THE OBSCURE PRECESSION OF MERCURY'S PERIHELION

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Abstract.

The Sun's motion around the Solar System barycentre produces a small quadrupole moment in the gravitational energy of Mercury. This moment has until now gone undiscovered, but it actually generates 7arcsec/cy precession of Mercury's perihelion. Consequently, the residual 43arcsec/cy previously explained by general relativity theory must account for this new component and only 36arcsec/cy for GR.

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1. Introduction

The orbit of planet Mercury has been calculated by several investigators; see Clemence (1947), Brouwer & Clemence (1961), and review in Pireaux & Rozelot (2003). In their calculations, the inverse square law has been applied to set up the differential equations of motion using the measured distances and velocities between Mercury, the Sun and planets. Then the observed precession of the perihelion of Mercury was explained as being due to general precession in longitude, perturbation by the planets, solar oblateness, and 43arcsec/cy for general relativity.

In this paper, an obscure contribution to precession has been identified due to the Sun orbiting around the barycentre, producing a small quadrupole moment in the energy of Mercury. The overall effect of this solar motion is similar to solar oblateness. The *orbiting* Sun drags Mercury around with it and does not allow Mercury to settle into a closed ellipse, so its binding energy differs in form from that for a stationary Sun. In fact, the curved trajectory of the Sun causes Mercury to actually orbit around a focal point just inside the Sun's trajectory, towards the barycentre. When averaged over an orbit, Mercury experiences this as a variation from Newton's law, even though the *instantaneous* force is always towards the Sun's centre and obeys an inverse square law. This results in 7arcsec/cy precession of Mercury's orbit, which has never been investigated.

To aid understanding, consider an analogous system of *coupled oscillators* such as a mass oscillating on a spring which is suspended from a periodically moving pivot. Displacement and kinetic energy of the mass involves the spring and pivot's movement. That is, a harmonic oscillator (cf. Mercury) is acted upon by an external periodic force (cf. Sun) which affects energy, amplitude, and phase of the oscillator. Similarly, a pendulum bob suspended from a moving pivot could not describe simple harmonic motion.

Consequently, previous investigators have calculated the orbit of Mercury with reference to a perfect ellipse around a stationary Sun, and then added 43arcsec/cy GR precession in order to get a fit to observations. By referring to the Sun, an obscure effect of solar circulation has been omitted from their calculations. It is necessary to refer the moving Sun and planets to the fixed barycentre; then 7arcsec/cy precession will result and only 36arcsec/cy be required from GR theory.

An exaggerated example of this phenomenon will now be explained to make it obvious, before transforming it to the real subtle system.

2. Derivation of precession due to the moving Sun

The absolute binding energy of Mercury, in the field of the Sun orbiting around the barycentre, may be calculated by using Newton's law. First, consider the *hypothetical system* shown in Figure 1. Let Mercury (mass M_1) be regarded as stationary at distance (r_{1C} = 57.9x10^{6}km) from the origin at barycentre C, while the Sun (mass M) travels rapidly around C at radius (r_{SC} = 7.43x10^{5}km). Then, for the Sun at instantaneous distance r_1 from Mercury we can write:

$$r_1^{\ 2} = r_{1C}^{\ 2} + r_{SC}^{\ 2} - 2r_{1C} r_{SC} \cos\theta \qquad . \qquad \text{(1a)}$$

The *instantaneous* gravitational force exerted by the Sun on Mercury is given by the inverse square law, ($F_1 = -GMM_1 / r_1^2$), and the force directed towards barycentre C is $F_1\cos\alpha$, where α is given by:

$$r_{SC}^{\ 2} = r_{1C}^{\ 2} + r_1^{\ 2} - 2r_{1C} r_1 \cos\alpha \ . \qquad \text{(1b)}$$

Upon eliminating $\cos\alpha$, this force towards C is:

$$F = F_1 \cos\alpha = -\left(\frac{GMM_1}{r_1^{\ 2}}\right)\left(\frac{r_{1C} - r_{SC}\cos\theta}{r_1}\right) \ . \qquad \text{(2a)}$$

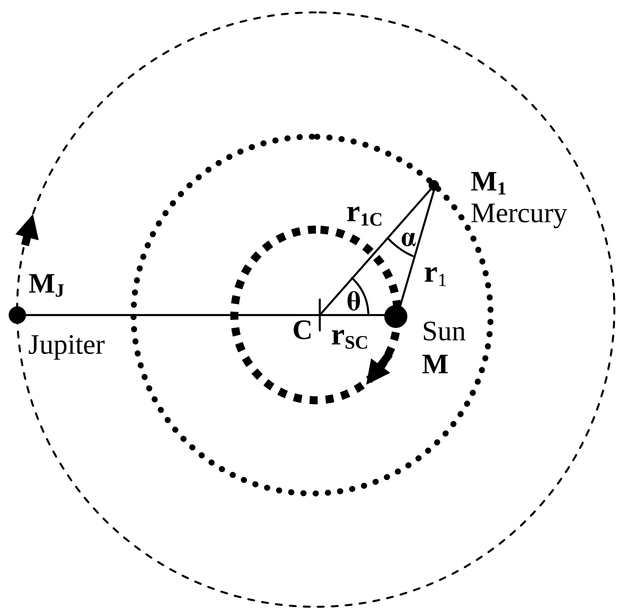

Fig. 1 Schematic diagram showing Jupiter and the Sun moving around their barycentre C. Theoretically, Mercury is here considered to be stationary during one orbit of the Sun

Now eliminate variable r_1 and get all the $\cos\theta$ terms in the numerator:

$$F \approx -\left(\frac{GMM_1}{r_{1C}^{\ 2}}\right)\left(1 - \frac{3}{2}\frac{r_{SC}^{\ 2}}{r_{1C}^{\ 2}} + \frac{2r_{SC}}{r_{1C}}\cos\theta + \frac{9}{2}\frac{r_{SC}^{\ 2}}{r_{1C}^{\ 2}}\cos^2\theta\right). \qquad \text{(2b)}$$

After averaging θ over a complete orbit of the Sun, the average force towards C becomes:

$$\tilde{F} \approx -\left(\frac{GMM_1}{r_{1C}{}^2}\right)\left[1+\frac{3}{4}\left(\frac{r_{SC}{}^2}{r_{1C}{}^2}\right)\right] . \qquad (2c)$$

This is the *average* force that a stationary Mercury would respond to; although the *instantaneous* force is actually towards the Sun according to Newton's inverse square law. The force is slightly stronger than an inverse square law for a stationary Sun located at C. By integrating from r_{1C} to infinity, the absolute potential energy of Mercury in this system would be:

$$PE_{av} \approx -\left(\frac{GMM_1}{r_{1C}}\right)\left[1+\frac{1}{4}\left(\frac{r_{SC}{}^2}{r_{1C}{}^2}\right)\right] . \qquad (3)$$

These two expressions would apply to the equation of motion for Mercury *around C*, with angular momentum being greater than that around a fixed Sun at C. Inertial Mercury can only focus on the centre of gravity at C, even though the instantaneous gravitational force is towards the Sun's centre. We are interested in the way that the small quadrupole moment should produce precession, just like an oblate Sun.

Apparently, received wisdom does not allow a moving Sun, like the pseudo-toroidal Sun described above, to affect the absolute potential of Mercury. However, logically, the Sun's circulation perturbs Mercury so its orbit cannot settle and its energy should be different *in form* from normal. Therefore these expressions will now be modified for a slowly moving Sun, to prove that a remnant quadrupole moment will remain, as follows.

The real Sun orbits the barycentre at $7.43\text{x}10^5$km radius over 11.86 years due to Jupiter. Its slow motion around C means the effective centre of gravity now lies between the Sun and C, which is where inertial Mercury focuses so a small quadrupole moment should be expected. Therefore r_{SC} in these equations will be replaced by a compensated value of the order ($r'_{SC} \approx r_{SC}/100$), such that Eq.(2c) is modified to give the actual average acceleration of Mercury:

$$\tilde{a} \approx -\frac{GM}{{r_{1C}}^2}\left[1+\frac{3}{4}\left(\frac{r'_{SC}}{r_{1C}}\right)^2\right] \quad . \tag{4}$$

After substituting ($u = 1/r_{1C}$), plus Mercury's specific angular momentum [$h \approx (GMr_{1C})^{1/2}$], then orbit theory yields a differential equation for the trajectory:

$$\frac{d^2u}{d\phi^2}+u=\frac{-\tilde{a}}{h^2u^2}=\frac{GM}{h^2}+\left(\frac{GM}{h^2}\frac{3}{4}(r'_{SC})^2\right)u^2 \quad . \tag{5}$$

This type of equation has previously been solved because general relativity theory gives a similar expression for the trajectory of Mercury, (see Rindler, 2001):

$$\frac{d^2u}{d\phi^2}+u=\frac{GM}{h^2}+\left(\frac{3GM}{c^2}\right)u^2 \quad , \tag{6}$$

where the final term accounts for 43arcsec/cy precession of Mercury's orbit. Hence, by direct comparison, we can calculate the precession to expect from the quadrupole moment in Eq.(5):

$$\delta\omega \approx \left(\frac{c}{2h}r'_{SC}\right)^2 \times 43 arc\sec/cy \quad . \tag{7}$$

Here, the small effect of orbit eccentricity is included in the 43arcsec/cy term.

The value of r'_{SC} can be found by considering Figure 2, wherein Mercury (period $\boldsymbol{\tau}_1$) is allowed to orbit and track the Sun as it moves slowly around C with period ($\boldsymbol{\tau}_{SC} = 49.2\boldsymbol{\tau}_1$). Mercury's orbit is now focussed on a moving centre of gravity P, at radius r_{PC} from C towards the Sun. This distance is derivable from an action principle which describes the effective action seen from Mercury orbiting around P. By defining *action* as (Kinetic energy x Time), it is conserved for any value of $\boldsymbol{\tau}_1$ when:

$$\tfrac{1}{2}M{v_{PC}}^2(\boldsymbol{\tau}_{PC}+\boldsymbol{\tau}_1)=\tfrac{1}{2}M{v_{SC}}^2\boldsymbol{\tau}_{SC} \quad , \tag{8a}$$

where ($\boldsymbol{\tau}_{PC} = \boldsymbol{\tau}_{SC}$) and ($v_{PC} / v_{SC} = r_{PC} / r_{SC}$). Here, the lower equivalent KE of the centre of gravity is compensated by an extension in time of one more period of Mercury, $\boldsymbol{\tau}_1$.

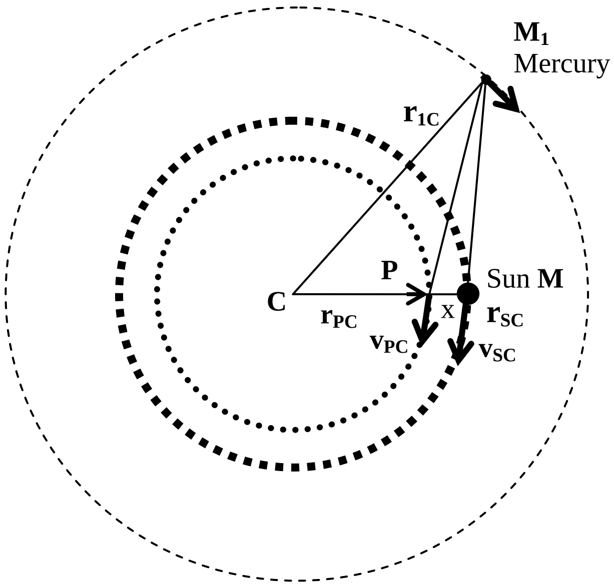

Fig. 2 Schematic diagram showing the Sun moving slowly around the barycentre C. Mercury is considered to be orbiting the centre of gravity, focal point P, between C and the Sun

An alternative expression for action is (Angular momentum x Angle), thus:

$$Mv_{PC}r_{PC} \times 2\pi\left(1+\frac{\tau_1}{\tau_{SC}}\right) = Mv_{SC}r_{SC} \times 2\pi \quad . \qquad (8b)$$

On the left, the lower equivalent angular momentum is compensated by extra angle, due to an extra period of Mercury. Simplification gives:

$$r_{PC} = r_{SC}\left(\frac{\tau_{SC}}{\tau_{SC}+\tau_1}\right)^{1/2} \quad . \qquad (9)$$

From the viewpoint of an observer on Mercury: after 49.2 orbits, the Sun has moved in a circle around P of radius *x* given by:

$$x = r_{SC} - r_{PC} = r_{SC}\left[1-\left(\frac{\tau_{SC}}{\tau_{SC}+\tau_1}\right)^{1/2}\right] \quad . \qquad (10)$$

It is this circular motion of the Sun *around P* which now determines the quadrupole moment operating on Mercury because radial component *x* is equivalent to r'_{SC} in Eq.(7). Thus for (τ_1 = 88.0days), (τ_{SC} = 4331days) and (r_{SC} = 7.43x10^5km), it evaluates to ($x \equiv r'_{SC} = 7433$km). Substitution of this in Eq.(7), with ($h = 2.76\text{x}10^{15}\text{m}^2\text{s}^{-1}$), yields the expected precession due to the quadrupole moment:

$$\delta\omega \approx 0.162 \times 43 \approx 7.0 arc\sec/cy \quad . \tag{11}$$

This means that the well-known 43arcsec/cy residual precession, previously explained as GR precession, must be reduced by 7arcsec/cy to a residual of 36arcsec/cy for GR.

The absolute potential energy of Mercury in this approximately real system is finally:

$$PE_{av} \approx -\left(\frac{GMM_1}{r_{1C}}\right)\left[1 + \frac{1}{4}\left(\frac{r'^{\,2}_{SC}}{r_{1C}{}^2}\right)\right] \quad . \tag{12}$$

Precession due to other planets increasing the Sun's wobble is variable because together they cause great fluctuation in r_{SC} , with a long-term average at around $8.4\text{x}10^5$km (Landscheidt, 2007).

Precessions currently attributed to general relativity in the orbits of Venus, Earth and Icarus, will also be affected by the Sun's movement, (Shapiro et al (1968), Lieske & Null (1969), Sitarski (1992)).

3. Conclusion

Motion of the Sun around the Solar System barycentre produces a small quadrupole moment in the average gravitational energy of Mercury. The effect of this is to generate 7arcsec/cy precession in Mercury's orbit, just as an oblate Sun would. This has not been included previously, so only 36arcsec/cy precession due to general relativity theory is now required for a fit to the observations.

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