Modified Newtonian Dynamics and Dark Matter from a generalized gravitational theory

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Abstract

Vast amounts of data clearly demonstrate discrepancies between the observed dynamics, in large astronomical systems, and the predicted dynamics by Newtonian gravity and general relativity. The appearance of these discrepancies has two possible explanations: either these systems contain large quantities of a new kind of unseen matter –the Dark Matter (DM)– or the gravitational law has to be modified at this scale –as in MOdified Newtonian Dynamics (MOND)–. This dichotomy is not entirely new in the history of physics, with DM playing now the role of the old non-existent Vulcan planet.

We have shown how both (**i**) the MONDian form and (**ii**) Milgrom acceleration follow from an extended theory of gravity –characterized by a new kind of gravitational potentials $h_{\mu\nu}(R(t))$–, which (**iii**) was initially aimed to solve those deficiencies of general relativity shared with classical electrodynamics –and that were previously solved with new electromagnetic potentials $\Phi(R(t))$ and $\boldsymbol{A}(R(t))$–. We also show (**iv**) how the modified equation of motion can be cast into ordinary form, when an fictitious distribution of DM matter is added to the real mass. From our definition of DM, we obtain (**v**) the main properties traditionally attributed to it, in excellent agreement with the DM literature. Finally, (**vi**) we discuss further avenues of research opened by this new paradigm.

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1 Introduction

Over the last six decades, vast amounts of data clearly demonstrate discrepancies between the observed dynamics, in large astronomical systems, and the predicted dynamics when either Newtonian gravity or general relativity are applied to the directly observable distribution of mass.

The appearance of these discrepancies has two possible explanations: either these systems contain large quantities of a new kind of unseen matter –the Dark Matter (DM) [1]– or the gravitational law has to be modified at this scale –as in MOdified Newtonian Dynamics (MOND) [2]–.

This dichotomy is not entirely new in the history of physics. Astronomers already attributed to a new kind of unseen matter the discrepancies between the Newtonian predictions for the motion of Mercury and its observed motion. A new planet was supposed to exist orbiting near the Sun. The confidence on the universal validity of Newtonian gravitational theory was so high that to «*the people of the late 19th century, Vulcan was real. It was a planet. It had theoretical credibility and had actually been seen. Even textbooks accorded it a chapter*» [3].

The first discovery of Vulcan was announced on 2 January 1860 during a meeting of the *Académie des Sciences* in Paris. Several re-discoveries and confirmations were done in posterior decades [3], somehow as discoveries of the hypothetical DM are announced in our days [4]. All of us know now that Vulcan does not exist and that the motion of Mercury includes gravitational effects which cannot be accounted by Newtonian gravity alone.

In a striking parallelism with the Vulcan case, the hypothetical DM has never been directly detected despite much experimental and observational effort during several decades [5]. The situation has not changed in recent years, with *Xenon10* excluding previously unexplored parameter space [6] and *Fermi* finishing with another null detection of any sign of the existence of the hypothetical DM [7].

In the meanwhile, MOND has gained reputation among astronomers and astrophysicists thanks to its successful explanations of many phenomena in galaxies, including (**i**) the prediction of the shape of rotation curves of low surface-brightness (LSB) galaxies, before any of them had ever been measured; (**ii**) the baryonic Tully-Fisher relation, one of the tightest observed relations in astrophysics, is a natural consequence of MOND; (**iii**) the prediction of non-Newtonian dynamics in tidal dwarf galaxies (TDG), which should be devoid of collisionless DM; (**iv**) the first realistic simulations of galaxy merging in MOND were recently carried out, notably reproducing the morphology of the Antennae galaxies; and (**v**) the prediction of a finite mass for pressure-supported systems that are nearly isothermal. This list is by no means exhaustive [2, 8, 9, 10].

MOND has been also useful in gravitational lensing, providing a constant deflecting angle at large distance from the center of the gravitational field, which is consistent with the value presently used in the study of gravitational lensing by galaxies and clusters of galaxies [8, 11].

Moreover, it is fair to emphasize that a simple model motivated by MOND provided the *only successful a priori prediction* of the first-to-second peak amplitude ratio of the acoustic peaks of the cosmic background radiation [14].

MOND precise predictions continue to be well-verified in recent years [12, 13, 14]. By contrast, the DM model either does not address or gets wrong systematic aspects of galaxy photometry and kinematics [10, 14, 15]. And when the DM model works, it cannot rival with MOND. Consider the case of flat rotation curves. In the alternative DM model –which casts no doubt on standard gravity theory–, they are explained, *a posteriori*, by assuming that every disk galaxy is nested inside a roundish spherical halo of DM with special properties. However, when the observational data is already at hand, the DM models still requires one or two free parameters to approximate, *a posteriori*, the success of the MOND *predictions* [16].

The next section introduces the theoretical foundations for an extension of the conventional gravitational theory. We derive MOND, reproducing its main pros and correcting its limitations and perceived shortcomings, in the following section. In a posterior section, we will show how the new theory can be cast into Newtonian form plus a fictitious distribution of DM.

2 Foundations of the new gravitational theory

Returning briefly to the Vulcan case, it is relevant to recall that different *ad hoc* modifications of Newtonian theory were invented to describe the observed Mercury orbit, but the final solution became as a bonus from a new theory of gravitation –general relativity– which had been initially designed to overcome other deficiencies of the Newtonian theory –such as its incompatibility with, the then recently developed, special relativity–.

A modification of Maxwell & Lorentz classical electrodynamics has been recently proposed for removing its traditional difficulties and inconsistencies such as the indefiniteness in the field energy location, self-forces, divergent self-energy, the concept of electromagnetic mass, and the radiation irreversibility in time with respect to the time symmetry of Maxwell equations [17, 18].

This modification is characterized by the introduction of new potentials $\Phi(R(t))$ and $\boldsymbol{A}(R(t))$, which are *mathematically and physically irreducible* to the traditional field potentials $\Phi^{[F]}(r,t)$ and $\boldsymbol{A}^{[F]}(r,t)$; this modification satisfies the continuous transition between D'Alembert's and Poisson's equation solutions, which was violated in the traditional Maxwell & Lorentz formalism [17].

Essentially, the same kind of criticism applies to general relativity. Indeed, Chubykalo & Smirnov-Rueda main remark [17]:

> «*Nevertheless, the conventional theory does not explain in detail how the function $g(r,t)$ is converted into implicit time-dependent function $f(R(t))$ (and* vice versa*) when the steady-state problems are studied.*»

applies verbatim to the relation between the metric potential $\Phi^{[GR]}(r,t)$ of general relativity and the Newtonian potential $\Phi(R(t))$.

Therefore, in close parallelism with the electromagnetic case [17, 18], our modern modification of the classical gravitational theory will be driven by the introduction of a new kind of gravitational potentials $h_{\mu\nu}(R(t))$ [19], which *are not reducible* to the metric potentials $h_{\mu\nu}^{[\mathrm{GR}]}(r,t)$ associated to general relativity [20].

Applying Hamiltonian methods –see also [17]–, we will obtain the equation of motion that corresponds to the special weak gravity case $h_{00} = 2\Phi/c^2 + \mathcal{O}(c^{-4})$, $h_{0i} = -4A_i/c^2 + \mathcal{O}(c^{-5})$, and $h_{ij} = 2\Phi\delta_{ij}/c^2 + \mathcal{O}(c^{-4})$, for a test body with velocity $v \ll c$. By simplicity, we will consider a single massive source m' [21]

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(m\boldsymbol{v} + m\frac{2\Phi}{c^2}\boldsymbol{v}'\right) = m\frac{\Phi}{\boldsymbol{R}}. \tag{1}$$

This equation is a generalization of geodesic and Newtonian equations [19]. The new term in the sum corrects the purely kinetic momentum $m\boldsymbol{v}$ and leads to new predictions beyond general relativity. Applying $v' \ll c$ and working out gives

$$a = -\frac{Gm'}{R^2} + \frac{2Gm'a'}{Rc^2}. \tag{2}$$

We can observe that the time derivative of the non-kinetic part of the momentum gives a $1/R$ correction to the Newtonian force; moreover, this corrective force is acceleration scaled. In the next section, we will show how (2) can be used to derive MOND.

3 Derivation of MOND

For a direct comparison with MOND, a non-negative acceleration scale is introduced

$$a_0 \equiv 4Gm'\left(\frac{a'}{c^2}\right)^2. \tag{3}$$

Using this scale, the generalized equation of motion (2) can be written as

$$a = -\frac{Gm'}{R^2} - \frac{\sqrt{Gm'a_0}}{|R|}, \tag{4}$$

which (**i**) reduces to Newtonian form when the second term in the right-hand-side can be neglected and (**ii**) reduces to MONDian form $|a| = \sqrt{Gm'a_0}/|R|$ in the contrary case. The scale a_0 can be identified with Milgrom acceleration [2, 8, 9].

We have obtained the MONDian form as an unexpected bonus from a generalized gravitational theory initially aimed to solve those deficiencies of general relativity which are shared with classical electrodynamics.

MOND proposes the following empirical formula [2, 8, 9]

$$\mu a = -\frac{Gm'}{R^2}, \tag{5}$$

with $\mu = \mu(|a|/a_0)$ defined by $\mu(x) \approx x$ for $x \ll 1$ and $\mu(x) \to 1$ for $x \gg 1$.

MOND does not unambiguously define the function μ for intermediate values of its argument; however, (4) is also valid for them. Let us analyze the special case –subindex c– when the strengths of the Newtonian force and the $1/R$ force in (4) are comparable; i.e., when $|a| \sim a_0$. Equating both forces, we obtain the following relationship

$$Gm'_c = R_c^2 a_0. \tag{6}$$

Following an analysis similar to that in [16], the relationship (6) will *almost* predict, for a fixed radius R_c, MONDian behavior for $m' \gg m'_c$ and Newtonian behavior for low masses. Taking $a_0 = 1.21 \times 10^{-10}$ ms^{-2} [14] and $R_c = 5.2 \times 10^{17}$ m [16] –of the order of the typical size of globular clusters–, we obtain $m'_c \simeq 2 \times 10^5$ M$_\odot$. This is about right: globular star clusters at $(10^4 - 10^5)$ M$_\odot$ show no missing mass problem [16].

It is empirically established that MOND acceleration parameter $a_0 \sim cH_0$ [2, 23]. Sanders & McGaugh use $a_0 \approx cH_0/6$ [8]. This numerical coincidence has always been taken as symptom of some deep link between MOND and cosmology [2, 8, 23]. However, a_0 plays no role in the standard cosmological model based in general relativity plus DM plus dark energy. Moreover, even in relativistic MOND theories as TeVeS [16], a_0 does not appear to arise in a natural way but must be inserted by hand. «*The near numerical coincidence of a_0 with cH_0 remains unexplained*» in TeVeS [23]. We will show how the value of a_0 can be derived from cosmology.

We start by applying the definition (3) to the Universe as a whole and obtaining

$$a_0 \cong 4GM \left(\frac{\ddot{R}}{c^2}\right)^2, \tag{7}$$

where M is the mass of the Universe and $\ddot{R}$ the Milne & Friedmann acceleration. Using the Newtonian form of the Friedmann equations [23] and working out

$$a_0 \approx GM \left(\frac{1}{R_H}\right)^2 \approx \frac{1}{2}\frac{c^2}{R_H}, \tag{8}$$

where R_H denotes the Hubble radius. This expression can be finally written as

$$a_0 \approx \frac{1}{2} cH_0, \tag{9}$$

explaining the «*near numerical coincidence of a_0 with cH_0*» [23].

Within the framework of our extended gravitational theory, a_0 is not a fundamental constant as c or G, but a parameter; a_0 can vary locally –according to the definition (3)–, but remains approximately constant in a global scale –according to (9)–, thanks to the cosmological principle of homogeneity. The novel interpretation of the Milgrom acceleration as a parameter, which can vary locally, explains the variations in the value of a_0 found in some specific systems.

4 DM as a fictitious distribution of matter

As has been shown in the previous section, several hundred of observations done in last decades are well-explained by our extended gravitational theory, without any need for the original DM hypothesis [1]. Nevertheless, considering the popularity that DM still has among cosmologists and some astrophysicists, it will be interesting to derive, from our new theory, the properties traditionally attributed to DM.

The modified equation of motion (4) can be rewritten as

$$a = -\frac{G(m' + m_{\mathrm{DM}})}{R^2}, \tag{10}$$

with the following definition of a *fictitious distribution of matter*

$$m_{\mathrm{DM}} \equiv |R|\sqrt{m' a_0/G}. \tag{11}$$

Evidently, using the ordinary expression (10) with the fictitious component m_{DM} added to the real mass m' is totally equivalent to the use of the modified equation of motion (4).

Of course, no definition (11) for m_{DM} is available in the DM models, because DM was assumed to be a new kind of matter to be found. In these models, the existence or no existence of DM, its distribution, and its properties are all inferred, *a posteriori*, from the difference between the observations and the predictions done by using the real mass m' in Newtonian and general relativistic expressions.

For instance, in ZWICKY's original analysis of rotation curves of galaxies [1], the assumed distribution of $m_{\mathrm{DM}}^{[ASS]}$ was obtained, *a posteriori*, from the difference between the observed acceleration and the ordinary Gm'/R^2. Consider gravitational lensing in the DM paradigm [4] as another example; also here the assumed distribution of $m_{\mathrm{DM}}^{[ASS]}$ is obtained, *a posteriori*, from the difference between the observed lensing and the lensing predicted by general relativity.

Using the definition (11), we can predict the main properties associated to this *fictitious matter*; our predictions are in excellent agreement with the properties attributed to DM in the traditional DM paradigm, as well as with direct experiments performed up to the date:

- DM cannot be ordinary baryonic matter –stars, planets, gas, dwarfs, neutron stars, and others– [1, 4].
- Does not carry any electric charge.
- The so-named 'evidence' for DM is *exclusively* based on the assumption that Newtonian and general relativistic expressions apply.
- All direct searches for DM give null results –as corresponds to a truly fictitious kind of matter– [5, 6, 7].
- $m_{\mathrm{DM}} \propto R$ and its mass density drops as R^{-2} [5, 16].
- Neither emits nor scatters light [4] or other electromagnetic radiation.
- Will gravitationally deflect light [4, 16].
- Can be spatially segregated from the observable baryonic mass [4].
- There are regions where DM is totally absent: the solar system, small globular clusters –see discussion in section 3–, and the cores of galaxies [5].

- Does not interact with other matter except through gravity; e.g., via (10).
- It behaves like a perfect fluid, without any internal resistance or viscosity.

Although the definition (11) has been obtained from an equation of motion (4), valid under certain restrictions, the above list of properties for the fictitious DM coincide very well with the properties hypothesized in the DM literature.

In this paper, we have only started to explore this interesting topic. For instance, in the derivation of the generalized equation of motion (4) we have not considered high-velocity neither strong-gravity corrections. Additionally, we have worked in a pure state approximation of the underlying Liouvillian dynamics, where dissipation and chaos were totally neglected as well. Relaxing those approximations would lead to new predictions. In a future work, we will address the problem of dark energy from this new perspective [24].

Summarizing, we have shown in this paper how both (**i**) the MONDian form and (**ii**) Milgrom acceleration follow, in a natural way, from an extended theory of gravity –characterized by a new kind of gravitational potentials $h_{\mu\nu}(R(t))$–, which (**iii**) was initially aimed to solve those deficiencies of general relativity shared with classical electrodynamics –and that were previously solved with new electromagnetic potentials $\Phi(R(t))$ and $\boldsymbol{A}(R(t))$–. We also show (**iv**) how the modified equation of motion can be cast into ordinary form, when an fictitious distribution of DM matter is added to the real mass. From our definition of DM, we obtain (**v**) the main properties traditionally attributed to it, in excellent agreement with the DM literature. Finally, (**vi**) we discuss further avenues of research opened by this new paradigm.

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