{C(Z_{2p}), *, (1, 2); p$ an odd prime$\}$ be the complex modulo integer groupoid. G is a Smarandache groupoid.*

THEOREM 2.16: *Let*

$G = \{C(Z_{3p}), *, (1, 3), p$ an odd prime, $p \neq 3\}$ be the complex modulo integer groupoid. G is a Smarandache groupoid.

THEOREM 2.17: *Let*

$G = \{C(Z_{p_1p_2}), *, (1, p_1), p_1 \neq p_2\}$ and $H = \{C(Z_{p_1p_2}), *, (1,p_2), p_1 \neq p_2\}$ be two complex modulo integer groupoids. H and G are Smarandache groupoids.

THEOREM 2.18: *Let $G = \{C(Z_n), *, (1, p); p/n\}$ be a complex modulo integer groupoid. G is a Smarandache groupoid.*

THEOREM 2.19: *Let $G = \{C(Z_n), *, (t, u), t + u \equiv 1 \pmod n\}$ be a complex modulo integer groupoid. G is a Smarandache idempotent groupoid.*

THEOREM 2.20: *Let $G = \{C(Z_n), *, (t, u), t + u \equiv 1 \pmod n\}$ be a complex modulo integer groupoid. G is a Smarandache p-groupoid if and only if $t^2 \equiv t \pmod n$ and $u^2 \equiv u \pmod n$.*

The proofs are left as an exercise to the reader.

THEOREM 2.21: *Let*

$G = \{C(Z_n), *, (t, u), t + u \equiv 1 \pmod n\}$ be a complex modulo integer groupoid, G is a Smarandache alternative groupoid if and only if $t^2 \equiv t \pmod n$ and $u^2 \equiv u \pmod n$.

THEOREM 2.22: *Let*

$G = \{C(Z_n), *, (t, u), t + u \equiv 1 \pmod n\}$ be a Smarandache complex modulo integer groupoid. G is a Smarandache strong Bol groupoid if and only if $t^3 = t \pmod n$ and $u^2 \equiv u \pmod n$.

THEOREM 2.23: *Let*

$G = \{C(Z_n), *, t + u \equiv 1 \pmod n\}$ be a Smarandache complex modulo integer groupoid. G is a Smarandache strong Moufang groupoid if and only if $t^2 = t \pmod n$ and $u^2 \equiv u \pmod n$.

The proof is direct and is left as an exercise to the reader.

Example 2.52: Let $G = \{C(Z_6), *, (3, 4)\}$ be a Smarandache complex modulo integer groupoid. G is a Smarandache strong Bol groupoid.

THEOREM 2.24: *Let*

$G = \{C(Z_p), *, \left(\frac{p+1}{2}, \frac{p+1}{2}\right)$, p a prime$\}$ be a complex modulo integer groupoid. G is a Smarandache groupoid.

THEOREM 2.25: *Let*

$G = \{C(Z_p), *, \left(\frac{p+1}{2}, \frac{p+1}{2}\right)$, p a prime$\}$ be a complex modulo integer groupoid. G is a Smarandache groupoid.

Example 2.53: Let $G = \{C(Z_{12}), *, (6, 6)\}$ be a complex modulo integer groupoid. $S = \{0, 6, 6i_F, 6 + 6i_F\} \subseteq G$ is a semigroup. So G is a Smarandache groupoid.

Example 2.54: Let $G = \{C(Z_9), *, (7, 7)\}$ be a complex modulo integer groupoid. $S = \{0, 7, 7i_F, 7 + 7i_F\} \subseteq G$ is a semigroup, so G is a Smarandache groupoid.

In view of this we have the following theorem.

THEOREM 2.26: *Let $G = \{C(Z_n), *, (m, m)\}$, n even $m^2 \equiv m \pmod{n}$ and $m + m \equiv 0 \pmod{n}$ be a complex modulo integer groupoid. Then G is a Smarandache complex modulo integer groupoid of order n^2.*

Proof follows from the simple fact $S = \{0, m, mi_F, m+mi_F\} \subseteq G$ is a semigroup.

In general we have the following result, which is first illustrated by an example.

Example 2.55: Let $G = \{C(Z_9), *, (5, 5)\}$ be a complex modulo integer groupoid.

Hence G is a Smarandache complex modulo integer groupoid as every element r is such that $r * r = r \pmod{n}$.

THEOREM 2.27: *Let $G = \{C(Z_n), *, (m, m)\}$ be a complex modulo integer groupoid. G is a Smarandache complex modulo integer groupoid only if $m + m = 1 \pmod{n}$.*

Proof is direct and hence left as an exercise to the reader.

Example 2.56: Let $G = \{C(Z_{15}), *, (8, 8)\}$ be a complex modulo integer groupoid.

Consider $a + bi_F$ in G, now

$$\begin{aligned}(a + bi_F) * (a + bi_F) &= a * a + b * bi_F. \\ &= (8a + 8a) + (8b + 8b)i_F \\ &= 16a \pmod{15} + 16bi_F \pmod{15} \\ &= a + bi_F.\end{aligned}$$

Thus every element is also a semigroup. Thus G is a Smarandache complex modulo integer groupoid. We know every odd number m is such that $\frac{(m+1)}{2}+\frac{(m+1)}{2} = 1 \pmod m$ thus the proof of the theorem trivial.

The following theorem is straight forward and uses only number theoretic techniques.

THEOREM 2.28: *Let* $G = \{C(Z_n), *, (m, m); m+m \equiv 1 \pmod n \text{ and } m^2 = m \pmod n\}$ *be the Smarandache complex modulo integer groupoid.*

(i) G is a Smarandache idempotent groupoid of complex modulo integers.
(ii) G is a Smarandache strong P-groupoid.
(iii) G is a Smarandache strong Bol-groupoid.
(iv) G is a Smarandache strong Moufang groupoid.
(v) G is a Smarandache strong alternative groupoid.

It is interesting and important to mention here that, $G = \{C(Z_n), *, (t, u)\}$ is a complex modulo integer.

We can build complex modulo integer groupoids with identity as follows:

We known $C(Z_n) = \{a + bi_F \mid a, b \in Z_n, i_F^2 = n-1\}$ is the complex modulo integers.

Now we adjoin an element e with $C(Z_n)$ as follows:
$C(Z_n) \cup \{e\} = \{a + bi_F \mid a, b \in C(Z_n) \cup \{e\}\}$.

We define a binary non associative closed operation * on
$C(Z_n \cup e) = C(Z_n) \cup \{e\}$ as follows:
$a * a = e$
$a * e = e * a = a$ for all $a \in C(Z_n \cup e)$.

For $a \neq b$; $a * b = ta + ub \pmod{n}$
where $a, b \in C(Z_n \cup e)$ and $t, u \in Z_n \setminus \{0\}$.

$\{C(Z_n \in e), *, (t, u)\}$ is a groupoid called the groupoid with identity.

Example 2.57: Let $G = \{C(Z_4 \cup e), *, (t, u)\}$ be a groupoid with identity of order 5^2.

Every $H = \{e, g\} \subseteq G$ is a semigroup for every $g \in C(Z_4 \cup e)$.

Thus G is a Smarandache groupoid.

THEOREM 2.29: *Let $G = \{C(Z_n \cup e\}, *, (t, u), t, u \in Z_n \setminus \{0\}\}$ be a complex modulo integer groupoid with identity. G is a Smarandache groupoid.*

Proof is straight forward as every pair of the form $\{e, g\} = S$ for every $g \in C(Z_n \cup e)$ is a semigroup, hence the claim.

Thus we say we can get a class of such groupoids for every fixed n as $t, u \in Z_n \setminus \{0\}$ can vary.

THEOREM 2.30: *No groupoid in the class of groupoids $G = \{C(Z_n \cup e), *, (t, u), t, u \in Z_n \setminus \{0\}\}$ is a Smarandache complex modulo integer idempotent groupoid.*

The proof is direct by using the fact in G, $a*a = e$ for every $a \in G$, so no element in G can be an idempotent.

Example 2.58: Let $G = \{C(Z_6 \cup e), *, (5, 3)\}$ be a complex modulo integer groupoid. It is easily verified that G is a Smarandache strong complex modulo integer right alternative groupoid. Further it is important to note that G is not even a Smarandache modulo integer left alternative groupoid.

Example 2.59: Let $G = \{C(Z_6 \cup e), *, (4, 5)\}$ be a complex modulo integer groupoid. Clearly G is a Smarandache strong left alternative groupoid.

We have the following interesting theorems, the proof of which is direct [14, 19].

THEOREM 2.31: *Let $G = \{C(Z_n \cup e), *, (t, u)\}$ be a complex modulo integer groupoid with identity e. G is a Smarandache strong right alternative groupoid if and only if $t^2 \equiv 1 \pmod n$ and $tu + u \equiv 0 \pmod n$.*

THEOREM 2.32: *Let $G = \{C(Z_n \cup e), *, (t, u)\}$ be a Smarandache complex modulo integer groupoid with identity e. G is a Smarandache strong left alternative if and only if $u^2 \equiv 1 \pmod n$ and $(t + tu) \equiv 0 \pmod n$.*

THEOREM 2.33: *Let $G = \{C(Z_n \cup e), *, (t, u)\}$ denote the class of complex modulo integer groupoids n not a prime. G is a Smarandache strong Moufang groupoid and Smarandache strong P-groupoid only when $t^2 = t \pmod n$ and $u^2 = u \pmod n$.*

Now we proceed onto define complex modulo integer loops.

DEFINITION 2.5: *Let*

$C(Z_n) = \{a + bi_F \mid a, b \in \{1, 2, \ldots, n\}, i_F^2 = n-1\}$ be the complex modulo integers. We adjoin an element e called the identity with Z_n and choose $n > 3$ and n odd.

*Now consider $C(L_n^{(m)}) = \{0, a \in C(Z_n) \cup \{e\}$; with * a binary operation on $C(L_n(m))$ where $m \in Z_n \setminus \{0\}$ such that $(m, n) = 1$ and $(m-1, n) = 1$ with $m < n$ such that*

(i) $e * a = a * e = a$ for all $a \in C(L_n^{(m)})$.
(ii) $a * a = a^2 = e$ for all $a \in C(L_n^{(m)})$.
(iii) $a * b = (mb - (m-1)a) \pmod n$
for all $a, b \in C(L_n^{(m)})$.

$C(L_n^{(m)})$ is defined as the new class of loops of complex modulo integers.

We just show how operation is carried out.

Example 2.60: Let

$C(L_5(2)) = \{e, C(Z_5)\} = \{e, 1, 2, \ldots, 5, i_F, 2i_F, 3i_F, 4i_F, 5i_F, 1 + i_F, \ldots, 5 + 5i_F\}$. For $x = a + bi_F$ and $y = c+di_F$ in $C(Z_5) \cup \{e\}$; we have

$x * y = 2(c + di_F) - 1(a+bi_F) = (2c - a) + (2d - b)i_F$ is in $C(Z_5) \cup \{e\}$.

Suppose we take $2 + 5i_F$, $4 + 2i_F$ in $C(L_5(2))$ then

$$\begin{aligned}(2+5i_F) * (4+2i_F) &\\ &= 2(4+2i_F) - 1(2+5i_F)\\ &= 3 + 4i_F + 3 + 5i_F\\ &= 1 + 4i_F.\end{aligned}$$

Now we see the order of $C(L_5(2))$ is even and order of

$(C(L_5(2)) = 5^2 + 1 = 26$.

We see these loops are different from usual loops built using $\{e, 1, 2, \ldots, n\} \cong Z_n \cup \{e\}$.

It is important to mention here that in case of complex modulo integer groupoids with identity 'e' serves a different role.

We can only say e acts as the identity element equidistantly placed on a circle with e as its centre.

Example 2.61: Let

$C(L_7(3)) = \{0, 1, 2, \ldots, 7, i_F, 2i_F, \ldots, 7i_F, 1+i_F, 2+i_F, \ldots, 7+i_F\} \cup \{e\}$ be the complex modulo integer loop of order 64.

Example 2.62: Let

$C(L_{15}(2)) = \{(C(Z_{15}) \cup e\}, *, 2\}$ be the complex modulo integer quasi loop of order 256.

We see every loop built using the complex modulo integers is of even order.

THEOREM 2.34: *Let $C(L_n(m))$ be a complex modulo integer loop. ($n > 3$, n – odd). $2 / o\,(C(L_n(m))$ and $4 / o\,C(L_n(m))$.*

Proof follows from the very fact if n is odd $n = 2t + 1$ and

$$\begin{aligned} o\,(C(L_n(m)) &= (2t + 1)^2 + 1 \\ &= 4t^2 + 4t + 1 + 1 \\ &= 4t^2 + 4t + 2. \end{aligned}$$

If 2 divides $o\,(C(L_n(m))$ then $4t^2 + 4t + 2 / 2 = 2t^2 + 2t + 1$.

Clearly $2 \nmid 2t^2 + 2t + 1$ if t is odd or even. Hence the claim.

THEOREM 2.35: *Every complex modulo integer loop $C(L_n(m))$ is a Smarandache quasi loop.*

Proof follows from the simple fact every pair $(e, x) = H$ where $x \in C(L_n(m)) \setminus \{e\}$ is a group. Hence the claim.

We will denote by

$C(L_n) = \{C(L_n(m)); m < n, (m, n) = 1 \text{ and } (m - 1, n) = 1\}$; ($n > 3$ and n odd) the class of complex modulo integer loops.

THEOREM 2.36: *The class of complex modulo integer loops $C(L_n)$ contains exactly one left alternative quasi loop and one right alternative quasi loop and does not contain any alternative loop.*

The proof is direct by using simple number theoretic techniques, however we give examples of them.

Example 2.63: Let $C(L_{15}(2))$ be a complex modulo integer loop. $C(L_{15}(2))$ is the right alternative loop. Infact $C(L_{15}(2))$ is the only right alternative loop in $C(L_{15})$.

Example 2.64: Let $C(L_9(2))$ be a complex modulo integer loop in $C(L_9)$. Clearly $C(L_9(2))$ is a right alternative complex modulo integer loop.

Example 2.65: Let $C(L_{19}(18))$ be a complex modulo integer loop. $C(L_{19}(18))$ is only a left alternative loop in the class of loops $C(L_{19})$.

Example 2.66: Let $C(L_{23}(22))$ be a complex modulo integer loop. $C(L_{23}(22))$ is only a left alternative loop in the class of loops $C(L_{23})$.

These loops given in examples 2.66 and 2.67 are not right alternative.

The following theorem can be easily proved by using number theoretic techniques.

THEOREM 2.37: *Let $C(L_n)$ be the class of complex modulo integer loops.*

(i) $C(L_n)$ does not contain any Moufang loop.
(ii) $C(L_n)$ does not contain any Bol loop.
(iii) $C(L_n)$ does not contain any Bruck loop.

Example 2.67: Let $C(L_7(3))$ be a complex modulo integer loop. It is easily verified $C(L_7(3))$ is a weak inverse property loop.

It is important to observe. $3^2 - 3 + 1 = 9 - 3 + 1 \equiv 0 \pmod n$.

In view of this we have the following theorem.

THEOREM 2.38: *Let $C(L_n(m)) \in C(L_n)$ be the complex modulo integer loop. $C(L_n(m))$ is a weak inverse property loop (WIP-loop) if and only if $(m^2 - m + 1) = 0 \pmod n$.*

For proof refer [15, 19].

Recall the associator of a complex modulo integer loop $A(C(L_n(m))) = \langle\{t \in C(L_n(m) \mid t = (x, y, z)$ for some $x, y, z \in (L_n(m)\rangle$.

The following theorem can be easily proved using simple number theoretic techniques.

THEOREM 2.39: *Let $C(L_n(m)) \in C(L_n)$ be a complex modulo integer loop. The associator $A(C(L_n(m))) = C(L_n(m))$.*

Example 2.68: Let $C(L_{11}(6))$ be a complex modulo integer loop. $C(L_{11}(6))$ is a commutative loop.

Example 2.69: Let $C(L_{15}(8))$ be a complex modulo integer loop.
$C(L_{15}(8))$ is a commutative loop.

Example 2.70: Let $C(L_{21}(11))$ be a complex modulo integer loop. $C(L_{21}(11))$ is a commutative loop.

Inview of these examples we have the following theorem the proof of which can be derived using simple number theoretic techniques.

THEOREM 2.40: *Let*

$C(L_n) = \{C(L_n(m)) \mid (m, n) = 1$ and $(m-1, n) = 1, m < n\}$ be the class of complex modulo integer loop. $C(L_n)$ contains one and only one commutative loop.

This happens when $m = \frac{(n+1)}{2}$, clearly for this m we have $(m, n) = 1$ and $(m-1, n) = 1$.

Example 2.71: Let $C(L_5(2))$ be a complex modulo integer loop. $C(L_5^{(2)})$ has $\{e, 1\}$, $\{e, 4\}$, $\{e, 2\}$ and $\{e, 2i_F\}$ as subgroups.

Example 2.72: Let

$C(L_5(2)) = \{e, 1, 2, 3, 4, 5, i_F, 2i_F, 3i_F, 4i_F, 5i_F, 1+i_F, 2 + 2i_F, \ldots, 4i_F + 5, 5+5i_F\}$ be a complex modulo integer loop of order 36.

The number of subgroups of order two in $C(L_5(2))$ is 25. Consider $H = \{e, 2, 2i_F, 3i_F+4, 4+3i_F, 1+i_F\} \subseteq C(L_5(2))$.

*	e	2	$2i_F$	$3+4i_F$	$4+3i_F$	$1+i_F$
e	e	2	$2i_F$	$3+4i_F$	$4+3i_F$	$1+i_F$
2	2	e	$3+4i_F$	$4+3i_F$	$1+i_F$	$2i_F$
$2i_F$	$2i_F$	$4+3i_F$	e	$1+i_F$	$3+4i_F$	2
$3+4i_F$	$3+4i_F$	$1+i_F$	2	e	$2i_F$	$4+3i_F$
$4+3i_F$	$4+3i_F$	$2i_F$	$1+i_F$	2	e	$3+4i_F$
$1+i_F$	$1+i_F$	$3+4i_F$	$4+3i_F$	$2i_F$	2	e

Clearly H is a subloop and is non commutative. Consider

$P = \{e, i_F, 1, 2i_F + 4, 2 + 4i_F, 3i_F + 3\} \subseteq C(L_5(2))$, P is again a non commutative subloop of order 6.

Clearly $6 \nmid 26$.

$T = \{e, 3i_F, 3, i_F+2, 1+2i_F, 4+4i_F\} \subseteq C(L_5(2))$ is a non commutative subloop of order 6.

$W = \{e, 4i_F, 4, 3i_F + 1, 3+i_F, 2+2i_F\} \subseteq C(L_5(2))$ is again a non commutative subloop of order 6.

$M = \{e, i_F, 2i_F, 3i_F, 4i_F, 5i_F\} \subseteq C(L_5(2))$ is a subloop of order six given by the following table.

*	e	i_F	$2i_F$	$3i_F$	$4i_F$	$5i_F$
e	e	i_F	$2i_F$	$3i_F$	$4i_F$	$5i_F$
i_F	i_F	e	$3i_F$	$5i_F$	$2i_F$	$4i_F$
$2i_F$	$2i_F$	$5i_F$	e	$4i_F$	i_F	$3i_F$
$3i_F$	$3i_F$	$4i_F$	i_F	e	$5i_F$	$2i_F$
$4i_F$	$4i_F$	$3i_F$	$5i_F$	$2i_F$	e	i_F
$5i_F$	$5i_F$	$2i_F$	$4i_F$	i_F	$3i_F$	e

Clearly M is also a subloop of order 6 and is non commutative. Now we consider the subloop generated by

$R = \{2 + 3i_F, 1 + 4i_F\}$. We just construct the table for it.

*	e	$2+3i_F$	$1+4i_F$	$3+2i_F$	$5+5i_F$	$4+i_F$
e	e	$2+3i_F$	$1+4i_F$	$3+2i_F$	$5+5i_F$	$4+i_F$
$2+3i_F$	$2+3i_F$	e	$5+5i_F$	$4+i_F$	$3+2i_F$	$1+4i_F$
$1+4i_F$	$1+4i_F$	$3+2i_F$	e	$5+5i_F$	$4+i_F$	$2+3i_F$
$3+2i_F$	$3+2i_F$	$1+4i_F$	$4+i_F$	e	$2+3i_F$	$5+5i_F$
$5+5i_F$	$5+5i_F$	$4+i_F$	$2+3i_F$	$1+4i_F$	e	$3+2i_F$
$4+i_F$	$4+i_F$	$5+5i_F$	$3+2i_F$	$2+3i_F$	$1+4i_F$	e

We see R generates again a subloop of order six.

Consider W = $\{\langle 1 + 2i_F, 3+i_F\rangle\} \subseteq C(L_5(2))$, the subloop generated by these two elements.

*	e	$1+2i_F$	$3+i_F$	$4+3i_F$	$5+5i_F$	$2+4i_F$
e	e	$1+2i_F$	$3+i_F$	$4+3i_F$	$5+5i_F$	$2+4i_F$
$1+2i_F$	$1+2i_F$	e	$5+5i_F$	$2+4i_F$	$4+3i_F$	$3+i_F$
$3+i_F$	$3+i_F$	$4+3i_F$	e	$5+5i_F$	$2+4i_F$	$1+2i_F$
$4+3i_F$	$4+3i_F$	$3+i_F$	$2+4i_F$	e	$1+2i_F$	$5+5i_F$
$5+5i_F$	$5+5i_F$	$2+4i_F$	$1+2i_F$	$3+i_F$	e	$4+3i_F$
$2+4i_F$	$2+4i_F$	$5+5i_F$	$4+3i_F$	$1+2i_F$	$3+i_F$	e

is again a subloop of order 6. We see subloops in general are of order six. Further we see that so in general the Lagrange theorem for finite groups is not true in case of these finite loops.

Example 2.73: Let $C(L_5(4))$ be a complex modulo integer loop of order 36. To find subloops in $C(L_5(4))$. Consider

H = $\langle 2 + 2i_F, i_F + 3\rangle \subseteq C(L_5(4))$. The table for H is as follows:

*	e	$2+2i_F$	$3+i_F$	$1+3i_F$	$4+5i_F$	$5+4i_F$
e	e	$2+2i_F$	$3+i_F$	$1+3i_F$	$4+5i_F$	$5+4i_F$
$2+2i_F$	$2+2i_F$	e	$1+3i_F$	$3+i_F$	$5+4i_F$	$4+5i_F$
$3+i_F$	$3+i_F$	$4+5i_F$	e	$5+4i_F$	$2+2i_F$	$1+3i_F$
$1+3i_F$	$1+3i_F$	$5+4i_F$	$4+5i_F$	e	$3+i_F$	$2+2i_F$
$4+5i_F$	$4+5i_F$	$1+3i_F$	$5+4i_F$	$2+2i_F$	e	$3+i_F$
$5+4i_F$	$5+4i_F$	$3+i_F$	$2+2i_F$	$4+5i_F$	$1+3i_F$	e

H is a subloop of order 6. K = $\langle 1 + 2i_F, 2+i_F\rangle \subseteq C(L_5(4))$. The table for K is as follows:

*	e	$1+2i_F$	$2+i_F$	$5+3i_F$	$3+5i_F$	$4+4i_F$
e	e	$1+2i_F$	$2+i_F$	$5+3i_F$	$3+5i_F$	$4+4i_F$
$1+2i_F$	$1+2i_F$	e	$5+3i_F$	$2+i_F$	$4+4i_F$	$3+5i_F$
$2+i_F$	$2+i_F$	$3+5i_F$	e	$4+4i_F$	$1+2i_F$	$5+3i_F$
$5+3i_F$	$5+3i_F$	$4+4i_F$	$3+5i_F$	e	$2+i_F$	$1+2i_F$
$3+5i_F$	$3+5i_F$	$5+3i_F$	$4+4i_F$	$1+2i_F$	e	$2+i_F$
$4+4i_F$	$4+4i_F$	$2+i_F$	$1+2i_F$	$3+5i_F$	$5+3i_F$	e

Clearly K is a subloop of order 6. Consider the subloop generated by P = $\langle\{1 + 3i_F, 2+4i_F, 2i_F +1, 3+2i_F\}\rangle \subseteq C(L_5(4))$.

Consider the subloop generated by

M = $\{\langle 2+i_F, 1+3i_F, 4+2i_F\rangle\} \subseteq C(L_5(4))$. We see

M = $\{e, 2+i_F, 1+3i_F, 4+2i_F, 3+4i_F, 5+5i_F\} \subseteq C(L_5(4))$ is a subloop of $C(L_5 (4))$.

T = $\{e, 2+3i_F, 1+4i_F, 4+3i_F, 5+2i_F, 3+2i_F, 5+3i_F, 4+4i_F, 2+i_F, 2+4i_F, 4+2i_F, 1+5i_F, 3+4i_F, 3+i_F, 4+i_F, 4+5i_F, 3+3i_F, 5+5i_F, 5+i_F, 2+2i_F, 1+i_F, 1+2i_F, 3+5i_F, 1+3i_F, 2+5i_F, 5+4i_F\} \subseteq C(L_5(4))$; is a subloop of $C(L_5(4))$.

Example 2.74: Let $C(L_7(3))$ be a complex modulo integer loop. Clearly order of $C(L_7(3))$ is 64. Take

M = $\{\langle 2+5i_F, 3+2i_F\rangle\} \subseteq C(L_7(3))$; to calculate M.

M = $\{e, 2+5i_F, 3+2i_F, 7+4i_F, 5+3i_F, 6+7i_F, 1+i_F, 2+6i_F, 2+3i_F, 6+i_F, 6+5i_F, 1+5i_F, 7+5i_F, 3+7i_F, 5+6i_F, 7+2i_F, 7+7i_F, 5+5i_F, 6+6i_F, 3+3i_F, 2+i_F, 5+4i_F, 4+6i_F, 3+5i_F, 1+4i_F, \ldots\} \subseteq C(L_7(3))$ is a subloop of order 50 in $C(L_7(3))$.

Example 2.75: Let C $(L_{15}(2))$ be a complex modulo integer loop of order (225+1) + 30 = 256. Consider

$H = \{\langle 2 + 4i_F, 1+7i_F\rangle\} \subseteq L_{15}(2)$. The subloop generated by H is as follows:

$H = \{e, 2+4i_F, 1+7i_F, 15+10i_F, 3+i_F, 4+13i_F, 5+7i_F, 14+i_F, 8+10i_F, 12+7i_F, 7+10i_F, \ldots\} \subseteq C(L_{15}(2))$ is a subloop of $C(L_{15}(2))$.

Likewise one can construct subloops of a loop.

We can also study special identities in these loops.

We will call a complex modulo integer loop to be a Smarandache complex modulo integer loop if $C(L_n(m))$ has a proper subset which is a group. All loops $C(L_n(m))$ are Smarandache complex modulo integer loop as $H = \{e, x\}$ for every $x \in C(L_n(m) \setminus \{e\})$ is a group.

We can define the notion of Smarandache cosets in complex modulo integer loops when the loops are S-loops.

For instance take the complex modulo integer loop $C(L_5(2))$. We know $A = \{e, 1\}$ is a subgroup of the S-loop $C(L_5(2))$.

The right coset of A is

$A \text{ o} 1 = \{e, 1\}$, $A \text{ o } 3 = \{3, 5\}$, $A \text{ o } 4 = \{4, 2\}$, $A \text{ o } 2 = \{2, 3\}$, $A \text{ o } 5 = \{5, 4\}$, $A \text{ o } i_F = \{i_F, 2i_F + 4\}$, $A \text{ o } 2i_F = \{2i_F , 4i_F + 4\}$, $A \text{ o } 3i_F = \{3i_F, i_F +4\}$, $A \text{ o } 4i_F = \{4i_F, 3i_F +4\}$, $A \text{ o } 5i_F = \{5i_F, 5i_F + 4\}$, $A\text{o } 1 + i_F = \{i_F + 1, 2i_F + 1\}$, $A \text{ o } 1+2i_F = \{1+2i_F, 1+4i_F\}$, $A \text{ o } 1+3i_F = \{1+3i_F , 1+i_F \}$, $A \text{ o } 1+4i_F = \{1+4i_F, 1+3i_F\}$, $A \text{ o } 1+5i_F = \{1 + 5i_F , 5+5i_F \}$ and so on.

Similarly one can find the left coset of A, $1 \text{ o } A = \{e, 1\}$, $2 \text{ o } A = \{5, 2\}$, $3 \text{ o } A = \{3, 4\}$, $4 \text{ o } A = \{4, 3\}$, $5 \text{ o } A = \{5, 2\}$, $i_F \text{ o } A = \{i_F, 2+4i_F \}$, $3i_F \text{ o } A = \{3i_F , 2+2i_F\}$ and so on.

Clearly as in case of usual loops we cannot define partition on them.

Example 2.76: Let $C(L_7(4))$ be a commutative complex modulo integer loop of order $7^2 + 1 + 14 = 7+2.7 + 1 = (7+1)^2 = 64$.

$A = \{e, 5\} \subseteq C(L_7(4))$ be the group of $C(L_7(4))$.

$1 \text{ o } A = \{1, 3\}$, $2 \text{ o } A = \{2, 7\}$ and so on. $7 \text{ o } A = \{6, 7\}$.

Example 2.77: Let $C(L_9(8))$ be a modulo complex integers. Take $B = \{e, 1, 4, 7\} \subseteq C(L_9(8))$ be a subgroup of $C(L_9(8))$.

The S-coset of B is as follows:

$B \text{ o } 2 = \{e, 9, 6, 3\}, \ldots, B \text{ o } 9 = \{9, 2, 8, 5\}$, We say $C(L_n(m))$ be a S-loop of level II if L has a normal subgroup, $A \subseteq C(L_n(m))$; A is a subgroup if for all $m \in C(L_n(m))$; we have $mA = Am$.

The reader is expected to give examples of such normal subgroups. Interested reader can analyse about the first normalizer and second normalizer of the subloops of the complex modulo integer loop $C(L_n(m))$; n odd, $n > 3$. $m < n$ with $(m, n) = (m, n - 1) = 1$. In general the first normalizer need not be equal to the second normalizer equal for every subloop of $C(L_n(m))$. It is easily verified that $C(L_n(m))$ is a Smarandache strong cyclic loop. The notion of Smarandache commutator subloop of a complex modulo integer loop is a matter of routine with appropriate changes.

We see $L_n(m)$ are loops of order n+1 where as the complex modulo integer loops $C(L_n(m))$ are of order $(n+1)^2$. Both are of even order as n is odd. Infact $2 / n+1$ and $2^2 / (n+1)^2$.

$C(L_n(m))$ has $L_n(m)$ as its subloop so $C(L_n(m))$ always contains subloops which are S-subloops. So we can say $C(L_n(m))$ has subloops P, such that

$$A(P) = L_n(m) = L_n^A(m) \qquad \ldots \text{I}$$

where $P = L_n(m) \subseteq C(L_n(m))$.

Also if $i_F\ L_n(m) = \{e, i_F, \ldots, i_Fn, *, (m, n) = (n-1, m) = 1\}$ then also

$$A\ (i_F\ L_n(m)) = i_F\ L_n^A\ (m) = i_F.L_n(m) \qquad \ldots\ \text{II}$$

So infact $C(L_n(m))$ has S-subloops $L_n(m)$ and $i_FL_n(m)$ such that I and II true respectively.

I and II are true only when $L_n(m)$ and $i_FL_n(m)$ has no S-subloops. Otherwise I and I are not true.

Further $C(L_n(m))$ for n a prime has atleast two subloops which are not S-associative complex modulo integer subloops of $C(L_n(m))$. Several other results enjoyed by the loops $L_n(m)$ can be easily extended in case of complex modulo integer loops with appropriate modifications.

Chapter Three

NON ASSOCIATIVE COMPLEX MODULO INTEGER RINGS

In this chapter we proceed onto introduce the new notion of non associative complex modulo integer rings using the non associative complex modulo integer structures like groupoids and loops or using complex modulo integer ring and usual groupoids or loops. This chapter has two sections. The first section deals with groupoids and loops over the complex modulo integer rings and complex rings. Second section deals with complex modulo integer groupoids (loops) over the complex ring or otherwise.

3.1 Groupoids and Loops over the Complex Modulo Integer Rings (Complex Rings).

In this section we use the groupoids (and loops) to construct non associative ring using the complex modulo ring $C(Z_n)$ or the usual complex numbers $C(Z)$ or $C(Q)$ or $C(R) = C$. The notational convention has been already discussed in chapter one.

DEFINITION 3.1.1: *Let G be any groupoid. $F = C(Z_n)$ (or $C(Z)$) be the complex modulo integer ring or field.*

$$FG = \left\{\sum_{i=1}^{n} a_i g_i \,\middle|\, n < \infty, a_i \in C(Z_n) \text{ or } C(Z); g_i \in G\right\}$$ *is the set of finite formal sums where addition and product are defined componentwise which is as follows:*

$$\textit{For } x = \sum_{i=1}^{n} a_i g_i \textit{ and } y = \sum_{i=1}^{n} b_i g_i \textit{ in } FG$$

we have

$$x + y = \sum_{i=1}^{n} a_i g_i + \sum_{i=1}^{n} b_i g_i$$

$$= \sum_{i=1}^{n} (a_i + b_i) g_i$$

$$\textit{and } x.y = \left(\sum_i a_i g_i\right)\left(\sum_j b_j h_j\right)$$

$$= \sum_k c_k p_k \, . \quad c_k = \sum a_i b_j$$

$$\textit{and } p_k = g_i \; h_j.$$

It is easily verified that '+' on FG is a commutative group. Further $\times$ is verified to be a non associative closed binary operation of FG. One can easily verify that product distributes over addition. Further (FG, +, $\times$) is a non associative ring which may be commutative or otherwise depending on G.

FG is defined as the complex modulo integer groupoid ring. When F is used as C(R) = C the complex ring of reals, that is

$C = \{a + bi \mid a, b \in R\}$ and $C(Q) = \{a + bi \mid a, b \in Q\}$ and

$C(Z) = \{a + bi \mid a, b \in Z\}$ are all complex commutative rings with unit 1. The groupoid rings are in general non commutative non associative ring with unit or otherwise.

We will first give some examples of them before we proceed to discuss about the properties enjoyed by them.

Example 3.1.1: Let G = {Z_5, *, (2, 3)} be a groupoid of order five. C(Z) = {a + bi | a, b ∈ Z} be the complex integers. C(Z)G be the groupoid complex ring.

Clearly C(Z)G is of infinite order and is a non commutative and a non associative ring.

We will illustrate this situation by some more examples.

Example 3.1.2: Let G = {Z_9, *, (3, 2)} be the groupoid. C(Z) = {a + bi | a, b ∈ Z} be the complex integer ring. C(Z)G is the complex integer groupoid ring.

We see C(Z)G is non associative and non commutative.

Example 3.1.3: Let G = {Z_{17}, *, (8, 9)} be the groupoid. C(Q) = {a + bi | a, b ∈ Q} be the complex integer ring. C(Q)G is the groupoid complex rational ring.

We see C(Q)G is a non associative ring.

Example 3.1.4: Let G = {Z_{11}, *, (0, 7)} be the groupoid.

C(Z) = {a + bi | a, b ∈ Z} be the complex integer ring. C(Z)G is the groupoid complex integer ring of the groupoid G over the complex integer ring C(Z).

Example 3.1.5: Let G = {Z_{16}, *, (3, 7)} be the groupoid. C be the complex field. CG is the groupoid complex ring.

Example 3.1.6: Let G = {Z_{18}, *, (11, 3)} be the groupoid of order 18. C(Z_{11}) be the complex modulo integer ring. C(Z_{11})G is the groupoid complex modulo integer ring of finite order.

We just show how product is defined in C (Z_{11})G.

Suppose $\alpha = 3g_0 + 5g_7 + 8g_5 + g_3$ and

$\beta = 4g_1 + 5g_3 + 9g_8 + g_9$ (where $g_0, g_7, g_1, g_9, g_3, g_5, g_8 \in Z_{17}$ with $g_n = n \in Z_{17}$; so $g_7 = 7 \pmod{17}$ and so on) are in $C(Z_{11})G$; to find

$$\begin{aligned}\alpha + \beta &= (3g_0 + 5g_7 + 8g_5 + g_3) + (4g_1 + 5g_3 + 9g_8 + g_9)\\ &= 3g_0 + 4g_1 + (1+5)g_3 + 9g_8 + 5g7 + 8g_5 + g_9\\ &= 3g_0 + 4g_1 + 6g_3 + 9g_8 + 5g_7 + 8g_5 + g_9.\end{aligned}$$

Clearly $\alpha + \beta \in C(Z_{11})G$

Now we find $\alpha\beta = \alpha \times \beta = (3g_0 + 5g_7+8g_5 + g_3) \times (4g_1 + 5g_3 + 9g_8 + g_9) = 3.4\ (g_0 * g_1) + 3.5\ (g_0 * g_3) + 3.9\ (g_0 * g_8) + 3.1\ (g_0 * g_9) + 5.4\ (g_7 * g_1) + 5.5\ (g_7 * g_3) + 5.9\ (g_7 * g_8) + 5.1\ (g_7 * g_9) + 8.4\ (g_5 * g_1) + 8.5\ (g_5 * g_3) + 8.9\ (g_5 * g_8) + 8.1\ (g_5 * g_9) + 1.4\ (g_3 * g_1) + 1.5\ (g_3 * g_3) + 1.9\ (g_3 * g_8) + 1.1\ (g_3 * g_9)$

$$\begin{aligned}&= 12g_3 + 4g_9 + 5g_6 + 3g_9 + 9g_8 + 3g_{14} + g_{11} + 5g_{14} + 10g_4\\ &\quad + 7g_{10} + 6g_7 + 8g_{10} + 4g_0 + 5g_8 + 9g_3 + g_6\\ &= 10g_3 + 7g_9 + 6g_6 + 3g_8 + 8g_{14} + g_{11} + 10g_4 + 4g_{10} + 6g_7\\ &\quad + 4g_0 \in C(Z_{11})G.\end{aligned}$$

Consider $\beta\alpha = (4g_1 + 5g_3 + 9g_8 + g_4)(3g_0 + 5g_7 + 8g_5 + g_3)$

$$\begin{aligned}&= g_1 * g_0 + 9g_1 * g_7 + 10g_1 * g_5 + 4g_1 * g_3 + 4g_3 * g_0 + 3g_3\\ &\quad * g_7 + 7g_3 * g_5 + 5g_3 * g_3 + 5g_8 * g_0 + g_8 * g_7 + 6g_8 * g_5\\ &\quad + 9g_8 * g_3 + 3g_4 * g_0 + 5g_4 * g_7 + 8g_4 * g_5 + g_4 * g_3.\\ &= g_{11} + 9g_{14} + 10g_8 + 4g_2 + 4g_{15} + 3g_0 + 7g_{12} + 5g_6 + 5g_{16}\\ &\quad + g_1 + 6g_3 + 9g_7 + 3g_8 + 5g_{11} + 8g_5 + g_{17}\\ &= 6g_{11} + 9g_{14} + 2g_8 + 4g_2 + 4g_{15} + 3g_0 + 7g_2 + 5g_6 + 5g_{16} +\\ &\quad g_1 + 6g_3 + 9g_7 + 8g_5 + g_{17}.\end{aligned}$$

We see clearly $\alpha\beta \neq \beta\alpha$.

Thus $C(Z_{11})G$ is a non commutative groupoid ring of infinite order.

Clearly $G \subseteq C(Z_{11})G$ but $C(Z_{11}) \not\subseteq C(Z_{11})G$.

Example 3.1.7: Let $G = \{Z_6, *, (0, 3)\}$ be the groupoid and $C(Q) = \{a + bi \mid a, b \in Q\}$ be the ring of complex rationals.

C(Q)G be the groupoid ring clearly C(Q)G is a non associative ring of infinite order.

The table for G is as follows:

0	g_0	g_1	g_2	g_3	g_4	g_5
g_0	0	g_3	0	g_3	0	g_3
g_1	0	g_3	0	g_3	0	g_3
g_2	0	g_3	0	g_3	0	g_3
g_3	0	g_3	0	g_3	0	g_3
g_4	0	g_3	0	g_3	0	g_3
g_5	0	g_3	0	g_3	0	g_3

Thus C(Q)G has zero divisors.

For if $\alpha = g_1 + g_2 + g_4 \in$ C(Q)G then $\alpha^2 = 0$. If $\alpha_1 = g_3$ then $\alpha_1^2 = g_3$ so α_1 is an idempotent of C(Q)G.

Example 3.1.8: Let G = $\{Z_8, *, (3, 2)\}$ be a groupoid. F = C(Z) be the ring of complex integers. FG the groupoid ring.

We see for $\alpha = (g_0 + g_2 + g_4 + g_6)$ and

$$\beta = (g_0 - g_2 + g_4 - g_6);\ \alpha\beta = 0.$$

where $\alpha, \beta \in$ FG.

Further $g_2 \text{ o } g_2 = g_2$ is also an idempotent in FG. Likewise $g_4 \text{ o } g_4 = g_4$ and $g_6 \text{ o } g_6 = g_6$ are also idempotents of FG.

Example 3.1.9: Let G = $\{Z_4, (3, 1), *)$ be a groupoid given by the following table.

*	g_0	g_1	g_2	g_3
G_0	g_0	g_1	g_2	g_3
G_1	g_3	g_0	g_1	g_2
G_2	g_2	g_3	g_0	g_1
G_3	g_1	g_2	g_3	g_0

Let F = C(Q) = $\{a + bi \mid a, b \in Q\}$ be the complex rational ring. FG be the groupoid ring of G over F. Clearly

$\alpha = 1/4\ (g_0 + g_1 + g_2 + g_3)$ in FG is such that $\alpha^2 = \alpha$ that is this non associative ring has idempotents.

We just illustrate how the notion of groupoids using Z_n is represented symbolically.

Let $Z_n = \{0, 1, 2, \ldots, n–1\} = \{g_0, g_1, \ldots, g_{n-1}\}$ and $t, p \in Z_n$. Now for $g_i, g_j \in Z_n$, $0 \leq i, j \leq n$. $g_i * g_j = tg_i * pg_j = g_{ti+pj \text{ (mod n)}}$; thus $\{Z_n, (t, p), *\}$ is a groupoid; this will be the notation we will be using to make working with modulo integer groupoids in an abstract way.

Example 3.1.10: Let $G = \{Z_7, *, (3, 4)\}$ be a groupoid given by the following table.

*	g_0	g_1	g_2	g_3	g_4	g_5	g_6
g_0	g_0	g_4	g_1	g_5	g_2	g_6	g_3
g_1	g_3	g_0	g_4	g_1	g_5	g_2	g_6
g_2	g_6	g_3	g_0	g_4	g_1	g_5	g_2
g_3	g_2	g_6	g_3	g_0	g_4	g_1	g_5
g_4	g_5	g_2	g_6	g_3	g_0	g_4	g_1
g_5	g_1	g_5	g_2	g_6	g_3	g_0	g_4
g_6	g_4	g_1	g_5	g_2	g_6	g_3	g_0

We take F = C(Q) the complex rational field. FG is the groupoid complex non associative ring. Infact

$\alpha = 1/7\ (g_0 + g_1 + \ldots + g_6) \in$ FG is an idempotent of FG. If C(Q) is replaced by C(Z) then C(Z) has no idempotents.

Example 3.1.11: Let $G = \{Z_6, *, (2, 4)$ be a groupoid given by the following table.

0	g_0	g_1	g_2	g_3	g_4	g_5
g_0	g_0	g_4	g_2	g_0	g_4	g_2
g_1	g_2	g_0	g_4	g_2	g_0	g_4
g_2	g_4	g_2	g_0	g_4	g_2	g_0
g_3	g_0	g_4	g_2	g_0	g_4	g_2
g_4	g_2	g_0	g_4	g_2	g_0	g_4
g_5	g_4	g_2	g_0	g_4	g_2	g_0

Let F = C(Q) = {a + bi | a, b ∈ Q} be the complex rational ring. FG be the groupoid ring.

$\alpha = (g_0 + g_1 + g_2 + g_3 + g_4 + g_5)$ in FG is such that $\alpha^2 \neq \alpha$. If we take $\beta = 1/3\ (g_0 + g_2 + g_4)$ in FG then $\beta^2 = \beta$, thus β is an idempotent in FG.

Example 3.1.12 : Let G = {Z_8, *, (0, 4)} be a groupoid C(Q)G be the complex integer ring C(Q)G be the groupoid ring of G over the ring C(G). Clearly $\alpha = 1/2\ (1+g_4)$ is such that

$\alpha^2 = 1/4\ (1+g_4 * g_4 + 2g_4) = 1/4\ (1 + g_0 + 2g_4)$ is not an idempotent.

However at this juncture it is pertinent to mention that we identify g_0 with 0 so that 0 is the element in C(Q)G such that (a + bi) 0 = 0 for every a, b ∈ Q however $g_i \,.\, 0 = g_i \,.\, g_0 \neq 0$ in general. Infact this groupoid table has only zero and four.

Example 3.1.13: Let G = {Z_{12}, *, (3, 4)} be a groupoid and F = C(Z) be the complex integer ring FG is a groupoid complex ring with zero divisors.

THEOREM 3.1.1: *Let G be a groupoid F = C(Z) be the complex integer ring (F = C(Q) or C can also be taken). FG the groupoid complex ring. H be a normal subgroupoid of G. FH the subgroupoid ring is a subring of FG and is an ideal of FG.*

Proof is direct from the very definition of normal subgroupoid.

THEOREM 3.1.2: *Let G be a groupoid F = C(Z) (or F = C(Q) or F = C = C(R) be the complex ring; FG be the groupoid ring. In general {0} is not an ideal of FG.*

Proof is clear from the following examples.

Example 3.1.14: Let G = $\{Z_{15}, *, (7, 6)\}$ be a groupoid. F = C(Z) be the complex ring. FG be the groupoid ring; {0} is not an ideal of FG.

Example 3.1.15: Let G = $\{Z_{11}, *, (3, 4)\}$ be the groupoid. F = C(Q) be the rational complex ring. FG be the groupoid ring. {0} is not an ideal of FG.

Example 3.1.16: Let G = $\{Z_{36}, *, (7, 19)\}$ be a groupoid. F = C(Z) be the complex integer ring. FG is be the groupoid complex ring. {0} is not an ideal of FG.

THEOREM 3.1.3: *Let*

$G = \{Z_n, *, (t, u); t, u \in Z_n \setminus \{0, 1\}; (t, u) = 1\}$ be the groupoid F = C (Z) (or C (Q) or C) be the complex ring, FG be the groupoid complex ring. {0} is not an ideal of FG.

Proof follows from the fact {0} is not an ideal of G if G is of the form given in theorem.

Example 3.1.17: Let G = $\{Z_4, *, (2, 3)\}$ be the groupoid. F = C(Z) = {a + bi | a, b ∈ Z} be the complex integer ring. FG be the groupoid ring. P = $\{g_0, g_2\} \subseteq G$; FP is a left ideal of FG. Clearly FP is not a right ideal of FG.

Likewise for T = $\{g_1, g_2\} \subseteq G$; FT is only a left ideal of FG and FT is not a right ideal of FG.

Example 3.1.18: Let G′ = $\{Z_4, (3, 2), *)$ be a groupoid. F = C(Z) = {a + bi | a, b ∈ Z} be the complex integer ring.

P = $\{g_0, g_2\} \subseteq G'$ is such that FP is the right ideal of FG′; where FG′ is the groupoid complex ring of the groupoid G′ over the integer complex ring F. Also H = $\{g_1, g_3\} \subseteq G'$, is such that FH is a right ideal of FG′ and both FP and FH are not left ideals of FG′.

Inview of this we have the following theorem, the proof of which is uses simple number theoretic techniques.

THEOREM 3.1.4: *Let $G = \{Z_n, *, (t, u)\}$ be the groupoid, $F = C(Z)$ (or $C(Q)$ or $C(R) = C$) be the complex ring of integers (or complex ring of rationals or complex ring of real). FG the groupoid complex ring. P is a left ideal of FG if and only if P is the right ideal of FG′ where $G' = \{Z_n, *, (u, t)\}$.*

Here we give conditions for the groupoid ring FG with $G = \{Z_n, *, (t, u)\}$ to have left or right ideals.

We show this by an examples.

Example 3.1.19: Let $G = \{Z_{10}, *, (3, 7)\}$ be a groupoid. $F = C(Q)$ be the rational complex ring. FG the groupoid ring of G over F. FG has no left ideals or right ideals in it.

When the groupoid ring has no ideals (left or right) we call them as simple rings.

Example 3.1.20: Let $G = \{Z_{15}, *, (2, 13)\}$ be the groupoid, $F = C = C(R)$ be the complex ring. FG be the groupoid ring of G over F. FG is simple.

THEOREM 3.1.5: *Let $G = \{Z_n, *, (t, u)\}$ be a groupoid $F = C$ be the complex field of reals FG be the groupoid ring. If $t + u = n$ where both t and u are primes then FG is simple.*

THEOREM 3.1.6: *Let $G = \{Z_p, *, (t, u)\}$, p a prime, be a groupoid, $F = C$ be the complex field of reals.*

FG the groupoid ring;

if $t + u = p$ and $(t, u) = 1$ then FG is simple.

Example 3.1.21: Let $G = \{Z_{13}, *, (6, 7)\}$ be a groupoid, C be the complex field CG is a simple groupoid complex ring.

Example 3.1.22: Let $G = \{Z_{17}, *, (9, 6)\}$ be a groupoid, $F = C$ be the complex field. FG be the groupoid complex ring. FG in general is not simple as $(9, 6) \neq 1$.

Example 3.1.23: Let G = {Z_{11}, *, (7, 5)} be a groupoid C(Q) = F, be the rational complex ring. FG be the groupoid ring, FG is not a simple non associative ring.

We can define a Smarandache non associative ring to be a non associative ring which has a proper subset which is an associative ring.

We first give examples of this situation.

Example 3.1.24: Let G = {Z_6, *, (4, 5)} be a groupoid. F = C(Q) be the groupoid ring of G over F. FG is a Smarandache ring. P = {1, 3, 5} $\subseteq$ Z_6 is such that FS where S = {P, *, (4, 5)} is a subring of FG.

Example 3.1.25: Let G = {Z_8, *, (2, 8)} be a groupoid. F = C(Z) be the complex integer ring. FG the groupoid ring. Take FT (where T = {0, 2, 4, 6,, *, (2, 8)}) is a proper subset of FG and infact FT is the ideal of FS.

This is obvious from the fact that T is a normal subgroupoid of G.

In view of this we have the following theorem.

THEOREM 3.1.7: *Let G = {Z_n, *, (t, p)} be a groupoid. F = C(Z) (or C(Q) or C) be the complex ring. FG the groupoid complex ring. FG has an ideal if G has a normal subgroupoid.*

The proof is direct and hence left as an exercise to the reader. As in case of groupoids we can for groupoid ring of complex numbers define the notion of Moufang groupoid ring, Bol groupoid ring and so on.

Example 3.1.26: Let G = {Z_{12}, *, (3, 9)} be a groupoid. F = C(Z) = {a + bi | a, b $\in$ Z} be the ring of complex integers, FG be the groupoid ring.

FG is a Smarandache Moufang groupoid ring as G is a S-Moufang groupoid.

Thus we will say a groupoid ring FG is a Smarandache Moufang groupoid ring if the groupoid G is a Smarandache Moufang groupoid. Likewise FG is a Smarandache strong Moufang groupoid ring if G is a Smarandache strong Moufang groupoid.

Example 3.1.27: Let G = $\{Z_{10}, *, (5, 6)\}$ be a groupoid.

F = C(Q) = $\{a + ib \mid a, b \in Q\}$ be the complex rational ring. FG be the groupoid ring. FG is a Smarandache strong Moufang groupoid ring as G is a Smarandache strong Moufang groupoid.

We have seen examples of Smarandache strong Moufang groupoid ring and Smarandache Moufang groupoid ring. However we wish to state only subrings in FG which are of the form FH where H is the subgroupoid of G are taken as Smarandache rings while defining special identities in these rings. For we can have subrings which may not satisfy the Moufang identity. So based on the property of the groupoid only we define the Smarandache special identities.

THEOREM 3.1.8: *Let* $G = \{Z_n, *, (t, u)\}$ *be a groupoid.*

F = $C(Z) = \{a + bi \mid a, b \in Z\}$ *(or C(Q) or C(R) = C} be the complex ring. FG be the groupoid complex ring.*

If $(x * y) * (z * x) = (x* (y*z)) * x$ *is true for all* $x, y, z \in G$ *and G is a Smarandache groupoid, then FG is a Smarandache strong Moufang groupoid complex ring.*

The proof is obvious from the fact that every S-subring will satisfy the Moufang identity, hence the claim of the theorem.

THEOREM 3.1.9: *Let FG be a groupoid complex ring if G is a Smarandache strong Moufang groupoid ring then FG is a Smarandache moufang groupoid ring, however a Smarandache Moufang groupoid ring in general is not a Smarandache strong Moufang groupoid ring.*

We see the groupoid complex ring FG where

$G = \{Z_{12}, *, (3, 9)\}$ given in example 3.1.26 is only a Smarandache Moufang groupoid ring and is not a Smarandache strong Moufang groupoid ring.

A similar situation is true in case of Smarandache strong Bol groupoid ring.

This is exhibited by the following examples.

Example 3.1.28: Let $G = \{Z_{12}, *, (3, 4)\}$ be a groupoid, $F = C(Z)$ be the complex integer ring. FG be the complex groupoid integer ring. For every $x, y, z \in FG$. We see

$((x * y) * z)* y = 3x + 4y$ and $x * [(y * z) * y] = 3x + 4y$ so $((x * y) * z)* y = x * [(y * z) * y]$. Thus FG is a Smaradache strong Bol groupoid ring.

Example 3.1.29: Let $G = \{Z_4, *, (2, 3)\}$ be a groupoid.

$F = \{C(Q) = a + bi \mid a, b \in Q\}$ be the complex rational ring. FG the groupoid complex rational ring.

We see $A = \{0, 2\} \subseteq Z_4$, FA is a complex groupoid subring in FG. However in general for $x, y, z \in FG$;

$$((x * y) * z) * y = 2z + 3y \text{ and}$$

$$x * [(y * z) * y] = 2x + 2z + y. \text{ Since}$$

$2z + 3y \neq 2x + 2z + y$ for all choices of $x, y, z \in FG$. So FG is not a Smarandache strong Bol groupoid ring but only a Smarandache Bol groupoid ring.

Thus we have the following theorem.

THEOREM 3.1.10: *Let $G = \{Z_n, *, (t, u)\}$ be a groupoid. $F = C(Z)$ (or $C(Q)$ or R) be a complex ring. FG the groupoid complex ring. If FG is a Smarandache strong Bol groupoid ring then FG is a Smarandache Bol groupoid ring. However if FG is a Smarandache Bol groupoid ring then FG in general is not a Smarandache strong Bol groupoid ring.*

We give examples of Smarandache P-groupoid rings.

Example 3.1.30: Let G = $\{Z_6, *, (4, 3)\}$ be a groupoid. F = C(Z) = $\{a + ib \mid a, b \in Z\}$ be a complex integer ring. FG be the groupoid complex integer ring. For every x, y in FG.

$$
\begin{aligned}
(x * y) * x &= (4x + 3y) * x \\
&= 16x + 12y + 5x \\
&= x.
\end{aligned}
$$

$$
\begin{aligned}
\text{Also } x * (y * x) &= x * [4y + 3x] \\
&= 4x + 12y + 9x \\
&= x.
\end{aligned}
$$

We see (x * y) * x = x * (y * x) so FG is a Smarandache strong P-groupoid complex ring.

Example 3.1.31: Let G = $\{Z_4, *, (2, 3)\}$ be a groupoid.

F = C(Q) = $\{a + bi \mid a, b \in Q\}$ be a complex rational ring. FG be the groupoid complex rational ring.

For every x, y $\in$ FG we see

$$
\begin{aligned}
(x * y) * x &= [2x + 3y] * x \\
&= 4x + 6y + 3x \\
&= 2y + 3x.
\end{aligned}
$$

$$
\begin{aligned}
x * (y * x) &= x * [2y + 3x] \\
&= 2x + 6y + 9x \\
&= 3x + 2y.
\end{aligned}
$$

Since (x * y) * x = x * (y * x) for all x, y $\in$ FG is a Smarandache strong groupoid complex P-ring.

Recall we say a non associative R to be a P-ring if

$$x * (y*x) = (x*y)*x \text{ for all } x, y \in R.$$

Now we give an example of a Smarandache P-groupoid ring or a Smarandache groupoid P-ring which is not a Smarandache P-ring.

Example 3.1.32: Let G = $\{Z_6, *, (3, 5)\}$ be a groupoid.

F = C(Z) = $\{a + ib \mid a, b \in Z\}$ be a complex integer ring. FG be the groupoid complex integer ring. Clearly FG is only a

Smarandache P-groupoid ring as all elements in FG do not satisfy the P-groupoid identity.

Example 3.1.33: Let G = {Z_{12}, *, (5, 10)} be a groupoid.

F = C(Q) = {a + ib | a, b ∈ Q} be the complex rational ring. FG be the groupoid complex ring. Consider x, y ∈ FG; we see

$$\begin{aligned} (x * y) * x &= [5x + 10y] * x \\ &= 25x + 50y + 10x \\ &= 11x + 2x \quad \text{and} \end{aligned}$$

$$\begin{aligned} x * (y * x) &= x * [5y + 10x] \\ &= 5x + 50y + 100x \\ &= 9x + 2y. \end{aligned}$$

Thus in general

(x * y) * x ≠ x * (y * x) as 11x + 2y ≠ 9x + 2y for all x, y ∈ FG.

But if x, y ∈ FH where H = {{0, 6}, *, (5, 10)} ⊆ G then FH satisfies the P-identity. Thus FG is only a Smarandache complex P-groupoid and not a Smarandache strong P-ring.

We see a non associative ring R is right alternative if

$$(xy)\, y = x\, (yy) \text{ for all } x, y \in R.$$

Similarly left alternative if (xx)y = x (xy). R is said to be alternative ring if it is both right alternative and left alternative.

If the groupoid is alternative so will be the groupoid complex ring likewise for right alternative and left alternative identity; we provide examples of them.

Example 3.1.34: Let G = {Z_{14}, *, (7, 8)} be a groupoid.

C = {a + ib | a, b ∈ R} be the complex ring CG be the groupoid complex ring.

We see G is a Smarandache strong alternative groupoid, hence CG is a Smarandache strong alternative ring.

Example 3.1.35: Let G = $\{Z_{12}, *, (1, 6)\}$ be a groupoid.

F = C(Z) = {a + ib | a, b ∈ Z} be a complex integer ring, FG be the groupoid ring. FG is a Smarandache strong alternative groupoid ring as G is a Smarandache strong alternative groupoid.

Recall a non associative ring is a Smarandache ring if it has a subring which is associative.

We show by the following theorem we have a class of groupoid complex rings which are Smarandache rings.

THEOREM 3.1.11: *Let G = $\{Z_n, *, (t, u)\}$ be a groupoid (n > 5); F = C(Z) (or C(Q) or C(R) = C) be a complex ring. FG is a Smarandache ring if (t, u) = 1 and $t \neq u$ with $t + u \equiv 1 \pmod n$.*

All groupoid rings are not in general Smarandache. The following examples substantiate them.

Example 3.1.36: Let G = $\{Z_5, *, (1, 3)\}$ be a groupoid. F = C(G) be the complex integer ring. FG be the groupoid complex ring. FG is not a Smarandache ring.

Example 3.1.37: Let G = $\{Z_5, *, (2, 1)\}$ be a groupoid. F = C(Q) = {a + bi | a, b ∈ Q} be a complex rational ring. FG be the groupoid ring. FG is a Smarandache ring.

Example 3.1.38: Let G = $\{Z_9, *, (5, 3)\}$ be a groupoid. F = C be the complex field. FG the groupoid complex ring. FG is not a Smarandache ring.

THEOREM 3.1.12: *Let G = $\{Z_n, *, (1, p)$, p a prime, p / n} be a groupoid F = C(Z) (or C(Q) or R) be the complex ring FG the groupoid complex ring. FG is a Smarandache ring.*

The proof easily follows from the fact G is a S-groupoid.

THEOREM 3.1.13: *Let G = $\{Z_n, *, t + u \equiv 1 \bmod n, (t, u)\}$ be a groupoid. F = C(Z) (or C(Q) or C} be the complex integer ring. FG the groupoid complex ring. FG is a Smarandache groupoid*

P-ring (Smarandache P-groupoid ring) if and only if $t^2 \equiv t \pmod{n}$ and $u^2 \equiv u \pmod{n}$.

The proof is straight forward and hence left as an exercise to the reader.

THEOREM 3.1.14: *Let $G = \{Z_n, *, (t, u); t + u \equiv 1 \pmod{n}\}$ be a groupoid. $F = C(Z)$ (or $C(Q)$ or C) be the complex ring. FG be the groupoid ring. FG is a Smarandache alternative ring if and only if $t^2 \equiv t \pmod{n}$ and $u^2 \equiv u \pmod{n}$.*

This proof is simple for one can easily show

$(x * y) * y = (x) * (y * y)$ and $(x * x) * y = x * (x * y)$ for all $x, y \in FG$.

THEOREM 3.1.15: *Let*

$G = \{Z_n, *, (m, m); m + m \equiv 1 \pmod{n}$ and $m^2 = m \pmod{n}\}$ be a groupoid. $F = C(Z)$ (or $C(Q)$ or C) be the complex ring. FG be the groupoid complex ring.

(i) FG is a Smarandache strong P-groupoid ring (groupoid P-ring).
(ii) FG is a Smarandache strong Bol groupoid ring (groupoid Bol ring).
(iii) FG is a Smarandache strong Moufang groupoid ring (groupoid Moufang ring).
(iv) FG is a Smarandache strong alternative groupoid ring (groupoid alternative ring).

The proof is direct exploiting only number theoretic techniques.

Example 3.1.39: Let $G = \{Z_{2n}, *, (2, 0)\}$ be a groupoid. $F = C(Z)$ (or $C(Q)$ or R) be the complex ring. FG is a Smarandache ring.

In view of this we have a class of Smarandache rings.

THEOREM 3.1.16: *Let $G = \{Z_{2n}, *, (0, 2)\}$ be a groupoid and $F = C(Z)$ (or $C(Q)$ or C) be the complex ring. FG is a groupoid complex ring which is a S-ring.*

Example 3.1.40: Let G = {Z_{14}, *, (0, 7)} be a groupoid, F = C(Z) (or C(Q) or C) be the complex ring. FG be the groupoid ring, FG is a S-ring.

Inview of this we have the following theorem.

THEOREM 3.1.17: *Let G = {Z_{2m}, *, (0, m)} be a groupoid,. F = C(Z) (or C(Q) or C) be the complex ring, FG the groupoid complex ring is a S-ring.*

Now we can define finite complex modulo integer non associative rings using groupoids and complex modulo integers $C(Z_n) = \{a + bi_F \mid i_F^2 = n-1, a, b \in Z_n\}$. Let G = {$Z_m$, *, (t, u)} be a groupoid $C(Z_n)$ be the complex modulo integer

$C(Z_n)G = \left\{\sum_{i=1}^{t} a_i g_i \middle|\right.$ $t < \infty$, $a_i \in C(Z_n)$ and $g_i \in G$} is the groupoid complex modulo integer ring which is non associative and is of finite order.

We will give examples of them and discuss their properties.

Example 3.1.41: Let G = {Z_3, *, (1, 2)} be a groupoid given by the following table.

*	g_0	g_1	g_2
g_0	g_0	g_2	g_1
g_1	g_1	g_0	g_2
g_2	g_2	g_1	g_0

Let F = $C(Z_2)$ = {0, 1, i_F, $1+i_F$} be the complex modulo integer ring of characteristic two.

FG = {0, g_0, g_1, g_2, $g_0 + g_1$, $g_0 + g_2$, $g_1 + g_2$, $g_0 + g_1 + g_2$, i_Fg_0, i_Fg_1, i_Fg_2, $i_F(g_0 + g_1)$, $i_F(g_0 + g_2)$, $i_F(g_1 + g_2)$, $i_F(g_0 + g_1 + g_2)$, $g_0 + i_F g_1$, $i_Fg_0 + g_1$, $g_1 + i_F g_2$, $i_Fg_1 + g_2$, $i_Fg_0 + g_2$, $g_0 + i_F g_2$, $g_0 + g_1 + i_F g_2$, $g_0 + i_Fg_1 + g_2$, $i_F g_0 + g_1 + g_2$, $g_0 + i_Fg_1 + i_Fg_2$, $i_Fg_0 + i_Fg_1$ +

g_2, $i_F g_0 + g_1 + i_F\, g_2\, (1 + i_F)\, g_0$, $(1+i_F)\, g_1$, $1+i_F\, g_2$, …, $(1+i_F)\, (g_0 + g_1 + g_2)\}$ is the groupoid complex modulo integer ring.

Consider

$(g_0 + g_1 + g_2) = g_0^2 + g_1^2 + g_2^2 + g_0\, g_1 + g_0\, g_2 + g_1\, g_2 + g_2\, g_1 + g_1\, g_0 + g_2\, g_0 = g_0 + g_0 + g_0 + g_2 + g_1 + g_1 + g_2 + g_2 + g_1 = g_0 + g_1 + g_2$ is an idempotent in FG.

Further FG is non commutative and non associative of finite order.

$\alpha = 1 + g_0 \in$ FG, $(1 + g_0)^2 = 1 + g_0$ is again an idempotent in FG. $(1 + g_1)^2 = 1+g_0$ is not an idempotent in FG. Further $(1+g_2)^2 = 1+g_0$ is again not an idempotent of FG.

Let $\alpha = 1 + i_F\, g_0 \in$ FG,

$\alpha^2 = (1+i_F g_0)^2 = 1 + (i_F g_0)^2 = 1+g_0$ is not an idempotent in FG.

Consider $(1 + g_1 + g_2) = \alpha$ in FG.

$\alpha^2 = 1 + g_0 + g_0 + g_2 + g_1 = 1 + g_1 + g_2$ is again an idempotent in FG.

Let $\alpha = 1+g_1$ and $\beta = 1+g_2$ be in FG.

$$\begin{aligned} \alpha\beta &= (1+g_1)\,(1+g_2) \\ &= 1 + g_1 + g_2 + g_1\, g_2 \\ &= 1 + g_1 + g_2 + g_2 \\ &= (1 + g_1) \text{ is not a zero divisor.} \end{aligned}$$

$(1 + g_1)\, g_2 = 0$ is a zero divisors.

$g_2\, (1+g_1) \neq 0$ is not a zero divisors.

Consider $1 + g_2 = \alpha$ and $\beta = g_1 \in$ FG.

$$\begin{aligned} \beta\alpha = (1+ g_2) \quad &= g_1 + g_1\, g_2 \\ &= g_1 + g_2 \neq 0. \end{aligned}$$

$$\begin{aligned} \text{Consider } \alpha\beta \quad &= (1+g_2)\, g_1 \\ &= g_1 + g_2\, g_1 \\ &= g_1 + g_1 = 0. \end{aligned}$$

Thus $\alpha\beta = 0$ is a zero divisor and $\beta\alpha \neq 0$ so is not a zero divisor.

These are only either left zero divisor or right zero divisor.

Example 3.1.42: Let G = $\{Z_4, *, (3, 1)\}$ be the groupoid.

$F = C(Z_3) = \{a + bi_F \mid a, b \in Z_3, i_F^2 = 2\}$ be the complex modulo integer ring. FG be the groupoid complex modulo integer ring.

The table for G is as follows:

0	g_0	g_1	g_2	g_3
g_0	g_0	g_1	g_2	g_3
g_1	g_3	g_0	g_1	g_2
g_2	g_2	g_3	g_0	g_1
g_3	g_1	g_2	g_3	g_0

Consider $\alpha = 2g_0 + i_F\, g_1 \in$ FG; $\alpha^2 = (2g_0 + i_F\, g_1)^2 = g_0 + 2g_0 + 2i_F\, g_1 + 2i_F\, g_3 = 3g_0 + 2i_F\, g_1 + 2i_F\, g_3$ is in FG.

$$\alpha = g_0 + g_2 + g_1 + g_3 \in \text{FG}.$$

$$\begin{aligned}\alpha^2 &= (g_0 + g_2 + g_1 + g_3)^2 \\ &= g_0 + g_0 + g_0 + g_0 + g_1 + g_3 + g_2 + g_3 + g_1 + \\ &\quad g_2 + g_2 + g_3 + g_1 + g_1 + g_2 + g_3 = \alpha.\end{aligned}$$

Thus α is an idempotent element.

Consider $\alpha = g_0 + i_F\, g_1 + g_2 + i_F\, g_3$ and $\beta = g_0 + i_F\, g_2$ in FG.

$$\begin{aligned}\alpha\beta &= (g_0 + i_Fg_1 + g_2 + i_Fg_3)\,(g_0 + i_F\, g_2) \\ &= g_0 + i_Fg_3 + g_2 + i_F\, g_1 + i_Fg_2 + 2g_1 + i_Fg_0 + 2g_3 \text{ is in FG.}\end{aligned}$$

Let $\alpha = (1+i_F)g_0 + (2+i_F)g_1$ be in FG. To find α^2.

$$\alpha^2 = ((1 + i_F)g_0 + (2+i_F)g_1)^2$$

$= (1+i_F)^2 g_0 + (2+i_F)^2 g_0 + (i_F+1)(2+i_F)g_1 + 1+i_F)(2+i_F) g_3$
$= (1+2+2i_F)g_0 + (4+2+2i_F)g_0 + (2+2)g_1 + (2+2)g_3$
$= i_F g_0 + g_1 + g_3 \in FG.$

Let $\alpha = g_0 + g_1$ and $\beta = g_2 + g_3$ be in FG.

$$\begin{aligned} \alpha\beta &= (g_0 + g_1)(g_2 + g_3) \\ &= g_2 + g_1 + g_3 + g_2 = g_1 + g_3. \end{aligned}$$

Consider $\alpha = g_0$ and $\beta = g_1 + g_2 + g_3$ in FG.

$$\begin{aligned} \alpha\beta &= g_0 (g_1 + g_2 + g_3) \\ &= g_1 + g_2 + g_3 \in FG. \end{aligned}$$

Let $\beta = g_0 + 2$ and $\alpha = g_1 + g_2 + g_3$ then

$$\begin{aligned} \beta\alpha &= (g_0 + 2)(g_1 + g_2 + g_3) \\ &= (g_1 + g_2 + g_3 + 2g_1 + 2g_2 + 2g_3 \\ &= 0. \end{aligned}$$

So $\beta\alpha$ is a zero divisor.

Consider
$$\begin{aligned} \alpha\beta &= (g_1 + g_2 + g_3)(g_0 + 2) \\ &= g_3 + g_2 + g_1 + 2g_1 + 2g_2 + 2g_3 \\ &= 0. \end{aligned}$$

Thus $\alpha\beta = \beta\alpha = 0$ is a zero divisor in FG.

Let $\alpha = g_0 + g_1 + g_2 + g_3$ and

$\beta = g_0 + g_1 + g_2 + g_3 + 2$ be in FG.

$$\begin{aligned} \alpha\beta &= (g_0 + g_1 + g_2 + g_3)(g_0 + g_1 + g_3 + g_2 + 2) \\ &= g_0 + g_1 + g_3 + g_2 + 2g_0 + g_3 + g_0 + g_1 + g_2 + 2g_1 \\ &\quad + g_2 + g_3 + g_0 + g_1 + 2g_2 + g_1 + g_2 + g_3 + g_0 + 2g_3 = 0; \end{aligned}$$

thus $\alpha\beta$ is a zero divisor in FG.

Now we proceed onto describe some more properties about groupoid complex modulo integer rings. We can define the concept of S-rings, subrings and ideals of groupoid complex modulo integer rings.

Example 3.1.43: Let $G = \{Z_{12}, *, (2, 10)\}$ be the groupoid.

$F = C(Z_{11}) = \{a + bi_F \mid a, b \in Z_{11},\ i_F^2 = 11\}$ be a ring of complex modulo integers. FG the groupoid ring.

$H = \{\{0, 2, 4, 6, 8, 10\} \subseteq Z_{12}, *, (2, 10)\} \subseteq G$ be a subgroupoid of G. FH $\subseteq$ FG is a groupoid subring of FG.

Example 3.1.44: Let $G = \{Z_{12}, *, (3, 9)\}$ be a groupoid.

$F = C(Z_{12}) = \{a + bi_F \mid a, b \in Z_{12}, i_F^2 = 11\}$ be a complex modulo integer ring. FG the groupoid ring.

$H = \{\{0, 3, 6, 9\} \subseteq Z_{12}, *, (3, 9)\} \subseteq G$; be a subgroupoid. FH be the groupoid subring.

Example 3.1.45: Let $G = \{Z_{12}, *, (10, 8)\}$ be the groupoid.

$F = \{a + i_Fb \mid a, b \in Z_9, i_F^2 = 8\}$ be the complex modulo integer ring. FG be the groupoid ring.

$K = \{\{0, 4, 8\} \subseteq Z_{12}, *, (10, 8)\} \subseteq G$ be the subgroupoid of G.

FK $\subseteq$ FG is a groupoid subring of FG. Let

$H = \{\{2, 6, 10\} \subseteq Z_{12}, (10, 8)\} \subseteq G$ be the subgroupoid of G. FH $\subseteq$ FG; FH is a groupoid subring of FG.

We see number of elements of FH and FK are equal.

Example 3.1.46: Let $G = \{Z_4, *, (2, 3)\}$ be a groupoid.

$F = C(Z_5) = \{a + bi_F \mid a, b \in Z_5, i_F^2 = 4\}$ be the modulo integer complex ring. FG be the groupoid ring. Take

$H = \{\{1, 3\} \subseteq Z_4, *, (2, 3)\} \subseteq G$ be the subgroupoid of G.

FH be the groupoid ring. FH is a groupoid subring of FG.

Further FH is a left ideal of FG. Clearly FH is not a right ideal of FG.

Example 3.1.47: Let $G = \{Z_4, *, (3, 2)\}$ be the groupoid.

$F = C(Z_{12}) = \{a + bi_F \mid a, b \in Z_{12}, i_F^2 = 11\}$ be the modulo complex ring. FG be the groupoid ring.

Take $H = \{\{0, 2\} \subseteq Z_4, *, (3, 2)\} \subseteq G$ to be a subgroupoid. FH $\subseteq$ FG; FH is a groupoid subring of FG and infact right ideal of FG. Clearly FH is not a left ideal of FG.

We cannot always claim that every groupoid ring built using the complex modulo integer ring has right ideals and left ideals.

Thus is shown by some examples.

Example 3.1.48: Let G = $\{Z_{10}\ *, (3, 7)\}$ be the groupoid of order 10. $C(Z_2) = \{a + bi_F \mid a, b \in Z_2, i_F^2 = 1\} = F$ be the complex modulo integer ring. FG be the groupoid ring. FG has no ideals be it right or left. It has no left or right ideals.

In view of these facts we have the following theorem the proof of which is direct and simple.

THEOREM 3.1.18: *Let G = $\{Z_n, *, (t, u)\}$ be a groupoid and $F = C(Z_n) = \{a + bi_F \mid a, b \in Z_m, i_F^2 = m-1\}$ be a complex modulo integer ring. FG be the groupoid complex modulo integer ring of the groupoid G over the complex modulo integer ring F. P is a left ideal in FG if and only if P is right ideal in FG′ where G′ = $\{Z_n, *, (u, t)\}$ is the groupoid.*

We can also say when the groupoid rings do not have ideals that is they are simple.

Example 3.1.49: Let G = $\{Z_{12}, *, (5, 7)\}$ be a groupoid.

$F = C(Z_3) = \{a + bi_F \mid a, b \in Z_3, i_F^2 = 2\}$ be a complex modulo integer ring, FG be the groupoid ring. FG has no ideals hence simple.

Example 3.1.50: Let G = $\{Z_7, *, (2, 5)\}$ be a groupoid.

$F = C(Z_5) = \{a + bi_F \mid a, b \in Z_5, i_F^2 = 4\}$ be the complex modulo integer ring. FG be the groupoid ring. FG is simple for it has no ideals.

Example 3.1.51: Let G = $\{Z_{19}, *, (13, 6)\}$ be a groupoid.

$F = C(Z_7) = \{a + bi_F \mid a, b \in Z_7, i_F^2 = 6\}$ be the ring of complex modulo integer.

FG be the groupoid ring of G over F. FG is simple.

Inview of this we have the following theorem.

THEOREM 3.1.19: *Let $G = \{Z_n, *, (t, u)\}$ be a groupoid. $F = C(Z_m) = \{a + bi_F \mid a, b \in Z_m, i_F^2 = m-1\}$ be a complex modulo integer ring. FG be the groupoid modulo complex integer ring. FG is simple if $t + u = n$ and both t and u are primes.*

The proof is simple for one can verify the result using simple number theoretic techniques.

THEOREM 3.1.20: *Let $G = \{Z_p, *, (t, u)\}$ be a groupoid. $F = C(Z_n) = \{a + bi_F \mid a, b \in Z_n, i_F^2 = n-1\}$ be the complex modulo integer. FG be the groupoid ring. If $t + u = p$; $(t, u) = 1$ then also FG is a simple ring.*

The proof is simple and hence left as an exercise to the reader.

Now we proceed onto define and study groupoid rings for which the groupoid has identity.

Recall $G = Z_n \cup \{e\}$ be a modulo integers with $e \notin Z_n$. Define a operation * on G by $a_i * a_i = e$ for all $a_i \in Z_n$ and

$a_i * e = e * ai = ai$ for all $a_i \in Z_n$. For any $a_i, a_j \in Z_n$; $a_i * a_i = e$ and $a_i * a_j = ta_i + ua_j \pmod n$; $t, u \in Z_n$. $\{G, *, (t, u)\}$ is a groupoid with identity.

Let $C = \{a + bi \mid a, b \in R\}$ be the complex field. CG be the groupoid ring where $1.e = e.1 = 1$ is called the identity of CG.

Example 3.1.52: Let $G = \{Z_9 \cup \{e\}, *, (2, 3)\}$ be the groupoid with identity e.F = $C(Z) = \{a + bi \mid a, b \in Z\}$ be the complex integer ring.

$FG = \left\{ \sum_{i=1}^{n} a_i g_i \,\middle|\, a_i \in Z,\ g_i \in G \right\}$ be the groupoid ring. Clearly $1.G \subseteq FG$ and $F.e = F.1 \subseteq FG$.

All properties for FG can be derived as in case of usual groupoid ring when the groupoid does not contain the identity. Using complex modulo integers or complex field C or complex rational ring all properties can be derived.

Example 3.1.53: Let $G = \{Z_5 \cup \{e\}, *, (3, 2)\}$ be a groupoid ring with identity. $F = C(Z_2) = \{a + bi_F \mid a, b \in Z_2,\ i_F^2 = 1\}$ be the complex modulo integer. FG be the groupoid ring.

$FG = \{\sum a_i g_i \mid g_i \in G$ and $a_i \in F\}$. Consider $\alpha = 1+g_4$ in FG. $\alpha^2 = 1+2g_4 + g_0$.

Suppose $\alpha = 1 + g_0 + g_1 + g_2 + g_3 + g_4$ in FG.

$$
\begin{aligned}
\alpha^2 &= (1 + g_0 + g_1 + g_2 + g_3 + g_4)^2 \\
&= 1 + g_0 + g_1 + g_2 + g_3 + g_3 + g_0 + 1 + g_2 + g_4 + g_1 + g_3 \\
&\quad + g_1 + g_3 + 1 + g_2 + g_4 + g_1 + g_2 + g_1 + g_3 + 1 + g_2 + \\
&\quad g_4 + g_3 + g_4 + g_1 + g_3 + 1 + g_2 + g_4 + g_2 + g_4 + 1 + g_3 \\
&\quad + g_1 \\
&= g_1 + g_2 + g_3 + g_4.
\end{aligned}
$$

So α^2 is neither an idempotent nor a nilpotent element of FG.

More properties of groupoid rings using complex rings can be studied or analysed as a matter of routine. Now we proceed onto define loop rings where rings are complex modulo integer ring or complex modulo rational ring or complex ring and the loops are real loops built using $Z_n \cup \{e\}$.

DEFINITION 3.1.2: *Let $L_n(m) = \{e, 1, 2, \ldots, n\}$ be a set where $n > 3$, n odd and m is a positive integer such that $(m, n) = 1$ and $(m-1, n) = 1$ with $m < n$. Define on $L_n(m)$ a binary operation * such that*

(i) $e * i = i * e = i$ for all $i \in L_n(m)$
(ii) $i^2 = i*i = e$ for all $i \in L_n(m)$

(iii) $i * j = t$ where $t = (mj - (m-1)i) \pmod n$ for all $i, j \in L_n(m)$; $i \neq j$, $i \neq e$ loop under the binary operation $*$.

For more about these loops refer [15].

Example 3.1.54: Let $L_{13}(5)$ be a loop of order 14.

Example 3.1.55: Let $L_{17}(8)$ be a loop of order 18.

Example 3.1.56: Let $L_{15}(8)$ be a loop of order 16.

We shall be using only these types of loops to construct non associative complex rings.

DEFINITION 3.1.3: *Let $L = L_n(m)$ be a loop.*

$F = C(Z) = \{a + bi \mid a, b \in Z\}$ be the complex integer ring. $FL = \left\{\sum_{i=1}^{n} a_i g_i \,\middle|\, a_i \in C(Z) \text{ and } g_i \in L_n(m)\right\}$ denote the finite formal sums.

Addition is defined componentwise, for $a, b \in FL$ where

$$a = \sum_{i=0}^{n} a_i g_i \text{ and } b = \sum_{i=0}^{n} b_i g_i\ ;$$

$$a+b = \sum_{i=0}^{n} a_i g_i + \sum_{i=0}^{n} b_i g_i = \sum_{i=0}^{n} (a_i + b_i) g_i$$

we see $a + b \in FL$.

For a, b in FL, $ab = \sum_{i=0}^{n} a_i g_i \sum_{i=0}^{n} b_i g_i = \sum_{j=0}^{k} c_j h_j$

where $h_j = g_i\, g_t$ and $c_j = \sum a_i\, b_t$.

Clearly e.1 = 1.e = 1 acts as the multiplicative identity. We see (FL, +, ×) is a non associative ring with identity, known as the complex non associative loop ring of the loop L over the integer complex ring C(Z). Infact C(Z) can be replaced by

C(Q) = {a + ib | a, b ∈ Q} or C = {a + ib | a, b ∈ R} which are the complex rational ring or complex real ring or complex field respectively.

Still the loop ring would be a non associative complex ring. We will first illustrate this situation by some examples.

Example 3.1.57: Let L = $L_9(8)$ be the loop.

F = C(Z) = {a + bi | a, b ∈ Z} be the complex integer ring. FL be the loop complex ring.

Example 3.1.58: Let L = $L_5(2)$ = {e, 1, 2, 3, 4, 5} be the loop given by the following table.

We will denote g_i by i; $1 \leq i \leq 5$.

*	e	g_1	g_2	g_3	g_4	g_5
e	e	g_1	g_2	g_3	g_4	g_5
g_1	g_1	e	g_3	g_5	g_2	g_4
g_2	g_2	g_5	e	g_4	g_1	g_3
g_3	g_3	g_4	g_1	e	g_5	g_2
g_4	g_4	g_3	g_5	g_2	e	g_1
g_5	g_5	g_2	g_4	g_1	g_3	e

Clearly L is a non associative non commutative loop of order six. Consider F = C(Z) = {a + bi | a, b ∈ Z} be the complex integer ring.

$$FL = \left\{ \sum_{i=0}^{n} a_i g_i \,\middle|\, a_i \in C(Z) \text{ and } g_i \in L;\ g_o = e \in L \right\}.$$

Now we show how addition and multiplication are performed on FL.

Let $\alpha = 8e - 12g_1 + 14g_2 - 5g_5$ and
$b = 7e + 20g_1 - 3g_3 + 4g_4 + g_5$ be in FL.

To find $\alpha+\beta$; $\alpha+\beta = 8e - 12g_1 + 14g_2 - 5g_5 + 7e + 20g_1 - 3g_3 + 4g_4 + g_5 = 15e + 8g_1 + 14g_2 - 3g_3 + 4g_4 - 4g_5$;
$\alpha + \beta$ is in FL.

Now consider

$$\begin{aligned}\alpha\beta &= (8e - 12g_1 + 14g_2 - 5g_5)\,(7e + 20g_1 - 3g_3 + 4g_4 + g_5)\\ &= 56e - 84g_1 + 98g_2 - 35g5 + 160g_1 - 240e + 280g_5 - \\ &\quad 100g_2 - 24g_3 - 36g_5 - 42g_4 + 15g_1 + 32g_4 - 48g_2 + \\ &\quad 56g_1 - 20g_3 + 8g_5 - 12g_4 + 14g_2 - 5e.\\ &= -189e + 147g_1 - 50g_2 + 217g_5 - 30g_3 - 22g_4 \text{ is in FL.}\end{aligned}$$

Consider

$$\begin{aligned}\beta\alpha &= (7e + 20g_1 - 3g_3 + 4g_4 + g_5)\,(8e - 12g_1 + 14g_2 - 5g_5)\\ &= 56e + 160g_1 - 24g_3 + 32g_4 + 8g_5 - 84g_1 - 240e + 36g_4\\ &\quad - 48g_3 - 12g_2 + 98g_2 + 280g_3 - 42g_1 + 56g_5 + 14g_4 - \\ &\quad 35g_5 - 100g_4 + 15g_2 - 20g_1 - 5e\\ &= -189e + 14g_1 + 208g_3 - 18g_4 + 29g_5.\end{aligned}$$

We see $\alpha\beta \neq \beta\alpha$, but both $\alpha\beta$ and $\beta\alpha$ are in FL. Thus FL is a non commutative non associative loop ring of infinite order.

Thus we see the loop ring FL is non commutative if and only if L is a non commutative loop.

We now give examples of commutative loop rings.

Example 3.1.59: Let $L = L_5(3)$ be a loop.

$F = C(Q) = \{a + bi \mid a, b \in Q\}$ be a complex rational ring. FL be the loop ring. FL is a commutative complex loop ring as L is a commutative loop.

Example 3.1.60: Let $L = L_{13}(7)$ be a loop of order 14. F = C be the complex ring FL be the complex loop ring. FL is a commutative complex loop ring.

In view of this we have the following theorem.

THEOREM 3.1.21: *Let $L = L_n\left(\frac{n+1}{2}\right)$ be a loop. $F = C(Z)$ (or $C(Q)$ or C) be the complex ring FL be the complex loop ring. FL is a commutative complex loop ring.*

Proof follows from the fact that $L = L_n(m)$ is a commutative loop if and only if $m = \left(\frac{n+1}{2}\right)$.

We see using the above statement we can get a characterization theorem for a complex loop ring to be commutative.

We can also get a class of complex loop rings using the loops of order n+1, n > 3.

We know from [15]; $L_n = \{L_n(m) \mid m < n, (m, n) = 1, (m–1, n) = 1\}$ denotes the class of loops of order n+1.

Now $FL_n = \{FL_n(m) \mid L_n(m) \in L_n\}$ denotes the class of loop rings. We give the properties associated with this class of loop rings.

THEOREM 3.1.22: *The class of loop complex rings (complex non associative rings) FL_n contains one and only one commutative, non associative complex ring. This happens when $L_n(m)$ in L_n is such that $m = \left(\frac{n+1}{2}\right)$.*

Proof easily follows from the fact that the class of loops L_n contains one and only one commutative loop.

We say a loop ring FL is a left alternative loop ring if $(\alpha\alpha)\beta = \alpha(\alpha\beta)$ for all α, β in FL.

Likewise the loop ring FL is right alternative if and only if $(\alpha\beta)\beta = \alpha(\beta\beta)$ for all α, β in FL.

If in a loop ring both the identities left alternative identity as well as right alternative. Identity is satisfied then we say FL is an alternative non associative ring.

We call the loop ring FL where F is a complex modulo integer ring and L is a loop of the form $L_n(m)$. If (xy) z = e = 1 imply x (yz) = e = 1 for all x, y, z $\in$ FL, then FL is a weak inverse property loop ring.

We can also define a loop complex ring FL to be a Jordan ring if ab = ba; a^2 (ba = (a^2 b) a for all a, b $\in$ FL. We give in the following conditions for the loop L where

$L \in L_n = \{L_n(m) \mid m < n; (m, n) = 1, (m-1, n) = 1\}$.

We first give some examples of them.

Example 3.1.61: Let L = $L_7(5)$ be a loop and

F = C(Z) = {a + bi | a, b $\in$ Z} be a complex integer ring. FL be the loop complex integer ring, clearly FL satisfies the weak inverse property condition.

Example 3.1.62: Let L = $L_{43}(7)$ be a loop and

F = C(Q) = {a + bi | a, b $\in$ Q} be the complex rational ring. FL be the loop ring, FL is a weak inverse property ring.

Inview of this we give the following theorem.

THEOREM 3.1.23: *Let L = $L_n(m)$ be a loop in L_n.*

F = {C(Q) = a + bi; a, b $\in$ Q} be the complex rational ring. FL be the loop ring. FL is a weak inverse property complex non associative ring if and only if (m^2 – m + 1) = 0 (mod n).

Proof follows from the fact that a loop $L_n(m) \in L_n$ is a weak inverse property loop if and only if $(m^2 - m+1) \equiv 0 \pmod{n}$.

THEOREM 3.1.24: *Let L = $L_n(m)$ be a loop.*
F = {C(Z) (or C(Q) or C) be the complex integer ring. (complex rational ring or complex real field). FL be the loop complex ring. FL is a right alternative loop complex ring if m = 2.

COROLLARY 3.1.1: *Let L = $L_n(m)$ be a loop with m = 2 and F be the complex ring. FL is a right alternative complex loop ring.*

Thus we have the following characterization theorem.

THEOREM 3.1.25: *Let L = $L_n(m)$ be a loop of order n+1. F = C(Z) (or C(Q) or C); the complex loop ring is a right alternative loop ring if and only if m = 2.*

Example 3.1.63: Let L = $L_{19}(2)$ be the loop of order 20. F = C the complex field. FL the loop ring is a complex right alternative loop ring. Clearly FL is not a left alternative loop ring.

Example 3.1.64: Let L = $L_{23}(2)$ be the loop of order 24. F = C(Z) be the complex integer ring. FL be the complex loop ring which is right alternative but is not left alternative.

We now give examples of left alternative loop rings.

Further using L = $L_n(m) \in L_n$ we see no FL is alternative.

Example 3.1.65: Let L = $L_{13}(12)$ be a loop. F = C(Q) be the complex ring. FL be the complex loop ring. FL is a left alternative complex loop ring.

Example 3.1.66: Let L = $L_{25}(24)$ be a loop. F = C be the complex field. FL the complex loop ring is a left alternative loop ring.

Example 3.1.67: Let L = $L_{23}(22)$ be the loop. F = C(Q) be the complex ring. FL the complex loop ring is a left alternative ring which is clearly not a right alternative ring.

THEOREM 3.1.26: *Let L = $L_n(m)$ be a loop. F be a complex ring. FL the complex loop ring is left alternative if and only if m = n–1.*

The proof follows from the simple fact that a loop $L_n(m)$ is left alternative if and only if $m = n-1$.

Let R be a non associative ring, the associator
$A(R) = \langle\{x \in R \mid x = (t, u, v) \text{ for some } t, u, v \in R\}\rangle$.

Example 3.1.68: Let $L = L_9(5)$ be a loop. $C(Z) = F$ be the complex ring of integers. FL be the loop ring. The associator of FL denoted by $A(FL) = FL$.

Example 3.1.69: Let $L = L_{13}(7)$ be a loop. $C(Q) = F$ be the complex rational ring. FL be the loop ring. $A(FL) = FL$.
Inview of this we have the following theorem.

THEOREM 3.1.27: *Let $L_n(m) \in L_n$ be a loop. F be a complex ring. FL the complex loop ring. The associator of FL is FL; that is $A(FL) = FL$.*

Follows from the simple fact if $L_n(m) \in L_n$ is a loop of order n+1 then the associator $A(L_n(m)) = L_n(m)$.

Recall let L be a loop, H a subloop of L. H is a normal subloop of L is
(i) $xH = Hx$
(ii) $(Hx)\, y = H\,(xy)$ and
(iii) $y(xH) = (yx)H$ for all $x, y \in L$.

A loop is simple if it has no normal subloops.

Example 3.1.70: Let $L = L_5(2)$ be a loop of order six.
$F = C(Z) = \{a + ib \mid a, b \in Z\}$ be the complex integer ring. FL is the complex integer loop ring. Clearly FL is non associative.
Let $H = \{e, g_1\} \subseteq L$ be a subgroup of L. FH is a group complex ring and $FH \subseteq FL$; so FL is a S-ring.

Example 3.1.71: Let $L = L_9(7)$ be a loop.
$F = C(Q) = \{a + ib \mid a, b \in Q\}$ be the complex rational ring. FL be a complex rational loop ring. Consider KL where

$K = C(Z) = \{a + ib \mid a, b \in Z\} \subseteq F$; KL is a complex rational loop subring of FL. Clearly FL is also a S-ring.

Example 3.1.72: Let $L = L_{21}(11)$ be a loop.

$F = C(Q) = \{a + ib \mid a, b \in Q\}$ be a ring. FL is a loop complex ring of the loop L over the ring F. FL is a commutative loop complex ring. This ring is also a S-ring.

We want to study about ideals in these rings. Infact these rings have a class of subrings which are not complex but non associative and are just subrings and not ideals. First we give examples of them.

Example 3.1.73: Let $L = L_{25}(8)$ be a loop.

$F = C(Z) = \{a + ib \mid a, b \in Z\}$ be the complex integer ring. FL be the complex loop ring of the loop L over the complex ring F. Let $Z \subseteq F$, ZL be the loop ring. ZL is a subring of FL; ZL is not associative but is a non complex or real subring. Clearly ZL is not an ideal of FL.

Example 3.1.74: Let $L = L_{37}(7)$ be a loop.

$F = C(Q)$ be the complex ring. FL be the loop complex ring. Take $H = \{e, g_{12}\} \subseteq L$, FH is a complex loop ring which is associative. Clearly $FH \subseteq FL$ is an associative subring of FL. Further FH is not an ideal of FL but a complex associative subring. Thus the complex loop ring FL can have non complex non associative subrings that is real non associative subrings which are not complex, so are not ideal of FL.

FL can have associative complex subrings which are also not ideals of FL.

Example 3.1.75: Let $L = L_{15}(2)$ be a loop.

$F = C(Q) = \{a + ib \mid a, b \in Q\}$ be the complex rational ring. FL be the loop complex ring of the loop L over the complex ring F. FL is a non associative ring. Take $H = \{e, g_1, g_4, g_7, g_{10}, g_{13}\} \subseteq L$, H is a subloop of L so FH is a loop subring of the ring FL which is complex as well as non associative. FH is also not an ideal of FL.

THEOREM 3.1.28: *Let $L = L_n(m)$ be a loop in $F = C(Q)$ (or C) be the complex ring. FL be the loop complex ring. FL is simple.*

Follows from the fact every $L \in L_n$ is a simple loop. Further we see C(Q) has no ideal. However if in the theorem C(Z) is replaced by $K = C(2Z) = \{a + bi \mid a, b \in 2Z\}$. KL is a loop complex subring of FL. Clearly KL is also an ideal of FL. $P = C(3Z) = \{a + bi \mid a, b \in 3Z\}$ be a complex subring of F. PL is also an ideal of FL. Now we see FL is not simple if

$F = C(Z) = \{a + bi \mid a, b \in Z\}$. Further if $F = C(Q)$ or C then FL has loop complex subrings which are not ideals.

Take $P = C(Z) \subseteq F$, PL is a subring of FL but is not an ideal of FL.

Example 3.1.76: Let $L = L_{43}(7)$ be a loop. $F = C(Q)$ be the complex ring. $H_1(13)$ be its S-subloop. FL be the loop complex ring. $SN_1(FH_1(13)) = SN_2(FH_1(13))$ where $FH_i(13)$ is a loop complex subring of FL, $1 \leq i \leq 2$.

Example 3.1.77: Let $L = L_{45}(8)$ be a loop and

$F = C = \{a + ib \mid a, b \in R\}$ be a complex ring. FL the loop complex ring. $H_1(15) = \{e, 1, 16, 31\} \subseteq L$ be a subloop of L. $FH_1(15)$ is a loop complex subring of FL.

We see $SN_1(FH_1(15)) \neq SN_2(FH_1(15))$.

Similar results in this direction can be derived by any interested reader.

All results studied for loop rings and groupoid rings of non complex rings can be easily extended to the case of complex loop rings with appropriate modifications. Most of the results are a matter of routine and hence is left as an exercise to the reader to solve.

3.2 Complex Loops and Complex Groupoids over Real Rings.

In this section we for the first time introduce the new notion of complex loop rings, here complex loops are used in the place of loops. Likewise complex groupoid rings are those rings where complex groupoids are used in the place of groupoids. Both the notions of complex groupoids and complex loops are introduced in chapter II of this book. Properties about these two new structures are discussed.

DEFINITION 3.2.1: *Let $G = \{C(Z_n), *, (t, u)\}$ (where $C(Z_n) = \{a + bi_F \mid a, b \in Z_n,\ i_F^2 = n-1)$ be a complex groupoid. $F = R$ (or Z_n or Z or Q) be the field (or ring). FG be the groupoid ring of G over F. FG is a non associative complex ring.*

Groupoid ring have been defined earlier. Further these groupoid rings may or may not have identity. Also groupoid rings are non associative may or may not be commutative.

We first give examples of groupoid rings of all types and derive some properties related with them.

Example 3.2.1: Let $G = \{C(Z_{89}), *, (5, 4)\}$ be the complex groupoid. $Z = F$ be the ring of integers FG is a groupoid ring.

$G = C(Z_9) = \{g_i + i_F g_j \mid i, j \in Z_9\ \ i_F^2 = g_8\}$.

Now we show how product and addition of FG are made.

Let $\alpha = 9g_1 + 8g_3 - 5g_2 + 10g_7$

and $\beta = -19g_0 + g_1 - 2g_2 + 5g_3 + g_8$ be in FG.

$$\alpha + \beta = 9g_1 + 8g_3 - 5g_2 + 10g_7 + (-19g_0 + g_1 - 2g_2 + 5g_3 + g_8)$$
$$= -19g_0 + 10g_1 - 7g_2 + 13g_3 + 10g_7 + g_7 \text{ and}$$

$\alpha + \beta \in$ FG.

Let us now find

$$\alpha\beta = (9g_1 + 8g_3 - 5g_2 + 10g_7)(-19g_0 + g_1 - 2g_2 + 5g_3 + g_8)$$
$$= -171g_1g_0 - 152g_3g_0 - 95g_2g_0 - 190g_7g_0 + 9g_1g_1 + 8g_3g_1 + 5g_2g_1 + 10g_7g_1 - 18g_1g_2 - 16g_3g_2 - 10g_2g_2 - 20g_7g_2 + 45g_1g_3 + 40g_3g_3 + 25g_2g_3 + 50g_7g_3 + 9g_1g_8 + 8g_3g_8 + 5g_2g_8 + 10g_7g_8$$

$= -171g_5 - 152g_6 - 95g_1 - 190g_8 + 9g_0 + 8g_1 + 5g_5$
$+ 10g_7 - 18g_4 - 16g_5 - 10g_0 - 20g_7 + 45g_8 + 40g_0$
$+ 25g_4 + 50g_2 + 9g_1 + 8g_6 + 5g_7 + 10g_4$
$= -182g_5 - 144g_6 - 78g_1 - 145g_8 + 39g_0 - 5g_7 + 17g_4 + 50g_2$
is in FG.

That is $g_i * g_j = g_{5i+4j} \pmod 9$ where $0 \leq i, j \leq 8$ is the operation performed on G.

Example 3.2.2: Let G = {C(Z_8), *, (4, 5)} be a complex groupoid. F = Z_8 be the ring of integer modulo 8. FG is the complex modulo integer groupoid ring of finite order.

Example 3.2.3: Let G = {C(Z_7), *, (0, 4)} be a complex modulo integer groupoid. F = Q be the field of rationals FG be the groupoid rings. FG is a complex non associative ring of infinite order.

Example 3.2.4: Let G = {C(Z_{49}), *, (9, 9)} be a complex groupoid and F = R be the field of reals, FG be the complex groupoid ring of infinite order.

Example 3.2.5: Let G = {C(Z_{42}), *, (11, 22)} be a complex modulo integer groupoid. F = Z be the integer ring. FL be the complex groupoid ring which is of infinite order both non commutative and nonassociative.

Example 3.2.6: Let G = {C(Z_{11}), *, (3, 3)} be a complex groupoid. F = Z_{11} be the modulo integer ring. FG be the complex groupoid ring of finite order which is commutative but non associative.

Example 3.2.7: Let G = {C(Z_{14}), *, (8, 6)} be a complex modulo integer groupoid. L = Z_{10} be the ring of modulo integer 10. LG is a complex modulo integer groupoid ring of finite order.

Example 3.2.8: Let G = {C(Z_{40}), *, (7, 14)} be the complex modulo integer groupoid. L = Z_{10} be the ring of modulo integer. FG be the groupoid complex modulo integer ring.

Example 3.2.9: Let G = {$C(Z_{49})$, *, (9, 18)} be a complex modulo integer groupoid. F = R be the field of reals. FG is the complex groupoid (modulo integer) ring of infinite order non commutative and non associative.

Example 3.2.10: Let G = {$C(Z_{20})$, *, (10, 3)} be a complex modulo integer groupoid. F = Z_{25} be the ring of modulo integers. FG be the complex groupoid ring.

Example 3.2.11: Let G = {$C(Z_3)$, *, (2, 1)} be a complex modulo integer groupoid. F = Z_3 be the ring of modulo integers. FG be the complex modulo integer groupoid ring.

Example 3.2.12: Let G = {C(Z), *, (m, n)} be a complex groupoid. F = Z be the ring of integers. FG be the complex groupoid ring.

Example 3.2.13: Let G = {C(Q), *, (8, 9)} be a complex groupoid. F = Q be the ring of integers. FG be a complex groupoid ring.

Example 3.2.14: Let G = {C(Q), *, (2, 4)} be a complex groupoid. F = Z_{12} be the ring of modulo integers. FG be the complex groupoid ring.

We can define subrings of these complex groupoid rings.

DEFINITION 3.2.2: *Let G be a complex groupoid, F be any real field or a commutative ring. FG be the complex groupoid ring. Suppose H $\subseteq$ FG; and if H is itself a complex non associative ring then we define H to be a complex groupoid subring of FG.*

We will illustrate this situation by some examples.

Example 3.2.15: Let G = {$C(Z_{12})$, *, (8, 9)} be a complex groupoid. F = Z be the ring of integers. FG be the complex groupoid ring. Take H = 3ZG be the complex non associative ring. H $\subseteq$ FG so H is a subring of FG.

Example 3.2.16: Let G = {C(Z_4), *, (3, 1)} be a complex groupoid. F = Z_{12} be the ring of modulo integers. FG be the complex groupoid ring.

Consider K = {0, 2, 4, 6, 8, 10} $\subseteq$ F, KG is the complex groupoid subring of FG.

Example 3.2.17: Let G = {C (Z_{40}), *, (10, 3)} be a complex groupoid. R be the field of reals. RG be the complex groupoid ring.

Consider QG $\subseteq$ RG; QG is the complex groupoid subring of RG.

Example 3.2.18: Let G = {C (Z_{43}), *, (0, 13)} be a complex groupoid. F = Q, the field of rationals. QG be the complex groupoid ring. Consider ZG (Z $\subseteq$ Q the ring of integers); ZG is the complex groupoid subring of QG.

Now we see H = {Z_{43}, *, (0, 13)} $\subseteq$ G is also a subgroupoid of G and H is not a complex modulo integer groupoid. Consider QH; QH $\subseteq$ QG is a modulo integer groupoid ring which is not complex so we define QH to be a pseudo complex modulo integer groupoid subring of QG.

We give examples of them before we proceed to define other properties about these complex groupoid rings.

Example 3.2.19: Let G = {C (Z_{14}), *, (0, 7)} be a complex groupoid ring. Z be the ring of integers. ZG be the complex groupoid ring. Consider H = {Z_{14}, *, (0, 7)} $\subseteq$ G. ZH is the pseudo complex groupoid subring. 5ZG is the complex groupoid subring. Both the subrings are non associative and non commutative.

Example 3.2.20: Consider the complex groupoid

G = {C (Z_4), *, (2, 3)}. Take F = R the reals, RG be the complex groupoid ring. Take H = {g_0, g_2, g_{2i_F}, g_{2+2i_F} } $\subseteq$ G; RH is a complex groupoid subring of RG which is not an ideal of RG.

Example 3.2.21: Let G = {C (Z_{10}), *, (1, 5)} be a complex groupoid of modulo integers. F = Z_{12} be the ring of modulo integers. FG be the complex groupoid ring. FG is a non associative complex modulo integer ring. FG has both subrings, pseudo subrings and ideals.

Take SG $\subseteq$ FG where S = {0, 2, 4, 6, 8, 10} $\subseteq$ Z_{12} = F; clearly SG is a subring which is also an ideal of FG. Consider FT where T = {g_0, g_5} $\subseteq$ G, FT is only a subring which is a pseudo complex subring of FG and is not an ideal of FG. Infact FG is a S-ring. Further FT is an associative subring of FG.

Example 3.2.22: Let G = {C(Z_6), *, (4, 5)} be a groupoid of complex modulo integers, F = Z_{20} be the ring of modulo integers FG be the complex groupoid ring.

Let H = {g_0, g_2, g_4} $\subseteq$ G, FH be the non complex non associative subring of FG. FH is a pseudo complex non associative subring which is not an ideal of FG.

Example 3.2.23: Let G = {C(Z_{12}), *, (1, 3)} be a complex modulo integer groupoid. F = Z_5 be the ring of modulo integers. FG be the complex groupoid ring. A = {0, 3, 6, 9} $\subseteq$ G be a pseudo complex groupoid.

FA is the pseudo complex groupoid subring of FA. Clearly FG is not an ideal but FA, we see is an ideal over F.

This type of ideals we call as pseudo basic ring ideals for they are ideals over the basic ring F over which the structures is defined.

Example 3.2.24: Let G = {C(Z_{11}), *, (2, 3)} be a groupoid of complex modulo integers. F = Z_2 = {0, 1} be the field of characteristic two. FG be the groupoid ring. Clearly FG is a non commutative non associative complex groupoid ring of finite order.

Take FH where H = {0, 2} and FT where T = {1, 3} be pseudo groupoid subrings of FG. They are pseudo basic ring ideals. Both the rings are non commutative. Table for H is given below:

*	g_0	g_2
g_0	g_0	g_2
g_2	g_0	g_2

and the table for T is as follows:

*	g_1	g_3
g_1	g_1	g_3
g_3	g_1	g_3

Now FH = $\{0, 1, g_0, g_2, 1+g_0, 1+g_2, g_0 + g_2, 1+g_0 + g_2\}$ and

FT = $\{0, 1, g_1, g_3, g_1+1, g_3+1, g_1+g_3, 1+g_1 + g_3\}$. These are non commutative rings of order 8 of characteristic two.

They are also pseudo basic ring ideal of FG.

We can also define pseudo basic subring ideals if the ring is replaced by a subring and over the subring the structure is a ring. This has more relevance when we use our basic rings as Q or R.

We give only examples of these structures.

Example 3.2.25: Let G = $\{C(Z_{12})\ *, (1, 3)\}$ be a complex modulo integer groupoid. F = R be the field of reals. FG be the complex groupoid ring. Consider QG ($Q \subseteq R$, the field of rationals). QG is a subring of FG. However QG is also a pseudo basic subring ideal over the subring Q. We see QG is not an ideal over FG.

Also if ZG ($Z \subseteq R$, Z the ring of integer is taken), ZG is a subring of FG, but ZG is not an ideal in FG; however ZG is a pseudo basic subring ideal over Z.

Now consider A $\subseteq$ G the subgroupoid given by the following table.

*	g_0	g_3	g_6	g_9
g_0	g_0	g_9	g_6	g_3
g_3	g_3	g_0	g_9	g_6
g_6	g_6	g_3	g_0	g_9
g_9	g_9	g_6	g_3	g_0

Now FA is a subring which is a pseudo complex subring of FG. Clearly FA is also non associative FA is a pseudo basic ring ideal over R.

Now having seen examples of these structures we proceed onto relate other properties.

We have the following theorem.

THEOREM 3.2.1: *Let G = {C(Z_n), *, (m, m); 1 < m < n} be a complex modulo integer groupoid. R be the field of reals. RG is a commutative but non associative complex ring.*

Proof is direct using simple number theoretic techniques.

THEOREM 3.2.2: *Let G = {C(Z_n), *, (m, t); 0 < t < n} be a complex modulo integer groupoid. R the field of reals. RG the complex groupoid ring. If RG has a S-subring then RG is a S-ring. Further if RG is a S-ring every subring of RG need not be a S-subring.*

The converse part can be proved by using counter examples. All results studied for usual groupoids over ring / complex rings can be easily extended in case of complex groupoids over rings with appropriate modifications. This task is also left as an exercise. We now proceed onto define the notion of complex loop ring FL where L is the complex loop and F is a real field or a real commutative ring with identity.

DEFINITION 3.2.3: *Let*

$L = C(L_n(m)) = \{a + i_F b \mid a, b \in L_n = \{e, g_1, g_2, \ldots, g_n\}, i_F^2 = g_{n-1}\}$ be a complex loop. F = R (or Q or Z or Z_n) be the ring

or a field. FL is the complex loop ring consisting of finite formal sums of the form $\sum_i \alpha_i g_i$ *; i - varies over a finite index* $\alpha_i \in F$ *and* $g_i \in L$*; satisfying the following conditions.*

(i) *If* $\alpha = \sum_i \alpha_i g_i$ *and* $\beta = \sum_i \beta_i g_i$ *are in FL,*
$\alpha = \beta$ *if and only if* $\alpha_i = \beta_i$ *for every i.*

$$\text{(ii) } \alpha + \beta = \sum_i \alpha_i g_i + \sum_i \beta_i g_i = \sum_i (\alpha_i + \beta_i) g_i \text{ is in FL.}$$

$$\text{(iii) } \alpha\beta = \sum_i \alpha_i g_i \sum_i \beta_i g_i = \sum_i \gamma_k h_k \quad g_i\, g_j = h_k$$

with $h_k \in L$ *and* $\gamma_k \in F$.

(iv) $g.1 = 1.g = g$ *for all* $g \in L$ *and* $1 \in F$.

(v) $e.\ \alpha = \alpha.e = \alpha$ *as* $1.e = e.1 = 1$ *for all* $\alpha \in F$.

FL is a non associative ring called the loop ring.
We give examples of them.

Example 3.2.26: Let

$L = \{C(L_9(5)\} = \{a + bi_F \mid a, b \in \{e, g_1, g_2, \ldots, g_9\},\ i_F^2 = g_8\}$ be the complex loop. F = Z be the ring. FL is the loop ring.

If $\alpha = 7\,(g_1 + i_F\, g_2) + 3\,(g_3 + i_F\, g_6) - 2\,(g_7 + i_F\, g_1)$ and
$\beta = 3(g_3 + i_F\, g_6) - 2\,(g_7 + i_F\, g_1) + 3\,(g_3 + g_4\, i_F) - 4\,(g_6 + i_F\, g_3)$ is in FL.

$$\alpha\beta = (7\,(g_1 + i_F g_2) + 3\,(g_3 + i_F g_6) - 2\,(g_7 + i_F g_1)) \times (3\,(g_3 + g_3 i_F) - 4\,(g_6 + g_3 i_F))$$

$= 21\,(g_1 + i_Fg_2) * (g_3 + g_3\, i_F) + 9\,(g_3 + g_6i_F)\,(g_3 + g_4\, i_F)$
$\quad – 6\,(g_7 + i_Fg_1)\,(g_3 + g_4i_F) – 28\,(g_1 + i_Fg_2)\,(g_6 + g_3i_F)$
$\quad – 12\,(g_3 + i_Fg_6)\,(g_6 + g_3i_F) + 8\,(g_7+i_Fg_1)\,(g_1 + g_3\, i_F)$

$= 21\,[5\,(g_3 + g_4i_F) – 4\,(g_1 + i_Fg_2)] + 9\,[5\,(g_3 + g_4i_F)$
$\quad – 4\,(g_3 + g_6i_F)] – 6\,[5\,(g_3 + g_4\, i_F) – 4\,(g_7 + i_F\, g_1)]$
$\quad – 28\,[5\,(g_6 + g_3i_F) – 4\,(g_1 + i_Fg_2)] – 12\,[5\,(g_6 + g_3i_F)$
$\quad – 4\,(g_3 + i_Fg_6)] + 8\,[5\,(g_6 + g_3i_F) – 4\,(g_7 + i_Fg_1)]$

$= 21\,[g_6 + g_2i_F + g_5 + g_8i_F] + 9\,[g_6 + g_2i_F + g_6 + g_3i_F]$
$\quad – 6\,[g_6 + g_2i_F + g_8 + g_5i_F] – 28\,[g_3 + g_6i_F + g_6 + g_3i_F]$
$\quad + 8\,[g_3 + g_6i_F + g_8 + g_5\, i_F]$

$= 21\,[g_2 + g_1i_F] + 9\,[g_3 + g_5i_F] – 6\,[g_5 + g_7i_F]$
$\quad – 28\,[g_8 + g_5i_F] – 12\,[g_9 + g_9i_F] + 8\,[g_2 + g_2\, i_F].$

Example 3.2.27: Let $L = C(L_{13}(7))$ be a loop of complex modulo integers. $F = Z_{13}$ be the ring of modulo integers. FL be the loop ring. FL is a commutative and non associative ring.

Example 3.2.28: Let $L = C(Z_{25}(8))$ be a complex modulo integer loop. $F = Z_{25}$ be the ring of modulo integers. FL be the loop ring.

Example 3.2.29: Let $L = C(L_{27}(14))$ be the complex modulo integer loop. $F = Z$ be the ring of integers, FL is the complex loop ring of infinite order.

Example 3.2.30: Let $L = C(Z_{29}(17))$ be a complex modulo integer loop. $Z_{17} = F$ be the ring of modulo integers. FL is the complex loop ring.

Example 3.2.31: Let $C(Z_{21}(11)) = L$ be the loop. $F = Z_{15}$ be the ring of modulo integers. FL be the loop ring of L over F.

Example 3.2.32: Let $L = C(Z_{11}(7))$ be the complex loop. $F = Z_{11}$ be the field of modulo integers. FL be the complex modulo integer loop ring of finite order.

Example 3.2.33: Let L = $C(Z_{13}(5))$ be the complex modulo integer loop. F = Z_{13} be the modulo integer field. FL is the complex modulo integer loop ring of finite order which is non commutative.

Example 3.2.34: Let L = $C(Z_{43}(2))$ be a complex modulo integer loop. Z_2 = F be the ring of modulo integers. FL is the loop ring of finite order.

Example 3.2.35: Let L = $C(Z_{43}(2))$ be a complex modulo integer loop. FL is the complex loop ring of finite order.

Example 3.2.36: Let L = $C(Z_{43}(2))$ be the complex modulo integer loop. F = Z be the ring of integers. FL is the loop ring of infinite order.

THEOREM 3.2.3: *Let L = $C(L_n(m))$ be a complex modulo integer loop. Z = F the ring of integers, FL the complex loop ring is a S-loop ring.*

Proof is direct and hence left as an exercise to the reader.

THEOREM 3.2.4: *Let L be a finite complex modulo integer loop. R be the field of reals. RL be the loop ring.*

Then J (RL $\underset{\neq}{\subset}$ W (RL).

All concepts related with non associative rings can be defined and derived in case of complex non associative rings also with simple modifications. Now we proceed onto define the notion of double complex non associative rings.

DEFINITION 3.2.4: *Let*

$G = C(Z_n) = \{a + bi_F \mid i_F^2 = n-1,\ a, b \in Z_n, (t, u);\ t, u \in Z_n, *\}$ be the complex modulo integer groupoid. F = C(Z) (or C(Q) or C(R) = C) be the ring of complex integers (or ring of rational complex numbers or complex field). The double complex groupoid ring FG consists of all finite formal sums of the form $\sum r_i g_i$ (i- running over finite index) where $r_i \in F$ and $g_i \in G$ satisfying the following conditions.

(i) $\sum_{i=1}^{n} r_i g_i = \sum_{i=1}^{n} s_i g_i \Leftrightarrow r_i = s_i$ *for i=1, 2, …, n.*

(ii) $\sum_{i=1}^{n} r_i g_i + \sum_{i=1}^{n} s_i g_i = \sum_{i=1}^{n} (r_i + s_i) g_i$.

(iii) $\left(\sum_{i=1}^{n} r_i g_i\right) \left(\sum_{j=1}^{n} s_j g_j\right) = \sum t_k\, g_k$ *where* $g_k = g_i\, g_j$ *where*

$$t_k = \sum r_i\, s_j.$$

(iv) $r_i g_i = g_i\, r_i$ *for all* $g_i \in G$ *and* $r_i \in F$.

(v) $r (\sum r_i g_i) = \sum r r_i g_i$ *for* $r, r_i \in F$ *and* $g_i \in G$ *as* $1 \in F$ *and* $m_i \in G$ *we have* $1.G \subseteq FG$ *but* $F \not\subseteq FG$ *and* $F \subseteq FG$ *if and only if G has identity.*

Clearly FG is a non associative ring with $0 \in F$ *as the additive identity.*

The groupoid ring FG is defined to be a alternative ring if and only if (xx) y = x(xy) and x (yy) = (xy)y for all x, y ∈ FG.

We first proceed onto give examples of such ring.

Example 3.2.37: Let

$G = C(Z_7) = \{a + bi_F \mid i_F^2 = 6 = g_6,\ a, b \in Z_7, *, (3, 4)\}$ be the complex groupoid of modulo integers, C(Z) = F be the integer complex ring FG is the double complex groupoid ring.

For we will use the following notation.

$G = \{g_0, \ldots, g_6, g_{i_F}, g_{2i_F}, \ldots, g_{6i_F}, g_{2+i_F}, g_{1+i_F}, \ldots, g_{6+6i_F}, *, (3, 4)\}$.

For any $g_t, g_p \in G$ we see $g_t * g_p = g_t = g_{3t+4p (\text{mod } 7)}$.

That is if g_{3+4i_F} and g_{2+i_F} are in G, then

$$\begin{aligned} g_{3+4i_F} * g_{2+i_F} &= g_{3(3+4i_F)+4(2+i_F)(\text{mod } 7)} \\ &= g_{(9+12i_F+8+4i_F)(\text{mod } 7)} \\ &= g_{(3+2i_F)} \in G. \end{aligned}$$

Now if $\alpha = (3+2i)\ g_{3+i_F} + (2-i)g_3 + 4ig_0$ and

$\beta = (7-4i)\ g_{2+i_F} + (7+4i)\ g_{3+i_F} + (9+2i)\ g_3$ is in FG then

$$\begin{aligned}
\alpha+\beta = &\ ((3+2i)\ g_{3+i_F} + (2-i)g_3 + 4ig_0) + ((7-4i)\ g_{2-i_F} \\
&+ (7+4i)\ g_{3+i_F} + (9+2i)\ g_3) \\
= &\ (2-i + 9+2i)g_3 + 4ig_0 + (7-4i)\ g_{2+i_F} \\
&+ (3+2i+7+4i)\ g_{3+i_F} \\
= &\ 4ig_0 + (7-4i)\ g_{2+i_F} + (11+i)g_3 + (10+6i)\ g_{3+i_F}.
\end{aligned}$$

Now we see $\alpha+\beta \in$ FG.

Consider

$$\begin{aligned}
\alpha\beta = &\ ((3+2i)\ g_{3+i_F} + (2-i)g_3 + 4ig_0)\ ((7-4i)\ g_{2+i_F} \\
&+ (7+4i)\ g_{3+i_F} + (9+2i)\ g_3) \\
= &\ (3+2i)\ (7-4i)\ g_{3+i_F}\ g_{2+i_F} + (3+2i)\ (7+4i)\ g_{3+i_F}\ g_{3+i_F} \\
&+ (3+2i)\ (9+2i)\ g_{3+i_F}\ g_3 + (2-i)\ (7-4i)g_3\ g_{2+i_F} \\
&+ (2-i)\ (7+4i)g_3,\ g_{3+i_F} + (2-i)\ (9+2i)g_3.\ g_3. \\
= &\ 4i\ (7-4i)g_0\ g_{2+i_F} + 4i\ (7+4i)g_0\ g_{3+i_F} + 4i\ (9+2i)g_0\ g_3 \\
= &\ (21 + 8 - 12i + 14i)\ g_{9+3i_F+8+4i_F} \\
&+ (21+14i+12i-8)\ g_{9+3i_F+12+4i_F} \\
&+ (27-4+6i+18i)\ g_{9+3i_F+12} + (14-4-7i-8i)\ g_{9+8+4i_F} \\
&+ (14-7i + 8i + 4)\ g_{9+12+4i_F} + (18+4i-9i+2)\ g_{8+4i_F} \\
&+ (28-16i)\ g_{12+4i_F} + (36i-8)g_{12}. \\
= &\ (29+2i)g_3 + (13+26i)\ g_0 + (23+24i)\ g_{3i_F} \\
&+ (10-15i)\ g_{3+4i_F} + (18+i)\ g_{4i_F} + (20-5i)g_0 \\
&+ (28i+16)\ g_{i+4i_F} + (28-16i)\ g_{5+4i_F} + (36i-8)g_5
\end{aligned}$$

is in FG.

Thus we have shown how product and sum are defined on FG.

Example 3.2.38: Let G = $\{C(Z_{40}), *, (3, 8)\}$ be a complex modulo integer groupoid. F = C(R) = C the complex field. FG be the double complex complex modulo integer groupoid ring.

Example 3.2.39: Let G = $\{C(Z_{27}), *, (7, 8), i_F^2 = 26\}$ be the complex groupoid. F = C (Z_{27}) be the complex ring of modulo integers. FG the double complex complex modulo integer groupoid ring.

Example 3.2.40: Let G = $\{C(Z_{10}), *, (3, 8), i_F^2 = 9\}$ be a complex modulo integer groupoid.

F = C (Z_2) = $\{a + bi_F \mid i_F^2 = 1, a, b \in Z_2\}$ be the complex modulo integer ring. FL be the double complex complex modulo integer groupoid ring.

Example 3.2.41: Let G = $\{C(Z_{13}), *, (3, 9), i_F^2 = 12\}$ be a complex modulo integer groupoid. $C(Z_{10})$ = F be the complex modulo integer ring FG be the double complex complex modulo integer groupoid ring.

We can get both finite and infinite double complex complex modulo integer groupoid rings.

Now we can define double complex complex modulo integer loop rings.

In the definition if we replace the groupoid $C(Z_n)$ by $C(L_n(m))$ the resulting ring is a non associative ring defined as the double complex complex modulo integer loop ring.

We give examples of this situation.

Example 3.2.42: Let $C(L_9(8))$ = G be a complex modulo integer loop ring. F = C(Z) be the complex modulo integer ring. FG is the double complex complex modulo integer loop ring of infinite order.

Example 3.2.43: Let $C(L_{17}(8)) = L$ be the complex loop ring of modulo integers. $F = C$ be the complex field. FL is the double complex complex modulo integer loop ring of infinite order.

Example 3.2.44: Let $C(L_{23}(7))$ be a complex modulo integer loop. $F = C(Z_5)$ be the complex ring of modulo integers. FL is the double complex complex loop ring of finite order.

Example 3.2.45: Let $C(L_{29}(2))$ be a complex modulo integer loop. $F = C(Z_2)$ be the complex modulo integer ring. FL is the double complex complex modulo integer loop ring of finite order.

Example 3.2.46: Let $C(L_{29}(28))$ be a complex modulo integer loop ring. $F = C(Z_2)$ be the complex modulo integer ring. FL is the double complex complex modulo integer loop ring of finite order.

Example 3.2.47: Let $C(L_{43}(2))$ be a complex modulo integer loop. $F = C(Z)$ be the complex modulo integer ring. FL is the double complex complex modulo integer loop ring of infinite order.

Example 3.2.48: Let $C(L_{43}(42))$ be a complex modulo integer loop. $F = C(Z_{20})$ be the complex modulo integer ring. FL is the double complex complex modulo integer loop ring of finite order.

Example 3.2.49: Let $C(L_{25}(9))$ be the complex modulo integer loop ring. $F = C(Z_{20})$ be the complex modulo integer ring. FL is the double complex complex modulo integer loop ring of finite order.

All properties true in case of usual non associative rings / loop rings can be easily extended to the case of these double complex complex loop rings with simple modifications. Also interested reader can study the special identities satisfied by the double complex complex loop ring depending on the nature of the loop under consideration. These are left as simple extensions and exercises to the reader.

Finally we proceed onto define complex modulo integer matrix groupoids. These also only form a special class of groupoids built using complex modulo integers.

Throughout our discussion

$G = C(Z_n) = \{a + bi_F \mid i_F^2 = n-1, a, b \in Z_n, (t, u), *\}$ will denote a complex modulo integer groupoid t, u $\in Z_n$.

Let

$M = \{g_1, \ldots, g_m) \mid g_i = a + bi_F;\ a, b \in Z_n, i_F^2 = n-1, *, (t, u)\}$. We define for $x = (g_1, g_2, \ldots, g_m)$ and $y = (h_1, h_2, \ldots, h_m) \in M$. $x*y = (g_1, g_2, \ldots, g_m) * (h_1, h_2, \ldots, h_m) = (g_1 * h_1, g_2 * h_2, \ldots, g_m * h_m)$

$= (tg_1 + uh_1 \pmod n), \ldots, tg_m + uh_m \pmod n)$; $x*y \in M$. M is defined as the complex modulo integer row matrix groupoid.

We will first provide some examples of them.

Example 3.2.50: Let

$$G = \{(g_1, g_2, g_3, g_4, g_5) \mid g_i \in C(Z_{10}), *, (3, 8)\}$$

be the complex modulo integer row matrix groupoid.

Take $x = (3+2i_F, i_F, 7, 3i_F+1, 2)$ and $y = (0, 2+5i_F, 8i_F, 9, 0) \in G$.

$$x*y = (3+2i_F, i_F, 7, 3i_F+1, 2)\ 8\ (0, 2+5i_F, 8i_F, 9, 0)$$

$$= (3+2i_F * 0, i_F*2+5i_F, 7*8i_F, 3i_F+1*9, 2*0)$$

$$= (9+6i_F, 3i_F + 16 + 40i_F, 21 + 74i_F, 3+9i_F + 72, 6)$$

$$= (9+6i_F, 3i_F + 6, 1+4i_F, 5+9i_F, 6) \in G.$$

In this way G is a non associative structure and is a groupoid of finite order.

Example 3.2.51: Let

$$G = \{(g_1, g_2, g_3) \mid g_i \in C(Z_{40}), 1 \leq i \leq 3, *, (12, 0)\}$$

be the complex modulo integer row matrix groupoid of finite order.

Example 3.2.52: Let

$$G = \{(g_1, g_2, \ldots, g_{16}) \mid g_i \in C(Z_{240}), 1 \leq i \leq 16, *, (28, 28)\}$$

be the complex modulo integer row matrix groupoid of finite order.

We have subgroupoids, ideals and special identities satisfied by these groupoids also. This is a matter of routine and the reader is requested to refer [14].

Example 3.2.53: Let

$$G = \{(g_1, g_2, \ldots, g_{28}) \mid g_i \in C(Z), 1 \leq i \leq 28, *, (20, -17)\}$$

be the complex modulo integer row matrix groupoid of infinite order.

Example 3.2.54: Let

$G = \{(g_1, g_2, \ldots, g_{10}) \mid g_i \in C(R) = a+ib$ with $a, b \in R, i^2 = -1, *, (20, -\sqrt{3}, 19+\sqrt{5})\}$ be a complex modulo integer row vector (matrix) groupoid of infinite order.

Example 3.2.55: Let

$$G = \{(g_1, g_2, g_3, \ldots, g_{45}) \mid g_i \in C(Q), *, (3/7, 19/10)\}$$

be a complex modulo integer row matrix groupoid of infinite order.

Example 3.2.56: Let

$$G = \{(g_1, g_2, g_3, g_4) \mid g_i \in C(Z), *, (26, -43), 1 \leq i \leq 4\}$$

be a complex modulo integer row matrix groupoid of infinite order.

Now let L = $\left\{ \begin{bmatrix} g_1 \\ g_2 \\ \vdots \\ g_9 \end{bmatrix} \middle| \; g_m \in C(Z_{41}), *, (3, 8), 1 \leq i \leq m \right\}$, L

can be defined as the complex groupoid of column vectors (matrix).

We see if x = $\begin{bmatrix} g_1 \\ g_2 \\ \vdots \\ g_9 \end{bmatrix}$ and y = $\begin{bmatrix} h_1 \\ h_2 \\ \vdots \\ h_9 \end{bmatrix}$ are in L then

$$x*y = \begin{bmatrix} g_1 \\ g_2 \\ \vdots \\ g_9 \end{bmatrix} * \begin{bmatrix} h_1 \\ h_2 \\ \vdots \\ h_9 \end{bmatrix} = \begin{bmatrix} g_1 * h_1 \\ g_2 * h_2 \\ \vdots \\ g_9 * h_9 \end{bmatrix}$$

$$= \begin{bmatrix} 3g_1 + 8h_1 (\text{mod } 41) \\ 3g_2 + 8h_2 (\text{mod } 41) \\ \vdots \\ g_9 + 8h_9 (\text{mod } 41) \end{bmatrix}.$$

L is a complex column matrix groupoid of finite order.

Example 3.2.57: Let

$$P = \left\{ \begin{bmatrix} g_1 \\ g_2 \\ \vdots \\ g_{12} \end{bmatrix} \middle| \; g_i \in C(Z_{12}), *, 1 \leq i \leq 12, (3, 3) \right\}$$

be a complex modulo integer column matrix groupoid of finite order. P is commutative.

Example 3.2.58: Let

$$P = \left\{ \begin{bmatrix} g_1 \\ g_2 \\ \vdots \\ g_{20} \end{bmatrix} \middle| \; g_i \in C\,(Z_5) \right.$$

$$= \{a + bi_F \mid a, b \in Z_5,\; i_F^2 = 4, *, (3, 0), 1 \leq i \leq 20\}\}$$

be a complex column matrix groupoid. Clearly P is non commutative and is of finite order.

Example 3.2.59: Let

$$P = \left\{ \begin{bmatrix} g_1 \\ g_2 \\ \vdots \\ g_7 \end{bmatrix} \middle| \; g_i \in C\,(Z_{25}) \right.$$

$$= \{a + bi_F \mid a, b \in Z_{25},\; i_F^2 = 24, *, (20, 6)\}, 1 \leq i \leq 7\}\}$$

be a complex column matrix groupoid of finite order.

Now we can just indicate how complex $m \times n$ matrix groupoids of finite order is defined.

Let

$G = \{A = (m_{ij})$ where A is $m \times n$ matrix groupoid with $m_{ij} \in C(Z_n) = \{a + bi_F \mid a, b \in Z_n,\; i_F^2 = n-1\}, (t, u), *\}$
be a complex $m \times n$ matrix groupoid ($t, u \in Z_n$).

Example 3.2.60: Let

$$P = \left\{ \begin{pmatrix} a_1 & a_2 \\ a_3 & a_4 \end{pmatrix} \middle| \; a_i \in C\,(Z_{40}) \right.$$

$$= \{a + bi_F \mid a, b \in Z_{40},\; i_F^2 = 39, *, (7, 19)\}\}$$

be a complex modulo integer 2×2 groupoid.

We show how operations are carried out on G.

Let x = $\begin{pmatrix} a_1 & a_2 \\ a_3 & a_4 \end{pmatrix}$ and y = $\begin{pmatrix} b_1 & b_2 \\ b_3 & b_4 \end{pmatrix}$ be in G,

$$x*y = \begin{pmatrix} a_1 & a_2 \\ a_3 & a_4 \end{pmatrix} * \begin{pmatrix} b_1 & b_2 \\ b_3 & b_4 \end{pmatrix}$$

$$= \begin{pmatrix} a_1 * b_1 & a_2 * b_2 \\ a_3 * b_3 & a_4 * b_4 \end{pmatrix}$$

$$= \begin{pmatrix} 7a_1 + 19b_1 (\text{mod } 40) & 7a_2 + 19b_2 (\text{mod } 40) \\ 7a_3 + 19b_3 (\text{mod } 40) & 7a_4 + 19b_4 (\text{mod } 40) \end{pmatrix} \text{ is in G.}$$

We can use also usual multiplication only when the matrices are square matrices.

If x = $\begin{pmatrix} a_1 & a_2 \\ a_3 & a_4 \end{pmatrix}$ and y = $\begin{pmatrix} b_1 & b_2 \\ b_3 & b_4 \end{pmatrix}$

$$x*y = \begin{pmatrix} a_1 * a_1 + a_2 * b_3 (\text{mod } n) & (a_1 * b_2 + a_2 + b_4)(\text{mod } n) \\ a_3 * b_1 + a_4 * b_3 (\text{mod } n) & a_3 * b_2 + a_4 * b_4 (\text{mod } n) \end{pmatrix}$$

x*y $\in$ G; this operation will have meaning only when they are square matrices). Thus according to need one can use any type of operation in case of square matrices.

Example 3.2.61: Let

$$G = \left\{ \begin{pmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \\ a_{10} & a_{11} & a_{12} \\ a_{13} & a_{14} & a_{15} \end{pmatrix} \middle| \; a_i \in C(Z_{45}) \right.$$

$= \{a + bi_F \mid a, b \in Z_{45},\ i_F^2 = 44\}; 1 \leq i \leq 15; (3, 17), *\}\}$,
be a complex modulo integer matrix groupoid of finite order.

Example 3.2.62: Let

$$G = \left\{ \begin{pmatrix} a_1 & a_2 & a_3 & a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 & a_{10} & a_{11} & a_{12} \end{pmatrix} \middle| \; a_i \in C(Z_{20}) = \right.$$

$\{a + bi_F; a, b \in Z_{20}, 1 \leq i \leq 12;\ i_F^2 = 19, (13, 0), *\}\}$
be a complex modulo integer matrix groupoid of finite order.

Example 3.2.63: Let

$$G = \left\{ \begin{pmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \\ a_{10} & a_{11} & a_{12} \end{pmatrix} \middle| \; a_i \in C(Z_{23}) = \right.$$

$\{a + bi_F \mid a, b \in Z_{23},\ i_F^2 = 22; (9, 0), *\}\}$
be a complex modulo integer matrix groupoid of finite order.

Example 3.2.64: Let

$$G = \left\{ \begin{pmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \end{pmatrix} \middle| \; a_i \in C(Z_{28}) = \right.$$

$\{a + bi_F \mid a, b \in Z_{28},\ i_F^2 = 27; (11, 11), *\}\}$
be a complex modulo integer matrix groupoid of finite order.

Example 3.2.65: Let

$$G = \left\{ \begin{pmatrix} a_1 & a_2 & a_3 & a_4 \\ a_5 & a_6 & a_7 & a_8 \\ a_9 & a_{10} & a_{11} & a_{12} \\ a_{13} & a_{14} & a_{15} & a_{16} \\ a_{17} & a_{18} & a_{19} & a_{20} \end{pmatrix} \right| a_i \in C(Z_{120}) =$$

$$\{a + bi_F \mid a, b \in Z_{120}, \ i_F^2 = 119; (49, 0), *\}\}$$

be a complex modulo integer matrix groupoid of finite order.

Example 3.2.66: Let

$$G = \left\{ \begin{bmatrix} a_1 & a_2 \\ a_3 & a_4 \\ a_5 & a_6 \\ a_7 & a_8 \\ a_9 & a_{10} \end{bmatrix} \right| a_i \in C(Z_{30}) =$$

$$\{a + bi_F \mid a, b \in Z_{30}, \ i_F^2 = 29; (8, 16), *\}\}$$

be a complex modulo integer matrix groupoid of finite order.

Now having seen examples of them we can now proceed onto define complex modulo integer polynomial groupoids.

Let $G = C(Z_n) = \{a + bi_F \mid a, b \in Z_n, \ i_F^2 = n-1\}$ be the collection of complex modulo integers.

Let $G[x] = \left\{ \sum_{i=0}^{\infty} g_i x_i \right| g_i \in G, *, (t, u)\}$ be the complex polynomial modulo integer groupoid.

Here if p(x) and q(x) are in G[x] then

$p(x) * q(x) = (g_0 + g_1x + \ldots + g_n x^n) * (h_0 + \ldots + h_m x^m)$
$(g_i, h_j \in G, 1 \leq i \leq n, 1 \leq j \leq m).$

$= \quad g_0 * h_0 + g_1 * h_0x + g_0 * h_1 x + g_n * h_m x^{m+n}$
$= \quad (tg_0 + uh_0) + (tg_1 + uh_0)x + (tg_0 + uh_1)x + \ldots +$
$(tg_n+ uh_m)x^{m+n}$
$= \quad (tg_0 + uh_0) + (tg_1 + uh_0 + tg_0 + uh_{12})x + \ldots +$
$(tg_n + uh_m)x^{m+n} \in G[x].$

We give some examples of them.

Example 3.2.67: Let

$G = \left\{ \sum_{i=0}^{\infty} g_i x_i \;\middle|\; g_i \in C(Z_{20}) = \{a + bi_F \mid a, b \in Z_{20}, i_F^2 = 19\} *, (8, 7)\right\}$ be a complex modulo integer polynomial groupoid.

Example 3.2.68: Let

$G[x] = \left\{ \sum_{i=0}^{\infty} g_i x^i \;\middle|\; g_i \in C(Z_4) = \{a + bi_F \mid a, b \in Z_4, i_F^2 = 3\} *, (2, 1)\right\}$ be a complex modulo integer polynomial groupoid.

Let $p(x) = (2+i_F)x^3 + 3x^2 + 1$ and
$q(x) = (3+2i_F)x^7 + (3i_F+1)x^2 + i_F$ be in G [x].

$p(x) * q(x) = ((2+i_F)x^3 + 3x^2 + 2i_Fx + 1) * ((3+2i_F)x^7 + (3i_F+1)x^2 + i_F)$
$= (2+i_F) * (3+2i_F)x^{10} + (2+i_F) * (3i_F + 1)x^5 + (2+i_F) * (i_F)x^3 + 3 * (3+2i_F)x^9 + 3 * (3i_F+1)x^4 + 3* i_F x^2 + 1* (3+2i_F)x^7 + 1 * (3i_F+1)x^2 + 1*i_F$

$= [2 (2+i_F) + (3+2i_F)]x^{10} + (4+2i_F + 3i_F + 1)x^5 + (4+2i_F + i_F)x^3 + (6+3+i_F)x^9 + (6 + 3i_F+1)x^4 + (6+i_F)x^2 + (3+3+2i_F)x^7 + (3+3i_F+1)x^2 + 3+i_F$

$= 3x_{10} + (i_F+1)x^5 + 3i_F x^3 + (1+i_F)x^9 + (3+3i_F) x^4 + (2+i_F) x^2 + (2+2i_F) x^7 + 3i_F x^2 + 3+i_F \in G[x].$

This is the way the product of two polynomials are determined.

Example 3.2.69: Let

$$G[x] = \left\{ \sum_{i=0}^{\infty} g_i x^i \;\middle|\; g_i \in C(Z_{14}) = \{a + bi_F \mid a, b \in Z_{14}, \right.$$

$$\left. i_F^2 = 13\}, 0 \leq i \leq \infty, *, (9, 9) \right\}$$

be a complex modulo integer polynomial groupoid of infinite order. G [x] is a commutative groupoid.

Example 3.2.70: Let

$$G[x] = \left\{ \sum_{i=0}^{\infty} g_i x^i \;\middle|\; g_i \in C(Z_{10}) = \{a + bi_F \mid a, b \in Z_{10}, \right.$$

$$\left. i_F^2 = 9\}, *, (5, 7) \right\}$$

be a complex modulo integer polynomial groupoid.

Example 3.2.71: Let

$$G[x] = \left\{ \sum_{i=0}^{\infty} g_i x^i \;\middle|\; g_i \in C(Z_{19}) = \{a + bi_F \mid a, b \in Z_{19}, \right.$$

$$\left. i_F^2 = 18\}, (11, 8), * \right\}$$

be a complex modulo integer polynomial groupoid.

Example 3.2.72: Let

$$G[x] = \left\{ \sum_{i=0}^{\infty} g_i x^i \;\middle|\; g_i \in C(Z_8) \right.$$

$$\left. = \{a + bi_F \mid a, b \in Z_8, i_F^2 = 7\}, (2, 4), * \right\}$$

be a complex modulo integer polynomial groupoid.

We have seen several types of complex modulo integer polynomial groupoids of infinite order. Finding or defining subgroupoids, ideals, zero divisors, idempotents or S-zero divisors etc. are a matter of routine for these modulo integer complex polynomial groupoids and hence is left as an exercise to the reader.

Likewise study of substructures and special elements in case of complex modulo integer matrix groupoids is also matter of routine and hence is left as an exercise to the reader. However we just give some examples of them.

Example 3.2.73: Let

$$G = \left\{ \begin{bmatrix} a_1 & a_2 \\ a_3 & a_4 \\ a_5 & a_6 \\ a_7 & a_8 \end{bmatrix} \middle| \; a_i \in C(Z_{10}) \right.$$

$$= \{a + bi_F \mid a, b \in Z_{10},\ i_F^2 = 9\};\ 1 \leq i \leq 8, *, (3, 10)\}$$

be a complex modulo integer matrix groupoid.

Take

$$W = \left\{ \begin{bmatrix} a_1 & a_2 \\ 0 & 0 \\ a_3 & a_4 \\ 0 & 0 \end{bmatrix} \middle| \; a_i \in C(Z_{10}) = \{a + bi_F \mid a, b \in Z_{10}, \right.$$

$$i_F^2 = 9\};\ 1 \leq i \leq 4, *, (3, 10)\} \subseteq G$$

is a complex modulo integer matrix subgroupoid of G and is not an ideal of G.

Example 3.2.74: Let

$$G = \left\{ \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_{10} \end{bmatrix} \middle| \; a_i \in C(Z_{19}) \right.$$

$$=\{a + bi_F \mid a, b \in Z_{19},\ i_F^2 = 18\};\ 1 \leq i \leq 10, *, (10, 0)\}$$

be a complex modulo integer column matrix groupoid.
Consider

$$P = \left\{ \begin{bmatrix} a_1 \\ 0 \\ 0 \\ a_2 \\ 0 \\ a_3 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \middle| \; a_i \in C(Z_{19});\ 1 \leq i \leq 3,\ *,\ (10, 0)\right\} \subseteq G;$$

P is only a complex modulo integer column matrix subgroupoid of G; which is not an ideal of G.

Example 3.2.75: Let

$$G = \left\{ \begin{pmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \\ a_{10} & a_{11} & a_{12} \end{pmatrix} \middle| \; a_i \in C(Z_{11}) \right.$$

$= \{a + bi_F \mid a, b \in Z_{11},\ i_F^2 = 10\},\ 1 \leq i \leq 12,\ (9, 9), *\}$
be a complex modulo integer groupoid of finite order.

Take

$$X = \left\{ \begin{pmatrix} 0 & a_1 & 0 \\ a_2 & 0 & a_3 \\ 0 & a_4 & 0 \\ a_5 & 0 & a_6 \end{pmatrix} \middle| \; a_i \in C(Z_{11});\ 1 \leq i \leq 6,\ (9, 9), *\right\} \subseteq G$$

is a complex modulo integer subgroupoid of G.

Example 3.2.76: Let

$$G = \left\{\begin{bmatrix} a_1 & a_2 \\ a_3 & a_4 \end{bmatrix} \middle| \; a_i \in C(Z_2)\right.$$

$= \{a + bi_F \mid a, b \in Z_2,\; i_F^2 = 1\},\ 1 \leq i \leq 4,\ (1, 0), *\}$
be a complex modulo integer groupoid.

Take

$$H = \left\{\begin{bmatrix} a_1 & 0 \\ 0 & a_2 \end{bmatrix} \middle| \; a_i \in C(Z_2);\ 1 \leq i \leq 2,\ (1, 0), *\right\} \subseteq G$$

is a complex modulo integer matrix subgroupoid of G. Clearly H is also only one sided ideal of G.

Clearly H is not a two sided ideal of G.

Inview of this we have the following theorem.

THEOREM 3.2.5: *Let*
*$G = \{A = (m_{ij}) \mid m_{ij} \in C(Z_n) = \{a + bi_F \mid a, b \in Z_n,\ i_F^2 = n{-}1\}$; A a $n \times n$ matrix with $1 \leq i, j \leq n$, *, (t, 0) (or (0, t))\} be a complex modulo integer matrix groupoid.*

Take $P = \{A = (m_{ii}) \mid m_{ii} \in C(Z_n)$, *, (t, 0) (or (0, t))\} $\subseteq$ G; P is only a one sided ideal of G.

We have an immediate corollary.

COROLLARY 3.2.1: *Let*
*$G = \{A = (m_{ij}) \mid m_{ij} \in C(Z_n) = \{a + bi_F \mid a, b \in Z_n, i_F^2 = n{-}1\}$; $1 \leq i, j \leq m\}$ be the complex matrix; *, (t, 0); $t \in Z_n \setminus \{0, 1\}\}$ be the complex modulo integer matrix groupoid.*

*Let $H = \{A = (m_{ii}) \mid m_{ii} \in C(Z_n)$, *, (t, 0)\} $\subseteq$ G be the collection of diagonal $m \times m$ matrices in G. H is a right ideal of G if and only if $P = \{B = (m_{ii}) \mid m_{ii} \in C(Z_n)$, *, (0, t)\} $\subseteq G'$*

$= \{A = (m_{ij}) \mid m_{ij} \in C(Z_n) \;\; 1 \le i, j \le m, *, (0, t)\}$ is a left ideal of G'.

This proof is also direct hence left as an exercise to the reader. However we will illustrate this situation by a simple example.

Example 3.2.77: Let

$$G = \left\{\begin{bmatrix} a_1 & a_2 \\ a_3 & a_4 \end{bmatrix} \middle|\; a_i \in C(Z_{10})\right.$$

$$= \{a + bi_F \mid a, b \in Z_{10},\; i_F^2 = 9\},\; 1 \le i \le 4,\;\; (3, 0), *\}$$

be a complex modulo integer matrix groupoid.

Consider

$$P = \left\{\begin{bmatrix} a_1 & 0 \\ 0 & a_2 \end{bmatrix} \middle|\; a_i \in C(Z_{10}); *, (3, 0), 1 \le i \le 2\right\} \subseteq G$$

be a complex modulo integer matrix subgroupoid.

Consider

$$x = \begin{bmatrix} b_1 & b_2 \\ b_3 & b_4 \end{bmatrix} \in G \text{ and } a = \begin{bmatrix} a_1 & 0 \\ 0 & a_2 \end{bmatrix} \in P;$$

$$\text{find } x * a = \begin{bmatrix} b_1 & b_2 \\ b_3 & b_4 \end{bmatrix} * \begin{bmatrix} a_1 & 0 \\ 0 & a_2 \end{bmatrix}$$

$$= \begin{bmatrix} b_1 * a_1 & b_2 * 0 \\ b_3 * 0 & b_4 * a_2 \end{bmatrix} = \begin{bmatrix} 3b_1 & 3b_2 \\ 3b_3 & 3b_4 \end{bmatrix}.$$

Thus $x * a \notin P$.

Consider a*x = $\begin{bmatrix} a_1 & 0 \\ 0 & a_2 \end{bmatrix} * \begin{bmatrix} b_1 & b_2 \\ b_3 & b_4 \end{bmatrix}$

$$= \begin{bmatrix} a_1 * b_1 & 0 * b_2 \\ 0 * b_3 & a_2 * b_4 \end{bmatrix} = \begin{bmatrix} 3a_1 & 0 \\ 0 & 3a_2 \end{bmatrix} \in P.$$

Thus we see P is not a two sided ideal only a right ideal of the groupoid.

Example 3.2.78: Let

$$G = \left\{ \left. \begin{pmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \end{pmatrix} \right| a_i \in C(Z_{40}); \right.$$

$$\{a + bi_F \mid a, b \in Z_{40},\ i_F^2 = 39\}; (0, 12), *\}$$

be a complex modulo integer matrix groupoid.

$$H = \left\{ \left. \begin{pmatrix} a_1 & 0 & 0 \\ 0 & a_2 & 0 \\ 0 & 0 & a_3 \end{pmatrix} \right| a_i \in C(Z_{40});\ 1 \leq i \leq 3,\ (0, 12), *\right\} \subseteq G;$$

H is only a complex modulo integer matrix subgroupoid and not a two sided ideal of G.

However H is a right ideal of G for take

$$x = \begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \end{bmatrix} \in G \text{ and}$$

$$b = \begin{bmatrix} b_1 & 0 & 0 \\ 0 & b_2 & 0 \\ 0 & 0 & b_3 \end{bmatrix} \text{ in H.}$$

$$\text{Consider } x*b = \begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \end{bmatrix} * \begin{bmatrix} b_1 & 0 & 0 \\ 0 & b_2 & 0 \\ 0 & 0 & b_3 \end{bmatrix}$$

$$= \begin{bmatrix} a_1 * b_1 & a_2 * 0 & a_3 * 0 \\ a_4 * 0 & a_5 * b_2 & a_6 * 0 \\ a_7 * 0 & a_8 * 0 & a_9 * b_3 \end{bmatrix} = \begin{bmatrix} 12b_1 & 0 & 0 \\ 0 & 12b_2 & 0 \\ 0 & 0 & 12b_3 \end{bmatrix} \in H;$$

so H is a left ideal of G.

$$\text{Consider } b * x = \begin{bmatrix} b_1 & 0 & 0 \\ 0 & b_2 & 0 \\ 0 & 0 & b_3 \end{bmatrix} * \begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \end{bmatrix}$$

$$= \begin{bmatrix} b_1 * a_1 & 0 * a_2 & 0 * a_3 \\ 0 * a_4 & b_2 * a_5 & 0 * a_6 \\ 0 * a_7 & 0 * a_8 & b_3 * a_9 \end{bmatrix}$$

$$= \begin{bmatrix} 12a_1 & 0 & 0 \\ 0 & 12a_5 & 0 \\ 0 & 0 & 12a_9 \end{bmatrix} \in H.$$

However it is easily verified H is not an ideal.

Example 3.2.79: Let

$$G = \left\{ \sum_{i=0}^{\infty} a_i x^i \;\middle|\; a_i \in C(Z_{20}) \right.$$

$$= \{a + bi_F \mid a, b \in Z_{20},\ i_F^2 = 1\}, *, (3, 2)\}$$

be a complex modulo integer polynomial groupoid of infinite order.

Take

$$H = \left\{ \sum_{i=0}^{\infty} a_i x^{2i} \;\middle|\; a_i \in C(Z_{20}), *, (3, 2)\right\} \subseteq G;$$

H is a complex modulo integer polynomial subgroupoid of G. Clearly H is not an ideal of G.

Example 3.2.80: Let

$$G = \left\{ \sum_{i=0}^{\infty} a_i x^{i} \;\middle|\; a_i \in C(Z_{40})\right.$$

$$= \{a + bi_F \mid a, b \in Z_{40},\ i_F^2 = 39\}, *, (5, 0)\}$$

be a complex modulo integer polynomial groupoid of infinite order.

Consider

$$H = \left\{ \sum_{i=0}^{\infty} a_i x^{3i} \;\middle|\; a_i \in C(Z_{40}), *, (5, 0)\right\} \subseteq G;$$

H is a complex modulo integer subgroupoid of infinite order of G. Clearly H is not an ideal of G.

For if $x = 5x^2 + (3+i_F)\, x + 7i_F \in G$

and $y \quad = (i_F + 3)\, x^3 + (7+i_F) \in H$

$$\begin{aligned} x*y &= (5x^2 + (3+i_F)x + 7i_F) * ((i_F+3)x^3 + 7+i_F) \\ &= 5\,(i_F+3)x^3 + 5\,(3+i_F)x = 5.7i_F) \\ &\quad + 0\,((i_F+3)x^3 + 7+i_F) \\ &= (5i_F+15)x^3 + (15 + 5i_F)x + 35i_F \in H. \end{aligned}$$

Now H is a right ideal of G and is not a left ideal of G.

We will give a related theorem.

THEOREM 3.2.6: *Let*

$$G = \left\{ \sum_{i=0}^{\infty} a_i x^{i} \;\middle|\; a_i \in C(Z_n)\right.$$

$$= \{a + bi_F \mid a, b \in Z_n,\ i_F^2 = n-1\}, *, (p, 0), p \in Z_n \setminus \{0\}\}$$

be a complex modulo integer polynomial groupoid.

Take

$$W = \left\{ \sum_{i=0}^{\infty} a_i x^{si} \;\middle|\; a_i \in C(Z_n),\ *,\ (p,\ 0),\ s \text{ a positive integer} \right\}$$

to be a complex modulo integer subgroupoid of G. Clearly W is not an ideal; however W is not a left ideal but W is only a right ideal.

Proof is obvious from the very definition and direct and hence left for the reader as an exercise.

THEOREM 3.2.7: *Let*

$$G = \left\{ \sum_{i=0}^{\infty} a_i x^i \;\middle|\; a_i \in C(Z_n) \right.$$

$$= \{a + bi_F \mid a, b \in Z_n,\ i_F^2 = n-1\},\ *,\ (0,\ t),\ t \in Z_n \setminus \{0\}\}$$

be a complex modulo integer polynomial groupoid.

Take

$$H = \left\{ \sum_{i=0}^{\infty} a_i x^{mi} \;\middle|\; a_i \in C(Z_n),\ *,\ (0,\ t),\ m \text{ a positive integer} \right\} \subseteq G;$$

H is a left ideal of G.

This proof is also simple and hence left as an exercise to the reader.

We see it is very difficult to find ideals of these complex modulo integer polynomial groupoids.

Now having seen these structure we see all these structures we can easily find zero divisors, idempotents etc in these complex groupoids.

We will give one or two examples.

Example 3.2.81: Let

$$G = \left\{ \sum_{i=0}^{\infty} a_i x^i \,\middle|\, a_i \in C(Z_{12}) \right.$$

$$= \{a + bi_F \mid a, b \in Z_{12},\ i_F^2 = 11\}, *, (3, 0)\}$$

be a complex modulo integer polynomial groupoid.

Take

$$p(x) = (4 + 8i_F)x^7 + (8+4i_F)x^3 + 4i_F \text{ and}$$

$$q(x) = 3i_F\, x^9 + (10+5i_F)x^8 + 3i_F\, x^4 + i_F x + (2+5i_F) \text{ in } G.$$

We see $p(x) * q(x) = 0$ is a zero divisor in G. We can have zero divisors in polynomial groupoids.

Example 3.2.82: Let

$$G = \left\{ \sum_{i=0}^{\infty} a_i x^i \,\middle|\, a_i \in C(Z_7) \right.$$

$$= \{a + bi_F \mid a, b \in Z_7,\ i_F^2 = 6\}, *, (3, 4)\}$$

be a complex polynomial groupoid.

Consider

$p(x) = x^3 + x + 1$ in G then

$$p(x) * p(x) = (x^3 + x + 1) * (x^3 + x + 1) = 3x^3 + 3x + 3 + 4x^3 + 4x + 1 = 0.$$

Thus this groupoid has nilpotents elements also.

In view of this we have the following theorem.

THEOREM 3.2.8: *Let*

$$G = \left\{ \sum_{i=0}^{\infty} a_i x^i \,\middle|\, a_i \in C(Z_p) = \{a + bi_F \mid a, b \in Z_p,\ i_F^2 = p-1\}, \right.$$

*p a prime, *, (t, u) such that $t + u \equiv p = 0 \pmod p$} be a complex modulo integer polynomial groupoid.*

Every polynomial $p(x) = \sum_{i=0}^{n} x^i$ *in G is such that*

$p(x) * p(x) = 0.$

This proof is also simplex and exploits only easy number theoretic techniques.

Example 3.2.83: Let

$$G = \left\{ \sum_{i=0}^{\infty} a_i x^i \;\middle|\; a_i \in C(Z_n) = \{a + bi_F \mid a, b \in Z_n,\; i_F^2 = n-1\}, \right.$$
$$\left. *, (t, u) \text{ such that } t + u \equiv 0 \pmod{n} \right\}$$

be a complex modulo integer polynomial groupoid.

G has non trivial nilponents for take $p(x) = \sum_{i=0}^{\infty} x^i$ in G and $p(x) * p(x) = 0$.

Example 3.2.84: Let

$$G = \left\{ \sum_{i=0}^{\infty} a_i x^i \;\middle|\; a_i \in C(Z_9) \right.$$
$$\left. = \{a + bi_F \mid a, b \in Z_n,\; i_F^2 = n-1\}, *, (6, 4) \right\}$$

be a complex modulo integer polynomial groupoid.

$$p(x) = \sum_{i=0}^{\infty} x^i \in G; \text{ is such that } p(x) * p(x) = p(x).$$

For take

$p(x) = x^5 + x^3 + x^2 + x + 1$ in G

$$\begin{aligned} p(x) * p(x) &= 6(x^5 + x^3 + x^2 + x + 1) + 4(x^5 + x^3 + x^2 + x + 1) \\ &= x^5 + x^3 + x^2 + x + 1 \\ &= p(x). \end{aligned}$$

In view of this we have the following nice theorem which guarantees idempotents in complex modulo integer polynomial groupoids.

THEOREM 3.2.9: *Let*

$$G = \left\{ \sum_{i=0}^{\infty} a_i x^i \;\middle|\; a_i \in C(Z_n) = \{a + bi_F \mid a, b \in Z_n,\ i_F^2 = n-1\}, *,\right.$$

(t, u) such that t + u ≡ 1 (mod n)} be a complex modulo integer polynomial groupoid.

Every polynomial $p(x) = \sum_{i=0}^{n} x^i$ (for every positive integer) in G is an idempotent in G.

Thus we have seen in case of complex modulo integer polynomial groupoids also G has zero divisors, nilponents and idempotents.

Now we can use these complex matrix modulo integer groupoids to build complex groupoids ring. The definition is a matter of routine and hence we give examples of them.

Example 3.2.85: Let

$$G = \{(a_1, a_2, a_3) \mid a_i \in C(Z_{14}) = \{a + bi_F \mid a, b \in Z_{14},\ i_F^2 = 13, 1 \leq i \leq 3; (9, 8), *\}$$

be the complex modulo integer row matrix groupoid.

Z_{10} be the ring. $Z_{10}G$ is the groupoid complex ring. Clearly G is non commutative non associative and is of finite order. G has subrings.

Example 3.2.86: Let

$$G = \{(a_1, a_2, \ldots, a_{10}) \mid a_i \in C(Z_5) = \{a + bi_F \mid a, b \in Z_5,\ i_F^2 = 4, 1 \leq i \leq 10; (3, 0), *\}$$

be the complex modulo integer row matrix groupoid.

F = Z be the ring. FG is the groupoid ring. FG is a non associative non commutative infinite ring having zero divisors and subrings.

Example 3.2.87: Let

$$G = \{(a_1, a_2, a_3, \ldots, a_{20}) \mid a_i \in C(Z_{40}) = \{a + bi_F \mid a, b \in Z_{40},$$
$$i_F^2 = 39, \;\; 1 \leq i \leq 20; (9, 9), *\}$$

be a groupoid.

F = Z_2 be the field of characteristics two. FG be the groupoid ring. FG is a commutative complex non associative ring of finite order.

Inview of this we have the following theorem the proof of which is simple.

THEOREM 3.2.10: *Let*

$$G = \{(a_1, \ldots, a_m) \mid a_i \in C(Z_n) = \{a + bi_F \mid a, b \in Z_n,$$
$$i_F^2 = n{-}1\}, \; 1 \leq i \leq m, \;\; (t, t), t \in Z_n \setminus \{0, 1\}, *\}$$

be a groupoid. R any commutative ring. RG is a non associative commutative complex groupoid ring, finite or infinite depending on R.

Example 3.2.88: Let

$$G = \{(a_1, a_2, a_3, \ldots, a_{10}) \mid a_i \in C(Z_{12}) = \{a + bi_F \mid a, b \in Z_{12},$$
$$i_F^2 = 11\}, \;\; 1 \leq i \leq 10; (8, 0), *\}$$

be a complex modulo integer row matrix groupoid of finite order.

Let F = Z_3 be the field of characteristic three FG is the groupoid ring of finite order has right ideals which are not left ideals. If (8, 0) is replaced by (0, 8) then if G′ be that groupoid ring. FG′ has left ideals that are not right ideals.

Inview of this we have the following theorem.

THEOREM 3.2.11: *Let*

$$G = \{(a_1, \ldots, a_m) \mid a_i \in C(Z_n) = \{a + bi_F \mid a, b \in Z_n,$$
$$i_F^2 = n{-}1\}, \;\; 1 \leq i \leq m, \;\; (t, 0), t \in Z_n \setminus \{0, 1\}\} \;\; and$$

$G' = \{(a_1, \ldots, a_m) \mid a_i \in C(Z_n) = \{a + bi_F \mid a, b \in Z_n, i_F^2 = n-1\}, 1 \leq i \leq n, (0, t), *, t \in Z_n \setminus \{0, 1\}\}$ a complex modulo integer groupoid. F be a field or a commutative ring. FG and FG′ be complex modulo integer groupoid rings. If H is a left ideal of FG then H′ (with H′ the same subset as that of H) is a right ideal of FG′.

Proof is simple and direct hence left as an exercise to the reader. Now these theorem are true in general when the row matrix complex groupoid is replaced by column matrix groupoid or any m × n complex matrix groupoid. We will illustrate these situations by some simple examples.

Example 3.2.89: Let

$$G = \left\{ \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_8 \end{bmatrix} \middle| \, a_i \in C(Z_7) = \{a + bi_F \mid a, b \in Z_7, \ i_F^2 = 6\}; \right.$$

$$\left. 1 \leq i \leq 8, \ *, (3, 4) \right\}$$

be a complex modulo integer groupoid of column matrices. F = Z_7 be the field of characteristic seven. FG is a complex modulo integer groupoid ring. FG is non commutative, non associative and is of finite order.

Example 3.2.90: Let

$$G = \left\{ \begin{bmatrix} a_1 & a_2 \\ a_3 & a_4 \\ \vdots & \vdots \\ a_{21} & a_{22} \end{bmatrix} \middle| \, a_i \in C(Z_{25}) \right.$$

$$\left. = \{a + bi_F \mid a, b \in Z_{25}, \ i_F^2 = 24\}; \ 1 \leq i \leq 22, \ *, (3, 3) \right\}$$

be a complex modulo integer matrix groupoid. Let F = Q be the rational field. FG be the complex groupoid ring. FG is a commutative complex groupoid ring of infinite order.

Example 3.2.91: Let

$$G = \left\{ \begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ \vdots & \vdots & \vdots \\ a_{22} & a_{23} & a_{24} \end{bmatrix} \middle| \; a_i \in C(Z_{198}) = \{a + bi_F \mid a, b \in Z_{198}, \right.$$

$$\left. i_F^2 = 197\}; 1 \leq i \leq 24, \; *, (13, 0) \right\}$$

be a complex modulo integer matrix groupoid. Let F = Z_{20} be the ring. FG is a complex groupoid ring which is finite but non commutative and non associative and has right ideals which are not left ideals.

Example 3.2.92: Let

$$G = \left\{ \begin{bmatrix} a_1 & a_2 \\ a_3 & a_4 \end{bmatrix} \middle| \; a_i \in C(Z_3) \right.$$

$$\left. = \{a + bi_F \mid a, b \in Z_3, \; i_F^2 = 2\}; 1 \leq i \leq 4, \; *, (2, 0) \right\}$$

be the complex modulo integer matrix groupoid. F = Z_3 be the finite field. FG is a non commutative, non associative complex ring of finite order. FG has left ideals which are not right ideals. Several results about these rings can be derived as in case if any non associative rings.

Example 3.2.93: Let

G = {all 7 × 7 upper triangular matrices with entries from $C(Z_{42}) = \{a + bi_F \mid a, b \in Z_{42}, \; i_F^2 = 41\}$; *, (21, 21)} be a complex modulo integer matrix groupoid of finite order. Let F = Z_{21} be the ring. FG is a complex commutative, non associative groupoid of finite order. G has subgroupoids which are not ideals.

Example 3.2.94: Let

G = {all 10×10 lower triangular matrices with entries from $C(Z_{23})$ = {a + bi_F | a, b $\in Z_{23}$, $i_F^2 = 22$}; *, (7, 15)} be the complex modulo integer groupoid.

F = Z_{23} be the field. FG is the complex groupoid ring which is both non commutative and non associative of finite order.

Example 3.2.95: Let

G = {all 8×8 matrices with entries from
$C(Z_5)$ = {a + bi_F | a, b $\in Z_5$, $i_F^2 = 4$}; *, (2, 4)}

be the complex modulo integer matrix groupoid. F = R be the field of reals. FG is a non commutative non associative ring of infinite order.

Example 3.2.96: Let

G = {all 5×5 matrices with entries from C(Z), *, (3, –7)} be a complex integer matrix groupoid. F = Z_2 be the finite field. FG is the groupoid ring which is non associative and non commutative of infinite order.

Example 3.2.97: Let

P = {all 7×10 matrices with entries from C, *, (12, 3)} be a complex matrix groupoid. F = Z be the ring. FP is an infinite non commutative non associative complex ring.

Example 3.2.98: Let

G = {all 3×11 matrices with entries from
C(Q) = a+ib, a, b $\in$ Q, $i^2 = -1$; (7, 7), *}

be a complex matrix groupoid. F = Z_3 be the finite field. FG be the complex groupoid ring. FG is non associative but commutative ring of infinite order.

Example 3.2.99: Let

G = {all 10×8 matrices with entries from

C(Z) = a+ib with a, b $\in$ Z, $i^2 = -1$, (8, 0), *} be the complex integer matrix groupoid. F = Z_4 be the ring. FG be the complex groupoid ring FG is non commutative non associative complex ring of infinite order. FG has infinite number of right ideals which are not left ideals. FG also has subrings which are not ideals.

Example 3.2.100: Let

G = {all 4×4 matrices with entries from C(Z) = a+ib with a, b $\in$ Z, $i^2 = -1$, (0, 12), *} be the complex matrix groupoid ring. F = Z_5 be the finite field FG be the complex matrix groupoid ring. FG has left ideals which are not right ideals. We can also define the notion of complex groupoid semiring for which we give some examples.

Example 3.2.101: Let

G = {all 2×3 matrices with entries from C(Q) = a+ib | a, b $\in$ Q, $i^2 = -1$, *, (12, −10)} be a complex rational groupoid. F = $Z^+ \cup \{0\}$ be the finite field. FG be the groupoid semiring which is a complex non associative, non commutative infinite semiring.

Example 3.2.102: Let

$$G = \left\{ \begin{bmatrix} a_1 & a_2 \\ a_3 & a_4 \end{bmatrix} \middle| \; a_i \in C(Z_9); 1 \le i \le 4, *, (3, 7) \right\}$$

be a complex modulo integer matrix groupoid. F = $Q^+ \cup \{0\}$ be the semifield. FG be the groupoid semiring. FG is a non associative non commutative complex semiring of infinite order.

Example 3.2.103: Let

G = {($a_1, a_2, \ldots, a_9$) where $a_i \in C(Z_{15})$; {$a + bi_F$ | a, b $\in Z_{15}$, *, $i_F^2 = 14$};$1 \leq i \leq 9$, *, (3, 3)} be a complex modulo integer groupoid. F = $Z^+ \cup \{0\}$ be the semifield. FG be the complex modulo integer groupoid semiring of infinite order.

Example 3.2.104: Let

G = {all 10 × 10 matrices with entries from $C(Z_7)$ = {$a + bi_F$ | a, b $\in Z_7$, *, $i_F^2 = 6$}, *, (4, 0)} be the complex modulo integer matrix groupoid. F = $R^+ \cup \{0\}$ be the semifield. FG be the complex groupoid ring. FG is a non associative non commutative ring which has right ideals which are not left ideals.

Example 3.2.105: Let

$$G = \left\{ \left. \begin{pmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \\ a_{10} & a_{11} & a_{12} \end{pmatrix} \right| a_i \in C(Z_{12}) = \right.$$

$$\{a + bi_F \mid a, b \in Z_{12},\ i_F^2 = 11\},\ 1 \leq i \leq 12,\ *,\ (0, 11)\}$$

be the complex modulo integer matrix groupoid. F = $Z^+ \cup \{0\}$ be the semifield FG be the groupoid ring. FG is a complex non commutative non associative semiring of infinite order having left ideals which are not right ideals.

Example 3.2.106: Let

$$G = \left\{ \left. \begin{bmatrix} a_1 & a_2 & \ldots & a_{15} \\ a_{16} & a_{17} & \ldots & a_{30} \\ a_{31} & a_{32} & \ldots & a_{45} \end{bmatrix} \right| a_i \in C(Z_8) = \{a + bi_F \mid a, b \in Z_8, \right.$$

$$i_F^2 = 7\},\ 1 \leq i \leq 45,\ *,\ (3, 3)\}$$

be the complex modulo integer matrix groupoid. $F = Z^+ \cup \{0\}$ be the semifield. FG be the groupoid ring. FG is a non associative, commutative complex ring of infinite order.

Example 3.2.107: Let

G = {all 10×10 matrices with entries from $C(Z_7)$; $\{a + bi_F \mid a, b \in Z_7, i_F^2 = 6\}$, *, (3, 3)} be the complex modulo integer groupoid. $F = C_5 = \{0 < a_1 < a_2 < a_3 < 1\}$ be the chain lattice which is a semiring. FG is a groupoid semiring of finite order.

This method gives us infinite number finite semirings which are non associative. Several result in this direction can be derived with no difficulty and is left for the reader as exercise. For more about semirings please refer [18].

Chapter Four

STRONG COMPLEX STRUCTURES

In this chapter we define groupoids and quasi loops in which we use complex modulo integers on which operation are performed using complex numbers. Also in this chapter we define complex modulo integer groupoids over complex ring which we call as strong complex rings. Also complex modulo loop over complex rings which are strong complex loop rings.

DEFINITION 4.1: *Let*

$S(C(G)) = \{a + bi_F \mid a, b \in Z_n,\ i_F^2 = n-1, (mi_F, ti_F); m, t \in Z_n, *\}$ be the set with $*$, a closed binary operation on $S(C(G))$ such that for any $x, y \in SC(G)$;

$x * y = xmi_F + yti_F \pmod n$. $S(C(G))$ is defined as the strong complex modulo integer groupoid of type I if $(mi_F, ti_F) = 1$.

We can denote $S(C(G))$ by $S(C(Z_n))$ also. Mostly we use $S(C(Z_n))$ as it gives the specific Z_n used.

Example 4.1: Let

$$S(C(Z_8)) = \{a + bi_F \mid a, b \in Z_8,\ i_F^2 = 7, *, (3i_F, 4i_F)\}$$

be the strong complex modulo integer groupoid of type I.

Example 4.2: Let

$$S(C(Z_7)) = \{a + i_F b \mid a, b \in Z_7,\ i_F^2 = 6, *, (3i_F, 2i_F)\}$$

be the strong complex modulo integer groupoid.

Take $x = 3 + 5i_F$ and $y = 2+i_F$ in $S(Z_7)$.

$$\begin{aligned} x*y &= (3+5i_F) * (2+i_F) \\ &= [3i_F (3+5i_F) + (2+i_F)\, 2i_F] \pmod 7 \\ &= [(9i_F + 15 i_F^2) + (4i_F + 2 i_F^2)] \pmod 7 \\ &= [(2i_F+6) + (4i_F+5)] \pmod 7 \\ &= 6i_F + 4 \text{ is in } S(C(Z_7)). \end{aligned}$$

We see $S(C(Z_7))$ is a strong complex a modulo integer groupoid of type I.

Example 4.3: Let

$$S(C(Z_{48})) = \{a + bi_F \mid a, b \in Z_8, *,\ i_F^2 = 47\}$$

be the strong complex modulo integer groupoid of type I.

Example 4.4: Let

$$S(C(Z_{91})) = \{a + bi_F \mid a, b \in Z_{91}, *,\ i_F^2 = 90\}$$

be the strong complex modulo integer groupoid of type I.

Example 4.5: Let

$$S(C(Z_{27})) = \{a + bi_F \mid a, b \in Z_{27}, *,\ i_F^2 = 26\}$$

be the strong complex modulo integer groupoid of type I.

Example 4.6: Let

$$S(C(Z_{120})) = \{a + bi_F \mid a, b \in Z_{120}, *,\ i_F^2 = 119\}$$

be the strong complex modulo integer groupoid of type I.

DEFINITION 4.2: *Let*

$$S(C(Z_n)) = \{a + bi_F \mid a, b \in Z_n, *,\ i_F^2 = n-1, (mi_F, ti_F)\}$$

be the strong complex modulo integer groupoid of type II if $(mi_F, ti_F) \neq 1$.

We give examples of them.

Example 4.7: Let

$$S(C(Z_{48})) = \{a + bi_F \mid a, b \in Z_{48}, *, \; i_F^2 = 47, (3i_F, 9i_F)\}$$

be the strong complex modulo integer groupoid of type II.

Example 4.8: Let

$$S(C(Z_{47})) = \{a + bi_F \mid a, b \in Z_{47}, *, \; i_F^2 = 46, (9i_F, 36i_F)\}$$

be the strong complex modulo integer groupoid of type II.

Example 4.9: Let

$$S(C(Z_{12})) = \{a + bi_F \mid a, b \in Z_{12}, *, \; i_F^2 = 11, (2i_F, 4i_F)\}$$

be the strong complex modulo integer groupoid of type II.

Example 4.10: Let

$$S(C(Z_{45})) = \{a + bi_F \mid a, b \in Z_{45}, *, \; i_F^2 = 44, (8i_F, 24i_F)\}$$

be the strong complex modulo integer groupoid of type II.

Now we proceed onto define strong complex modulo integer groupoid of type III.

DEFINITION 4.3: *Let*

$$G = S(C(Z_n)) = \{a + i_F b \mid a, b \in Z_n, \; i_F^2 = n-1, (mi_F, mi_F)\}$$

be a strong complex modulo integer groupoid of type III.

We call $S(C(Z_n))$ to be the type III groupoid if $mi_F = ti_F \in C(Z_n) \setminus \{0, 1\}$.

We will give some examples of them.

Example 4.11: Let

$$G = S(C(Z_{91})) = \{a + bi_F \mid a, b \in Z_{91}, \; i_F^2 = 90; (10i_F, 10i_F), *\}$$

be the strong complex modulo integer groupoid of type III.

Example 4.12: Let

$$G = S(C(Z_{49})) = \{a + bi_F \mid a, b \in Z_{49},\ i_F^2 = 48; (2i_F, 2i_F), *\}$$

be the strong complex modulo integer groupoid of type III.

Example 4.13: Let

$$G = S(C(Z_{25})) = \{a + bi_F \mid a, b \in Z_{25},\ i_F^2 = 24; (12i_F, 12i_F), *\}$$

be the strong complex modulo integer groupoid of type III.

Example 4.14: Let

$$G = S(C(Z_{27})) = \{a + bi_F \mid a, b \in Z_{27},\ i_F^2 = 26; (7i_F, 7i_F), *\}$$

be the strong complex modulo integer groupoid of type III.

Example 4.15: Let

$$G = S(C(Z_{29})) = \{a + bi_F \mid a, b \in Z_{29},\ i_F^2 = 28; (19i_F, 19i_F), *\}$$

be the strong complex modulo integer groupoid of type III.

Example 4.16: Let

$$G = S(C(Z_{12})) = \{a + bi_F \mid a, b \in Z_{12},\ i_F^2 = 11; (3i_F, 3i_F), *\}$$

be the strong complex modulo integer groupoid of type III.

Now we proceed onto define type IV strong complex modulo integer groupoid.

DEFINITION 4.4: *Let*

$$G = S(C(Z_n)) = \{a + bi_F \mid a, b \in Z_n,\ i_F^2 = n-1, (mi_F, 0)\}$$

be the groupoid. G is a strong complex modulo integer groupoid of type IV.

We give examples of them.

Example 4.17: Let

$$G = S(C(Z_{40})) = \{a + bi_F \mid a, b \in Z_{40},\ i_F^2 = 39; (3i_F, 0), *\}$$

be the strong complex modulo integer groupoid of type IV.

Example 4.18: Let

$$G = S(C(Z_{28})) = \{a + bi_F \mid a, b \in Z_{28},\ i_F^2 = 27; (8i_F, 0), *\}$$

be a strong complex modulo integer groupoid of type IV.

Example 4.19: Let

$$G = S(C(Z_{20})) = \{a + bi_F \mid a, b \in Z_{20},\ i_F^2 = 19; (0, 7i_F), *\}$$

be a strong complex modulo integer groupoid of type IV.

Example 4.20: Let

$$G = S(C(Z_{120})) = \{a + bi_F \mid a, b \in Z_{120},\ i_F^2 = 119; (8i_F, 0), *\}$$

be a strong complex modulo integer groupoid of type IV.

Now we proceed onto define type IV strong complex modulo integer groupoid of type V.

DEFINITION 4.5: *Let*

$$G = S(C(Z_n)) = \{a + bi_F \mid a, b \in Z_n,\ i_F^2 = n-1, (mi_F, t) \text{ where } (m, t) = 1; *\}$$

be a groupoid. G is defined as the strong complex modulo integer groupoid of type V.

We will give examples of such situations.

Example 4.21: Let

$$G = S(C(Z_{40})) = \{a + bi_F \mid a, b \in Z_{40},\ i_F^2 = 39; (8i_F, 7), *\}$$

be the strong complex modulo integer groupoid of type V.

Example 4.22: Let

$$G = S(C(Z_{42})) = \{a + bi_F \mid a, b \in Z_{42},\ i_F^2 = 41; (3, 13i_F), *\}$$

be a strong complex modulo integer groupoid of type V.

Example 4.23: Let

$$G = S(C(Z_{31})) = \{a + bi_F \mid a, b \in Z_{31},\ i_F^2 = 30; (29i_F, 2), *\}$$

be the strong complex modulo integer groupoid of type V.

Example 4.24: Let

$G = S(C(Z_{148})) = \{a + bi_F \mid a, b \in Z_{148}, i_F^2 = 147; (28i_F, 15), *\}$

be a strong complex modulo integer groupoid of type V.

Example 4.25: Let

$G = S(C(Z_{280})) = \{a + bi_F \mid a, b \in Z_{280}, i_F^2 = 279; (9i_F, 10), *\}$

be a strong complex modulo integer groupoid of type V.

If in the type V strong complex modulo integer groupoids $(mi_F, t) \neq 1$ then we define those groupoids to be type VI groupoids.

We give only examples of type VI groupoids.

Example 4.26: Let

$G = S(C(Z_{25})) = \{a + bi_F \mid a, b \in Z_{25}, *, i_F^2 = 24, (9i_F, 18)\}$

be a strong complex modulo integer groupoid of type VI.

Example 4.27: Let

$G = S(C(Z_{11})) = \{a + bi_F \mid a, b \in Z_{11}, *, i_F^2 = 10, (10i_F, 5)\}$

be the strong complex modulo integer groupoid of type VI.

Example 4.28: Let

$G = S(C(Z_{29})) = \{a + bi_F \mid a, b \in Z_{29}, *, i_F^2 = 28, (12i_F, 3)\}$

be the strong complex modulo integer groupoid of type VI.

Example 4.29: Let

$G = S(C(Z_{45})) = \{a + bi_F \mid a, b \in Z_{45}, *, i_F^2 = 44, (5i_F, 15)\}$

be the strong complex modulo integer groupoid of type VI.

Example 4.30: Let

$G = S(C(Z_{40})) = \{a + bi_F \mid a, b \in Z_{40}, i_F^2 =39, (10i_F, 30), *\}$

be the complex strong modulo integer groupoid of type VI.

Example 4.31: Let

$G = S(C(Z_{13})) = \{a + bi_F \mid a, b \in Z_{13}, *, i_F^2 = 12, (8i_F, 4)\}$

be the strong complex modulo integer groupoid of type VI.

We see we need not in general mention to which type the groupoid belongs to for by the pair of elements used as the operation on G one can easily find the type of the groupoid. All these six type of groupoids are of finite order.

We can now define infinite groupoids of these six types.

Let

$$G = S\ (C(Z))\ (\text{or } S\ (C(R))\ \text{or}$$

$S\ C(Q)) = \{a + ib \mid a, b \in Z, i^2 = -1, *, (mi, ni)\}$ be a strong complex groupoid of infinite order.

If (mi, ni) = 1 then we call G the type I groupoid.
If (mi, ni) = d ≠ 1 then we call G the type II groupoid.
If in (mi, ni); m = n we call G the type III groupoid.
If in (mi, ni) one of m or n is zero then we call G to be type IV groupoid.

If we replace (mi_F, n), $n \in Z$ (or Q or R) with $(mi_F, n) = 1$ then we say G is a type V groupoid.

If $(mi_F, n) = d \neq 0$ then we call those groupoids as type VI groupoid. We give two examples of each type of groupoids of infinite order.

Example 4.32: Let

$$G = S(C(Z)) = \{a + bi \mid a, b \in Z,\ i^2 = -1, (8i, 27i), *\}$$

be a strong complex type I groupoid of infinite order.

Example 4.33: Let

$$G = S(C(Z)) = \{a + bi \mid a, b \in Z,\ i^2 = -1, (9i, 25i), *\}$$

be a strong complex modulo integer groupoid of infinite order of type I.

If x = 3+4i and y = 2–7i then
$x*y = (3+4i) * (2-7i) = 9i\ (3+4i) + (2-7i)\ 25i$

$= 27i - 36 + 50i + 175 = 77i + 139$ is in G.

This is the way operation of G is performed.

If $x = 2$ and $y = -11$

then $x*y = 9i\,(2) + (25i)11$
$= 18i + 275i = 293$ I is in G.

Thus we see for every real x, y in G we get their product x*y to be a complex number.

Take $x = -5i$ and $y = 3i$ in G, now

$x*y = -5i * 3i = 9i\,(-5i) + 3i \times 25i = 45 - 75 = -30$ is in G. Thus the product of two complex numbers under * is a real number.

Let $x = -2i$ and $y = 7$ be two elements in G.

$x*y = 9i\,(-2i) + 25i \times 7 = +18 + 175i$ is a mixed complex number in G.

Example 4.34: Let

$$S\,(C\,(Q)) = G = \{a + bi \mid a, b \in Q, i^2 = -1, (9i, -18i), *\}$$

be the strong complex groupoid of infinite order of type II.

Example 4.35: Let

$$S(C(R)) = S\,(C) = \{a + bi \mid a, b \in R, i^2 = -1, (5\sqrt{3}\,i, 125\sqrt{2}\,i)\}$$

be the strong complex groupoid of type II of infinite order.

Example 4.36: Let

$$S(C(Q)) = \{a + bi \mid a, b \in Q, i^2 = -1, (13/9i, 65/12i), *\}$$

be the strong complex groupoid of infinite order of type II.

Example 4.37: Let

$$S(C(Z)) = \{a + bi \mid a, b \in Z, *, (6i, 6i)\}$$

be the strong complex groupoid of type III of infinite order.

Example 4.38: Let

$$S(C(Q)) = \{a + bi \mid a, b \in Q, *, (8/27i, 8/27i)\}$$

be the strong complex groupoid of type III of infinite order.

Example 4.39: Let

$$G = S(C) = S(C(R)) = \{a + bi \mid a, b \in R, i^2 = -1, \left(\frac{\sqrt{29}i}{7}, \frac{\sqrt{29}i}{7}\right), *\}$$

be a strong complex groupoid of type III of infinite order.

Example 4.40: Let

$$G = S\,(C(Z)) = \{a + bi \mid a, b \in Z, i^2 = -1, (7i, 7i), *\}$$

be the strong complex groupoid of infinite order of type III.

Clearly all type III groupoids are commutative but non associative.

For take x = 3, y = –2i, and z = 1+i in G in example 4.40

$$\begin{aligned} x*y = 3*2i &= 7i3 + (-2i)\,7i \\ &= 21i + (-14)i^2 \\ &= 21i + 14 \qquad \text{(i)} \end{aligned}$$

$$\begin{aligned} y*x = -2i*3 &= -2i \times 7i + 3.7i \\ &= -14i^2 + 21i \\ &= +14 + 21i \qquad \text{(ii)} \end{aligned}$$

I and II are the same so x*y = y*x

$$\begin{aligned} \text{Now } (x*y)*z &= (3*-2i) * (1+i) \\ &= (3 \times 7i = 7i \times -2i)\ \ (1+i) \\ &= (21i + 14) * (1+i) \\ &= 7i\,(21i + 14) + (1+i)\,7i \\ &= (-147 + 98i) + 7i - 7 \\ &= -154 + 105\,i \qquad \text{(i)} \end{aligned}$$

$$
\begin{aligned}
x*(y*z) &= 3*(-2i*(1+i)) \\
&= 3*(-2i \times 7i + 7i\,(1+i)) \\
&= 3*(14 + 7i - 7) \\
&= 3*(7 + 7i) \\
&= 7i \times 3 + (7 + 7i)\,7i \\
&= 21i + 49i - 49 \qquad \text{(ii)}
\end{aligned}
$$

Clearly (i) and (ii) are not equal so in general the * operation on G is non associative.

Example 4.41: Let

$G = S(C(Q)) = \{a + bi \mid a, b \in Q, i^2 = -1, (9/7i, 9/7i), *\}$

be a strong complex groupoid of infinite order of type III.

Now we proceed onto give examples of type IV groupoid.

Example 4.42: Let

$G = S(C(Z)) = \{a + bi \mid a, b \in Z, i^2 = -1, (8i, 0), *\}$

be a strong complex groupoid of infinite order of type IV.

Example 4.43: Let

$G = S(C(R)) = \{a + bi \mid a, b \in R, i^2 = -1, (9i, 0)\}$

be the strong complex groupoid of infinite order of type IV.

Example 4.44: Let

$G = S(C(Z)) = \{a + bi \mid a, b \in Z, i^2 = -1, (0, 4i)\}$

be the strong complex groupoid of infinite order.

Take $x = 3-4i$ and $y = -2+5i$.

$$
\begin{aligned}
x*y &= (3-4i)*(-2+5i) \\
&= (3-4i)\,0 + (-2+5i)\,4i \\
&= 0 + (-8i - 20) \quad (\because\ i^2 = -1) \\
&= -20 - 8i \ \text{ is in G.}
\end{aligned}
$$

Now we proceed onto give examples of strong complex groupoids of type V.

Example 4.45: Let

$$G = S\,(C\,(Z)) = \{a + bi \mid a, b \in Z, i^2 = -1, (9i, 7), *\}$$

be the strong complex groupoid of type V. Consider x = 17 and y = –2 in G.

$$\begin{aligned} x * y &= 17 * (-2) \\ &= 17 \times 9i + -2 \times 7 \\ &= 153\,i - 14 \\ &= -14 + 153i \text{ is in } G. \end{aligned}$$

Example 4.46: Let

$$G = S(C(Q)) = \{a + bi \mid a, b \in Q, i^2 = -1, \left(\frac{19i}{7}, \frac{29}{3}\right), *\}$$

be the strong complex infinite groupoid of type V.

Example 4.47: Let

$$G = S(C(Z)) = \{a + bi \mid a, b \in Z, i^2 = -1, (25i, 31), *\}$$

be the strong complex integer groupoid of type V.

Example 4.48: Let

$$G = S(C(Q)) = \{a + bi \mid a, b \in Q, i^2 = -1, \left(\frac{20i}{13}, \frac{19}{7}\right), *\}$$

be a strong complex groupoid of rationals of type V.

Example 4.49: Let

$$G = S(C(R)) = \{a + bi \mid a, b \in R, i^2 = -1, \left(\sqrt{3}i, \sqrt{7}/3\right), *\}$$

be the strong complex groupoid of reals of infinite order of type V.

Now we proceed onto give examples of the notion of strong complex infinite groupoids of type VI.

Example 4.50: Let

$$G = S(C(Z)) = \{a + bi \mid a, b \in Z, i^2 = -1, (20i, 45), *\}$$

be the strong complex infinite groupoid of type VI.

Example 4.51: Let

$$G = S(C(Q)) = \{a + bi \mid a, b \in Q, i^2 = -1, \left(\frac{30}{7}, \frac{15i}{8}\right), *\}$$

be the strong complex infinite groupoid of type VI.

Example 4.52: Let

$$G = S(C(Z)) = \{a + bi \mid a, b \in Z, i^2 = -1, (27, 9i), *\}$$

be the strong complex infinite groupoid of type VI.

Example 4.53: Let

$$G = S(C(Q)) = \{a + bi \mid a, b \in Q, i^2 = -1, (15, 40i), *\}$$

be the strong complex rational groupoid of type VI.

Now we define strong mixed complex groupoid of both infinite and finite order of type VII.

DEFINITION 4.6: *Let* $G = \{S(C(Z))$ *or* $S(C(Z_n))$ *or* $S(C(Q))$ *or* $S(C)$

$= S(C(R))\} = \{a + bi$ *(or* $a + bi_F$*)* $\mid a, b \in Z$ *or* Q *or* R *(or* $a, b \in Z_n$, $i_F^2 = n-1$*) with* $i^2 = -1$, $(m+ni, c+id)$ *where* $m, n, c, d, \in Z \setminus \{0\}$ *(or* $Q \setminus \{0\}$ *or* $R \setminus \{0\}$*),* $((m + ni_F, c+i_Fd) \mid m, n, c, d \in Z_n \setminus \{0\}), *\}$ *be a groupoid. G is defined as the strong mixed complex number groupoid or just strong complex groupoid of type VII.*

If in $(m+ni, c+id)$ *n or d is zero or used in the mutually exclusive sense then we call the groupoid to be a type VIII strong complex groupoid. If m or c is zero or used in the mutually exclusive sense we call G to be a type IX strong complex groupoid.*

If $m+ni = c+id$, we call G to be a type X strong complex groupoid. If $m + ni = 0$ or $c + id = 0$ we call G the type XI strong complex groupoid.

We will illustrate all these situations by some examples.

Example 4.54: Let

$$G = S(C(Z)) = \{a + bi \mid a, b \in Z, i^2 = -1, (3+4i, 8-3i), *\}$$

be the strong complex groupoid of type VII of infinite order.

Example 4.55: Let

$$G = S(C(Q)) = \{a + bi \mid a, b \in Q, i^2 = -1, (4+7/3i, 8/7+3/5i), *\}$$

be the strong complex groupoid of type VII.

Take x = 3 and y = 1–i in G.

$$\begin{aligned} x*y &= 3*(1-i) \\ &= 3(4+7/3i) + (1-i)(8/7 + 3/5i) \\ &= 12 + 7i + 8/7 + 3/5i - 8/7i + 3/5) \\ &= 13 \quad 26/35 + 226i/35 \\ &= 455/35 + 226/35i \in G. \end{aligned}$$

Example 4.56: Let

$$G = S(C(Z_{10})) = \{a + bi_F \mid a, b \in Z_{10}, i_F^2 = 9, *, (2+3i, 7+5i)\}$$

be the strong complex groupoid of type VII of finite order.

Let $x = 3 + i_F$ and $y = 2i_F$ be in G.

$$\begin{aligned} x*y &= (3+i_F) * 2i \pmod{10} \\ &= (2+3i_F)(3+i_F) + (7+5i_F) 2i_F \pmod{10} \\ &= 6 + 2i_F + 9i_F + 27 + 14i_F + 10 \times 9 \\ &= 3 + 5i_F \in G. \end{aligned}$$

Example 4.57: Let

$$G = S(C(R)) = \{a + bi \mid a, b \in R, i^2 = -1, (\sqrt{3} + \sqrt{7}/5\,i, 17 - 3\sqrt{5}\,i), *\}$$

be the strong complex groupoid of type VII of infinite order.

Now we proceed onto give examples of type VIII groupids.

Example 4.58: Let

$G = S(C(Z_{20})) = \{a + bi_F \mid a, b \in Z_{20},\ i_F^2 = 19, (10+3i_F, 3), *\}$

be the strong complex modulo integer groupoid of type VIII.

Suppose $x = 3$ and $y = 7+3i_F$ are in G then

$$\begin{aligned} x * y &= 3 * 7+3i_F \\ &= 3(10+3i_F) + (7+3i_F) \times 3 \\ &= 30 + 9i_F + 21 + 9i_F \\ &= 11 + 18i_F \text{ which is in G.} \end{aligned}$$

Example 4.59: Let

$G = S(C(Z)) = \{a + bi \mid a, b \in Z, i^2 = -1, *, (8, 3-4i)\}$

be the strong complex groupoid of type VIII.

Take $x = -7+i$ and $y = 3-4i$ in G.

$$\begin{aligned} \text{Now } x*y &= (-7+i) * (3-4i) \\ &= 8(-7+i) + (3-4i)(3-4i) \\ &= -56 + 8i + 9 - 12i - 12i \\ &= -63 - 16i \in G. \end{aligned}$$

Example 4.60: Let

$G = S(C(Q)) = \{a + bi \mid a, b \in Q, i^2 = -1, (7/9 + 3/8i)\ 12/7), *\}$

be the strong complex rational groupoid of type VIII.

Consider $x = 3$ and $y = i$ then

$$\begin{aligned} x*y &= 3(7/9 + 3/8i) + i\ \ 12/7 \\ &= 7/3 + 9/8i + 12/7\ i \\ &= 7/3 + 152/56i \in G. \end{aligned}$$

Example 4.61: Let

$$G = \{S\,(C) = S(C(R))\} = \{a + bi \mid a, b \in R, i^2 = -1, \left(\left(\frac{\sqrt{19}}{10}, \frac{\sqrt{12}}{7} + \frac{\sqrt{5}}{2}i\right), *\right\}$$

be the strong complex real groupoid of type VIII.

Example 4.62: Let

$$G = S(C(Z)) = \{a + bi \mid a, b \in Z, i^2 = -1, (30, 26 - 16i), *\}$$

be the strong complex modulo integer groupoid of type VIII.

Example 4.63: Let

$$G = S(C(Z_8)) = \{a + bi_F \mid a, b \in Z_8, i_F^2 = 7, (6, 3+2i_F), *\}$$

be the strong complex modulo integer groupoid of type VIII.

Let $x = 3 + i_F$ and $y = 4+3i_F \in G$.

$$\begin{aligned} x*y &= (3+i_F) * (4+3i_F) \\ &= 6\,(3+i_F) + (4+3i_F)\,(3+2i_F) \\ &= 18 + 6i_F + 12 + 9i_F + 8i_F + 6.7 \\ &= 2 + 6i_F + 4 + i_F + 2 \\ &= 7i_F \in G. \end{aligned}$$

This is the way the operation * on G is defined.

Now we proceed onto give examples of type IX groupoids.

Example 4.64: Let

$$G = S(C(Z)) = \{a + bi \mid a, b \in Z, i^2 = -1, (8i, 9-2i), *\}$$

be the strong complex integer groupoid of type IX. We show how product is carried out. Let $x = 3$ and $y = -7 \in G$.

$$\begin{aligned} x*y &= 3 * (-7) \\ &= 3 \times 8i + (-7)\,(9-2i) \\ &= 24i - 56 + 14i \\ &= 38i - 56 \\ &= -56 + 38i \in G. \end{aligned}$$

Example 4.65: Let

$G = S(C(Q)) = \{a + bi \mid a, b \in Q, i^2 = -1, (1+3/7i, -9/17i),*\}$ be the strong complex rational groupoid of type IX. Let x = 7/3i and y = 2 ∈ G.

$$
\begin{aligned}
x*y &= 7/3i * 2 \\
&= 7/3i\,(1+3/7i) + 2\,(-9/17i) \\
&= 7/3i + i^2 - 18/17i \\
&= -1 + i\left(\frac{7\times 17 - 3\times 18}{3\times 17}\right) \\
&= -1 + \frac{i65}{3\times 17} \in G.
\end{aligned}
$$

Example 4.66: Let

$G = S(C(Z_{13})) = \{a + bi_F \mid a, b \in Z_{13},\ i_F^2 = 12, (3i_F, 1+2i_F), *\}$ be a strong complex modulo integer groupoid of type IX.

For $x = 5i_F + 2$ and $y = 1+i_F \in G$

$$
\begin{aligned}
x*y &= (5i_F+2) * (i_F+1) \\
&= 3i_F\,(2+5i_F) + (1+i_F)\,(1+2i_F) \\
&= 6i_F + 15 \times 12 + 1 + 2i_F + i_F + 2 \times 12 \\
&= 10i_F + 9 \in G.
\end{aligned}
$$

Example 4.67: Let

$G = S(C(Q)) = \{a + bi \mid a, b \in Q, i^2 = -1, (8/3i, 3/7 + 2/5i),*\}$ be the strong complex rational groupoid of type IX.

Let x = 3/4 and y = 35i + 70 ∈ G.

$$
\begin{aligned}
x*y &= 3/4 * (35i + 70) = 8/3i \times ¾ + (35i + 70) \times (3/7 + 2/5i) \\
&= 6i + 35i \times 3/7 + 70\times 3/7 + 35i \times 2i / 5 + 70 \times 2i/5 \\
&= 6i + 15i + 30 + 14i^2 + 28i \\
&= 49i + 16 \in G.
\end{aligned}
$$

Example 4.68: Let

$G = S(C(R)) = \{a + bi \mid a, b \in R, i^2 = -1, (\sqrt{3}\,i, 3-4i),*\}$ be the strong complex real groupoid of type IX.

Let x $= -\sqrt{3}$ and y $= 3-\sqrt{7}\, i$

$$
\begin{aligned}
x*y &= \sqrt{3}\, i \times (-\sqrt{3}\,) + (3 - \sqrt{7}\, i)\,(3-4i) \\
&= -3i + (9 + 3\sqrt{7}\, i - 12i - 4\sqrt{7}\,) \\
&= (9-4\sqrt{7}\,) + i\,(15i - 3\sqrt{7}\,) \\
&= (9-4\sqrt{7}\,) - i\,(15 + 3\sqrt{7}\,) \in G.
\end{aligned}
$$

Now we proceed onto give type X groupoids.

Example 4.69: Let

$G = S(C(Z_7)) = \{a + bi_F \mid a, b \in Z_7,\ i_F^2 = 6, (3+4i_F, 0), *\}$

be a strong complex modulo integer groupoid of type X.

Let $x = 3+2i_F$ and $y = 4i_F \in G$

$$
\begin{aligned}
x*y &= (3+2i_F) * (4+i_F) \\
&= (3 +2i_F)\,(3+4i_F) + 0\,(4+i_F) \\
&= 9 + 8 \times 6 + 12i_F + 6i_F + 0 \\
&= 1 + 4i_F \in G.
\end{aligned}
$$

Example 4.70: Let

$G = S(C(Z_{15})) = \{a + bi_F \mid a, b \in Z_{15},\ i_F^2 = 14, (0, 2+10i_F), *\}$

be a strong complex modulo integer groupoid of type X.

Let $x = 3 + 4i_F$ and $y = 8 \in G$.

$$
\begin{aligned}
x*y &= 3+4i_F * 8 = 0\,(3+4i_F) + 8\,(2+10i_F) \\
&= 16 + 80i_F \\
&= 1 + 5i_F \in G.
\end{aligned}
$$

Example 4.71: Let

$G = S(C(Q)) = \{a + bi \mid a, b \in Q,\ i^2 = -1, (0, -3 + 8i), *\}$

be the strong complex modulo integer groupoid of type X.

Example 4.72: Let

$G = S(C(R)) = \{a + bi \mid a, b \in R, i^2 = -1, (0, \sqrt{3} - 5\sqrt{7}i), *\}$ be a strong complex groupoid of real of type X.

Let $x = 7i - 3$ and $y = \sqrt{5} + 4i \in G$ now

$$\begin{aligned} x*y = 7i - 3 * \sqrt{5} + 4i &= (7i\text{-}3)\,0 + (\sqrt{5} + 4i)(\sqrt{3} - 5\sqrt{7}\,i) \\ &= 0 + \sqrt{5}\sqrt{3} + 4i\sqrt{3} - 5\sqrt{5}\sqrt{7}\,i + 20\sqrt{7} \\ &= 20\sqrt{7} + \sqrt{15} + i\,(4\sqrt{3} - 5\sqrt{35}). \end{aligned}$$

Example 4.73: Let

$G = S(C(Z_{10})) = \{a + bi_F \mid a, b \in Z_{10}, i_F^2 = 9, (9+2i_F, 0), *\}$ be a strong complex modulo integer groupoid of type X.

Let $x = 8+4i_F$ and $y = 5i_F$ be in G.

$$\begin{aligned} x*y &= 8+4i_F * 5i_F \\ &= (8+4i_F)(9 + 2i_F) + 5i_F \times 0 \\ &= 72 + 36i_F + 16i_F + 8.9 + 0 \\ &= 4 + 6i_F \in G. \end{aligned}$$

Now having seen ten types of groupoids both infinite and finite we just leave the task to the reader of studying the related properties as it is considered as a matter of routine and simple number theoretic calculations will lead to easy characterizations and one can use [ ] as a reference.

We now proceed onto define the concept of quasi complex loops using the set

$$\{e, 1, 2, \ldots, n, ei_F, i_F, \ldots, ni_F, a + bi_F, a, b \in \{e, 1, 2, \ldots, n\}\}.$$

Our main motivation would be using these concepts in building non associative complex rings of both finite and infinite order, which will be carried out in the later part of this chapter.

DEFINITION 4.7: *Let*

$$L = S(C(L_n^{(m)})) = \{a + bi_F \mid a, b \in \{e, 1, 2, \ldots, n\};\ i_F^2 = n-1$$
$$\text{and } m = a+bi_F \text{ and } (m-1) = (a-1) + (b-1)\, i_F,\ *\}$$

*be the set with binary operation * defined as follows.*

(i) $e \text{ o } x = x \text{ o } e = x$.

(ii) $x \text{ o } x = x^2 = e$ for all $x \in L$.

(iii) for every x and y in $S(C(L_n(m))$

$x*y = [(a+bi_F)y - ((a-1)+(b-1)i_F)x\,]\ (\text{mod } n)$

$x \neq y$ *and* $x \neq e$ *or* $y \neq e$.

L is defined as the strong complex quasi loop.

We will illustrate this situation by some examples.

Example 4.74: Let

$$L = S(C(L_n^{(m)})) = S(C(L_{11}(7 + 4i_F))$$
$$= \{a + i_Fb \mid a, b \in L_{11}(7 + 4i_F),\ i_F^2 = 10, *\}$$

be a strong complex modulo integer quasi loop of finite order.

Let $x = 3 + 5i_F$ and $y = 9 + 8i_F$ in L.

$$\begin{aligned} x*y &= (9+8i_F) * (3+5i_F) \\ &= (7 + 4i_F)(3+5i_F) - 6+3i_F)(9 + 8i_F)\ (\text{mod } 11) \\ &= (21+12i_F + 35i_F + 20 \times 10) - \\ &\quad\ (54 + 27i_F + 48i_F + 24 \times 10)\ (\text{mod } 11) \\ &= 4+5i_F \in L. \end{aligned}$$

It is easily verified $x*y \neq y * x$ and $x * (y*z) \neq (x*y) * z$.

Example 4.75: Let

$$G = S(C(Z_{25}(2+5i_F)) = \{a + bi_F \mid a, b \in \{e, 1, 2, \ldots, 25\},$$
$$i_F^2 = 24,\ m = (2 + 5i_F)\}$$

be the strong complex modulo integer quasi loop of finite order.

Example 4.76: Let

$$G = S(C(Z_{27}\,(3+3i_F)) = \{a + bi_F \mid a, b \in \{e, 1, 2, \ldots, 27\}, \; i_F^2 = 26,\; m = (3 + 3i_F)\}$$

be the strong complex modulo integer quasi loop of finite order.

Example 4.77: Let

$$L = S(C(Z_{33}\,(4+i_F)) = \{a + bi_F \mid a, b \in \{e, 1, 2, \ldots, 33\}, \; i_F^2 = 32,\; m = (4+ i_F)\}$$

be the strong complex modulo integer quasi loop of finite order.

Example 4.78: Let

$$L = S(C(Z_{29}\,(3+3i_F)) = \{a + bi_F \mid a, b \in \{e, 1, 2, \ldots, 29\}, \; i_F^2 = 28,\; m = (3 + 3i_F)\}$$

be the complex strong modulo integer quasi loop of finite order.

Example 4.79: Let

$$L = S(C(Z_{21}(2+2i_F)) = \{a + bi_F \mid a, b \in \{e, 1, 2, \ldots, 21\}, \; *,\; m = (2+2i_F)\}$$

be the strong complex modulo integer quasi loop of finite order.

Example 4.80: Let

$$L = S(C(Z_{31}(30+30i_F)) = \{a + bi_F \mid a, b \in \{e, 1, 2, \ldots, 31\}, *, \; i_F^2 = 30,\; m = (30+30i_F)\}$$

be the strong complex modulo integer quasi loop of finite order.

We shall define now the notion of strong complex modulo integer complex matrix groupoids.

Let $G = \{(a_1, \ldots, a_n) \mid a_i \in C\,(Z_n)$
$= \{a + bi_F$ where $a, b \in Z_n,\ i_F^2 = n\text{-}1\}, *, (ti_F + u, ri_F + s)\}$ be defined as the strong complex modulo integer row matrix groupoid.

We will give examples of them.

Example 4.81: Let

$$G = \{(a_1, a_2, \ldots, a_{10}) \mid a_i \in C(Z_{40}) = \{a + bi_F \mid a, b \in Z_{40}, i_F^2 = 39\}; 1 \leq i \leq 10, *, (3 + 4i_F, 30 + 15i_F)\}$$

be the strong complex modulo integer row matrix groupoid.

Example 4.82: Let

$$G = \{(a_1, a_2, a_3, a_4) \mid a_i \in C(Z_{10}) = \{a + bi_F \mid a, b \in Z_{10}, i_F^2 = 9\}; 1 \leq i \leq 4, *, (3 + 4i_F, 8i_F)\}$$

be the strong complex modulo integer row matrix groupoid.

Example 4.83: Let

$$M = \{(a_1, a_2, \ldots, a_{10}) \mid a_i \in C(Z_9) = \{a + bi_F \mid a, b \in Z_9, i_F^2 = 8\}; 1 \leq i \leq 10, *, (4i_F+5, 5i_F)\}$$

be the strong complex modulo integer row matrix groupoid.

Example 4.84: Let

$$M = \{(a_1, a_2, \ldots, a_{15}) \mid a_i \in C(Z_{25}) = \{a + bi_F \mid a, b \in Z_{25}, i_F^2 = 24\}; 1 \leq i \leq 15, *, (6i_F+6, 6i_F)\}$$

be the strong complex modulo integer row matrix groupoid.

Example 4.85: Let

$$T = \{(a_1, a_2, a_3, a_4) \mid a_i \in C(Z_{15}) = \{a + bi_F \mid a, b \in Z_{15}, i_F^2 = 14\}; 1 \leq i \leq 4, *, (10+5i_F, 10)\}$$

be the strong complex modulo integer row matrix groupoid of finite order.

Example 4.86: Let

$$T = \{(a_1, a_2, \ldots, a_{28}) \mid a_i \in C(Z_{48}) = \{a + bi_F \mid a, b \in Z_{48}, i_F^2 = 47\}; 1 \leq i \leq 28, *, (47, 47i_F+1)\}$$

be the strong complex modulo integer row matrix groupoid of finite order.

Example 4.87: Let

$$P = \left\{ \begin{bmatrix} a_1 \\ \vdots \\ a_{10} \end{bmatrix} \middle| a_i \in C(Z_{50}) = \{a + bi_F \mid a, b \in Z_{50},\ i_F^2 = 49\}; \right.$$

$$\left. 1 \leq i \leq 10, *, (48i_F, 4) \right\}$$

be the strong complex modulo integer column matrix groupoid.

Example 4.88: Let

$$P = \left\{ \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_{50} \end{bmatrix} \middle| a_i \in C(Z_{24}) = \{a + bi_F \mid a, b \in Z_{24},\ i_F^2 = 23\}; \right.$$

$$\left. 1 \leq i \leq 50, *, (14, 10i_F+14) \right\}$$

be the strong complex modulo integer column matrix groupoid of finite order.

Example 4.89: Let

$$P = \left\{ \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ \vdots \\ a_{15} \end{bmatrix} \middle| a_i \in C(Z_{12}) = \{a + bi_F \mid a, b \in Z_{12},\ i_F^2 = 11\}; \right.$$

$$\left. 1 \leq i \leq 15, *, (10i_F, 10) \right\}$$

be the strong complex modulo integer column matrix groupoid.

Example 4.90: Let

$$P = \left\{ \begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \end{bmatrix} \middle| a_i \in C(Z_{14}) = \{a + bi_F \mid a, b \in Z_{14}, \right.$$

$$\left. i_F^2 = 13\}; 1 \leq i \leq 9, *, (8, 8i_F+8) \right\}$$

be the strong complex modulo integer square matrix groupoid.

Example 4.91: Let

$$M = \left\{ \begin{bmatrix} a_1 & a_2 & \ldots & a_{12} \\ a_{13} & a_{14} & \ldots & a_{24} \\ a_{25} & a_{26} & \ldots & a_{36} \\ a_{37} & a_{38} & \ldots & a_{48} \end{bmatrix} \middle| a_i \in C(Z_7) \right.$$

$$\left. = \{a + bi_F \mid a, b \in Z_7,\ i_F^2 = 6\}; 1 \leq i \leq 48, *, (3i_F, 2) \right\}$$

be the strong complex modulo integer 4×12 matrix groupoid of finite order.

Example 4.92: Let

$$M = \left\{ \begin{bmatrix} a_1 & a_2 & \ldots & a_6 \\ a_7 & a_8 & \ldots & a_{12} \\ a_{13} & a_{14} & \ldots & a_{18} \end{bmatrix} \middle| a_i \in C(Z_{43}) = \{a + bi_F \mid a, b \in Z_{43}, \right.$$

$$\left. i_F^2 = 42\}; 1 \leq i \leq 18, *, (3i_F+7, 0) \right\}$$

be the strong complex modulo integer 3×6 matrix groupoid of finite order.

Example 4.93: Let

$$M = \left\{ \begin{bmatrix} a_1 & a_2 & \dots & a_{10} \\ a_{11} & a_{12} & \dots & a_{20} \\ a_{21} & a_{22} & \dots & a_{30} \end{bmatrix} \middle| \, a_i \in C(Z_{10}) = \{a + bi_F \mid a, b \in Z_{10}, \right.$$

$$\left. i_F^2 = 9\}; 1 \le i \le 30, *, (9i_F, 2) \right\}$$

be the strong complex modulo integer 3×10 matrix groupoid of finite order.

Example 4.94: Let

$$M = \left\{ \begin{bmatrix} a_1 & a_2 & \dots & a_{10} \\ a_{11} & a_{12} & \dots & a_{20} \\ a_{21} & a_{22} & \dots & a_{30} \\ \vdots & \vdots & & \vdots \\ a_{91} & a_{92} & \dots & a_{100} \end{bmatrix} \middle| \, a_i \in C(Z_{25}) = \{a + bi_F \mid a, b \in Z_{25}, \right.$$

$$\left. i_F^2 = 24\}; 1 \le i \le 100, *, (22i_F, 0) \right\}$$

be the strong complex modulo integer square matrix groupoid. G has right ideals which are not left ideals.

We can also define strong complex modulo integer polynomial groupoids. The definition is a mater of routine we only give examples of them.

Example 4.95: Let

$$M = \left\{ \sum_{i=0}^{\infty} a_i x^i \,\middle|\, a_i \in C(Z_{27}) = \{a + bi_F \mid a, b \in Z_{27}, i_F^2 = 26\}; \right.$$

$$\left. (2i_F+3, 9i_F+20), * \right\}$$

be the strong complex modulo integer polynomial groupoid. Clearly G is non commutative and is of infinite order.

Example 4.96: Let

$$M = \left\{ \sum a_i x^i \,\middle|\, a_i \in C(Z_{41}) = \{a + bi_F \mid a, b \in Z_{41},\ i_F^2 = 40\}; \right.$$
$$\left. (8i_F+2,\ 25+23i_F),\ * \right\}$$

be the strong complex modulo integer polynomial groupoid of infinite order and is non commutative.

Example 4.97: Let

$$M = \left\{ \sum_{i=0}^{\infty} a_i x^i \,\middle|\, a_i \in C(Z_{12}) = \{a + bi_F \mid a, b \in Z_{12},\ i_F^2 = 11\}; \right.$$
$$\left. (2i_F+4,\ 4i_F+8),\ * \right\}$$

be the strong complex modulo integer polynomial groupoid of infinite order.

Example 4.98: Let

$$M = \left\{ \sum_{i=0}^{\infty} a_i x^i \,\middle|\, a_i \in C(Z_{250}) = \{a + bi_F \mid a, b \in Z_{250},\ i_F^2 = 249\}; \right.$$
$$\left. (15+15i_F,\ 15+15i_F),\ * \right\}$$

be the strong complex modulo integer polynomial groupoid which is commutative of infinite order.

Example 4.99: Let

$$M = \left\{ \sum_{i=0}^{\infty} a_i x^i \,\middle|\, a_i \in C(Z_{20}) = \{a + bi_F \mid a, b \in Z_{20},\ i_F^2 = 19\}; \right.$$
$$\left. (3i_F+17,\ 17i_F),\ * \right\}$$

be the strong complex modulo integer polynomial groupoid of infinite order.

Example 4.100: Let

$$M = \left\{ \sum_{i=0}^{\infty} a_i x^i \;\middle|\; a_i \in C(Z_{121}) = \{a + bi_F \mid a, b \in Z_{121},\ i_F^2 = 120\}; (19i_F, 120+3i_F), * \right\}$$

be the strong complex modulo integer polynomial groupoid of infinite order.

Example 4.101: Let

$$M = \left\{ \sum_{i=0}^{\infty} a_i x^i \;\middle|\; a_i \in C(Z_2) = \{a + bi_F \mid a, b \in Z_2,\ i_F^2 = 1\}; (i_F, 1+i_F), * \right\}$$

be the strong complex modulo integer polynomial groupoid of infinite order.

Example 4.102: Let

$$P = \left\{ \begin{bmatrix} a_1 & a_2 \\ a_3 & a_4 \\ \vdots & \vdots \\ a_{25} & a_{26} \end{bmatrix} \;\middle|\; a_i \in C(Z_7) = \{a + bi_F \mid a, b \in Z_7,\ i_F^2 = 6\}; 1 \le i \le 26, *, (3+4i_F, 2+5i_F) \right\}$$

be the strong complex modulo integer matrix groupoid of finite order.

Example 4.103: Let

$$G = \left\{ \begin{bmatrix} a_1 & a_2 & \dots & a_{12} \\ a_{13} & a_{14} & \dots & a_{24} \\ \vdots & \vdots & \dots & \vdots \\ a_{97} & a_{98} & \dots & a_{108} \end{bmatrix} \;\middle|\; a_i \in C(Z_{11}) = \{a + bi_F \mid a, b \in Z_{11}, i_F^2 = 10\}; 1 \le i \le 108, *, (6+6i_F, 4+8i_F) \right\}$$

be the strong complex modulo integer matrix groupoid of finite order.

Example 4.104: Let

$$G = \left\{ \begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \end{bmatrix} \middle| a_i \in C(Z_8) = \{a + bi_F \mid a, b \in Z_8,\ i_F^2 = 7\}; \right.$$

$$\left. 1 \leq i \leq 9, *, (3+5i_F, 3+5i_F) \right\}$$

be the strong complex modulo integer matrix groupoid of finite order.

Example 4.105: Let

$$G = \left\{ \begin{bmatrix} a_1 & a_2 & \ldots & a_{40} \\ a_{41} & a_{42} & \ldots & a_{80} \end{bmatrix} \middle| a_i \in C(Z_6) = \{a + bi_F \mid a, b \in Z_6, \right.$$

$$\left. i_F^2 = 5\}; 1 \leq i \leq 80, *, (2+3i_F, 5) \right\}$$

be the strong complex modulo integer matrix groupoid of finite order.

Example 4.106: Let

G = {all 6×6 matrices with entries from $C(Z_{18}) = \{a + bi_F \mid a, b \in Z_{18}, i_F^2 = 17\}$; $*$, $(3, 3i_F+3)$} be the strong complex modulo integer matrix groupoid of finite order.

Example 4.107: Let

G = {all 4×4 matrices with entries from $C(Z_{15}) = \{a + bi_F \mid a, b \in Z_{15}, i_F^2 = 14\}$; $*$, $(8i_F+7, 0)$} be the strong complex modulo integer matrix groupoid of finite order.

Example 4.108: Let

G = {all 8 × 5 matrices with entries from C (Z_{27}) = {a + bi_F | a, b ∈ Z_{27}, i_F^2 = 26}; *, (0, $14i_F$+25)} be the strong complex modulo integer matrix groupoid of finite order.

Example 4.109: Let

G = {all 8 × 16 matrices with entries from C (Z_8) = {a + bi_F| a, b ∈ Z_8, i_F^2 = 7}, *, ($3i_F$, $2i_F$)} be the strong complex modulo integer matrix groupoid of finite order.

Example 4.110: Let

G = {all 3 × 15 matrices with entries from C (Z_4) = {a + bi_F| a, b ∈ Z_4, i_F^2 = 3}, *, ($3i_F$, $3i_F$)} be the strong complex modulo integer matrix groupoid of finite order.

Example 4.111: Let

$$G = \left\{ \begin{bmatrix} a_1 & a_2 \\ a_3 & a_4 \end{bmatrix} \right| C\,(Z_{13}) = \{a + bi_F \mid a, b \in Z_{13},$$

$$i_F^2 = 12\}, *, (10i_F, 0)\}$$

be the strong complex modulo integer matrix groupoid of finite order. G has right ideals which are not left ideals.

Example 4.112: Let

G = {all 5×8 matrices with entries from C (Z_{19}) = {a + bi_F | a, b ∈ Z_{19}, i_F^2 = 18}, *, (0, $18i_F$)} be the strong complex modulo integer matrix groupoid of finite order. G has left ideals which are not right ideals.

Now we can define doubly strong complex groupoid rings which is a matter of routine. However we supply some examples of them.

Example 4.113: Let

$$G = \left\{ \begin{bmatrix} a_1 & a_2 \\ a_3 & a_4 \end{bmatrix} \middle| C(Z_3) = \{a + bi_F \mid a, b \in Z_3,\ i_F^2 = 2\}, \right.$$
$$\left. 1 \leq i \leq 4, *, (2+i_F, i_F+1) \right\}$$

be the strong complex matrix groupoid. F = Z_3 be the ring. FG is a groupoid ring which is a doubly strong complex matrix groupoid ring of finite order. FG is non commutative and non associative.

Example 4.114: Let

$$P = \left\{ \begin{bmatrix} a_1 & a_2 \\ a_3 & a_4 \\ \vdots & \vdots \\ a_{19} & a_{20} \end{bmatrix} \middle| a_i \in C(Z_{12}) = \{a + bi_F \mid a, b \in Z_{12},\ i_F^2 = 11\}; \right.$$
$$\left. 1 \leq i \leq 20, *, (3i_F, 4i_F) \right\}$$

be the strong complex modulo integer matrix groupoid. Z_4 = F be the ring. Z_4G be the strong complex modulo integer matrix groupoid ring.

(1) Find order of G.
(2) Prove G has zero divisors
(3) Prove G is non commutative
(4) Prove G has ideals.

Such study will make the reader under this concept. However we have given several problems for the reader in the final chapter.

Example 4.115: Let

G = {all 6 × 6 matrices with entries from C (Z_{48}) = {a + bi_F | a, b ∈ Z_{48}, i_F^2 = 47}; *, (19i_F, 12+36i_F)} be the strong complex modulo integer matrix groupoid.

$F = Z_{48}$ be the ring. FG is the doubly strong complex modulo integer groupoid ring of finite order which is both non commutative and non associative.

Example 4.116: Let

G = {all 3 × 12 matrices with entries from $C(Z_6) = \{a + bi_F \mid a, b \in Z_6, i_F^2 = 5\}$; *, $(4i_F, 1+3i_F)$} be the strong complex modulo integer matrix groupoid.

$F = Z_3$ be the finite field. FG be the groupoid ring of finite order which is both non commutative and non associative.

Example 4.117: Let

G = {all 5×2 matrices with entries from $C(Z_{30}) = \{a + bi_F \mid a, b \in Z_{30}, i_F^2 = 29\}$; *, $(3i_F, 0)$} be the strong complex groupoid. $F = Z_{10}$ be the ring. FG be the groupoid ring of G over F.

FG has right ideals. Now we can say the following result.

THEOREM 4.1: *Let*
*G = {n × m matrices with entries from $C(Z_s)$ $(ti_F, 0)$; $t \in Z_s \setminus \{0\}$, *} be the groupoid. $F = Z_{m_n}$ be the ring. FG be the groupoid ring. (i) FG has right ideals. (ii) FG has ideals.*

Proof is direct and hence left as an exercise. If $(ti_F, 0)$ is replaced by $(0, ti_F)$ FG has left ideals which are not right ideals.

Now we proceed onto give examples of strong quasi loop rings. The definition of strong quasi loop rings is direct hence left as an exercise.

Example 4.118: Let G = {S $(C(L_7(3i_F)))$} be the strong complex modulo integer quasi loop. $F = Z_7$ be the field. FG is the complex quasi loop ring of G over F of finite order. FG is a complex non associative non commutative ring.

Example 4.119: Let G = $\{S(C(L_9(7i_F+1))\}$ be the strong complex modulo integer quasi loop. F = Z_9 be the ring. FG is the strong complex modulo integer quasi loop ring which has ideals.

Example 4.120: Let G = $\{S(C(L_{43}\ (8i_F+17))\}$ be the strong complex modulo integer quasi loop. Z be the ring of integers. ZL be the strong complex quasi loop ring of infinite order.

Example 4.121: Let G = $\{S\ (C(L_{47}\ (46+i_F))\}$ be the strong complex modulo integer quasi loop. Z_{12} = F be the ring. FL be the strong complex modulo integer quasi loop. FL has zero divisors and ideals.

Inview of this we have the following theorem.

THEOREM 4.2: *Let L = $(C(L_t(a+bi_F)))$ with a, b $\in Z_t \setminus \{0\}$ be a strong complex quasi loop. F = Z_n (n a composite number) be a ring. FL be the strong complex modulo integer quasi loop. FL has zero divisors and ideals.*

Proof is direct and hence is left as an exercise to the reader.

Example 4.122: Let G = $\{S(C(L_{51}\ (27+30i_F))\}$ be the strong complex modulo integer quasi loop. Z_{210} = F be the ring. FL is the loop ring. Clearly FL is non associative non commutative and of finite order.

Clearly FL has zero divisors and ideals.

Example 4.123: Let G = $\{S(C(L_{31}\ (16+16i_F))\}$ be the strong complex modulo integer quasi loop. F = Z_3 be the ring. FL is the commutative quasi loop ring.

Having seen examples of non commutative / commutative non associative finite and infinite ring we can replace the real field / ring by complex ring / field and define super doubly strong complex non associative rings.

The definition is a matter of routine. However we give examples of them.

Example 4.124: Let M = {S(C(L_{12})), *, ($3i_F+4$, $8i_F+7$)} be the strong complex groupoid. F = C(Z_6) = {a + i_Fb | a, b ∈ Z_6, i_F^2 = 5} be the ring of complex modulo integers. FM the groupoid ring is defined as the super doubly strong complex non associative ring of finite order, which is also non commutative.

Example 4.125: Let G = {C(Z_{40}), *, ($8i_F$, $12i_F+4$)} be the strong complex modulo integer groupoid. F = C(Z_3) = {a + bi_F | a, b ∈ Z_3, i_F^2 = 2} be the complex modulo integers ring. FG is the super doubly strong complex groupoid ring.

Example 4.126: Let G = {C(Z_{19}), *, ($9i_F$, $8+3i_F$)} be the strong complex modulo integer groupoid. F = C(Z_{12}) = {a + bi_F | a, b ∈ Z_{12}, i_F^2 = 11} be the complex modulo integers ring. FG is the super special doubly strong groupoid ring. FG has zero divisors.

Example 4.127: Let G = {C(Z_{24}), *, (3, $17i_F+11$)} be the strong complex modulo integer groupoid. F = C(Z_{120}) = {a + bi_F | a, b ∈ Z_{120}, i_F^2 = 119} be the complex modulo integers ring. FG be the super doubly strong complex groupoid ring.

Example 4.128: Let G = {C(Z_{40}), *, ($8i_F$, $7i_F$)} be the strong complex groupoid. F = C(Z_{12}) be the complex modulo integer ring. FG be the super strong complex groupoid ring.

Example 4.129: Let G = {C(Z_{60}), *, ($9i_F+11$, $9i_F+11$)} be a strong complex groupoid. F = C(Z_{12}) be the complex modulo integer ring. FG is the super doubly strong complex modulo integer groupoid ring which is commutative but non associative.

Example 4.130: Let G = {C(Z_{12}), *, ($9i_F+7$, 0)} be the strong complex groupoid. F = C(Z_{12}) be the complex modulo integer ring. FG is the super doubly strong complex modulo integer groupoid ring.

This groupoid ring has right ideals which are not left ideals.

Example 4.131: Let G = $\{C(Z_{10}), *, (0, 7i_F+4)\}$ be the strong complex groupoid. F = $C(Z_{10})$ be the complex ring. FG is the super strong doubly complex groupoid ring. This non associative non commutative ring has left ideals which are not right ideals.

Now having seen examples of super strong doubly complex ring of finite / infinite order we can derive several related properties. This task is left as an exercise to the reader. Now we give examples of super strong complex matrix groupoid rings of finite / infinite order.

Example 4.132: Let

$$G = \{(a_1, a_2, \ldots, a_9) \mid a_i \in C(Z_{24}) = \{a + bi_F \mid a, b \in Z_{24}, i_F^2 = 23\}; 1 \leq i \leq 9, *, (3 + 4i_F, 10+8i_F)\}$$

be the strong complex modulo integer row matrix groupoid. F = $C(Z_2)$ be the complex modulo integer ring. FG is the super strong double complex row matrix groupoid ring. FG has zero divisors and subrings. Clearly FG is a non associative non commutative ring of finite order.

Example 4.133: Let

$$G = \{(a_1, \ldots, a_{12}) \mid a_i \in C(Z_{40}); *, (3i_F + 4, 3i_F+4)\}$$

be the strong complex row matrix groupoid. F = $C(Z_{10})$ be the complex ring, FG is the super complex doubly strong groupoid ring which is commutative but non associative and of finite order.

Example 4.134: Let

$$G = \{(a_1, \ldots, a_{40}) \mid a_i \in C(Z_3), *, (3i_F, 4), 1 \leq i \leq 40\}$$

be the strong complex row matrix groupoid. F = C (Z_2) be the ring, FG is the super doubly strong complex groupoid ring.

Example 4.135: Let

$$G = \left\{ \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_{40} \end{bmatrix} \middle| a_i \in C(Z_{25}), 1 \leq i \leq 40, *, (4i_F + 3, 0) \right\}$$

be the strong complex groupoid. $F = C(Z_{20})$ be the complex ring. FG is the super doubly strong complex column matrix groupoid ring, which has right ideals which are not left ideals.

Example 4.136: Let

$$G = \left\{ \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_{10} \end{bmatrix} \middle| a_i \in C(Z_{20}), 1 \leq i \leq 10, *, (0, 14i_F) \right\}$$

be the strong complex groupoid. $F = C(Z_{20})$ be the complex ring. FG is the super strong complex modulo integer groupoid ring, which has left ideals which are not right ideals.

Example 4.137: Let

$$G = \left\{ \begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \end{bmatrix} \middle| a_i \in C(Z_{21}), 1 \leq i \leq 9, *, (3i_F, 9i_F) \right\}$$

be the strong complex modulo integer groupoid. $F = C(Z_{21})$ be the complex ring. FG is the super complex doubly strong groupoid ring.

Example 4.138: Let

$G = \{$all 10×8 matrices with entries from $C(Z_{12})$, $*$, $(4i_F+3, 2)\}$ be the strong complex groupoid. $F = C(Z_{12})$ be the complex ring. FG is the super strong doubly complex modulo integer groupoid ring. FG has ideals and zero divisors.

We give two theorems the proof of which is straight forward.

THEOREM 4.3: *Let*

*G = {all $m \times t$ matrices with entries from $C(Z_n)$, *, $(r + si_F, 0)$} and G' = {all $m \times t$ matrices with entries from $C(Z_n)$, *, $(0, r+si_F)$} be strong complex groupoids. $F = C(Z_p)$ (or $C(Z)$ or $C(R)$ or $C(Q)$ or $C(Z_n)$) be the complex ring. FG and FG' are super strong doubly complex groupoid ring. If P is any right ideal in FG then P is a left ideal in FG' and vice versa.*

THEOREM 4.4: *Let*

*G = {all $m \times t$ matrices with entries from $C(Z_n)$, *, for a, b, c, d in Z_n, $(a + bi_F, c+di_F)$}
be the strong complex groupoid. $F = C(Z_s)$ (s a composite number). FG be the super complex doubly strong groupoid ring FG has subrings which are not ideals.*

Several other related results true in case of non associative rings [ ] can be derived in case of super strong double complex groupoid rings also with simple modifications.

Example 4.139: Let $G = \{S(L_{25}(3+2i_F))\}$ be the strong complex quasi loop of modulo integers. $F = C(Z_5)$ be the complex modulo integer ring. FG be the super strong doubly complex modulo integer ring.

Example 4.140: Let $G = S(C(L_{19}(3i_F)))$ be the strong complex quasi loop of modulo integers. $F = C(Z_{19})$ be the complex modulo integer ring. FG is the super strong doubly complex modulo integer quasi loop ring of finite order which is non commutative.

Example 4.141: Let $G = S(C(Z_{15}(8+8i_F)))$ be the strong complex modulo integers quasi loop. $F = C(Z)$ be the complex ring of integers. FG is the super strong complex modulo integer quasi loop ring of infinite order.

Example 4.142: Let G = $S(C(Z_7\ (4i_F)))$ be the strong complex modulo integers quasi loop. F = C(Q) be the complex rational ring. FG is the super strong quasi loop ring of infinite order.

Example 4.143: Let G = $S(C(Z_{121}\ (10i_F+111)))$ be the complex modulo integers quasi loop. F = $C(Z_{11})$ be the complex ring. FG is the super strong complex modulo integer quasi loop ring.

We can study all related properties of these structures and derive results as in case of non associative rings.

Chapter Five

APPLICATIONS OF COMPLEX NON ASSOCIATIVE STRUCTURES

We have studied complex non associative structures like complex modulo integer groupoids of several types of finite order and complex groupoids of infinite order. Certainly these structures will find applications in places of groupoids were some imaginary or complex value is involved. As the subject is very new only in due course of time we can certainly see researchers finding appropriate applications of these.

Likewise we have introduced the notion complex loops and strong complex quasi loops. These are non associative and also very specially constructed. It is a challenging problems to find appropriate applications as these complex quasi loops may be like loops only under special conditions. These also satisfy several special identities. We using these structures construct non associative rings of finite and infinite order. Several special identities are satisfied by these complex non associative rings.

Using these complex groupoids we built complex matrix groupoids of finite and infinite order and also complex polynomial groupoids of finite and infinite order. They also will find several applications once this field becomes popular.

Chapter Six

SUGGESTED PROBLEMS

In this chapter we suggest around 300 problems, some of them are easy, some difficult and some of them are at research level.

1. Give an example of a complex modulo integer groupoid of order 8^2.

2. Find subgroupoids (complex) of
$P = \{a + bi_F \mid a, b \in Z_{21}, (8, 3), i_F^2 = 20\}$
$= \{C(Z_{21}), *, (8, 3)\}$.
i) What is the order of P?
ii) Does the order of complex subgroupoid divide the order of P?
iii) Can $\{C(Z_{11}), *, (8, 3)\}$ have real subgroupoids?

3. Obtain some interesting properties enjoyed by complex groupoids by type I.

4. Let $G = \{C(Z_{50}), *, (8, 7)\}$ be a complex groupoid of type I.
   i) Find ideals of G.
   ii) What is the order of G?
   iii) Can G have complex subgroupoids which are not ideals of G?

5. Compare the usual groupoid $G = \{Z_n, *, (t, u)\}$ with $G' = \{C(Z_n), *, (t, u)\}$.

6. Does there exists a complex modulo integer Bol groupoid?

7. Give an example of a complex modulo integer Moufang groupoid (if it exists).

8. Does there exists a complex modulo integer P-groupoid?

9. Give an example of a Smarandache strong complex modulo integer Bol groupoid.

10. Obtain some interesting properties enjoyed by complex modulo integer groupoids of type II.

11. Give an example of a complex modulo integer groupoid of type II which is not Moufang.

12. Does there exists a complex modulo integer groupoid of type II which is Bol?

13. Does there exist a complex modulo integer groupoid of type II which is S-strong Moufang?

14. Does there exist a modulo integer groupoid of type I which is a Smarandache Bol and not S-strong Bol?

15. Give an example of a S-strong P-groupoid of complex modulo integers.

16. Characterize all those complex modulo integer groupoids which has ideal in them of type I and type II.

17. Is every complex modulo integer groupoid of type I a S-groupoid?

18. Does there exists a complex modulo integer groupoid of type II which is not a S-groupoid?

19. Can a complex modulo integer groupoid be a S-alternative groupoid of type II?

20. Can a complex modulo integer groupoid of type I have S-right ideals which are not S-left ideals?

21. Does there exist a complex modulo integer groupoid of type II (or type I) which has S-ideals?

22. Can type I complex modulo integer groupoids have zero divisors?

23. Can type II complex modulo integer groupoids have S-zero divisors?

24. Characterize all those complex modulo integer groupoids of type I (or type II) which have only zero divisors and no S-zero divisors.

25. Does there exists a modulo integer complex groupoid of type I (or type II) which has no zero divisors?

26. Characterize those complex modulo integer groupoids of type I (or type II) which has no S-subgroupoids.

27. Characterize all those complex modulo integer groupoids of type I (or type II) in which every subgroupoid is a S-subgroupoid.

28. Characterize all those complex modulo integer groupoids of type I (or type II) which are not S-groupoids.

29. Characterize all those complex modulo integer groupoids of type I (or type II) which are S-groupoids.

30. Characterize all those complex modulo integer groupoids of type I (or type II) which are S-strong P-groupoids.

31. Does there exist a complex modulo integer groupoid which is a S-strong idempotent groupoid?

32. What are the special / distinct features between complex modulo integer groupoids of type I and type II?

33. Obtain some special properties related with type III complex modulo integer groupoids.

34. Let G = $\{C(Z_{42}), *, (13, 0)\}$ be a complex modulo integer groupoid of type III.

    i) Find all subgroupoids of G.
    ii) Find S- subgroupoids of G.
    iii) Can G have S-ideals?
    iv) Does G contain zero divisors?
    v) Does G satisfy any of the special identities?
    vi) Can G have right ideals which are not left ideals?

35. Prove the class of type III complex modulo integer groupoids are never commutative but with zero divisors if n is not a prime (n^2 is the order of groupoid).

36. What are the special features enjoyed by type IV complex modulo integer groupoids?

37. Let $G = \{Z_{53}, *, (11, 11)\}$ be a complex modulo integer groupoid of order 53^2 of type IV.

    i) Find subgroupoids in G.
    ii) Is G a S-groupoid?
    iii) Can G have zero divisors?
    iv) Is G a commutative groupoid?
    v) Does G satisfy the associative law?
    vi) Can G have S-ideals?
    vii) Does G satisfy any of the special identities?
    viii) Is G a S-strong P-groupoid?

38. Characterize those complex modulo integer groupoids of type III and type IV which are S-groupoids.

39. Characterize those complex modulo integer groupoids of type III and type IV which are S-strong alternative groupoids.

40. Prove or disprove in case complex modulo integer groupoids of type IV every right ideal is also a left ideal.

41. Does type IV complex modulo integer groupoids enjoy any special or distinct properties from type I, type II and type III groupoids?

42. Obtain some interesting properties about Smarandache semi conjugate subgroupoids of complex modulo integer type IV groupoids.

43. Does every complex modulo integer groupoid of all types have semi conjugate subgroupoids? Justify.

44. Obtain some special properties enjoyed by complex modulo integer Smarandache conjgate subgroupoids of type I, II, III and IV.

45. Is every type IV groupoid of complex modulo integers a P-groupoid? Justify your claim.

46. Characterize those complex modulo integer simple groupoids of type I, type II, type III and type IV.

47. Characterize those complex modulo integer groupoids of type IV which are alternative groupoids.

48. Prove G = $\{C(Z_{40}), *, (11, 11)\}$ is not an alternative complex modulo integer groupoid.

49. Prove G = $\{C(Z_{12}), *, (4, 4)\}$ is an alternative complex modulo integer groupoid.

50. Prove G = $\{C(Z_{43}), *, (3, 3)\}$ is not an alternative complex modulo integer groupoid.

51. Is G =$\{C(Z_{53}), *, (13, 13)\}$ a complex modulo integer normal groupoid?

52. Characterize complex modulo integer groupoids of all types which are simple.

53. Is the complex modulo integer groupoid G = $\{C(Z_{10}), *, (4, 0)\}$ a P-groupoid?

54. Can G = $\{C(Z_{43}), *, (0, 9)\}$, the complex modulo integer groupoid be a P-groupoid?

55. Let G = $\{C(Z_{10}), *, (5, 6)\}$ be a complex modulo integer groupoid.

    i) What is the order of G?
    ii) Is G simple?
    iii) Is G a S-groupoid?
    iv) Is G a S-moufang groupoid?
    v) Is G a S-strong Moufang groupoid?
    vi) Is G a S-strong Bol groupoid?

vii) Is G a S-strong P-groupoid?
viii) Can G have S-subgroupoids?

56. Let G = {C(Z_{12}), *, (3, 4)} be a complex modulo integer groupoid.

i) Is G a G-groupoid?
ii) Find subgroupoids of G.
iii) Does G have subgroupoids which are not S-subgroupoids?
iv) Find S-ideals if any in G.
v) Find S-zero divisors if any in G.
vi) Is G a S-Moufang groupoid?
vii) Is G a S-strong Bol groupoid?
viii) Find any other interesting properties enjoyed by G.

57. Let G = {C(Z_{49}), *, (3, 12)} be a complex modulo integer groupoid.

i) Is G a S-groupoid?
ii) Does G have S-subgroupoids which divides the order of G?
iii) Is G a S-strong Bol groupoid?
iv) Can G hae S-zero divisors?
v) What are the special features enjoyed by G?
vi) Can G have zero divisor which are not S-zero divisors?
vii) Is G a normal subgroupoid?

58. Obtain condition for a complex modulo integer groupoid to be a Smarandache idempotent groupoid.

59. Let G = {C(Z_{19}), *, (12, 8)} be a groupoid of complex modulo integers.

i) Find order of G.
ii) Is G a S-groupoid?
iii) Is G a S-idempotent groupoid?
iv) Can G have S-subgroupoid?

v) Is G a S-P- groupoid?
vi) Can G have zero divisors?
vii) Can G have ideals?

60. Let G ={$C(Z_{12})$, *, (4, 9)} be a complex modulo integer groupoid.

i) Is G a S-strong Bol groupoid?
ii) Is G a S-strong Moufang groupoid?
iii) Can G have S-zero divisors?
iv) Does G contain subgroupoids which are not S-subgroupoids?
v) Does G contain normal subgroupoids?
vi) Can G have S-ideals?
vii) Is G simple?

61. Let G = {$C(Z_{13})$, *, (7, 7)} be a complex modulo integer groupoid.

i) Is G a Smarandache idempotent groupoid?
ii) Does G have S-subgroupoids H such that o(H)/o(G)?
iii) Is G a S-strong Bol groupoid?
iv) Can G be a simple?
v) Is G a S-alternative groupoid?
vi) Can G have zero divisors?
vii) Can G have normal S-subgroupoids?

62. Characterize those complex modulo integer groupoids which are S-idempotent groupoids.

63. Describe some unique properties enjoyed by complex modulo integer groupoids with identity {e} adjoined to it.

64. Are these new class of groupoids of complex modulo integers S- groupoids?

65. Can these groupoids G be S-idempotent groupoids?
(G = {$C(Z_n \cup \{e\})$, *, (t, u)})

66. Find condition on $G = \{C(Z_n \cup e), *, (t, u)\}$ be a Smarandache strong right alternative modulo complex integer groupoid.

67. If n in problem (66) is a prime number, can G be a Smarandache strong right alternative complex modulo integer groupoid?

68. Find some properties enjoyed by $C(L_n(m))$; complex modulo integer loops.

69. Find the order of the complex modulo integer loop $C(L_n(m))$.

70. Let $C(L_{19}(3))$ be a complex modulo integer loop.

    i) Find order of L.
    ii) Is L a S-loop?
    iii) Can L have S-subloops?
    iv) Is L a WIP loop?
    v) Can L have normal subloops?
    vi) Does L contain subloops which are not S-subloops?
    vii) Is L simple?

71. Show $C(L_n) = \{C(L_n(m) \mid n \text{ odd}, n < m, (m-1, n) = 1, (m, n) = 1\}$ the class of complex modulo integer loops has one and only one commutative loop when, $m = \frac{n+1}{2}$.

72. Obtain the number of complex modulo integer loops in the class $C(L_n)$.

73. Is every complex modulo integer loop in $C(L_n)$ simple?

74. Is every complex modulo integer loop in $C(L_n)$ a S-loop?

75. Let L = $C(L_{43}(42))$ be a complex modulo integer loop.

i) Find order of $C(L_{43}(42))$.
ii) Is L left alternative?
iii) Is L S-simple?
iv) Can L be right alternative?
v) Can L have proper S-subloops?
vi) Is L a S-loop?
vii) Can L have subloops other than subgroups of order four or two?

76. Prove $C(L_n)$ cannot contain any Bol loop.

77. Prove in a complex modulo integer loop L = $C(L_n(m))$, the order of a subloop of L in general need not divide order of L.

78. Let L = $C(L_n(m))$ be a complex modulo integer loop. Is L a S-strong cyclic loop?

79. Let L = $C(L_{13}(2))$ be a complex modulo integer loop.

i) Is L a right alternative loop?
ii) Is L a left alternative loop?
iii) Is L a S-loop?
iv) Is L a simple loop?

80. Let $C(L_9(8))$ = L be a complex modulo integer loop.

i) Does L contain S-subloops?
ii) Does L satisfy any special identity?
iii) Is L S-simple?
iv) Is L a S-strong Bol loop?
v) Can L be S-strong Moufang loop?
vi) Find in L two conjugate S-subloops. (Is it possible? Justify).
vii) Can L have normal subloops?
viii) Is it possible to verify Lagranges theorem for this loop?

81. Let L = $C(L_7(3))$ be a loop of complex modulo integers.

i) Is L a S-commutative loop?
ii) Is L a S-strongly commutative loop?
iii) Is L a S-strongly cyclic loop?
iv) Is L a power associative loop?
v) Is L S-pseudo commutative?
vi) Is L a S-strongly pseudo commutative?
vii) Find S-commutator subloop of L.
viii) Is L a S-weakly Lagrange loop?

82. Give an example of a S-weakly Lagrange loop of complex modulo integers.

83. Give an example of a complex modulo integer loop which is a S-Lagrange loop.

84. Give an example of a complex modulo integer loop which is a S-Cauchy loop.

85. Give an example of a Smarandache strong 2-Sylow loop.

86. Give an example of a complex modulo integer loop which is left semi alternative.

87. Find the Moufang centre of $C(L_{17}(5))$.

88. Find the commutator subloop of $C(L_{17}(2))$.

89. Give an example of a strictly non commutative loop in $C(L_{325})$

90. Show $C(L_{19}(8))$ is not a Bruck loop.

91. Prove $C(L_{43}(2))$ is not a Moufang loop.

92. Find all subloops of $C(L_{27}(8))$.

93. Find all S-subloops of $C(L_{125}(2))$.

94. Find all cyclic groups in $C(L_{13}(4))$.

95. Is $C(L_{25}(2))$ a S-diassociative loop?

96. Find all S-centres of the complex modulo integer loop $C(L_{55}(13))$.

97. Find SN_1 and SN_2 for all subloops in $C(L_{59}(10))$.

98. Find for the complex modulo integer loop $L = C(L_5(4))$ the S-left and S-right coset decomposition relative to any subgroup in L.

99. Find for the group $A = \{e, 10\}$ in $C(L_{19}(2))$ the S-right coset representation.

100. Find a S-hyperloop of $C(L_{17}(9))$.

101. Is it true for a complex modulo integer loop $C(L_{23}(7))$; $NZ (L_{23}(7)) = Z (L_{23}(7)) = e$?

102. Obtain some nice properties enjoyed by $C(Z_9)G$ where $G = \{Z_{10}, *, (3, 7)$, the complex modulo integer groupoid ring.

103. Let $F = C(Z_{12}) = \{a + bi_F \mid a, b \in Z_{12}, i_F^2 = 11\}$ be the complex modulo integer ring. $G = \{Z_{19}, *, (7, 2)\}$ be the groupoid. FG be the groupoid ring.

   i) Find the number of elements of FG.
   ii) Does FG have ideals?
   iii) Is FG a S-ring?
   iv) Can FG have subrings which are not S-subrings?
   v) Can FG have S-ideals?
   vi) Does FG contain S-zero divisors?
   vii) Find right ideals in FG which are not left ideals.
   viii) Using a two sided ideal find the quotient ring.

104. Let $G = \{Z_{25}, *, (8, 0)\}$ be a groupoid
$F = C(Z_{15}) = \{a + bi_F \mid i_F^2 = 14\}$ be the complex modulo integer ring. FG be the groupoid ring. FG is a non associative complex ring of finite order.

i) Is FG a S-ring?
ii) Is FG a S-strong Bol ring?
iii) Does FG satisfy any special identity?
iv) Can FG have zero divisors which are not S-zero divisors?
v) Can FG have ideals which are not S-ideals?

105. Let $G = \{Z_9, *, (0, 5)\}$ be a groupoid.
$F = C(Z) = \{a + bi \mid a, b \in Z\}$ be the complex field. FG be the groupoid ring. FG is the non associative complex ring of characteristic zero.

i) Find ideals in FG.
ii) Is FG have zero divisors?
iii) Can FG have zero divisors?
iv) Can FG have idempotents?
v) Can FG have S-ideals?
vi) Can FG have subrings which are not ideals?
vii) Does FG satisfy any of the special identities?

106. Let $G = \{Z_{40}, *, (3, 3)\}$ be a groupoid.
$F = \{a + ib \mid a, b \in Q\} = C(Q)$ be the rational complex ring. FG be the groupoid ring.

i) Is FG a S-ring?
ii) Can FG have zero divisors?
iii) Is FG a commutative ring?
iv) Can FG have ideals which are not S-ideals?
v) Can FG have subrings which are not ideals?
vi) Can FG have S-dempotents?
vii) Can FG have nilpotent elements.
viii) Does FG satisfy any special identity?

107. Let $G = \{Z_{140}, *, (9, 0)\}$ be a groupoid.
$F = \{a + bi_F \mid a, b \in Z_2, i_F^2 = 1\}$ be the complex modulo integer ring. FG be the groupoid ring.

i) Is FG have S-ideals?
ii) Can FG have S-ideals?
iii) What is the order of FG?
iv) Find zero divisors in FG which are not S-zero divisors.
v) Can FG have ideals which are not S-ideals?
vi) Find S-idempotents if any in FG.

108. Let $G = \{Z_{17}, *, (9, 8)\}$ be a groupoid. F = C the complex field. FG be the groupoid complex ring.

i) Show FG is non associative.
ii) Prove FG is non commutative.
iii) Find ideals if any in FG.
iv) Prove FG satisfies some special identities.
v) Is FG a S-strong right alternative ring?
vi) Is FG a S-strong idempotent ring?
vii) Can FG have S-zero divisors?

109. Let $G = \{Z_6, *, (5, 2)\}$ be a groupoid.
$F = C(Z) = \{a + bi \mid a, b \in Z\}$ be a complex integer ring. FG the groupoid ring.

i) Prove FG has left ideals which are not right ideals.
ii) Can FG have two sided ideals?
iii) Is FG a S-ring?
iv) Does FG satisfy any of the special identities?
v) Prove FG is a non associative ring.
vi) Can FG have zero divisors and S-zero divisors?

110. Let $L = L_9(8)$ be a loop of order 10.
$F = C(Z) = \{a + bi \mid a, b \in Z\}$ be the complex integer ring. FL be the loop ring.

i) Is FL a S-ring?

ii) Prove FL is non associative?
iii) Is FL a commutative ring?
iv) Can FL be a S-commutative ring?
v) Can FL have zero divisors?
vi) Can FL have idempotents?
vii) Can FL have ideals?
viii) Can FL be simple?

111. Let $L = L_{21}(20)$ be a loop and $F = \{C(Z_3) = \{a + bi_F \mid a, b \in Z_3, i_F^2 = 2\}$ be the complex modulo integer ring. FL be the loop ring.

i) Prove FL is non associative.
ii) Prove FL satisfies some special identities.
iii) Is FL a S-ring?
iv) Can FL have subrings which are not ideals?
v) Prove FL is non commutative.
vi) Prove FL has associative subrings.
vii) Can FL have S-ideals?

112. Let $L = L_{12}(2)$ be a loop. $F = \{C(Z_3) = \{a + bi_F \mid a, b \in Z_2, i_F^2 = 2\}$ be the complex modulo integer ring. FL be the loop ring.

i) Prove FL is non associative.
ii) Prove FL is not alternative.
iii) Is FL right alternative?
iv) Can FL be left alternative?
v) Can FL have zero divisors?
vi) Is FL simple?
vii) Can FL have S-ideals?
viii) Is FL a S-ring?
ix) Can FL have associative subrings?

113. Give an example of a right alternative ring using the loops in L_{19} over any complex field F.

114. Give an example of a left alternative ring using the loop in $L_{43} = \{L_{43}(m) \mid m < 43, (m, 43) = 1$ and $(m–1, 43) = 1\}$ and any complex ring F.

115. Let $L = L_{25}(8)$ be a loop and
$F = \{C(Z_{25}) = \{a + bi_F \mid a, b \in Z_{25}, i_F^2 = 24\}$ be a complex modulo integer ring. FL be the loop ring.

i) Prove FL is non associative.
ii) Prove FL is non commutative.
iii) Prove FL is a S-ring.
iv) Find ideals in FS.
v) Show FL has atleast 24 complex subrings which are associative.
vi) Can FL have ideals?
vii) Show FL has subrings which are not subideals?
viii) Does FL satisfy any of the special identities?

116. Let $L = L_{11}(6)$ be the loop.
$F = \{C(Z_5) = \{a + bi_F \mid a, b \in Z_5, i_F^2 = 4\}$ be a complex modulo integer ring. FL be the complex modulo integer loop ring.

i) Prove FL is commutative.
ii) Prove FL is non associative
iii) Is FL a S-ring?
iv) Can FL have S-zero divisors?
v) Can FL have ideals?
vi) Does FL satisfy any of the special identities?
vii) Show FL has associative subrings.
viii) Can FL have subrings which are not S-subrings?

117. Let $L = L_7(2)$ be a loop.
$F = \{a + bi_F \mid a, b \in Z_7, i_F^2 = 6\}$ be the complex modulo integer ring. FL be the complex modulo integer loop ring.

i) What is the order of FL?
ii) Is FL a S-ring?

iii) Can FL have ideals?
iv) Can FL have S-subrings?
v) Can FL have zero divisors?
vi) Show FL cannot have S-zero divisor. (verify)
vii) Can FL have S-idempotents?

118. Let $L = L_n(m)$ be a loop.
$F = C(Z_n) = \{a + bi_F \mid a, b \in Z_n, i_F^2 = n-1\}$ be the complex modulo integer ring. FL be the loop ring.

i) What is the order of FL?
ii) When is FL a left alternative ring? (that is for what value of m)
iii) For what value of m, is FL a right alternative ring?
iv) Does there exist a m for which FL is alternative?
v) For what value of m; FL is commutative?
vi) For what values of m, FL is WIP ring?

119. Let $L = L_{17}(3)$ be a loop.
$F = C(Z_3) = \{a + bi_F \mid a, b \in Z_3, i_F^2 = 2\}$ be a complex modulo integer ring. FL be the complex modulo integer loop ring.

i) Find order of FL
ii) Is FL a S-ring?
iii) Can FL have zero divisors?
iv) Is FL simple?

120. Let $L = L_n(m)$ be a loop of order n+1.
$F = C(Z_m) = \{a + bi_F \mid a, b \in Z_m, i_F^2 = m-1\}$ be a complex modulo integer ring. FL be the loop ring.

i) Prove FL is non associative.
ii) What is the order of FL?
iii) Does FL have ideals?
iv) Is FL a S-ring?
v) Can FL have zero divisors?

vi) Study (1) to (v) for $n = 5$ and $m = 2$ and $F = C(Z_2)$.
vii) If $n = 5$ and $m = 3$, what is the speciality about FL.

121. Let $L = L_9(5)$ be a loop and $F = C(R) = \{a + ib \mid a, b \in R\}$ be the complex field. FL be the complex loop ring.

i) Prove FL is non associative.
ii) Can FL have zero divisors?
iii) Is FL simple?
iv) Prove FL have subrings which are not ideal.
v) Prove FL has real loop subring.
vi) Is FL a S-ring?
vii) Does FL satisfy any of the special identities?
viii) Is FL commutative?

122. Let $L = L_{13}(12)$ be a loop. $F = C(Q) = \{a + bi \mid a, b \in Q\}$ be the rational complex ring. FL be the loop ring.

i) Is FL associative?
ii) Is FL commutative?
iii) Can FL be right alternative?
iv) Find zero divisors if any in FL.
v) Is FL a S-ring?
vi) Can FL have ideals?
vii) Is FL simple?
viii) Can FL have subrings which are not ideals?

123. Let $L = L_{11}(3)$ be a loop. $F = \{C(Z_{11}) = \{a + bi_F \mid a, b \in Z_{11}, i_F^2 = 10\}$ be a complex modulo integer ring. FL be the complex modulo integer ring.

i) Prove FL is non associative.
ii) Is FL a S-ring?
iii) Find in FL zero divisors or S-zero divisors.
iv) What is the order of FL?

v) Can FL have ideals?
vi) Can FL have subrings which are non associative?

124. Let G = {C(Z_3), *, (1, 2)} be a complex groupoid. F = Z_3 be the ring of modulo integer FG be the complex groupoid ring.

i) Prove G is non associative.
ii) Find the number of elements in FG.
iii) Is FG a S-ring?
iv) Can FG have S-ideals?
v) Can FG have zero divisors?
vi) Find S-subrings in any in FG.
vii) Does FG contain pseudo basic ring ideals?
viii) Does FG satisfy any of the special identities?

125. Let G = {C(Z_8), *, (7, 7)} be a complex modulo integer groupoid. F = R be the field of reals. FG be the complex groupoid ring.

i) Prove FG is non associative.
ii) Prove FG is commutative.
iii) Find S-ideals if any in FG.
iv) Can FG have S-subrings which one not ideals in FG?
v) Prove FG has pseudo basic ring ideals.
vi) Can FG have subrings which are not ideals?
vii) Can FG have S-zero divisors and S-idempotents?

126. Let G = {C(Z_{12}), *, (6, 3)} be a complex modulo integer groupoid. F = Z_{10} be the ring of modulo integers. FG be the complex modulo integer groupoid ring.

i) Prove FG is non associative.
ii) Find the number of elements in FG.
iii) Can FG have zero divisors?
iv) Does FG have S-subrings?
v) Is FG a S-ring?
vi) Can FG have S-ideals?

vii) Is PG where P = {0, 2, 4, 6, 8} = Z_{15} be a subring an ideal of FG?

viii) Can FH where H = {Z_{12}, *, (6, 3)} be an ideal of FG?

127. Obtain some interesting properties enjoyed by the compelx modulo integer groupoid ring, FG where F is real and G is a complex modulo integer groupoid.

128. Distinguish between the complex modulo integer groupoid rings and groupoid complex rings.

129. Let G = {C(Z_{10}), *, (5, 8)} be a complex modulo integer groupoid and F = Z_{10} be the real ring. FG the complex groupoid ring. Let H = {Z_{10}, *, (5, 8)} be a groupoid. K = {C(Z_{10}) = {a + bi_F | a, b $\in$ Z_{10}, i_F^2 = 9} be the complex ring of modulo integer KH be the groupoid complex ring.

i) Find order of KH.
ii) Find order of FG.
iii) Is FG $\cong$ KH?
iv) Give any striking differences between KH and FG.
v) Prove both KH and FG are associative.
vi) Can both KH and FG have ideals?
vii) Are these two rings, S-rings?
viii) Can these rings have S-zero divisors?
ix) Determine any special features enjoyed in common between them.

130. Let G = {C(Z_n), *, (t, u)} be a complex modulo integer groupoid. F = R the field of reals. FG be the complex groupoid ring. FG is a Smarandache strong Moufang groupoid ring if and only if $t^2 \equiv t \pmod n$ and $u^2 = u \pmod n$, n a non prime.

i) Is the claim true?
ii) Is FG Smarandache Moufang groupoid ring?
iii) Can FG have zero divisors?

iv) Prove FG is a S-ring.
v) Can FG have S-ideals?

131. Let $G = \{C(Z_6), *, (3, 4)\}$ be the complex groupoid ring of modulo integers. F = R, the field of reals. FG be the groupoid ring.

i) Prove FG is non associative.
ii) Prove FG is non commutative.
iii) Is FG a S-ring?
iv) Does FG have S-zero divisors?
v) Can FG have S-ideals?
vi) Prove FG have S-subrings which are not S-ideals?
vii) Prove FG has a Smarandache strong Moufang subring.
viii) Can FG satisfy any other special identity?

132. Let $G = \{C(Z_{17}), *, (8, 8)\}$ be a complex modulo integer groupoid. R = F be the reals. FG be the complex groupoid ring.

i) Prove FG is non associative.
ii) Is FG a commutative ring?
iii) Can FG have a-S-idempotents subring?
iv) Is FG a S-ring?
v) Can FG have S-ideals?
vi) Can FG have S-subrings which are not S-ideals?
vii) Is FG simple?
viii) Can FG have S-zero divisors?
ix) Can FG be isomorphic with any other ring?

133. Let $G = \{C(Z_{40}), *, (10, 3)\}$ be a complex modulo integer groupoid. $F = Z_{40}$ be the ring of modulo integers FG be the complex groupoid ring.

i) Find order of FG.
ii) Prove FG is non associative.
iii) Prove FG is non commutative.
iv) Is FG a S-ring?

v) Can FG have S-ideals?
vi) Can FG has S-zero divisors?
vii) Can FG have S-subrings which are not ideals?
viii) Can FG have pseudo basic ring ideals?

134. Let G = {C(Z_{25}), *, (12, 13)} be the complex modulo integer groupoid. F = Z_5 be the field of characteristic Z_5 be the field of characteristic five. FG the complex groupoid ring.
i) Answer the questions (1) to (viii) of problem (133) for this FG.

135. Let G = {C(Z_n), *, (m, t)} be the complex modulo integer groupoid. F = Z_t when t/n, n = tp be the ring of modulo integers. FG be the groupoid ring.

i) Find order of FG.
ii) Prove FG is non associative.
iii) Can FG be a S-strong Moufang ring?
iv) Find condition on m and t so that FG is a S-strong Bol ring.
v) If n = t^2 find order of FG.
vi) If n = t find order of FG.
vii) If n = 3t find order of FG
viii) If n = 6t find order of FG.
ix) Compare the properties enjoyed by FG for n = 2t, n = t and n = 6t.

136. Let G = {C(Z_n), *, (m, m)} be a complex modulo integer groupoid ring. F = Z_t where n/t be the ring of modulo integers. FG be the complex groupoid ring.

i) Find order of FG if t = 3n.
ii) Find order of FG if t = n^2.
iii) Find order of FG if t = 10n.
iv) Is FG a commutative ring?
v) Is FG a S-ring?
vi) Find S-ideals if any in FG.
vii) Does FG contain S-subrings which are not ideals?

137. Let G = $\{C(Z_{36}), *, (7, 7)\}$ be a complex modulo integer groupoid. $F = Z_3$ be the field of characteristic three. FG be the complex groupoid ring.

i) Find order of FG.
ii) Is FG commutative?.
iii) Prove FG is associative.
iv) Is FG a S-ring?
v) Is FG simple?
vi) Does FG satisfy any of the special identities?
vii) Can FG have S-ideals?
viii) Can FG have pseudo basic ring ideals?

138. Give an example of Smarandache strong Bol complex modulo integer groupoid ring.

139. Give an example of a Smarandache strong Moufang complex modulo integer ring.

140. Give an example of a Smarancahe strong alternative complex modulo integer groupoid ring.

141. Give an example of a Smarandache strong right alternative complex modulo integer groupoid ring which is not a Smarandache strong left alternative complex modulo integer ring.

142. Given an example of Smarandache idempotent complex modulo integer groupoid ring.

143. Give an example of a Smarandache Moufang complex modulo integer groupoid ring.

144. Give examples of Smarandache strong complex modulo integer groupoid P-rings.

145. Give an example of a complex modulo integer groupoid ring which is not a S-ring.

146. Give an example of a complex modulo integer groupoid ring which has no ideals.

147. Give an example of a complex modulo integer groupoid ring in which every subring is a S-subring.

148. Give an example of a complex modulo integer groupoid ring FG in which no subring is a S-subring but FG is a S-ring.

149. Give an example of a complex modulo integer groupoid ring FG in which every subring in an ideal.

150. Give an example of a complex modulo integer groupoid ring FG which has only right ideals and no left ideals.

151. Does there exists a complex modulo integer groupoid ring FG in which every zero divisor is a S-zero divisor?

152. Does there exists a complex modulo integer groupoid ring FG with no zero divisors?

153. Does there exists a complex modulo integer groupoid ring FG with no idempotents?

154. Does there exist a complex modulo integer groupoid ring FG which has principal ideals?

155. Find any interesting properties enjoyed by complex modulo integer ring FG where F = R or Q or Z.

156. Let FG be a complex modulo integer groupoid ring where $F = Z_n$.
Determine the special properties enjoyed by this $FG = Z_nG$.

157. Let $L = C(L_{11}(8))$ be a complex modulo integer loop. $F = Z_{11}$ be the field of characteristic eleven. FL be the

complex modulo integer loop ring. Find the special features enjoyed by FL.

158. Let L = $C(L_{13}(11))$ be a complex modulo integer loop. F = Z_{13} be the field of characteristic thirteen. FL be the complex modulo integer loop ring. Determine the special properties satisfied by FL.

159. Let L = $C(L_7(3))$ be a complex modulo integer loop. F = Z_7 be the field of characteristic seven. FL the complex modulo integer loop ring.

i) Find order of FL.
ii) Prove FL is a non associative ring.
iii) Prove FL is a non commutative finite ring.
iv) Is FL a S-ring?
v) Is FL S-Moufang ring?
vi) Is FL a S-alternative ring?
vii) Can FL have S-ideals?
viii) Can FL have zero divisors which are not S-zero divisors?

160. Let L = $C(L_7(3))$ be a complex modulo integer loop. F = Z_3 be the finite field with three elements. FL the complex modulo integer loop ring.

i) Find order of FL.
ii) Compare it with order of FL in problem (159).
iii) Can FL be S-commutative?
iv) Can FL have subrings?
v) Can FL have S-ideals?
vi) Find zero divisors if any in FL.
vii) Can FL have S-idempotents?
viii) Is FL simple?
ix) Find the special identity satisfied by FL.

161. Let L = $Z(L_7(4))$ be a loop of complex modulo integers. F = Z_4 be the ring of modulo integers. FL be the loop ring.

    i) Find order of FL.
    ii) Prove FL is non associative.
    iii) Prove FL is commutative.
    iv) Prove FL has ideals.
    v) Can FL have S-ideals?
    vi) Can FL have S-subrings which are not S-ideals?
    vii) Does FL satisfy any of the special identities?

162. Give an example of a complex modulo integer loop ring of finite order which is right alternative.

163. Give an example of a complex modulo integer loop ring of infinite order which is left alternative.

164. Does there exists a non associative alternative complex loop ring of finite order?

165. Give an example of a complex loop ring which satisfies the Moufang identity.

166. Give an example of a complex loop ring FL of finite order which satisfies the Bol identity (L is a complex modulo integer loop).

167. Obtain any striking property enjoyed by complex loop rings FL where L is a complex modulo integer loop.

168. Is every complex modulo integer loop ring FL a S-ring?

169. Give an example of a complex modulo integer loop L over the real field F = R, (RL the loop ring) in which every subring is a S-ring.

170. Let L = C($L_{15}(8)$) be the complex modulo integer loop. F = Z_{15} be the ring of characteristic fifteen. FL be the complex modulo integer loop ring.

i) Find order of FL.
ii) Prove FL is commutative.
iii) Does FL satisfy any of the special identities?
iv) Find S-ideals if any in FL.
v) Find S-subrings if any in FL which are not S-ideals.
vi) Find S-zero divisors in FL.
vii) Prove FL has ideals.

171. Let L = C($Z_{45}(8)$) be a complex modulo integer loop. F = Z_2 be the field of characteristic two. FL be the loop ring.

i) Find order of FL.
ii) Is FL a S-ring?
iii) Find ideals if any in FL.
iv) Find S-subring if any in FL.
v) Is FL simple?
vi) Can FL have S-subrings?
vii) Does FL satisfy any special identity?
viii) Is FL commutative?

172. Let L = C($Z_{55}(11)$) be a complex modulo integer loop. F = Z_{11} be the field. FL the complex modulo integer loop ring.

i) Show FL is finite order.
ii) Find S-zero divisors if any.
iii) Find S-ideals if any FL.
iv) Find S-subrings in FL.
v) Is FL a S-ring?
vi) Does FL satisfy any special identities?
vii) Is FL simple?
viii) Find S-idempotents if any in FL.
ix) Prove FL is non associative.

173. Let $L = C(L_{21}(11)\}$ be a complex loop. $F = Z_{11}$ be the field of characteristic eleven. FL the complex modulo integer ring.

i) Is FL non associative?
ii) Find order of FL.
iii) Find S-zero divisors in FL
iv) Find S-ideals.
v) Find S-subrings if any in FL.
vi) Can FL satisfy any special identity?
vii) Prove FL is non commutative.

174. Let $L = C(L_{19}(9))$ be a complex modulo integer ring. $F = Z_{19}$ be the field of characteristic zero. FL be the complex loop ring.

i) Find some interesting properties associated with FL
ii) Is FL commutative?
iii) Prove FL is non associative.
iv) Find order of FL.
v) Since basically both the loop $L_{19}(9)$ and the loop ring, where the ring is also Z_{19}, the loop ring enjoy any special properties. Hence or otherwise what is the special property enjoyed by C $(L_p(m))$ the complex modulo integer loop built over the ring Z_p?

175. Let $L = C\,(Z_9(8))$ be a complex modulo integer loop. $F = Z_8$ be the ring. FL be the complex loop ring.

i) Find order of FL.
ii) What is the special property enjoyed by FL as both $m = 8$ and the ring F over which L is defined is also Z_8. Hence or otherwise derive some special properties associated with the complex modulo integer loop ring. FL where $L = C(Z_n(m))$ is the complex modulo integer loop and $F = Z_m$.

176. Let L = {C(L_{27}(11)} be a complex modulo integer loop. F = Z_{11} be the field. FL be the complex modulo integer loop ring.

i) Find order of FL.
ii) Is FL a S-ring?
iii) Can FL have S-zero divisors?
iv) Is FL simple?
v) Prove FL is non associative.
vi) Can FL have S-subrings which are not S-ideals?
vii) Can FL have subrings which are not S-subring?
viii) Prove FL is non commutative.
ix) Does FL enjoy any special property because L = C(L_{27}(11)) and F = Z_{11}?

177. Let L = C(L_{13}(7)) be a complex modulo integer loop. F = Z_{13} be the ring. FL the complex loop ring.

i) Is FL a commutative loop ring?
ii) Find order of FL.
iii) Can FL be a S-ring?
iv) Can FL have subrings S such that o (S) / o (FL)?
v) Is FL simple?
vi) Can FL have subrings which are not ideals?
vii) What will be order of FL if Z_{13} = F is replaced by Z_7?
viii) Answer all the questions (i) to (vi)

178. Let L = C(L_{21}(11)) be a complex modulo integer loop. F = Z be the ring. FL the complex modulo integer loop ring.

i) Prove FL is non associative.
ii) Is FL a S-ring?
iii) Can FL be simple?
iv) Can FL satisfy a.c.c. condition on ideals?
v) Can FL have idempotents?
vi) Can FL have zero divisors?
vii) Can FL have S-subrings which are not ideals?
viii) Can FL have subrings which are not S-subrings?

179. Enumerate some special properties enjoyed by strong complex modulo integer groupoids of type I.

180. Let G = $S(C(Z_{11})) = \{a + bi_F \mid a, b \in Z_{11},\ i_F^2 = 10, *, (8i_F, 3i_F)\}$ be the strong complex modulo integer groupoid of type I.

i) Find order of $S(C(Z_{11}))$.
ii) Is G a commutative groupoid?
iii) Can G have S-zero divisors?
iv) Is G a S-groupoid?
v) Compare G with H = $\{C(Z_{11}), *, (8, 3)\}$.
vi) What is the main difference between H and G?

181. Obtain some interesting properties of strong complex modulo integer groupoids of type II.

182. Let G = $(C(Z_{29})) = \{a + bi_F \mid a, b \in Z_{29},\ i_F^2 = 28, *, (10i_F, 12i_F)\}$ be the strong complex integer groupoid of type II.

i) Is G = S $(C(Z_{29}))$ a S-groupoid?
ii) Find order of G.
iii) Find subgroupoids if any in G.
iv) Does G have S-subgroupoids?
v) Is G simple?
vi) Is G a commutative groupoid?
vii) Does G satisfy any special identity?

183. Prove all strong complex modulo integer groupoids of type IV are commutative.

184. Obtain some special properties enjoyed by strong complex modulo integer groupoids of type V.

185. Characterize those strong complex modulo integer groupoids of type I which are Moufang.

186. Obtain those strong complex modulo integer groupoids which are S-strong Moufang groupoids.

187. Does there exists Bol groupoids from the class of strong complex modulo integer groupoids of type III?

188. Characterize strong complex Smarandache strong Moufang groupoid built using $C(Z_n)$.

189. Give a characterization theorem for strong complex Smarandache strong Bol groupoids.

190. Obtain conditions for a strong complex modulo integer groupoids to be a Smarandache strong P-groupoid.

191. Give an example of a strong complex modulo integer groupoid which is a Smarandache strong right alternative groupoid.

192. Give an example of a Smarandache Bol groupoid from the class of strong complex modulo integer groupoids.

193. Give an example of a Smarandache Moufang groupoid from the class of strong complex modulo integer groupoids.

194. Does there exists a Smarandache strong Bol groupoid from the class of strong complex modulo integer groupoids of type IV?

195. Give some interesting properties enjoyed by strong complex modulo integer groupoids of type VI.

196. Distinguish the properties enjoyed by type IX and type X strong complex groupoids of modulo integers.

197. Can type X strong complex modulo integer groupoids be commutative?

198. Can type X strong complex modulo integer groupoids have zero divisors?

199. Can type IX strong complex modulo integer groupoids have ideals?

200. Will every strong complex modulo integer groupoid of type IX be simple?

201. Can all type VIII strong complex modulo integer groupoids be Smarandache P-groupoids?

202. Can type VII strong complex modulo integer groupoids be S-strong idempotent groupoids?

203. Establish some special features enjoyed by type VI strong complex modulo integer groupoids.

204. Compare type II and type IV strong complex modulo integer groupoids.

205. Which type of strong complex groupoids satisfy several special identities?

206. What type of groupoids are simple strong complex groupoids?

207. Does there exist any type of strong complex groupoids of modulo integers which has no S-subgroupoids?

208. Characterize those type of strong complex groupoids which are S-groupoids.

209. Characterize those type of strong complex groupoids which are normal.

210. Characterize those types of strong complex groupoids which are simple.

211. Characterize those type of strong complex groupoids which has S-ideals.

212. Characterize those types of strong complex groupoids which has S-zero divisors.

213. Characterize those types of strong complex groupoids which has no zero divisors.

214. Characterize those types of strong complex groupoids has no S-idempotents.

215. Characterize those strong complex groupoids which are Smarandache strong idempotent groupiods.

216. Does there exists much difference between a complex groupoid and a strong complex groupoid?

217. Does any of the strong complex groupoid of infinite order satisfy any of the special identities?

218. Can we say strong complex groupoids of infinite order are S-groupoids?

219. Can we say strong complex groupoids of infinite order are simple?

220. Does strong complex groupoids of infinite order have zero divisors?

221. Can strong complex groupoids of infinite order have S-idempotents?

222. Let G = $\{S(C(Z_{20}), * (8i_F, 3i_F+1), *\}$ be a strong complex modulo integer groupoid.

   i) Find order of G.
   ii) Is G simple?
   iii) Is G a S-groupoid?

iv) Find S-zero divisors if any in G.
v) Find S-ideals if any in G.
vi) Find S-subgroupoids in any in G.
vii) Find the types of identities satisfied by G.

223. Let
$G = S(C(R)) = \{a + bi \mid a, b \in R, i^2 = -1, (3i, -\sqrt{7}, 4-\sqrt{19i}), *\}$ be a strong complex groupoid of infinite order.

i) Is G commutative?
ii) Prove G is non associative.
iii) Is G simple?
iv) Can G have S-ideals?
v) Can G have S-subgroupoids?
vi) Can G have S-zero divisors?
vii) Can G satisfy any of the special identities?

224. Let
$G = \{S(C(Z)) = \{a + bi \mid a, b \in R, i^2 = -1,$
$(3i_F, 20 - 7i_F), *\}$ be the strong complex integer groupoid of infinite order.

i) Find S-ideals if any in G.
ii) Is G a S-groupoid?
iii) Can G have S-idempotents?
iv) Will G satisfy Bol identity?
v) Can G be simple?
vi) Will G be Smarandache strong Moufang groupoid?
vii) Can G have zero divisors?

225. Let
$G = S(C(Z_{40})) = \{a + bi_F \mid a, b \in Z_{40}, i_F^2 = 39,$
$(3i_F, 20+19i_F), *\}$ be the strong complex modulo integer groupoid.

i) Find order of G.
ii) Prove G is non commutative.

iii) Prove G is non associative.
iv) Is G simple?
v) Is G S-simple?
vi) Can G have S-ideals?
vii) Can G have S-zero divisors?
viii) Can G have S-idempotents?

226. Let $G = S(C(Z_{29})) = \{a + bi_F \mid a, b \in Z_{29},\ i_F^2 = 28, (3 + 2i_F, 14i_F), *\}$ be the strong complex modulo integer groupoid.

i) Find order of G.
ii) Can G have S-subgroupoids?
iii) Is G a S-groupoid?
iv) Is G simple?
v) Does G have S-ideals?
vi) Can G have S-zero divisors?
vii) Find S-idempotents if any in G.
viii) Is G a S-strong Bol groupoid?
ix) Is G a S-strong Moufang groupoid?

227. Obtain some interesting properties enjoyed by strong complex modulo integer groupoids.

228. Distinguish between the strong complex modulo integer groupoid
$G = \{a + bi_F \mid a, b \in Z_{13},\ i_F^2 = 12, (10i_F + 3, 3i_F + 10), *\}$
and $H = \{a + bi_F \mid a, b \in Z_{13},\ i_F^2 = 12, (10, 3)\ *\}$, the complex modulo integer groupoid.

229. Let
$G = S(C(Z_{43})) = \{a + bi_F \mid a, b \in Z_{43},\ i_F^2 = 42, (40i_F\ , 3), *\}$ be a complex strong groupoid. Does G satisfy any of the stricking properties?

230. If in problem (229) $(40i_F, 3)$ is replaced by $(40, 3i_F)$; what are different properties $S(C(Z_{43}))$ enjoys or it does not give any distinct properties? (Justify your claim).

231. Let $G = S(C(Z_{15})) = \{a + bi_F \mid a, b \in Z_{15}, i_F^2 = 14,$ $(3i_F, 12i_F), *\}$ be a strong complex groupoid. $H = S(C(Z_{15})) = \{a + bi_F \mid a, b \in Z_{15}, i_F^2 = 14, (3, 2), *\}$ be the complex groupoid.

i) Is $G \cong H$?
ii) Does both satisfy same set of special identities?
iii) Distinguish between G and H.
iv) In what ways G is different from H?
v) Is $o(G) = o(H)$?
vi) Find subgroupiods of same order in both H and G.
vii) Can G and h have S-ideals of same order?

232. Let
$G = \{(a_1, a_2, \ldots, a_9) \mid a_i \in S(Z_{20}))$
$= \{a + bi_F \mid a, b \in Z_{20}, i_F^2 = 19, (3i_F , 10+7i_F), *\}$
be a strong complex modulo integer groupoid.

i) Find order of G.
ii) Is G a S-groupoid?
iii) Is G simple?
iv) Find S-subgroupoids.
v) Does G satisfy any special identities?
vi) Does G contain S-subgroupoids which are not S-ideals?

233. Let
$G = S(C(Z_{49}))) = \{a + bi_F \mid a, b \in Z_{49}, i_F^2 = 48,$
$(3i_F + 46, 40i_F + 7), *\}$ be a strong complex modulo integer finite groupoid.

i) Find order of G.
ii) Prove G is non commutative.
iii) Prove G is non associative.
iv) Does G have S groupoids?
v) Does the order of every subgroupoids divide order of G?
vi) Does G contain S-zero divisors?

234. Let

$G = S(C(Z_{11})) = \{a + bi_F \mid a, b \in Z_{11}, i_F^2 = 10, (7i_F, 3), *\}$ be a strong complex modulo integer groupoid.

i) Find order of G.
ii) Is G commutative?
iii) Find some special properties associated with G.
iv) If $G = \{a + bi_F \mid a, b \in Z_{11}, *, (7, 3)\}$ is $H \cong G$ (where H is a complex groupoid)?
v) Can G be a Smarandache P-groupoid?
vi) Is G a S-groupoid?

235. Let

$G = S(C(Z_p)) = \{a + bi_F \mid a, b \in Z_p$, p a prime, $*$, $i_F^2 = p-1$, $(p+1/2\ i_F,\ p+1/2)\}$ be a strong complex modulo integer groupoid.

i) Find o (G).
ii) Is G commutative?
iii) Does G satisfy any special identities?
iv) Can G be a S-groupoid?
v) Can G have S-ideals?
vi) Show G is non associative.
vii) Find S-zero divisors if any in G.

236. Let

$G = S(C(Z_n)) = \{a + bi_F \mid a, b \in Z_n, i_F^2 = n-1, (pi_F, qi_F)$, where p/n and q/n. p and q primes, $*\}$

be a strong complex modulo integer groupoid.

i) Find order of G.
ii) Is G a S-Moufang groupoid?
iii) Does G enjoy any special property?
iv) Can G be a S-groupoid?
v) Is G -S simple?
vi) If p and q are non primes with $(p, q) = d \neq 1$, what will be the nature of G?

237. Let

$G = S(C(Z_{120})) = \{a + bi_F \mid a, b \in Z_{120},\ i_F^2 = 119,$
$(60, 60i_F), *\}$ be a strong complex groupoid.

i) Find order of G.
ii) Prove G is non commutative.
iii) Is G a S-groupoid?
iv) If $(60, 60i_F)$ is replaced by $(20i_F, 81i_F+20)$; what will be the special property enjoyed by G.
v) Can G be simple?
vi) Can G have left ideals which are not right ideals?

238. Let

$$M = \left\{\left[\begin{array}{ccccc} a_1 & a_2 & a_5 & a_7 & a_9 \\ a_3 & a_4 & a_6 & a_8 & a_{10} \end{array}\right] \middle|\ a_i \in C(Z_{10})\right.$$

$= \{a + bi_F \mid a, b \in Z_{10},\ i_F^2 = 9, *, (3, 5)\}$
be a complex modulo integer matrix groupoid.

i) Find order of G.
ii) Is M commutative?
iii) Show M can have subgroupoids which are not ideals?
iv) Can M have S-ideals?
v) Is M simple?
vi) Can M satisfy any of the special identities?

239. Let

$$M = \left\{\left[\begin{array}{ccccc} a_1 & a_2 & a_3 & a_4 & a_5 \\ a_6 & a_7 & a_8 & a_9 & a_{10} \\ a_{11} & a_{12} & a_{13} & a_{14} & a_{15} \\ a_{16} & a_{17} & a_{18} & a_{19} & a_{20} \\ a_{21} & a_{22} & a_{23} & a_{24} & a_{25} \end{array}\right] \middle|\ a_i \in C(Z_3)\right.$$

$= \{a + bi_F \mid a, b \in Z_3,\ i_F^2 = 2; 1 \le i \le 25; *, (2, 0)\}$
be a complex modulo integer matrix groupoid.

i) Prove T is finite
ii) Prove T has left ideals.
iii) Can T have S-zero divisors?
iv) Can T be simple?
v) Can T have S-ideals?
vi) Prove T has zero divisors.

240. Does there exist a complex modulo integer matrix groupoid which has no ideals?

241. Does there exists a complex modulo integer matrix groupoid which has no S-ideals?

242. Give an example of a complex modulo integer matrix groupoid which has S-ideals.

243. Does there exist complex modulo integer matrix groupoids which has S-zero divisors?

244. Give an example of a complex modulo integer matrix groupoid which has no S-subgroupoids.

245. Let

$$G = \left\{\left[\begin{array}{cccc} a_1 & a_2 & \ldots & a_{10} \\ a_{11} & a_{12} & \ldots & a_{20} \\ a_{21} & a_{22} & \ldots & a_{30} \\ a_{31} & a_{32} & \ldots & a_{40} \end{array}\right] \middle| \; a_i \in C(Z_{43}) \right.$$

$= \{a + bi_F \mid a, b \in Z_{43},\ i_F^2 = 42;\ 1 \leq i \leq 40;\ *, (10, 0)\}$
be a complex modulo integer matrix groupoid.

i) Find order of G.
ii) Is G a S-groupoid?
iii) Does G contain right ideals or left ideals?

iv) Is $M = \left\{ \begin{bmatrix} a_1 & 0 & \ldots & 0 \\ a_2 & 0 & \ldots & 0 \\ a_3 & 0 & \ldots & 0 \\ a_4 & 0 & \ldots & 0 \end{bmatrix} \middle| \; a_i \in C(Z_{43}), *, (10, 0)\right\} \subseteq G$

a left ideal or a right ideal?

246. Prove all complex modulo integer matrix groupoids $G = \{M \mid M$ is a $n \times m$ matrix with entries from $C(Z_t)$; $i_F^2 = t-1$, $*$, $(s, 0)$ (or $(0, r)$)$\}$ always has only one sided ideals. Can G have two sided ideals?

247. Let

$$M = \left\{ \sum_{i=0}^{\infty} a_i x^i \;\middle|\; a_i \in C(Z_{25}) = \{a + bi_F \mid a, b \in Z_{25}, i_F^2 = 24; \right.$$

$*$, $(10, 9)\}$ be a complex modulo integer polynomial groupoid.

i) Can M have ideals?
ii) Can M be a S-groupoid?
iii) Does M have subgroupoids?
iv) Can M have zero divisors?
v) Prove M is a non associative groupoids.
vi) Can M have nilpotent elements?
vii) Does M satisfy any of the special identities?
viii) Is M a S-strong Bol groupoid?
ix) Can M ever be a S-Moufamg groupoid?
x) What are the special features enjoyed by this polynomial groupoid M?

248. Study the special properties enjoyed by complex modulo integer polynomial groupoids.

249. Obtain some special properties enjoyed by complex modulo integer matrix groupoids.

250. Characterize those complex modulo integer polynomial groupoids which has ideals.

251. Characterize those complex modulo integer polynomial groupoids which has no ideals.

252. Characterize those complex modulo integer matrix groupoids which has S-ideals.

253. Let

$$G = \left\{ \begin{bmatrix} a_1 & a_2 & \ldots & a_{10} \\ a_{11} & a_{12} & \ldots & a_{20} \end{bmatrix} \middle| a_i \in C(Z_{10}) \right.$$

$= \{a + bi_F \mid a, b \in Z_{10},\ i_F^2 = 9\};\ 1 \leq i \leq 20;\ *, (9, 7)\}$
be a complex matrix groupoid.

i) Find number of elements in G.
ii) Prove G is non commutative.
iii) Prove G is non associative.
iv) Does G contain ideals?
v) Is G a S-groupoid?
vi) Can G have zero divisors?
vii) Does G satisfy any of the special identities?

254. Let

$$G = \left\{ \begin{bmatrix} a_1 & a_2 & a_3 & a_4 & a_5 \\ a_6 & a_7 & a_8 & a_9 & a_{10} \\ a_{11} & a_{12} & a_{13} & a_{14} & a_{15} \\ a_{16} & a_{17} & a_{18} & a_{19} & a_{20} \end{bmatrix} \middle| a_i \in C(Z_{40}) \right.$$

$= \{a + bi_F \mid a, b \in Z_{40},\ i_F^2 = 39\}, (3+5i_F, 8+10i_F), *\}$
be a strong complex modulo integer matrix groupoid of finite order.

i) Prove G is non commutative.
ii) Find the number of elements in G.
iii) Is G a S-groupoid?
iv) Prove G is non associative.

v) Can G have S-ideals?
vi) Is S simple?
vii) Can G have S-zero divisors?
viii) Find right ideals if any in G.

255. Let

$$G = \left\{\left.\begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \end{bmatrix}\right| a_i \in C(Z_{12}) = \{a + bi_F \mid a, b \in Z_{12},\right.$$

$i_F^2 = 11\}$, $1 \leq i \leq 6$, $(3i_F+7, 7+3i_F)$, $*\}$ be a strong complex modulo integer matrix groupoid.

i) Find order of G.
ii) Prove G is non commutative.
iii) Prove G is non associative.
iv) Is G a S-groupoid?
v) Does G have S-ideals?
vi) Can G have zero divisors?
vii) Is G simple?
viii) Does G satisfy any of the special identities?

256. Let $G = \left\{\left.\begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \end{bmatrix}\right| a_i \in C(Z_{10}), *, (3i_F, 9+5i_F)\right\}$ be a strong complex modulo integer matrix groupoid. Take F = $C(Z_5)$ be the complex modulo integer ring. FG be the super complex double strong groupoid ring of G over F.

i) Find order of G.
ii) Prove FG is non associative.
iii) Is FG non commutative?
iv) Is FG a S-groupoid ring?
v) Find ideals of FG.
vi) Can FG have S-ideals?

257. Let
$G = S(C(Z_{35})) = \{a + bi_F \mid a, b \in Z_{35}, i_F^2 = 34\}$,
$(3i_F+7, 21+14i_F), *\}$
be the strong complex groupoid. $F = C(Z_7)$ be the prime complex ring of characteristic seven. FG be the super doubly strong complex groupoid ring.

i) Find order of FG.
ii) Does FG satisfy any special property as o (F) / o (G)?
iii) Is FG a S-groupoid?
iv) Can FG have S-zero divisors?
v) Can FG satisfy any of the special identities?

258. Let $G = \{S(C(Z_{12})) = (8 + 4i_F, 3+9i_F), *\}$ be the strong complex groupoid. $F = C(Z_{48})$ be the complex modulo integer ring. FG be the super doubly strong complex groupoid ring.

i) Find order of FG.
ii) Since o (G) / o (F) does FG enjoy any nice algebraic structure?
iii) Is FG a S-ring?
iv) Is FG S-simple?
v) Prove FG has zero divisors.

259. Let $G = \{S(C(Z_n)), (a+bi_F, c+di_F), *$, such that $a+b = 0 \pmod n$ $c+d = 0 \pmod n$ $a, b, c, d \in Z_n \setminus \{0\})$
be a strong complex groupoid of modulo integers. $F = C(Z_m)$ such that o (F) / o (G) be the complex modulo integer ring FG be the super complex doubly strong groupoid ring.

i) Characterize FG.
ii) If o (G) / o (F) distinguish between thoso two rings.
iii) If $a+b \equiv 1 \pmod n$ and $c+d = 1 \pmod n$;what are the changes in properties of FG?

260. Let $G = \{(a_1, \ldots, a_{10})/ a_i \in S(C(Z_{24})), *, (12i_F, 12)\}$ be the strong complex modulo integer groupoid. $F = C(Z_{12})$ be

the complex modulo integer ring. FG be the super strong doubly comply modulo integer groupoid ring.

i) Find order of FG.
ii) Is FG commutative?
iii) Prove FG is associative.
iv) Find zero divisors if any in FG.
v) Can FG be simple?
vi) Is FG a S-ring?

261. Let

$$G = \left\{ \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_{20} \end{bmatrix} \middle| \; a_i \in C(Z_{19}),\ 1 \leq i \leq 20,\ *,\ (3i_F+9,\ 0) \right\}$$ be a

strong complex modulo integer groupoid. $F = C(Z_{19})$ be the complex modulo integer ring. FG be the super doubly strong complex modulo integer groupoid ring.

i) Find order of FG.
ii) Is FG a S-ring?
iii) Find any special property enjoyed by FG.
iv) Prove FG has one sided ideals (right ideals).
v) Can FG have zero divisors?
vi) Can FG have S-subrings?
vii) Prove FG has several subrings.
viii) Is $H = \left\{ \begin{bmatrix} a_1 \\ a_2 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \middle| \; a_i \in C\,(Z_{19}),\ 1 \leq i \leq 2,\ *,\ (3i_F+9,\ 0) \right\}$

$\subseteq$ G is such that FH is a subring?
Can FH be an ideal?

262. Let G = {S(C(Z_{28})), *, (0, $7i_F$)} be the strong complex groupoid. F = C(Z_7) be the complex ring. FG be the super strong doubly a complex groupoid ring.

i) Find order of G.
ii) Since o (F) / o (G) what is the special property enjoyed by FG?
iii) Is FG a S-ring?
iv) Is FG S-simple?
v) Can FG have S-zero divisors?
vi) Can FG have S-subrings which are not S-ideals?
vii) Can FG satisfy any type of special identity?

263. Let G = {3×3 matrices with entries from S(C(Z_{12})), *, ($3i_F$, 4)} be the complex matrix groupoid. F = C(Z_{12}) be the complex modulo integer ring. FG be the super doubly complex modulo integer groupoid ring.

i) Find order of FG.
ii) Prove FG has zero divisors.
iii) Is FG a S-ring?
iv) Can FG have S-ideals?

264. Obtain some special features enjoyed by super doubly complex groupoid rings of finite order.

265. Can these rings mentioned in (264) be S-simple?

266. Does these rings mentioned in (264) be S-rings?

267. Give a ring mentioned in problem (264) which is a S-strong Bol ring.

268. Can rings mentioned in problem 264 be S-strong Moufang ring?

269. Obtain some special properties enjoyed by quasi loops SC(L_{2n+1} ($a+bi_F$)).

270. Can these quasi loops mentioned in problem 269 be S-quasi loops?

271. What are the special features of these quasi loops which are different from usual loops?

272. Let H = {All 3×7 matrices with entries from $S(C(Z_{23})),. *, (22i_F+1, i_F+22)$} be the strong complex modulo integer groupoid. $F = C(Z_{23})$ be the complex ring. FH be the super doubly strong complex modulo integer groupoid ring.

i) Find order of FH.
ii) Is FH a S-ring?
iii) Is FH S-simple?
iv) Is FH a S-strong Bol ring?
v) Can FH be a S-strong alternative?

273. Let $G = SC(L_{27}\ (3+11i_F))$ be the complex modulo integer quasi loop. Enumerate the properties enjoyed by G.

274. Let G = $\{SC(L_{2n+1}\ (t+ui_F))\ n > 2, (t, u) \in Z_{2n+1} \setminus \{0\}, t+u = 2n+1\}$ be a complex modulo integer quasi loop. What is the special identity satisfied by G?

275. Characterize those complex modulo integer quasi loops which are loops.

276. Make a comparative study between complex quasi loops and complex loops.

277. Can any of the complex quasi loops be S-Moufang quasi loops?

278. Can any of the strong complex quasi loop be alternative?

279. Can any of the strong complex groupoids be Moufang groupoids?

280. Does there exists a strong complex quasi loop which is not a S-quasi loop?

281. Does there exists a strong complex groupoid which is not a S-groupoid?

282. Does there exists a super strong doubly complex groupoid ring which is not a S-ring?

283. Does there exist a super doubly strong complex groupoid ring which is S-simple?

284. Characterize those super doubly strong complex groupoid ring which has S-ideals.

285. Let P = SC(Z(3+13i_F)) be a strong complex groupoid.

   i) Does P satisfy any of the special identities?
   ii) Is P a S-groupoid?

286. Let S = SC(R $\sqrt{3} + \sqrt{7}$ i_F)) be a strong complex groupoid.

   i) Is S a S-groupoid?
   ii) Prove S-is non associative.
   iii) Does S-satisfy any of the special identities?
   iv) Is S, S-simple?

287. Let

$$G = \left\{\begin{bmatrix} a_1 & a_2 & a_3 & a_4 & a_5 \\ a_6 & a_7 & a_8 & a_9 & a_{10} \end{bmatrix} \middle| \; a_i \in C(Z_4);\ 1 \leq i \leq 10,\right.$$

$(3i_F, 2), *\}$
be a strong complex groupoid. F = C(Z_4) be the complex ring. FG be the super doubly strong complex groupoid ring.

   i) Find order of FG.
   ii) Is FG simple?
   iii) Is FG a S-ring?

iv) Is $P = \left\{\begin{bmatrix} a_1 & 0 & a_2 & 0 & a_3 \\ 0 & a_4 & 0 & a_5 & 0 \end{bmatrix} \middle| \; a_i \in C(Z_4); 1 \leq i \leq 5,\right.$ $(3i_F, 2), *\} \subseteq G$ an ideal of G?

Will FP $\subseteq$ FG the groupoid subring of FG be an ideal?

v) Does FG have ideals?

vi) Dies FG contain zero divisors?

288. Let P = $\{S(C(L_{23}\ (2+5i_F)))$ be a strong complex quasi loop.

i) Find order of P.
ii) Is P a S-quasi loop?
iii) Can P have subsets which are S-loops?
iv) Does P satisfy any of the special identities?
v) Can P have normal subloops?

289. Let R = {all 7 × 7 matrices with entries from $C(Z_2)$, *, $(1+i_F, 0)\}$ be a strong complex groupoid. F = Z_2 be the finite field. FR be the complex groupoid ring.

i) Find order of FR.
ii) Is FR a S-ring?
iii) Can FR have ideals?
iv) Prove FR has right ideals.
v) Can FR have S-zero divisors?

290. Obtain some nice applications of complex quasi loops.

291. Can a complex quasi loop be a Bruck quasi loop?

292. Can a complex quasi loop contain a subloop which is Moufang?

293. Obtain some nice applications of complex matrix groupoids.

294. Does there exist a compelx matrix groupoid which is a P-groupoid?

295. Let

$$G = \left\{ \begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ \vdots & \vdots & \vdots \\ a_{28} & a_{29} & a_{30} \end{bmatrix} \middle| a_i \in C(Z_{12});\ 1 \le i \le 30,\ (3i_F, 0), * \right\}$$

be a complex modulo integer groupoid. F = Z be the ring of integers. FG be a complex groupoid ring.

i) Is FG S-simple?
ii) Is FG a S-ring?
iii) Does FG satisfy any of the special identities?
iv) Can FG have zero divisors?
v) Prove FG has right ideals.
vi) Can FG have S-idempotents?

296. Let G = $C(L_{23}(8))$ be a complex loop of modulo integers F = Z_{23} be the field of characteristic 23. FG be the complex loop ring.

i) Find order of FG.
ii) Is FG a S-ring?
iii) Is FG S-simple?
iv) Does FG satisfy any of the special identities?

297. Let G = $C(L_p(t))$ be a complex loop of modulo integers p a prime F = Z_p be the finite filed. FG be the complex loop ring.

i) Is FG a S-ring?
ii) Find order of FG.
iii) If t = p–1, does FG satisfy any of the special

identities?

iv) If t = 2 does FG satisfy any of the special identities?

298. Let G = $\{C(Z_n(t))$, n a compositve odd number greater than 30 < n such that (t, n) = (t–1, n) = 1$\}$ be a complex loop. F = Z_n be the ring of modulo integers FG be the complex loop ring.

i) Find order of FG.
ii) Is FG a S-ring?
iii) If t = 2, does FG satisfy any of the special identities?
iv) If t = n–1, does FG satisfy any of the identities?
v) If Z_n is replaced by Z_m with o (Z_m) / o (Z_n); what are the special features enjoyed by Z_m G?

299. Let

$$G = \left\{\left.\begin{bmatrix} a_1 & a_2 \\ a_3 & a_4 \end{bmatrix}\right| a_i \in Z_{12}; 1 \leq i \leq 12, (2i_F, 10), *\right\}$$

be a strong complex modulo integer groupoid. F = Z_3 be the ring FG be the complex groupoid ring.

i) Find order of FG.
ii) Is FG a S-ring?
iii) Does FG have S-ideals?
iv) Is FG simple?
v) Can FG satisfy any of the special identities?
vi) If Z_3 is replaced by Z_4 study the problems (i) to (v)?
vii) If Z_3 is replace by Z_{12} study the question (i) to (v)?
viii) If Z_3 is replaced by Z_{24} study the question (i) to (v)?

300. Let G = $\{C(Z_{40}), *, (20i_F, 0)\}$ be the complex groupoid. F = Z_{20} be the ring of modulo integers. FG be the complex groupoid ring.

i) Is FG a S-ring?
ii) Find order of FG.
iii) Prove FG has right ideals.
iv) Can FG have left ideals?

v) Is FG simple?

301. Let G = {$C(Z_{11})$, *, $(0, 8i_F)$} be the complex groupoid, G = Z_{20} be the ring of modulo integers FG be the complex groupoid ring. Study questions (i) (ii) (iv) and (v) given in problem 300 for this FG.

302. Find some special properties enjoyed by complex loops $C(L_n(m))$.

303. Find some applications of polynomials of complex groupoids.

304. Let G = $\left\{ \sum_{i=0}^{\infty} a_i x^i \;\middle|\; a_i \in C(Z_3), *, (2i_F, 1+i_F) \right\}$ be the complex groupiod polynomials.

i) Find ideals if any in G.
ii) Is G a S-groupoid?
iii) Can G has zero divisors?

305. If $C(Z_3)$ is replaced by $C(Z_{12})$ in problem (304) prove G has zero divisors.

306. Let G = $\left\{ \sum_{i=0}^{\infty} a_i x^i \;\middle|\; a_i \in C(R), (\sqrt{3}\, i_F, 2), * \right\}$ be a complex polynomial groupoid.

i) Can G have S-ideals?
ii) Is G a S-groupoid?
iii) Find any special features enjoyed by G.

307. Let G = $\left\{\sum_{i=0}^{\infty} a_i x^i \,\middle|\, a_i \in C(Z_{24}), *, (0, 8i_F)\right\}$ be a complex polynomial groupoid.

i) Prove G has zero divisors.
ii) Can G have S-ideals?
iii) Is G a S-groupoid?
iv) Is a S-Moufang groupoid?
v) Is G simple?
vi) Prove G has left ideals.
vii) Can G have subgroupoids which are not ideals?

FURTHER READINGS

1. Birkhoff, G. and Bartee, T.C. *Modern Applied Algebra,* Mc-Graw Hill, New York, (1970).

2. Castillo J., *The Smarandache Semigroup*, International Conference on Combinatorial Methods in Mathematics, II Meeting of the project 'Algebra, Geometria e Combinatoria', Faculdade de Ciencias da Universidade do Porto, Portugal, 9-11 July 1998.

3. Chang Quan, Zhang, *Inner commutative rings*, Sictiuan Dascue Xuebao (Special issue), **26**, 95-97 (1989).

4. Hall, Marshall, *Theory of Groups*. The Macmillan Company, New York, (1961).

5. Herstein., I.N., *Topics in Algebra*, Wiley Eastern Limited, (1975).

6. Ivan, Nivan and Zukerman. H. S., *Introduction to number theory*, Wiley Eastern Limited, (1984).

7. Lang, S., *Algebra*, Addison Wesley, (1967).

8. Raul, Padilla, *Smarandache Algebraic Structures*, Smarandache Notions Journal, **9**, 36-38 (1998).

9. Smarandache, Florentin, *Special Algebraic Structures*, in Collected Papers, Abaddaba, Oradea, **3**, 78-81 (2000).

10. Smarandache Florentin, Multi structures and Multi spaces, (1969) www.gallup.unm.edu/~smarandache/transdis.txt

11. Vasantha Kandasamy, W. B., *On Smarandache Cosets*, (2002). http://www.gallup.unm.edu/~smaranandache/pseudoideals.pdf

12. Vasantha Kandasamy, W. B., *Smarandache Semigroups*, American Research Press, Rehoboth, NM, (2002).

13. Vasantha Kandasamy, W. B. and Smarandache, F., N-algebraic structures and S-N-algebraic structures, Hexis, Phoenix, Arizona, (2005).

14. Vasantha Kandasamy, W. B., Groupoids and Smarandache Groupoids, American Research Press, Rehoboth, NM, (2002).

15. Vasantha Kandasamy, W. B., Smarandache Loops, American Research Press, Rehoboth, NM, (2002).

16. Vasantha Kandasamy, W. B., Smarandache Rings, American Research Press, Rehoboth, NM, (2002).

17. Vasantha Kandasamy, W. B., Smarandache Non associative Rings, American Research Press, Rehoboth, NM, (2002).

18. Vasantha Kandasamy, W. B., Smarandache Semirings and semifields and semivector spaces, American Research Press, Rehoboth, NM, (2002).

19. Vasantha Kandasamy, W. B. and Smarandache, Florentin, Finite Neutrosophic Complex Numbers, Zip Publishing, Ohio, (2011).

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On India's 60th Independence Day, Dr.Vasantha was conferred the Kalpana Chawla Award for Courage and Daring Enterprise by the State Government of Tamil Nadu in recognition of her sustained fight for social justice in the Indian Institute of Technology (IIT) Madras and for her contribution to mathematics. The award, instituted in the memory of Indian-American astronaut Kalpana Chawla who died aboard Space Shuttle Columbia, carried a cash prize of five lakh rupees (the highest prize-money for any Indian award) and a gold medal.
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The authors have used the concept of finite complex modulo integers to construct non associative algebraic structures like groupoids, loops, and quasi-loops. Using these structures we built non associative complex matrix groupoids and complex polynomial groupoids. The authors suggest over 300 problems and some are at the research level.

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