Motives and Infinite Primes

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Abstract

In this chapter the goal is to find whether the general mathematical structures associated with twistor approach, superstring models and M-theory could have a generalization or a modification in TGD framework. The contents of the chapter is an outcome of a rather spontaneous process, and represents rather unexpected new insights about TGD resulting as outcome of the comparisons.

1. Infinite primes, Galois groups, algebraic geometry, and TGD

In algebraic geometry the notion of variety defined by algebraic equation is very general: all number fields are allowed. One of the challenges is to define the counterparts of homology and cohomology groups for them. The notion of cohomology giving rise also to homology if Poincare duality holds true is central. The number of various cohomology theories has inflated and one of the basic challenges to find a sufficiently general approach allowing to interpret various cohomology theories as variations of the same motive as Grothendieck, who is the pioneer of the field responsible for many of the basic notions and visions, expressed it.

Cohomology requires a definition of integral for forms for all number fields. In p-adic context the lack of well-ordering of p-adic numbers implies difficulties both in homology and cohomology since the notion of boundary does not exist in topological sense. The notion of definite integral is problematic for the same reason. This has led to a proposal of reducing integration to Fourier analysis working for symmetric spaces but requiring algebraic extensions of p-adic numbers and an appropriate definition of the p-adic symmetric space. The definition is not unique and the interpretation is in terms of the varying measurement resolution.

The notion of infinite has gradually turned out to be more and more important for quantum TGD. Infinite primes, integers, and rationals form a hierarchy completely analogous to a hierarchy of second quantization for a super-symmetric arithmetic quantum field theory. The simplest infinite primes representing elementary particles at given level are in one-one correspondence with many-particle states of the previous level. More complex infinite primes have interpretation in terms of bound states.

1. What makes infinite primes interesting from the point of view of algebraic geometry is that infinite primes, integers and rationals at the n:th level of the hierarchy are in 1-1 correspondence with rational functions of n arguments. One can solve the roots of associated polynomials and perform a root decomposition of infinite primes at various levels of the hierarchy and assign to them Galois groups acting as automorphisms of the field extensions of polynomials defined by the roots coming as restrictions of the basic polynomial to planes $x_n = 0$, $x_n = x_{n-1} = 0$, etc...

2. These Galois groups are suggested to define non-commutative generalization of homotopy and homology theories and non-linear boundary operation for which a geometric interpretation in terms of the restriction to lower-dimensional plane is proposed. The Galois group G_k would be analogous to the relative homology group relative to the plane $x_{k-1} = 0$ representing boundary and makes sense for all number fields also geometrically. One can ask whether the invariance of the complex of groups under the permutations of the orders of variables in the reduction process is necessary. Physical interpretation suggests that this is not the case and that all the groups obtained by the permutations are needed for a full description.

3. The algebraic counterpart of boundary map would map the elements of G_k identified as analog of homotopy group to the commutator group $[G_{k-2}, G_{k-2}]$ and therefore to the unit

element of the abelianized group defining cohomology group. In order to obtains something analogous to the ordinary homology and cohomology groups one must however replaces Galois groups by their group algebras with values in some field or ring. This allows to define the analogs of homotopy and homology groups as their abelianizations. Cohomotopy, and cohomology would emerge as duals of homotopy and homology in the dual of the group algebra.

4. That the algebraic representation of the boundary operation is not expected to be unique turns into blessing when on keeps the TGD as almost topological QFT vision as the guide line. One can include all boundary homomorphisms subject to the condition that the anticommutator $\delta_k^i\delta_{k-1}^j + \delta_k^j\delta_{k-1}^i$ maps to the group algebra of the commutator group $[G_{k-2}, G_{k-2}]$. By adding dual generators one obtains what looks like a generalization of anticommutative fermionic algebra and what comes in mind is the spectrum of quantum states of a SUSY algebra spanned by bosonic states realized as group algebra elements and fermionic states realized in terms of homotopy and cohomotopy and in abelianized version in terms of homology and cohomology. Galois group action allows to organize quantum states into multiplets of Galois groups acting as symmetry groups of physics. Poincare duality would map the analogs of fermionic creation operators to annihilation operators and vice versa and the counterpart of pairing of k:th and $n-k$:th homology groups would be inner product analogous to that given by Grassmann integration. The interpretation in terms of fermions turns however to be wrong and the more appropriate interpretation is in terms of Dolbeault cohomology applying to forms with homomorphic and antiholomorphic indices.

5. The intuitive idea that the Galois group is analogous to 1-D homotopy group which is the only non-commutative homotopy group, the structure of infinite primes analogous to the braids of braids of braids of ... structure, the fact that Galois group is a subgroup of permutation group, and the possibility to lift permutation group to a braid group suggests a representation as flows of 2-D plane with punctures giving a direct connection with topological quantum field theories for braids, knots and links. The natural assumption is that the flows are induced from transformations of the symplectic group acting on $\delta M^2_\pm \times CP_2$ representing quantum fluctuating degrees of freedom associated with WCW ("world of classical worlds"). Discretization of WCW and cutoff in the number of modes would be due to the finite measurement resolution. The outcome would be rather far reaching: finite measurement resolution would allow to construct WCW spinor fields explicitly using the machinery of number theory and algebraic geometry.

6. A connection with operads is highly suggestive. What is nice from TGD perspective is that the non-commutative generalization homology and homotopy has direct connection to the basic structure of quantum TGD almost topological quantum theory where braids are basic objects and also to hyper-finite factors of type II_1. This notion of Galois group makes sense only for the algebraic varieties for which coefficient field is algebraic extension of some number field. Braid group approach however allows to generalize the approach to completely general polynomials since the braid group make sense also when the ends points for the braid are not algebraic points (roots of the polynomial).

This construction would realize the number theoretical, algebraic geometrical, and topological content in the construction of quantum states in TGD framework in accordance with TGD as almost TQFT philosophy, TGD as infinite-D geometry, and TGD as generalized number theory visions.

2. p-Adic integration and cohomology

This picture leads also to a proposal how p-adic integrals could be defined in TGD framework.

1. The calculation of twistorial amplitudes reduces to multi-dimensional residue calculus. Motivic integration gives excellent hopes for the p-adic existence of this calculus and braid representation would give space-time representation for the residue integrals in terms of the braid points representing poles of the integrand: this would conform with quantum classical correspondence. The power of 2π appearing in multiple residue integral is problematic unless it disappears from scattering amplitudes. Otherwise one must allow an extension of p-adic numbers to a ring containing powers of 2π.

2. Weak form of electric-magnetic duality and the general solution ansatz for preferred extremals reduce the Kähler action defining the Kähler function for WCW to the integral of Chern-Simons 3-form. Hence the reduction to cohomology takes places at space-time level and since p-adic cohomology exists there are excellent hopes about the existence of p-adic variant of Kähler action. The existence of the exponent of Kähler gives additional powerful

constraints on the value of the Kähler fuction in the intersection of real and p-adic worlds consisting of algebraic partonic 2-surfaces and allows to guess the general form of the Kähler action in p-adic context.

3. One also should define p-adic integration for vacuum functional at the level of WCW. p-Adic thermodynamics serves as a guideline leading to the condition that in p-adic sector exponent of Kähler action is of form $(m/n)^r$, where m/n is divisible by a positive power of p-adic prime p. This implies that one has sum over contributions coming as powers of p and the challenge is to calculate the integral for K= constant surfaces using the integration measure defined by an infinite power of Kähler form of WCW reducing the integral to cohomology which should make sense also p-adically. The p-adicization of the WCW integrals has been discussed already earlier using an approach based on harmonic analysis in symmetric spaces and these two approaches should be equivalent. One could also consider a more general quantization of Kähler action as sum $K = K_1 + K_2$ where $K_1 = rlog(m/n)$ and $K_2 = n$, with n divisible by p since $exp(n)$ exists in this case and one has $exp(K) = (m/n)^r \times exp(n)$. Also transcendental extensions of p-adic numbers involving $n + p - 2$ powers of $e^{1/n}$ can be considered.

4. If the Galois group algebras indeed define a representation for WCW spinor fields in finite measurement resolution, also WCW integration would reduce to summations over the Galois groups involved so that integrals would be well-defined in all number fields.

3. Floer homology, Gromov-Witten invariants, and TGD

Floer homology defines a generalization of Morse theory allowing to deduce symplectic homology groups by studying Morse theory in loop space of the symplectic manifold. Since the symplectic transformations of the boundary of $\delta M^4_\pm \times CP_2$ define isometry group of WCW, it is very natural to expect that Kähler action defines a generalization of the Floer homology allowing to understand the symplectic aspects of quantum TGD. The hierarchy of Planck constants implied by the one-to-many correspondence between canonical momentum densities and time derivatives of the imbedding space coordinates leads naturally to singular coverings of the imbedding space and the resulting symplectic Morse theory could characterize the homology of these coverings.

One ends up to a more precise definition of vacuum functional: Kähler action reduces Chern-Simons terms (imaginary in Minkowskian regions and real in Euclidian regions) so that it has both phase and real exponent which makes the functional integral well-defined. Both the phase factor and its conjugate must be allowed and the resulting degeneracy of ground state could allow to understand qualitatively the delicacies of CP breaking and its sensitivity to the parameters of the system. The critical points with respect to zero modes correspond to those for Kähler function. The critical points with respect to complex coordinates associated with quantum fluctuating degrees of freedom are not allowed by the positive definiteness of Kähler metric of WCW. One can say that Kähler and Morse functions define the real and imaginary parts of the exponent of vacuum functional.

The generalization of Floer homology inspires several new insights. In particular, space-time surface as hyper-quaternionic surface could define the 4-D counterpart for pseudo-holomorphic 2-surfaces in Floer homology. Holomorphic partonic 2-surfaces could in turn correspond to the extrema of Kähler function with respect to zero modes and holomorphy would be accompanied by super-symmetry.

Gromov-Witten invariants appear in Floer homology and topological string theories and this inspires the attempt to build an overall view about their role in TGD. Generalization of topological string theories of type A and B to TGD framework is proposed. The TGD counterpart of the mirror symmetry would be the equivalence of formulations of TGD in $H = M^4 \times CP_2$ and in $CP_3 \times CP_3$ with space-time surfaces replaced with 6-D sphere bundles.

4. K-theory, branes, and TGD

K-theory and its generalizations play a fundamental role in super-string models and M-theory since they allow a topological classification of branes. After representing some physical objections against the notion of brane more technical problems of this approach are discussed briefly and it is proposed how TGD allows to overcome these problems. A more precise formulation of the weak form of electric-magnetic duality emerges: the original formulation was not quite correct for space-time regions with Euclidian signature of the induced metric. The question about possible TGD counterparts of R-R and NS-NS fields and S, T, and U dualities is discussed.

5. p-Adic space-time sheets as correlates for Boolean cognition

p-Adic physics is interpreted as physical correlate for cognition. The so called Stone spaces are in one-one correspondence with Boolean algebras and have typically 2-adic topologies. A generalization to p-adic case with the interpretation of p pinary digits as physically representable Boolean statements of a Boolean algebra with $2^n > p > p^{n-1}$ statements is encouraged by p-adic length scale hypothesis. Stone spaces are synonymous with profinite spaces about which both finite and infinite Galois groups represent basic examples. This provides a strong support for the connection between Boolean cognition and p-adic space-time physics. The Stone space character of Galois groups suggests also a deep connection between number theory and cognition and some arguments providing support for this vision are discussed.

1 Introduction

The construction of twistor amplitudes has led to the realization that the work of Grothendieck related to motivic cohomology simplifies enormously the calculation of the integrals of holomorphic forms over sub-varieties of the projective spaces involved. What one obtains are integrals of multivalued functions known as Grassmannian poly-logarithms generalizing the notion of poly-logarithm [3] and Goncharov has given a simple formula for these integrals [10] using methods of motivic cohomology [23] in terms of classical polylogarithms $Li_k(x)$, $k = 1, 2, 3, ...$ This suggests that motivic cohomology might have applications in quantum physics also as a a conceptual tool. One could even hope that quantum physics could provide fresh insights algebraic geometry and topology.

Ordinary theoretical physicist probably does not encounter the notions of homotopy, homology, and cohomology in his daily work and Grothendieck's work looks to him (or at least me!) like a horrible abstraction going completely over the head. Perhaps it is after all good to at least try to understand what this all is about. The association of new ideas with TGD is for me the most effective manner to gain at least the impression that I have managed to understand something and I will apply this method also now. If anything else, this strategy makes the learning of new concepts an intellectual adventure producing genuine surprises, reckless speculations, and in some cases perhaps even genuine output. I do not pretend of being a real mathematician and I present my humble apologies for all misunderstandings unavoidable in this kind enterprise. One should take the summary about the basics of cohomology theory just as a summary of a journalist. I still hope that these scribblings could stimulate mathematical imagination of a real mathematician.

While trying to understand Wikipedia summaries about the notions related to the motivic cohomology I was surprised in discovering how similar the goals and basic ideas about how to achieve them of quantum TGD and motive theory are despite the fact that we work at totally different levels of mathematical abstraction and technicality. I am however convinced that TGD as a physical theory represents similar high level of abstraction and therefore dare hope that the interaction of the these ideas might produce something useful. As a matter fact, I was also surprised that TGD indeed provides a radically new approach to the problem of constructing topological invariants for algebraic and even more general surfaces.

1.1 What are the deep problems?

In motivic cohomology one wants to relate and unify various cohomologies defined for a given number field and its extensions and even for different number fields if I have understood correctly. In TGD one would like to fuse together real and various p-adic physics and this would suggest that one must relate also the cohomology theories defined in different number fields. Number theoretical universality [18] allowing to relate physics in different number fields is one of the key ideas involved.

Why the generalization of homology [16] and cohomology [5] to p-adic context is so non-trivial? Is it the failure of the notion of boundary does not allow to define homology in geometric sense in p-adic context using geometric approach. The lack of definite integral in turn does not allow to define p-adic counterparts of forms except as a purely local notion so that one cannot speak about values of forms for sub-varieties. Residue calculus provides one way out and various cohomology theories defined in finite and p-adic number fields actually define integration for forms over closed surfaces (so that the troublesome boundaries are not needed), which is however much less than genuine integration. In twistor approach to scattering amplitudes one indeed encounters integrals of forms for varieties in projective spaces.

Galois group [13] is defined as the group leaving invariant the rational functions of roots of polynomial having values in the original field. A modern definition is as the automorphism group of the algebraic extension of number field generated by roots with the property that it acts trivially in the original field.

1. Some examples Galois group in the field or rationals are in order. The simplest example is second order polynomial in the field of rationals for which the group is Z_2 if roots are not rational numbers. Second example is $P(x) = x^n - 1$ for which the group is cyclic group $S(n)$ permuting the roots of unity which appear in the elementary symmetric functions of the roots which are rational. When the roots are such that all their products except the product of all roots are irrational numbers, the situation is same since all symmetric functions appearing in the polynomial must be rational valued. Group is smaller if the product for two or more subsets of roots is real. Galois group generalizes to the situation when one has a polynomial of many variables: in this case one obtains for the first variable ordinary roots but polynomials appearing as arguments. Now one must consider algebraic functions as extension of the algebra of polynomial functions with rational coefficients.

2. Galois group permutes branches of the graph $x = (P_n^{-1})(y, ...)$ of the inverse function of the polynomial analogous to the group permuting sheets of the covering space. Galois group is therefore analogous to first homotopy group. Since Galois group is subgroup of permutation group, since permutation group can be lifted to braid group acting as the first homotopy group on plane with punctures, and since the homotopies of plane can be induced by flows, this analogy can be made more precise and leads to a connection with topological quantum field theories for braid groups.

3. Galois group makes sense also in padic context and for finite fields and its abelianization by mapping commutator group to unit element gives rise to the analog of homology group and by Poincare duality to cohomology group. One can also construct p-adic and finite field representations of Galois groups.

These observations motivate the following questions. Could Galois group be generalized to so that they would give rise to the analogs of homotopy groups and homology and cohomology groups as their abelianizations? Could one find a geometric representation for boundary operation making sense also in p-adic context?

1.2 TGD background

The visions about physics as geometry and physics as generalized number theory suggest that number theoretical formulation of homotopy-, homology-, and cohomology groups might be possible in terms of a generalization of the notion of Galois group, which is the unifying notion of number theory. Already the observations of Andre Weil suggesting a deep connection between topological characteristics of a variety and its number theoretic properties indicate this kind of connection and this is what seems to emerge and led to Weil cohomology formulated. The notion of motivic Galois group is an attempt to realize this idea.

Physics as a generalized number theory involves three threads.

1. The fusion of real and p-adic number fields to a larger structure requires number theoretical universality in some sense and leads to a generalization of the notion of number by fusion reals and p-adic number fields together along common rationals (roughly) [18].

2. There are good hopes that the classical number fields could allow to understand standard model symmetries and there are good hopes of understanding $M^4 \times CP_2$ and the classical dynamics of space-time number theoretically [19].

3. The construction of infinite primes having interpretation as a repeated second quantization of an supersymmetric arithmetic QFT having very direct connections with physics is the third thread [17]. The hierarchy has many interpretations: as a hierarchy of space-time sheets for many-sheeted space with each level of hierarchy giving rise to elementary fermions and bosons as bound states of lower level bosons and fermions, hierarchy of logics of various orders realized

as statements about statements about..., or a hierarchy of polynomials of several variables with a natural ordering of the arguments.

This approach leads also to a generalization of the notion of number by giving it an infinitely complex number theoretical anatomy implied by the existence of real units defined by the ratios of infinite primes reducing to real units in real topology. Depending one one's tastes one can speak about number theoretic Brahman=Atman identity or algebraic holography. This picture generalizes to the level of quaternionic and octonionic primes and leads to the proposal that standard model quantum numbers could be understand number theoretically. The proposal is that the number theoretic anatomy could allow to represent the "world of classical worlds" (WCW) as sub-manifolds of the infinite-dimensional space of units assignable to single point of space-time and also WCW spinor fields as quantum superpositions of the units. One also ends up with he idea that there is an evolution associated with the points of the imbedding space as an increase of number theoretical complexity. One could perhaps say that this space represents "Platonia".

1.3 Homology and cohomology theories based on groups algebras for a hierarchy of Galois groups assigned to polynomials defined by infinite primes

The basic philosophy is that the elements of homology and cohomology should have interpretation as states of supersymmetric quantum field theory just as the infinite primes do have. Even more, TGD as almost topological QFT requires that these groups should define quantum states in the Universe predicted by quantum TGD. The basic ideas of the proposal are simple.

1. One can assign to infinite prime at n:th level of hierarchy of second quantizations a rational function and solve its polynomial roots by restricting the rational function to the planes $x_n, ...x_k = 0$. At the lowest level one obtains ordinary roots as algebraic number. At each level one can assign Galois group and to this hierarchy of Galois groups one wants to assign homology and cohomology theories. Geometrically boundary operation would correspond to the restriction to the plane $x_k = 0$. Different permutations for the restrictions would define non-equivalent sequences of Galois groups and the physical picture suggests that all these are needed to characterize the algebraic variety in question.

2. The boundary operation applied to G_k gives element in the commutator subgroup $[G_{k-2}, G_{k-2}]$. In abelianization this element goes to zero and one obtains ordinary homology theory. Therefore one has the algebraic analog of homotopy theory,

3. In order to obtain both homotopy and cohomotopy and cohomology and homology as their abelizations plus a resemblance with ordinary cohomology one must replace Galois groups by their group algebras. The elements of the group algebras have a natural interpretation as bosonic wave functions. The dual of group algebra defines naturally cohomotopy and cohomology theories. One expects that there is a large number of boundary homomorphisms and the assumption is that these homomorphisms satisfy anticommutation relations with anticommutor equal to an element of commutator subgroup $[G_{k-2}, G_{k-2},]$ so that in abelianization one obtains ordinary anticommutation relations. The interpretation for the boundary and coboundary operators would be in terms of fermionic annihilation (creation) operators is suggesti so that homology and cohomology would represent quantum states of super-symmetric QFT. Poincare duality would correspond to hermitian conjugation mapping fermionic creation operators to annihilation operators and vice versa. It however turns out that the analogy with Dolbeault cohomology with several exterior derivatives is more approrpriate.

4. In quantum TGD states are realized as many-fermion states assignable to intersections of braids with partonic 2-surfaces. Braid picture is implied by the finite measurement resolution implying discretization at space-time level. Symplectic transformations in turn act as fundamental symmetries of quantum TGD and given sector of WCW corresponds to symplectic group as far as quantum fluctuating degrees of freedom are considered. This encourages the hypothesis that the hierarchy of Galois groups assignable to infinite prime (integer/rational) having interpretation in terms of repeated second quantization can be mapped to a braid of braids of ....

The Galois group elements lifted to braid group elements would be realized as symplectic flows and boundary homomorphism would correspond to symplectic flow induced at given level in the interior of sub-braids and inducing action of braid group. In this framework the braided Galois group cohhomology would correspond to the states of WCW spinor fields in "orbital" degrees of freedom in finite measurement resolution realized in terms of number theoretical discretization.

If this vision is correct, the construction of quantum states in finite measurement resolution would have purely number theoretic interpretation and would conform with the interpretation of quantum TGD as almost topological QFT. That the groups characterize algebraic geometry than mere topology would give a concrete content to the overall important "almost" and would be in accordance with physics as infinite-dimensional geometry vision.

1.4 p-Adic integration and cohomology

This picture leads also to a proposal how p-adic integrals could be defined in TGD framework.

1. The calculation of twistorial amplitudes reduces to multi-dimensional residue calculus. Motivic integration gives excellent hopes for the p-adic existence of this calculus and braid representation would give space-time representation for the residue integrals in terms of the braid points representing poles of the integrand: this would conform with quantum classical correspondence. The power of 2π appearing in multiple residue integral is problematic unless it disappears from scattering amplitudes. Otherwise one must allow an extension of p-adic numbers to a ring containing powers of 2π.

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1.5 Topics related to TGD-string theory correspondence

Although M-theory has not been successful as a physical theory it has led to a creation of enormously powerful mathematics and there are all reasons to expect that this mathematics applies also in TGD framework.

1.5.1 Floer homology, Gromov-Witten invariants, and TGD

Floer homology defines a generalization of Morse theory allowing to deduce symplectic homology groups by studying Morse theory in loop space of the symplectic manifold. Since the symplectic transformations of the boundary of $\delta M^4_\pm \times CP_2$ define isometry group of WCW, it is very natural to expect that Kähler action defines a generalization of the Floer homology allowing to understand the symplectic aspects of quantum TGD. The hierarchy of Planck constants implied by the one-to-many correspondence between canonical momentum densities and time derivatives of the imbedding space coordinates leads naturally to singular coverings of the imbedding space and the resulting symplectic Morse theory could characterize the homology of these coverings.

One ends up to a more precise definition of vacuum functional: Kähler action reduces Chern-Simons terms (imaginary in Minkowskian regions and real in Euclidian regions) so that it has both phase and real exponent which makes the functional integral well-defined. Both the phase factor and its conjugate must be allowed and the resulting degeneracy of ground state could allow to understand qualitatively the delicacies of CP breaking and its sensitivity to the parameters of the system. The critical points with respect to zero modes correspond to those for Kähler function. The critical points with respect to complex coordinates associated with quantum fluctuating degrees of freedom are not allowed by the positive definiteness of Kähler metric of WCW. One can say that Kähler and Morse functions define the real and imaginary parts of the exponent of vacuum functional.

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Gromov-Witten invariants appear in Floer homology and topological string theories and this inspires the attempt to build an overall view about their role in TGD. Generalization of topological string theories of type A and B to TGD framework is proposed. The TGD counterpart of the mirror symmetry would be the equivalence of formulations of TGD in $H = M^4 \times CP_2$ and in $CP_3 \times CP_3$ with space-time surfaces replaced with 6-D sphere bundles.

1.5.2 K-theory, branes, and TGD

K-theory and its generalizations play a fundamental role in super-string models and M-theory since they allow a topological classification of branes. After representing some physical objections against the notion of brane more technical problems of this approach are discussed briefly and it is proposed how TGD allows to overcome these problems. A more precise formulation of the weak form of electric-magnetic duality emerges: the original formulation was not quite correct for space-time regions with Euclidian signature of the induced metric. The question about possible TGD counterparts of R-R and NS-NS fields and S, T, and U dualities is discussed.

1.6 p-Adic space-time sheets as correlates for Boolean cognition

p-Adic physics is interpreted as physical correlate for cognition. The so called Stone spaces are in one-one correspondence with Boolean algebras and have typically 2-adic topologies. A generalization to p-adic case with the interpretation of p pinary digits as physically representable Boolean statements of a Boolean algebra with $2^n > p > p^{n-1}$ statements is encouraged by p-adic length scale hypothesis. Stone spaces are synonymous with profinite spaces about which both finite and infinite Galois groups represent basic examples. This provides a strong support for the connection between Boolean cognition and p-adic space-time physics. The Stone space character of Galois groups suggests also a deep connection between number theory and cognition and some arguments providing support for this vision are discussed.

2 Some backgbround about homology and cohomology

Before representing layman's summary about the motivations for the motivic cohomology it is good to introduce some basic ideas of algebraic geometry [46].

2.1 Basic ideas of algebraic geometry

In algebraic geometry one considers surfaces defined as common zero locus for some number $m \leq n$ of functions in n-dimensional space and therefore having dimension $n - m$ in the generic case and one wants to find homotopy invariants for these surfaces: the notion of variety is more precise concept in algebraic geometry than surface. The goal is to classify algebraic surfaces represented as zero loci of collections of polynomials.

The properties of the graph of the map $y = P(x)$ in (x,y)-plane serve as an elementary example. Physicists is basically interested on the number of roots x for a given value of y. For polynomials one can solve the roots easily using computer and the resulting numbers are in the generic case algebraic numbers. Galois group is the basic object and permutes the roots with each other. It is analogous to the first homotopy group permuting the points of the covering space of graph having various branches of the many-valued inverse function $x = P^{-1}(y)$ its sheets. Clearly, Galois group has topological meaning but the topology is that of the imbedding or immersion.

There are invariants related to the internal topology of the surface as well as invariants related to the external topology such as Galois group. The generalization of the Galois group for polynomials of single variable to polynomials of several variables looks like an attractive idea. This would require an assignment of sequence of sub-varieties to a given variety. One can assign algebraic extensions also to polynomials and it would seem that these groups must be involved. For instance, the absolute Galois group associated with the algebraic closure of polynomials in algebraically closed field is free group of rank equal to the cardinality of the field (rank is the cardinality of the minimal generating set).

Homotopy [17], homology [17], and cohomology [17] characterize algebraically the shape of the surface as invariant not affected by continuous transformations and by homotopies. The notion of continuity depends on context and in the most general case there is no need to restrict the consideration to rational functions or polynomials or make restrictions on the coefficient field of these functions. For algebraic surfaces one poses restrictictions on coefficient field of polynimials and the ordinary real number based topology is replaced with much rougher Zariski topology for which algebraic surfaces define closed sets. Physicists might see homology and cohomology theories as linearizations of nonlinear notions of manifold and surface obtained by gluing together linear manifolds. This linearization allows to gain information about the topology of manifolds in terms of linear spaces assignable to surfaces of various dimensions.

In homology one considers formal sums for these surfaces with coefficients in some field and basically algebraizes the statement that boundary has no boundary. Cohomology is kind of dual of homology and in differential geometry based cohomology forms having values as their integrals over surfaces of various dimensions realize this notion.

Betti cohomology or singular cohomology [1] defined in terms of simplicial complexes is probably familiar for physicists and even more so the de Rham cohomology [7] defined by n-forms as also the Dolbeault cohomology [8] using forms characterized by m holomorphic and n antiholomorphic indices. In this case the role of continuous maps is taken by holomorphic maps. For instance, the classification of the moduli of 2-D Riemann surfaces involves in an essential manner the periods of one forms on 2-surfaces and plays important role in the TGD based explanation of family replication phenomenon [5].

In category theoretical framework homology theory can be seen as a http://en.wikipedia.org/wiki/functorfunctor [12] that assigns to a variety (or manifold) a sequence of homology groups characterized by the dimension of corresponding sub-manifolds. One considers formal sums of surfaces. The basic operation is that of taking boundary which has operation δ as algebraic counterpart. One identifies cycles as those sums of surfaces for which algebraic boundary vanishes. This is identically true for exact cycles defined as a boundaries of cycles since boundary of boundary is empty. Only those cycles with are not exact matter and the homology group is defines as the coset space of the kernel at n:th level with respect to the image of the $n+1$:th level two spaces. Cohomology groups can be defined in a formally similar manner and for de Rham cohomology Poincare duality maps homology group H_k to H^{n-k}. The correspondence between covariant with vanishing exterior derivative and contravariant antisymmetric tensors with vanishing divergence is the counterpart of homology-cohomology correspondence in Riemann manifolds.

The calculation of homology and cohomology groups relies on general theorems which are often raised to the status of axioms in generalizations of cohomology theory.

1. Exact sequences [10] of Abelian groups define an important calculational tool. So called short exact sequence $0 \rightarrow B \rightarrow C \rightarrow 0$ of chain complexes gives rise to long exact sequence $H_n(A) \rightarrow H_n(B) \rightarrow H_n(C) \rightarrow H_{n-1}(A) \rightarrow H_{n-1}(B) \rightarrow H_{n-1}(C)$....

   One example of short exact sequence is $0 \rightarrow H \rightarrow G \rightarrow G/H \rightarrow 0$ holding true when H is normal subgroup so that also G/H is group. This condition allows to express the homology groups of G as direct sums of those for H and G/H. In relative cohomology inclusion and δ define exact sequences allowing to express relative cohomology groups [30] $H_n(X, A \subset X)$ in terms of those for X and A. Mayer-Vietoris sequence relates the cohomologies of sets A, B and $X = A \cup B$.

2. Künneth theorem [21] allows to calculated homology groups for Cartesian product as convolution of those for the factors with respect to direct sum.

Steenrod-Eilenberg axioms [33] axiomatize cohomology theory in the category of topological spaces: cohomology theory in this category is a functor to graded abelian groups, satisfying the Eilenberg-Steenrod axioms: functoriality, naturality of the boundary homomorphism, long exact sequence, homotopy invariance, and excision. In algebraic cohomology the category is much more restricted: algebraic varieties defined in terms of polynomial equations and these axioms are not enough. In this case Weil cohomology [40] defines a possible axiomatization consisting of finite generation, vanishing outside the range $[0, dim(X)]$, Poincare duality, Künneth product formula, a cycle class map, and the weak and strong Lefschetz axioms.

In p-adic context sets do not have boundaries since p-adic numbers are not well-ordered so that the statement that boundary has vanishing boundary should be formulated using purely algebraic language. Also cohomology is problematic since definite integral is ill-defined for the same reason. This forces to question either the notion of cohomology and homology groups or the definition of geometric boundary operation and inspires the question whether Galois groups might be a more appropriate notion.

Perhaps it is partially due to the lack of a geometric realization of the boundary operation in the case of general number field that there are very many cohomology theories: the brief summary by Andreas Holmstrom written when he started to work with his thesis, gives some idea about how many!

2.2 Algebraization of intersections and unions of varieties

There are several rather abstract notions involved with cohomology theories: categories, functoriality, sheaves, schemes, abelian rings. Abelian ring is essentially the ring of polynomial functions generated by the coordinates in the open subset of the variety.

1. The spectrum of ring consists of its proper prime ideals of this function algebra. Ideal is subset of functions s closed under sum and multiplication by any element of the algebra and proper ideal is subspace of the entire algebra. In the case of the abelian ring defined on algebraic variety maximal ideals correspond to functions vanishing at some point. Prime ideals correspond to functions vanishing in some sub-variety, which does not reduce to a union of sub-varieties (meaning that one has product of two functions of ring which can separately vanish). Thus the points in spectrum correspond to sub-varieties and product of functions correspond to a union of sub-varieties.

2. What is extremely nice that the product of functions represents in general union of disjoint surfaces: for physicist this brings in mind many boson states created by bosonic creation operators with particles identified as surfaces. Therefore union corresponds to a product of ideals defining a non-prime ideal. The notion of ideal is needed since there is enormous gauge invariance involved in the sense that one can multiply the function defining the surface by any everywhere non-vanishing function.

3. The intersection of varieties in turn corresponds to the condition that the functions defining the varieties vanish separately. If one requires that all sums of the functions belonging to the corresponding ideals vanish one obtains the same condition so that one can say that intersection corresponds to vanishing condition for the sum for ideals. The product of cohomology elements corresponds by Poincare duality [26] the intersection of corresponding homology elements interpreted as algebraic cycles so that a beautiful geometric interpretation is possible in real context at least.

Remark: For fermionic statistics the functions would be anti-commutative and this would prevent automatically the powers of ideals. In fact, the possibility of multiple roots for polynomials of several variables implying what is known as ramification [29] represents a non-generic situation and one of the technical problems of algebraic geometry. For ordinary integers ramification means that integer contains in its composition to primes a power of prime which is higher than one. For the extensions of rationals this means that rational prime is product of primes of extension with some roots having multiplicity larger than one. One can of course ask whether higher multiplicity could be interpreted in terms of many-boson state becoming possible at criticality: in quantum physics bosonic excitations (Goldstone bosons) indeed emerge at criticality and give rise to long range interactions. In fact, for infinite primes allowing interpretation in terms of quantum states of arithmetic QFT boson many particle states corresponds to powers of primes so that the analogy is precise.

2.3 Motivations for motives

In the following I try to clarify for myself the motivations for the motivic cohomology which as a general theory is still only partially existent. There is of course no attempt to say anything about the horrible technicalities involved. I just try to translate the general ideas as I have understood (or misunderstood) them to the simple language of mathematically simple minded physicist.

Grothendieck has carried out a monumental work in algebraizing cohomology which only mathematician can appreciate enough. The outcome is a powerful vision and mathematical tools allowing to develop among other things the algebraic variant of de Rham cohomology, etale cohomology having values in p-adic fields different from the p-adic field defining the values of cohomology, and crystalline cohomology [6].

As the grand unifier of mathematics Grothendieck posed the question whether there good exists a more general theory allowing to deduce various cohomologies from single grand cohomology. These cohomology theories would be like variations of the same them having some fundamental core element -motive- in common.

Category theory [4] and the notion of scheme [32], which assigns to open sets of manifold abelian rings - roughly algebras of polynomial functions- consistent with the algebra of open sets, provide the backbone for this approach. To the mind of physicist the notion of scheme brings abelian gauge theory with non-trivial bundle structure requiring several patches and gauge transformations between them. A basic challenge is to relate to each other the cohomologies associated with algebraic varieties with given number field k manifolds. Category theory is the basic starting point: cohomology theory assigns to each category of varieties category of corresponding cohomologies and functors between these categories allow to map the cohomologies to each other and compare different cohomology theories.

One of the basic ideas underlying the motivic cohomology seems is that one should be able perform a local lifting of a scheme from characteristic p (algebraic variety in p-adic number field or its algebraic extension) to that in characteristic 0 (characteristic is the integer n for which the sum of n units is zero, for rational numbers, p-adic number fields and their extensions characteristic is zero and p for finite fields) that is real or complex algebraic variety, to calculate various cohomologies here as algebraic de Rham cohomology and using the lifting to induce the cohomology to p-adic context. One expects that the ring in which cohomology has naturally values consists of ordinary or p-adic integers or extension of p-adic integers. In the case of crystalline cohomology this is however not enough.

The lifting of the scheme is far from trivial since number fields are different and real cohomology has naturally Z or Q as coefficient ring whereas p-adic cohomology has p-adic integers as coefficient ring. This lift must bring in analytic continuation which is lacking at p-adic side since n particular in p-adic topology two spheres with same radius are either non-intersecting or identical. Analytical continuation using a net of overlapping open sets is not possible.

One could even dream of relating the cohomologies associated with different number fields. I do not know to what extend this challenge is taken or whether it is regarded as sensible at all. In TGD framework this kind of map is needed and leads ot the generalization of the number field obtained by glueing together reals and p-adic numbers among rationals and common algebraic numbers. This glueing together makes sense also for the space of surfaces by identifying the surfaces which correspond to zero loci of rational functions with rational coefficients. Similar glueing makes sense for the spaces of polynomials and rational functions.

Remarks::

1. The possibility of p-adic pseudo-constants in the solutions of p-adic differential and p-adic differential equations reflects this difficulty. This lifting should remove this non-uniqueness in

analytical continuation. One can of course ask whether the idea is good: maybe the p-adic pseudo constants have some deep meaning. A possible interpretation would be in terms of non-deterministic character of cognition for which p-adic space-time sheets would be correlates. The p-adic space-time sheets would represent intentions which can be transformed to actions in quantum jumps. If one works in the intersection of real and p-adic worlds in which one allows only rational functions with coefficients in the field or rationals or possibly in some algebraic extension of rationals situation changes and non-uniqueness disappears in the intersection of real and p-adic worlds and one might argue that it is here where the universal cohomology applies or that real and p-adic cohomologies are obtained by some kind of algebraic continuation from this cohomology.

2. The universal cohomology theory brings in mind the challenge encountered in the construction of quantum TGD. The goal is to fuse real physics and various p-adic physics to single coherent whole so that one would have kind of algebraic universality. To achieve this I have been forced to introduce a heuristic generalization of number field by fusing together reals and various p-adic number fields among rationals and common algebraic numbers. The notion of infinite primes is second key notion. The hierarchy of Planck constants involving extensions of p-adic numbers by roots of unity is closely related to p-adic length scale hierarchy and seems to be an essential part of the number theoretical vision.

3 Examples of cohomologies

In the following some examples of cohomologies are briefly discussed in hope of giving some idea about the problems involved. Probably the discussion reflects the gaps in my understanding rather than my understanding.

3.1 Etale cohomology and l-adic cohomology

Etale cohomology [9] is defined for algebraic varieties as analogues of ordinary cohomology groups of topological space. They are defined purely algebraically and make sense also for finite fields. The notion of definite integral fails in p-adic context so that also the notion of form makes sense only locally but not as a map assigning numbers to surfaces. This is cohomological counterpart for the non-existence of boundaries in p-adic realm. Etale cohomology allows to define cohomology groups also in p-adic context as l-adic cohomology groups.

In Zariski topology closed sets correspond to surfaces defined as zero loci for polynomials in given field. The number of functions is restricted only by the dimension of the space. In the real case this topology is much rougher than real topology. In etale cohomology Zariski topology is too rough. One needs more open sets but one does not want to give up Zariski topology.

The category of etale maps is the structure needed and actually generalizes the notion of topology. Instead of open sets one considers maps to the space and effectively replaces the open sets with their inverse images in another space. Etale maps -idempotent are essentially projections from coverings of the variety to variety. One can say that open sets are replaced with open sets for the covering of the space and mapping is replaced with a correspondence (for algebraic surfaces X and Y the correspondence is given by algebraic equations in $X \times Y$) which in general is multi-valued and this leads to the notion of etale topology. The etale condition is formulated in the Wikipedia article in a rather tricky manner telling not much to a physicist trying to assign some meaning to this word. Etale requirement is the condition that would allow one to apply the implicit function theorem if it were true in algebraic geometry: it is not true since the inverse of rational map is not in general rational map except in the ase of birational maps to which one assigns birational geometry [2].

Remarks:

1. In TGD framework field as a map from M^4 to some target space is replaced with a surface in space $M^4 \times CP_2$ and the roles of fields and space are permuted for the regions of space-time representing lines of generalized Feynman diagrams. Therefore the relation between M^4 and CP_2 coordinates is given by correspondence. Many-sheeted space-time is locally a many-sheeted covering of Minkowski space.

2. Also the hierarchy of Planck constant involving hierarchy of coverings defined by same values of canonical momentum densities but different values of time derivatives of imbedding space coordinates. The enormous vacuum degeneracy of Kähler action is responsible for this many-valuedness.

3. Implicit function theorem indeed gives several values for time derivatives of imbedding space coordinates as roots to the conditions fixing the values of canonical momentum densities.

The second heuristic idea is that certain basic cases corresponding to dimensions 0 and 1 and abelian varieties which are also algebraic groups obeying group law defined by regular (analytic and single valued) functions are special and same results should follow in these cases.

Etale cohomologies satisfy Poincare duality and Künneth formula stating that homology groups for Cartesian product are convolutions of homology groups with respect to tensor product. l-adic cohomology groups have values in the ring of l-adic integers and are acted on by the absolute Galois group of rational numbers for which no direct description is known.

3.2 Crystalline cohomology

Crystalline cohomology represents such level of technicality that it is very difficult for physicists without the needed background to understand what is in question. I however make a brave attempt by comparing with analogous problems encountered in the realization of number theoretic universality in TGD framework. The problem is however something like follows.

1. For an algebraically closed field with characteristic p it is not possible to have a cohomology in the ring Z_p of p-adic integers. This relates to the fact that the equation for $x^n = x$ in finite field has only complex roots of unity as its solutions when n is not divisible by p whereas for he integers n divisible by p are exceptional due to the fact that $x^p = x$ holds true for all elements of finite field $G(p)$. This implies that $x^p = x$ has p solutions which are ordinary p-adic numbers rather than numbers in an algebraic extension by a root of unity. p-Adic numbers indeed contain n:th cyclotomic field only if n divides $p-1$. On the other hand, any finite field has order $q = p^n$ and can be obtained as an algebraic extension of finite field $G(p)$ with p elements. Its elements satisfy the Frobenius condition $x^{q=p^n} = x$. This condition cannot be satisfied if the extension contains p:th root of unity satisgying $u^p = 1$ since one would have $(xu)^{p^n} = x \neq xu$. Therefore finite fields do not allow an algebraic extensions allowing p :th root of unity so the extension of p-adic numbers containing p:th root of unity cannot be not induced by the extension of $G(p)$. As a consequence one cannot lift cohomology in finite field $G(p^n)$ to p-adic cohomology.

2. Also in TGD inspired vision about integration $p-1$:th and possibly also p:th roots are problematic. p-Adic cohomology is about integration of forms and the reason why integration necessitates various roots of unity can be understood as follows in TGD framework. The idea is to reduce integration to Fourier analysis which makes sense even for the p-adic variant of the space in the case that it is symmetric space. The only reasonable definition of Fourier analysis is in terms of discrete plane waves which come as powers of n:th root of unity. This notion makes sense if n is not divisible by p. This leads to a construction of p-adic variants of symmetric spaces G/H obtained by discretizing the groups to some algebraic subgroup and replacing the discretized points by p-adic continuum. Certainly the n:th roots of unity with n dividing $p-1$ are problematic since they do not corresponds to phase factors. It seems however clear that one can construct an extension of p-adic numbers containing p:th roots of unity. If it is however necesssary to assume that the extension of p-adic numbers is induced by that for a finite field, situation changes. Only roots of unity for n not divisible by factors of $p-1$ and possibly also by p can appear in the discretizations. There is infinite number extensions and the interpretation is in terms of a varying finite measurement resolution.

3. In TGD framework one ends up with roots of unity also when one wants to realize p-adic variants of various finite group representations. The simplest case is p-adic representations of angular momentum eigenstates and plane waves. In the construction of p-adic variants of symmetric spaces one is also forced to introduce roots of unity. One obtains a hierarchy of extensions involving increasing number of roots of unity and the interpretation is in terms of

number theoretic evolution of cognition involving both the increase of maximal value of n and the largest prime involved. Witt ring could be seen as an idealization in which all roots of unity possible are present.

For $l = p$ l-adic cohomology fails for characteristic p. Crystalline cohomology fills in this gap. Roughly speaking crystalline cohomology is de Rham cohomology of a smooth lift of X over a field k with with characteristic p to a variety so called ring of Witt vectors with characteristic 0 consisting of infinite sequences of the elements of k while de Rham cohomology of X is the crystalline cohomology reduced modulo p.

The ring of Witt vectors for characteristic p is particular example of ring of Witt vectors [41] assignable to any ring as infinite sequences of elements of ring. For finite field G_p the Witt vectors define the ring of p-adic integers. For extensions of finite field one has extensions of p-adic numbres. The algebraically closed extension of finite field contains n:th roots of unity for all n not divisible by p so that one has algebraic closure of finite field with p elements. For maximal extension of the finite field G_p the Witt ring is thus a completion of the maximal unramified extension of p-adic integers and contains n:th roots of unity for n not divisible by p. "Unramified" [29] means that p defining prime for p-adic integers splits in extension to primes in such a manner that each prime of extension occurs only once: the analogy is a polynomial whose roots have multiplicity one. This ring is much larger than the ring of p-adic integers. The algebraic variety is lifted to a variety in Witt ring with characteristic 0 and one calculates de Rham cohomology using Witt ring as a coefficient field.

3.3 Motivic cohomology

Motivic cohomology is a attempt to unify various cohomologies as variations of the same motive common to all of them. In motivic cohomology [23] one encounters pure motives and mixed motives. Pure motives is a category associated with algebraic varieties in a given number field k with a contravariant functor from varieties to the category assigning to the variety its cohomology groups. Only smooth projective varieties are considered. For mixed motives more general varieties are allowed. For instance, the condition that projective variety meaning that one considers only homogenous polynomials is given up.

Chow motives [24] is an example of this kind of cohomology theory and relies on very geometric notion of Chow ring with equivalence of algebraic varieties understood as rational equivalence. One can replace rational equivalence with many variants: birational, algebraic, homological, numerical, etc...

The vision about rationals as common points of reals and p-adic number fields leads to ask whether the intersection of these cohomologies corresponds to the cohomology associated with varieties defined by rational functions with rational coefficients. In both p-adic and real cases the number of varieties is larger but the equivalences are stronger than in the intersection. For a non-professional it is impossible to say whether the idea about rational cohomology in the intersection of these cohomologies makes sense.

Homology and cohomology theories rely in an essential manner to the idea of regarding varieties with same shape equivalent. This inspires the idea that the polynomials or rational functions with rational coefficients could correspond to something analogous to a gauge choice without losing relevant information or bringing in information which is irrelevant. If this gauge choice is correct then real and p-adic cohomologies and homologies would be equivalent apart from modifications coming from the different topology for the real and p-adic integers.

4 Infinite rationals define rational functions of several variables: a possible number theoretic generalization for the notions of homotopy, homology, and cohomology

This section represents my modest proposal for how the generalization of number theory based on infinite integers might contribute to the construction of topological and number theoretic invariants of varieties. I can represent only the primitive formulation using the language of second year math student. The construction is motivated by the notion of infinite prime but applies to ordinary polynomials in which case however the motivation is not so obvious. The visions about TGD as almost topological QFT, about TGD as generalized number theory, and about TGD as infinite-dimensional geometry serve as the main guidelines and allow to resolve the problems that plagued the first version of the theory.

4.1 Infinite rationals and rational functions of several variables

Infinite rationals correspond in natural manner to rational functions of several variables.

1. If the number of variables is 1 one has infinite primes at the first level of the hierarchy as formal rational functions of variable X having as its value as product of all finite primes and one can decompose the polynomial to prime polynomial factors. This amounts to solving the roots of the polynomial by obtained by replacing X with formal variable x which is real variable for ordinary rationals. For Gaussian rationals one can use complex variable.

2. If the roots are not rationals one has infinite prime. Physically this state is the analog of bound state whereas first order polynomials correspond to free many-particle states of supersymmetric arithmetic QFT.

3. Galois group permuting the roots has geometric interpretation as the analog of the group of deck transformations permuting the roots of the covering of the graph of the polynomial y=f(x) at origin. Galois group is analogous to fundamental group whose abelianization obtained as a coset group by dividing with the commutator group gives first homology group. The finiteness of the Galois group does not conform with the view about cohomology and homology, which suggests that it is the group algebra of Galois group which is the correct mathematical structure to consider.

One can find the roots also at the higher levels of the hierarchy of infinite primes. One proceeds by finding the roots at the highest level as roots which are algebraic functions. In other words finds the decomposition

$$P(x_n, ...) = \prod_k (x_n - R_k(x_{n-1}, ...)$$

with R_k expanded in powers series with respect to x_{n-1}. This expansion is the only manner to make sense about the root if x_{n-1} corresponds to infinite prime. At the next step one puts $x_n = 0$ and obtains a product of R_k and performs the same procedure for x_{n-1} and continues down to $n = 1$ giving ordinary algebraic numbers as roots. One therefore obtains a sequence of sub-varieties by restricting the polynomial to various planes $x_i = 0$, $i = k, ..., n$ of dimension $k - 1$. The invariants associated with the intersections with these planes define the Galois groups characterizing the polynomial and therefore also infinite prime itself.

1. The process takes place in a sequential manner. One interprets first the infinite primes at level n+1 as as polynomial function in the variable X_{n+1} with coefficients depending on X_k, $k < n + 1$. One expands the roots R in power series in the variable X_n. In p-adic topology this series converges for all primes of the previous levels and the deviation from the value at $X_n = 0$ is infinitesimal in infinite-P p-adic topology.

2. What is new as compared to the ordinary situation is that the necessity of Taylor expansion, which might not even make sense for ordinary polynomials. One can find the roots and one can assign a Galois group to them.

3. One obtains a hierarchy of Galois groups permuting the roots and at the lowest level on obtains roots as ordinary algebraic numbers and can assign ordinary Galois group to them. The Galois group assigned to the collection of roots is direct sum of the Galois groups associated with the individual roots. The roots can be regarded as a power series in the variables X and the deviation from algebraic number is infinitesimal in infinite-p p-adic topology.

4. The interesting possibility is that the infinitesimal deformations of algebraic numbers could be interpreted as a generalization of real numbers. In the construction of motivic cohomology the idea is to lift varieties defined for surfaces in field of characteristic p (finite fields and their

extensions) to surfaces in characteristic 0 field (p-adic numbers) in some sense to infinitesimal thickenings of their characteristic 0 counterparts. Something analogous is encountered in the proposed scenario since the roots of the polynomials are algebraic numbers plus multi-p p-adic expansion in terms of infinite-p p-adic numbers representing infinitesimal in infinite-p p-adic topology.

4.2 Galois groups as non-commutative analogs of homotopy groups

What one obtains is a hierarchy of Galois groups and varieties of $n+1$-dimensional space with dimensions $n, n-1, ..., 1, 0$.

1. A suggestive geometric interpretation would be as an analog of first homotopy group permuting the roots which are now surfaces of given dimension k on one hand and as a higher homotopy group π_k on the on the other hand. This and the analogy with ordinary homology groups suggests the replacement of Galois group with their group algebras. Homology groups would be obtained by abelianization of the analogs of homotopy groups with the square of the boundary homomorphism mapping the group element to commutator sub-group. Group algebra allows also definition of cohomotopy and cohomology groups by assigning them to the dual of the group algebra.

2. The boundary operation is very probably not unique and the natural proposal inspired by physical intuition is that the boundary operations form an anticommutative algebra having interpretation in terms of fermionic creation (say) operators. Cohomology would in turn correspond to annilation operators. Poincare duality would be hermitian conjugation mapping fermionic creation operators to annihilation operators and vice versa. Number theoretic vision combined with the braid representation of the infinite primes in turn suggests that the construction actually reduces the construction of quantum TGD to the construction of these homology and cohomology theories.

3. The Galois analogs of homotopy groups and their duals up to the dimension of the algebraic surface would be obtained but not the higher ones. Note that for ordinary homotopy groups all homotopy group π_n, $n > 1$ are Abelian so that the analogy is not complete. The abelianizations of these Galois groups could in turn give rise to higher homology groups. Since the rational functions involved make sense in all number fields this could provide a possible solution to the challenge of constructing universal cohomology theory.

The hierarchy of infinite primes and the hierarchy of Galois groups associated with the corresponding polynomials have as an obvious analogy the hierarchy of loop groups and corresponding homotopy groups.

1. The construction brings in mind the reduction of n-dimensional homotopy to a 1-D homotopy of n-1-D homotopy. Intuitively n-dimensional homotopy indeed looks like a 1-D homotopy of n-1-D homotopy so that everything should reduce to iterated 1-dimensional homotopies by replacing the original space with the space of maps to it.

2. The hierarchical ordering of the variables plays an essential role. The ordering brings strongly in mind loop groups. Loop group $L(X^m, G)$ defined by the maps from space X^n to group G can be also regarded as a loop group from space X^m to the loop group $L(X^{n-m}, G)$ and one obtains $L(X^n, G) = L(X^1, L(X^{n-1})$.

   The homotopy equivalence classes of these maps define homotopy groups using the spaces X^n instead of spheres. Infinite primes at level n would correspond to $L(X^n, G)$. Locally the fundamental loop group is defined by $X = S^1$ which would suggest that homotopy theory using tori might be more natural then the one using spheres. Naively one might hope that this kind of groups could code for all homotopic information about space. As a matter fact, even more general identity $L(X \times Y, G) = L(X, L(Y, G))$ seems to hold true.

3. Note that one can consider also many variants of homotopy theories since one can replace the image of the sphere in manifold with the image of any manyfold and construct corresponding homotopy theory. Sphere and tori define only the simplest homotopy theories.

4.3 Generalization of the boundary operation

The algebraic realization of boundary operation should have a geometric counterpart at least in real case and it would be even better if this were the case also p-adically and even for finite fields.

1. The geometric analog of the boundary operation would replace the k-dimensional variety with its intersection with $x_k = 0$ hyperplane producing a union of $k-1$-dimensional varieties. This operation would make sense in all number fields. The components in the union of the surface would be very much analogous to the lower-dimensional edges of k-simplex so that boundary operation might make sense. What comes in mind is relative homology $H(X, A)$ in which the intersection of X with $A \subset X$ is equivalent with boundary so that its boundary vanishes. Maybe one should interpret the homology groups as being associated with the sequence of relative homologies defined by the sequence of varieties involved as $A_0 \subset A_1 \subset ..$ and relativizing for each pair in the sequence. The ordinary geometric boundary operation is ill-defined in p-adic context but its analog defined in this manner would be number theoretically universal notion making sense also for finite fields.

2. The geometric idea about boundary of boundary as empty set should be realized somehow- at least in the real context. If the boundary operation is consistent with the ordinary homology, it should give rise to a surface which as an element of H_{n-2} is homologically trivial. In relative homology interpretation this is indeed the case. In real context the condition is satisfied if the intersection of the n-dimensional surface with the $x_{n-1} = 0$ hyper-plane consists of closed surface so that the boundary indeed vanishes. This is indeed the case as simplest visualizations in 3-D case demonstrate. Therefore the key geometric idea would be that that the intersection of the surface defined by zeros of polynomial with lower dimensional plane is a closed surface in real context and that this generalizes to p-adic context as algebraic statement at the level of homology.

3. The sequence of slicings could be defined by any permutation of coordinates. The question is whether the permutations lead to identical homologies and cohomologies. The physical interpretation does not encourage this expection so that different permutation would all be needed to characterize the variety using the proposed homology groups.

4.4 Could Galois groups lead to number theoretical generalizations of homology and cohomology groups?

My own humble proposal for a number theoretic approach to algebraic topology is motivated by the above questions. The notion of infinite primes leads to a proposal of how one might assign to a variety a sequence of Galois group [13] algebras defining analogs of homotopy groups assignable to the algebraic extensions of polynomials of many variables obtained by putting the variables of a polynomial of n-variable polynomial one by one to zero and finding the Galois groups of the resulting lower dimensional varieties as Galois groups of corresponding extensions of polynomial fields. The construction of the roots is discussed in detail [15], where infinite primes are compared with non-standard numbers. The earlier idea about the possibility to lift Galois groups to braid groups is also essential and implies a connection with several key notions of quantum TGD.

1. One can assign to infinite primes at the n:th level of hierarchy (n is the number of second quantizations) polynomials of n variables with variables ordered according to the level of the hierarchy by replacing the products $X_k = \pi_i P_i$ of all primes at k:th level with formal variables x_n to obtain polynomial in x_n with coefficients which are rational functions of x_k, $k < n$. Note that X_k is finite in p-adic topologies and infinitesimal in their infinite-P variants.

2. One can construct the root decomposition of infinite prime at n:th level as the decomposition of the corresponding polynomial to a product of roots which are algebraic functions in the extensions of polynomials. One starts from highest level and derives the decomposition by expanding the roots as powers series with respect to x_n. The process can be done without ever mentioning infinite primes. After this one puts $x_n = 0$ to obtain a product of roots at $x_n = 0$ expressible as rational functions of remaining variables. One performs the decomposition with

respect to x_{n-1} for all the roots and continues down to $n = 1$ to obtain ordinary algebraic numbers.

3. One obtains a collection of varieties in n-dimensional space. At the highest level one obtains $n-1$-D variety referred to as divisor in the standard terminology, $n-2$-D variety in $x_n = 0$ hyperplane, $n-3$-D surface in $(x_n, x_{n-1}) = (0,0)$ plane and so on. To each root at given level one can assign polynomial Galois group permuting the polynomial roots at various levels of the hierarchy of infinite primes in correspondence with the branches of surfaces of a many-valued map. At the lowest level one obtains ordinary Galois group relating the roots of an ordinary polynomial. The outcome is a collection of sequences of Galois groups $\{(G_n, G_{n,i}, G_{n,i,j}...)\}$ corresponding to all sequences of roots from $k = n$ to $k = 1$.

   One can also say that at given level one has just one Galois group which is Cartesian product of the Galois groups associated with the roots. Similar situation is encountered when one has a product of irreducible polynomials so that one has two independent sets of roots.

The next question is how to induce the boundary operation. The boundary operation for the analogs of homology groups should be be induced in some sense by the projection map putting one of the coordinates x_k to zero. This suggests a geometric interpretation in terms of a hierarchy of relative homologies $H_k(S_k, S_{k-1})$ defined by the hierarchy of surfaces S_k. Boundary map would map S_k to is intersection at $(x_n = 0, ..., x_k = 0)$ plane. This map makes sense also p-adically. The square of boundary operation would produce an intersection of this surface in $x_{k-1} = 0$ plane and this should correspond to boundary sense for Galois groups.

4.4.1 Algebraic representation of boundary operations in terms of group homomorphisms

The challenge is to find algebraic realizations for the boundary operation or operations in terms of group homomorphisms $G_k \rightarrow G_{k-1}$. One can end up with the final proposal through heuristic ideas and counter arguments and relying on the idea that algebraic geometry should have interpretation in terms of quantum physics as it is described by TGD as almost topological QFT.

1. n-dimensional Galois group is somewhat like a fundamental group acting in the space of n-1-dimensional homotopies so that Grothendieck's intuition that 1-D homotopies are somehow fundamental is realized. The abelianizations of these Galois groups would define excellent candidates for homology groups and Poincare duality would give cohomology groups. The homotopy aspects becomes clearer if one interprets Galois group for n:th order polynomials as subgroup of permutation group and lifts the Galois group to a subgroup of corresponding braid group. Galois groups are also stable againt small changes of the coefficients of the polynomial so that topological invariance is guaranteed.

2. Non-abelian boundary operations $G_k \rightarrow G_{k-1}$ must reduce to their abelian counterparts in abelianization so that they their squares defining homomorphisms from level k to $k-2$ must be maps of G_k to the commutator subgroup $[G_{k-2}, G_{k-2}]$.

3. There is however a grave objection. Finite abelianized Galois groups contain only elements with finite order so that in this sense the analogy with ordinary homotopy and homology groups fails. On the other hand, if Galois group is replaced with its group algebra and group algebra is defined by (say) integer valued maps, one obtains something very much analogous to homotopy and homology groups. Also group algebras in other rings or fields can be considered. This replacement would provide the basis of the homotopy and homology groups with an additional multiplicative structure induced by group operation allowing the interpretation as representations of Galois group acting as symmetry groups. The tentative physical interpretation would in terms of quantum states defined by wave functions in groups. Coboundary operation in the dual of group algebra would be induced by the action of boundary operation in group algebra. Homotopy and homology would be associated with the group algebra and and cohomotopy and cohomology with its dual.

4. A further grave objection against the analog of homology theory is there is no reason to expect that the boundary homomorphism is unique. For instance, one can always have a trivial solution

mapping G_k to unit element of G_{k-1}. Isomorphism theorem [20] implies that the image of the group G_k in G_{k-1} under homomorphism h_k is $G_k/ker(h_k)$, where $ker(h_k)$ is a normal subgroup of G_k as is easy to see. One must have $h_{k-1}(G_k/ker(h_k)) \subset [G_{k-2}, G_{k-2}]$, which is also a normal subgroup.

The only reasonable option is to accept all boundary homomorphisms. This collection of boundary homomorphisms would satisfy anticommutation relations inducing similar anticommutation relations in cohomology. Putting all together, one would would obtain the analog of fermionic oscillator algebra. In particular, Poincare duality would correspond to the mapping exchanging fermionic creation and annihilation operators. It however turns out that tis interpretation fails. Rather, braided Galois homology could represent the states of WCW spinor fields in "orbital" degrees of freedom of WCW in finite measurement resolution. A better analogy for braided Galois cohomology is provided by Dolbeault cohomology which also allows complex conjugation.

If this picture makes sense, one would clearly have what category theorist would have suggested from the beginning. TGD as almost topological QFT indeed suggests strongly the interpretation of quantum states in terms of homology and cohomology theories.

4.4.2 Lift of Galois groups to braid groups and induction of braidings by symplectic flows

One can build a tighter connection with quantum TGD by developing the idea about the analogy between homotopy groups and Galois groups.

1. The only homotopy groups [17], which are non-commutative are first homotopy groups π_1 and plane with punctures provides the minimal realization for them. The lift of permutation groups to `http://en.wikipedia.org/wiki/Braid_group`braid groups [3] by giving up the condition that the squares of generating permutations satisfy $s_i^2 = 1$ defines a projective representation for them and should apply also now. There is also analogy with Wilson loops. This leads to topological QFTs for knots and braids [56, 57].

2. In TGD framework light-like 3-surfaces (and also space-like at the ends of causal diamonds) carry braids beginning at partonic 2-surfaces and ending at partonic 2-surfaces at the boundaries of causal diamonds. This realization is highly suggestive now. This also conforms with the general TGD inspired vision about absolute Galois group of rationals as permutation group S_∞ lifted to braiding groups such that its representation always reduce to finite-dimensional ones [22]. This also conforms with the view about the role of hyper-finite factors of type II_1 and the idea about finite measurement resolution and one would obtain a new connection between various mathematical structure of TGD.

3. The physical interpretation of infinite primes represented by polynomials as bound states suggests that infinite prime at level n corresponds to a braid of braids of ... braids such that at given level of hierachy braid group acting on the physical states is associated with covering group realized as subgroup of the permutation group for the objects whose number is the number of roots. This gives also a connection with the the notion of operad [25, 53, 44] which involves also a hierarchy of discrete structures with the action of permutation group inside each and appears also in quantum TGD as a natural notion [3, 6].

4. The assumption that the braidings are induced by flows of the partonic 2-surface could glue the actions of different Galois groups to single coherent whole was originally motivated by the hope that boundary homomorphism could be made unique in this manner. This restriction is however un-necessary and the physical picture does not support it. The basic motivation for the braid representation indeed comes from TGD as an almost topological QFT vision.

5. The role of symplectic transformations in TGD suggests the identification of flows as symplectic flows induced by those of $\delta M^2 \times CP_2$. These flows should map the area enclosed by the sub-braid (of braids) to itself and corresponding Hamiltonian should be constant at the boundary of the area and induce a flow horizontal to the boundary and also continuous at the boundary. The flow would in general be non-trivial inside the area and induce the braiding of the sub-braid of braids. One could assign "Galois spin" to the sub-braids with respect to the higher Galois

group and boundary homomorphism would realize unitary action of G_k as spin rotation at k_1:th level. At k_2:th level the "Galois spin" rotation would reduce to that in commutator subgroup and in homology theory would become trivial. The interpretation of the commutator group as the analog of gauge group might make sense. This would conform with an old idea of quantum TGD that the commutator subgroup of symplectic group acts as gauge transformations.

6. It is not necessary to assign the braids at various level of the hierarchy to the same partonic 2-surface. Since the symplectic transformations act on $\delta M^4_\pm \times CP_2$, one can consider also the projections of the braids to the homologically non-trivial 2-sphere of CP_2 or to the 2-sphere at light-cone boundary: both of these spheres play important part in the formulation of quantum TGD and I have indeed assigned the braidings to these surfaces [10].

7. The representation of the hierarchy of Galois groups acting on the braid of braids of... can be understood in terms of the replacement of symplectic group of $\delta M^4_\pm \times CP_2$ -call it G- permuting the points of the braids with its discrete subgroup obtained as a factor group G/H, where H is a normal subgroup of G leaving the endpoints of braids fixed. One must also consider subgroups of the permutation group for the points of the triangulation since Galois group for n:th order polynomial is in general subgroup of S_n One can also consider flows with these properties to get braided variant of G/H.

The braid group representation works also for ordinary polynomials with continuous coefficients in all number fields as also finite fields. One therefore achieves number theoretical universality. The values of the variables x_i appearing in the polynomials can belong to any numer field and the representation spaces of the Galois groups correspond to any number field. Since the Galois groups are stable against small perturbations of coefficients one obtains topological invariance in both real and p-adic sense. Also the representation in all number fields are possible for the Galois groups.

The construction is universal but infinite primes provide the motivation for it and can be regarded as a representation of the generalized cohomology group for surfaces which belong to the intersection of real and p-adic worlds (rational coefficients). In particular, the expansion of the roots in powers series is the only manner to make sense about the roots when x_n is identified with X_n so that convergence takes place if some of the lower level infinite primes appearing in the product defining X_n is interpreted as infinite p-adic prime. All higher powers are infinitesimal in infinite-P p-adic norm. At the lowest level one obtains expansion in X_1 for which X_1^n has norm p^{-n} with respect to any prime p. The value of the product of primes different from p is however not well-defined for given p-adic topology. If it makes sense to speak about multi-p p-adic expansion all powers X_1^n, $n > 0$ would be infinitesimal.

4.4.3 What can one say about the lifting to braid groups?

The generators of symmetry group are given by permutations s_i permuting i:th and $i+1$:th element of n-element set. The permutations s_i and s_j obviously commute for $|i-j| > 2$. It is also easy to see that the identity $s_i s_{i+1} s_i = s_{i+1} s_i s_{i+1}$ holds true. Besides this the identity $s_i^2 = 1$ holds true.

Braid group B_n [3] is obtained by dropping the condition $s_i^2 = 1$ and can be regarded as an infinite covering group of the permutation group. For instance, for the simplest non-trivial case $n = 3$ the braid group is universal central extension of the modular group $PSL(2, Z)$. In the general case the braid group is isomorphic to the mapping class group of a punctured disk with n punctures and the realization of the braidings as a symplectic transformations would mean additional restriction to the allowed isotopies inducing the braid group action.

One can decompose any element of braid group B_n to a product of element of symmetric group S_n and of pure braid group P_n consisting of braidings which correspond to trivial permutations. P_n is a normal subgroup of braid group and the following short exact sequence $1 \to F_{n-1} \to P_n \to P_{n-1} \to 1$ allows to decompose P_n to a product of image of free group F_{n-1} and of the image of P_n in P_{n-1}. This leads to a decomposition to a representation of P_n as an iterated semidirect product of free groups.

Concerning the lifting of Galois groups to subgroups of braid groups following observations are relevant.

1. For n:th order polynomial of single variable Galois group can be regarded as a subgroup of permutation group S_n. The identification is probably not completely unique (at least inner automorphisms make the identification non-unique) but I am unable to say whether this has significance in the recent context.

2. The natural lifting of Galois group to its braided version is as a product of corresponding subgroup of S_n with with pure braid group of n braids so that pure braidings would allow also braidings of all permutations as intermediate stages. Pure braid group is normal subgroup trivially. Whether also more restricted braidings are possible is not clear to me. Braid group has a subgroup obtained by coloring braid strands with a finite number of colors and allowing only the braidings which induce permutations of braids of same color. Clearly this group is a good candidate for the minimal group decomposable to a product of subgroups of symmetric subgroups containing braided Galois group. Different colors would correspond to the decomposition of S_n to a product of permutation groups. Note that one can have cyclic subgroups of permutation sub-groups.

One might hope that it is enough to lift the boundary homomorphisms between Galois groups G_k and G_{k-1} to homomorphisms between corresponding braided groups. Life does not look so simple.

1. The group algebra of Galois group is replaced with an infinite-dimensional group algebra of braid groups so that the number of physical states is expected to become much larger and the interpretation could be in terms of many-boson states.

2. The square of the boundary homomorphism must map braided Galois group $B(G_k)$ to $[B(G_{k-2}), B(G_{k-2})]$. The obvious question is whether this conditions reduces to corresponding conditions for Galois group and pure braided groups. In other words, does the braiding commute with the formation of commutator sub-group: $[B(G_k), B(G_k)] = B([G_k, G_k])$? In this case the decomposition of the braided Galois group to a product of Galois group and pure braid group would allow to realize the braided counterpart of boundary homomorphism as a product of Galois group homomorphism and homomorphism acting on the pure braid group. Direct calculation however shows that this is not the case so that the problem is considerably more complicated.

4.4.4 More detailed view about braided Galois homology

Consider next a more detailed view about the braided Galois homology.

1. One can wonder whether the application of only single boundary operator creates a state which represents gauge degree of freedom or whether boundaries correspond to "full" boundaries obtained by applying maximum number of boundary operations, which k:th level is k. "Full boundary" would correspond to what one obtains by applying at most k boundary operators to the state, and many combinations are possible if the number of boundary homomorphisms is larger than k. The physical states as elements of homology group would be analogous many-fermion states bu would differ from them in the sense that they would be annihilated by all fermionic creation operators. In particular, full Fermi spheres at k:th level would represent gauge degrees of freedom.

   Homologically non-trivial states are expected to be rather rare, especially so if already single boundary operation creates gauge degree of freedom. Certainly the existence of constraints is natural since infinite primes corresponding to irreducible polynomials of degree higher are interpreted as bound states. Homological non-triviality would most naturally express bound state property in bosonic degrees of freedom. In any case, one can argue that fermionic analogy is not complete and that a more natural interpretation is as an analog of cohomology with several exterior derivatives.

2. The analogy with fermionic oscillator algebra makes also the realization of bosonic oscillator operator algebra suggestive. Pointwise multiplication of group algebra elements regarded as functions in group looks the most plausible option since for continuous groups like $U(1)$ this implies additivity of quantum numbers. Many boson states for given mode would correspond to powers of group algebra element with respect to pointwise multiplication. If the commutator for the analogs of the bosonic oscillator operators is defined as

$$[B_1, B_2] \equiv \sum_{g_1,g_2} B_1(g_1)B_2(g_2)[g_1, g_2] \quad , [g_1, g_2] \equiv g_1 g_2 g_1^{-1} g_2^{-1} \quad ,$$

it is automatically in the commutator sub-group. This condition is not consistent with fermionic anti-commutation relations. The consistency requires that the commutator is defined as

$$[B_1, B_2] \equiv \sum_{g_1,g_2} (B_1(g_1)B_2(g_2)[[g_1, g_2] \ \ , [g_1, g_2] \equiv g_1g_2 - g_2g_1 \ \ . \tag{4.1}$$

The commutator must belong to the group algebra of the commutator subgroup. In this case the commutativity conditions are non-trivial. Bosonic commutation relations would put further constraints on the homology.

A delicacy related to commutation and anti-commutation relations should be noticed. One could fermionic creation (annihilation) operators as elements in the dual of group algebra. If group algebra and its dual are not identified (this might not be possible) then the anti-commutator is element of the field of ring in which group algebra elements have values. In the bosonic case the conjugate of the bosonic group algebra element should be treated in the same manner as a pointwise multiplication operator instead of an exterior derivative like operator.

3. One could perhaps interpret the commutation and anti-commutation relations modulo commutator subgroup in terms of finite measurement resolution realized by the transition to homology implying that observables commute in the standard sense. The connection of finite measurement resolution with inclusions of hyper-finite factors of type II_1 implying a connection with quantum groups and non-commutative geometry conforms also with the vision that finite measurement resolution means commutativity modulo commutator group.

4. The alert reader has probably already asked why one could not define also diagonal homology for G_k via diagonal boundary operators $\delta_k : G_k \rightarrow H_k$, where H_k is subgroup of G_k. The above argument would suggest interpretation for this cohomology in terms of finite measurement resolution. If one allows this the Galois cohomology groups would be labelled by two integers. Similar situation is encountered in motivic cohomology [23].

4.4.5 Some remarks

Some remarks about the proposal are in order.

1. The proposal makes as such sense if the polynomials with rational coefficients define a subset of more general function space able to catch the non-commutative homotopy and homology and their duals terms of Galois groups associated with rational functions with coefficients. One could however abstract the construction so that it applies to polynomials with coefficients in real and p-adic fields and forget infinite primes altogether. One can even consider the replacement of algebraic surfaces with more general surfaces as along as the notion of Galois group makes sense since braiding makes sense also in more general situation. This picture would conform with the idea of number theoretical universality based on algebraic continuation from rationals to various number fields. In this case infinite primes would characterize the rational sector in the intersection of real and p-adic worlds.

2. The above discussion is for the rational primes only. Each algebraic extension of rationals however gives rise to its own primes. In particular, one obtains also complex integers and Gaussian primes. Each algebraic extension gives to its own notion of infinite prime. One can also consider quaternionic and octonionic primes and their generalization to infinite primes and this generalization is indeed one of the key ideas of the number theoretic vision [17]. Note that already for quaternions Galois group defined by the automorphisms of the arithmetics is continuous Lie group.

3. The decomposition of infinite primes to primes in extension of rational or polynomials is analogous to the decomposition of hadron to quarks in higher resolution and suggests that reduction of the quantum system to its basic building bricks could correspond number theoretically to the introduction of higher algebraic extensions of various kinds of number fields. The emergence of higher extensions would mean emergence of algebraic complexity and have interpretation as evolution of cognition in TGD inspired theory of consciousness.

This picture conforms with the basic visions of quantum TGD about physics as infinite-dimensional geometry on one hand and physics as generalized number theory on one hand implying that algebraic geometry reduces in some sense to number theory and one can also regard quantum states as representations of algebraic geometric invariants in accordance with the vision about TGD as almost topological QFT.

Infinite primes form a discrete set since all the coefficients are rational (unless one allows even algebraic extensions of infinite rationals). Physically infinite primes correspond to elementary particle like states so that elementary particle property corresponds to number theoretic primeness. Infinite integers define unions of sub-varieties identifiable physically as many particle states. Rational functions are in turn interpreted in zero energy ontology as surfaces assignable to initial and final states of physical event such that positive energy states correspond to the numerator and negative energy states to the denominator of the polynomial. One also poses the additional condition that the ratio equals to real unit in real sense so that real units in this sense are able to represent zero energy state and the number theoretic anatomy of single space-time point might be able to represent arbitrary complex quantum states.

The generalization of the notion of real point has been already mentioned as also the fact that the number theoretic anatomy could in principle allow to code for zero energy states if they correspond to infinite rationals reducing to unit in real sense. Also space-time surfaces could by quantum classical correspondence represent in terms of this anatomy as I have proposed. Single space-time point could code in its structure not only the basic algebraic structure of topology as proposed but represent Platonia. If the above arguments really maks sense then this number theoretic Brahman=Atman identify would not be a mere beautiful philosophical vision but would have also practical consequences for mathematics.

4.5 What is the physical interpretation of the braided Galois homology

The resulting cohomology suggests either the interpretation in terms of many-fermion states or as a generalization of de Rham cohomology involving several exterior derivative operators. The arguments below show that fermionic interpretation does not make sense and that the more plausible interpretation in concordance with finite measurement resolution is in terms of "orbital" WCW degrees of freedom represented by the symplectic group assignable to the product of light-cone boundary and CP_2.

4.5.1 What the restriction to the plane $x_k = 0$ could correspond physically?

The best manner to gain a more detailed connection between physics and homology is through an attempt to understand what operation putting $x_k = 0$ could mean physically.

1. Given infinite prime at level n corresponds to single particle state characterized by Galois group G_n. The fermionic part of the state corresponds to its small part and purely bosonic part multiplies X_{n-1} factors as powers of primes not dividing the fermionic part of the state. Therefore the finite part of the state contains information about fermions and bosons labelled by fermionic primes. When one puts $x_n = 0$, the information about the bosonic part is lost.

   One can of course divide the polynomial by a suitable infinite integer of previous level so that its highest term is just power of X_n with a unit coefficient. Bosonic part appears in this case in the denominator of the finite part of the infinite prime and does not contribute to zeros of the resulting rational function at $n-1$:th level: it of course affects the zeros at n:th level. Hence the information about bosons at $n-1$:th level is lost also now unless one considers also the Galois groups assignable to the poles of the resulting rational function at $n-1$:th level.

2. What could this loss of information about bosons correspond geometrically and physically? To answer this question must understand how the polynomial of many variables can be represented physically in TGD Universe.

   The proposal has been that a union of hierarchically ordered partonic 2-surfaces gives rise to a local representation of n-fold Cartesian power for a piece of complex plane. A more concrete realization would be in terms of wormhole throats at the end of causal diamond at 3-surfaces topologically condensed at each other. The operation $x_n = 0$ would corresponding to the basic reductionistic step destroying the bound state by removing the largest space-time sheets so that one would have many-particle state rather than elementary particle at the lower level of the hierarchy of space-time sheet. This loss of information would be unavoidable outcome of the reductionistic analysis.

One can consider two alternative geometric interpretations depending on whether one identifies to infinite primes connected 3-surfaces or connected 2-surfaces.

1. If infinite primes correspond to connected 3-surfaces having hierarchical structure of topological condensate the disappearing bosons could correspond to the wormhole throats connecting smaller space-time sheet to the largest space-time sheet involved. Wormhole throats would carry bosonic quantum numbers and would be removed when the largest space-time sheet disappears. Many-fermion state at highest level represented by the "finite" part of the infinite prime would correspond to "half" wormhole throats- CP_2 type vacuum extremals topological condensed at smaller space-time sheets but not at the highest one.

2. If elementary particles/infinite primes correspond to connected partonic 2-surfaces (this is not quite not the case since tangent space data about partonic 2-surfaces matters), one must replace 3-D topological condensation by its 2-dimensional version. Infinite prime would correspond to single wormhole throat asa partonic 2-surface at which smaller wormhole throats would have suffered topological condensation. Topological condensation would correspond to a formation of a connection by flux tube like structure between the 2-surfaces considered. The disappearance of this highest level would mean decay to a many particle state containing several wormhole contacts. The formation of anyonic many-particle states could be interpreted in terms of build-up of higher level infinite primes.

3. What ever the interpretation is, it should be consistent with the idea that braiding as induced by symplectic flow. If the symplectic flow is defined by the inherent symplectic structure of the partonic 2-surface only the latter option works. If the symplectic flow acts at the level of the imbedding space - as is natural to assume- both interpretations make sense.

4.5.2 The restriction to $x_k = 0$ plane cannot correspond to homological boundary operation

Can one model the restriction to $x_k = 0$ plane as boundary operation in the sense of generalized homology? There are several objections.

1. There are probably several homological boundary operations δ_i at given level whereas the restriction $x_k = 0$ is a unique operation (recall however the possibility to permute the arguments in the case of polynomial).

2. The homology is expected to contain large number of generators whereas the state defined by infinite prime is unique as are also the states resulting via restriction operations.

3. It is not possible to assign fermion number to $x_k = 0$ operation since fermion number is not affected: this would not allow to assign fermion number to the homological boundary operators.

Although the interpretation as many-fermion states does not make sense, one must notice that the structure of homology is highly analogous to the space of states of super-symmetric QFT and of the set of infinite primes. Only the infinite primes $X_n \pm 1$, where X_n is the product of all primes at level n, correspond to states containing no fermions and have interpretation as Dirac sea and vacuum state. In the same manner the elements of braided Galois homology in general are obtained by applying the analogs of fermionic annihilation (creation) operators to a full Fermi sphere (Fock vacuum). Also the identification of *all* physical states as many-fermion states in quantum TGD where all known elementary bosons are identified as fermion pairs conforms with this picture.

A more natural interpretation of the restriction operation is as an operation making possible to assign to a given state in fermionic sector the space of possible states in WCW degrees of freedom characterized in terms of Galois cohomology represented in terms of the symplectic group of acting as isometries of WCW. The transition from Lie algebra description natural for continuum situation to discrete subgroup is natural due to the discretization realizing the finite measurement resolution.

One cannot however avoid a nasty question. What about the lower level bosonic primes associated with the infinite prime? What is their interpretation if they do not correspond to WCW degrees of freedom? Maybe one could identify the bosonic parts of infinite prime as super-partners of fermions behaving like bosons. The addition of a right handed neutrino to a given quantum state could represent this supersymmetry.

4.5.3 Braided Galois group homology and construction of quantum states in WCW degrees of freedom in finite measurement resolution

The above arguments fix the physical interpretation of infinite primes and corresponding group cohomology to quite high degree.

1. From above it is clear that the restriction operation cannot correspond directly to homological boundary operation. Single infinite prime corresponds to an entire spectrum of states. Hence the assignment of fermion number to the boundary operators is not correct thing to do and one must interpret the coboundary operations as analogs of exterior derivatives and various states as bosonic excitations of a given state analogous to states assignable to closed forms of various degrees in topological or conformal quantum field theories.

2. The natural interpretation of Galois homology is as a homology assignable to a discrete subgroup hierarchy of the symplectic group acting as isometries of WCW and therefore as the space of wave functions in WCW degrees of freedom in finite measurement resolution. Infinite primes would code for fermionic degrees of freedom identifiable as spinor degrees of freedom at the level of WCW.

3. The connection between infinite primes and braided Galois homology would basically reflect the supersymmetry relating these degrees of freedom at the level of WCW geometry where WCW Hamiltonians correspond to bosonic generators and contractions of WCW gamma matrices with symplectic currents to the fermionic generators of the super-symmetry algebra. If this identification is correct, it would solve the problem of constructing the modes of WCW spinor fields in finite measurement resolution. An especially well-come feature would be the reduction of WCW integration to summations in braided Galois group algebra allowing an easy realization of number theoretical universality. If the picture is correct it should also have connections to the realization of finite measurement resolution in terms of inclusions of hyper-finite factors of type II_1 [7] for which fermionic oscillator algebra provides the basic realization.

4. Of course, it is far from clear whether it is really possible to reduce spin, color and electroweak quantum numbers to number theoretic characteristics of infinite primes and it might well be that the proposed construction does not apply to center of mass degrees of freedom of the partonic 2-surface. I have considered these questions for the octonionic generalization of infinite primes and suggested how standard model quantum numbers could be understood in terms of subset of infinite octonionic primes [17].

4.6 Is there a connection with the motivic Galois group?

The proposed generalized of Galois group brings in mind he notion of motivic Galois group, which is one possible generalization for the notion of zero-dimensional Galois group associated with algebraic extensions of number fields to the level of algebraic varieties.

One of the many technical challenges of the motivic cohomology theory is the non-uniqueness of the imbedding of the algebraic extension as a subfield in the algebraic closure of k. The number of these imbeddings is however finite and absolute Galois group associated with the algebraic closure of k acts in the set of the imbeddings. Which of them one should choose?

Quantum physicist would solve this problem by saying that there is no need to choose: one could introduce quantum superpositions of different choices and "Galois spin" regarding the different imbeddings as analogs of different spin components. Absolute Galois group would act on the quantum states regarded as superpositions of different imbeddings by permuting them. In TGD framework this kind of representation could emerge in p-adic context raise Galois group to a role of symmetry group acting on quantum states: indeed absolute Galois group is very natural notion in TGD framework.

I have proposed this kind of interpretation for some years ago in a chapter [11] about Langlands program [48, 22, 49, 47].

If I have understood correctly, the idea of the motivic Galois theory is to generalize this correspondence so that the varieties in field k are replaced the varieties in the extension of k imbedded to the algebraic closure of k, the number of which is finite. Whether the number of the lifts for varieties is finite seems to depends on the situation.

1. If the imbedding is assumed to be same for all points of the variety the situation seems to reduce to the imbeddings of k to the algebraic completion of rationals and one would have quantum superposition of varieties in the union of finite number of representatives of the algebraic extension to which the absolute Galois group acts.

2. Physicist could however ask whether the invariance under the action of Galois group could be local in some sense. The selection of separable extension could indeed be only pseudo-constant in p-adic case and thus depend on finite number of pinary digits of the k-valued coordinates of the point of the algebraic variety. Local gauge invariance would say that any pseudo constant element of local absolute Galois group acts as a symmetry. This would suggest that one can introduce Galois connection. Since Lie algebra is not defined now one should introduce the connection as parallel translations by Galois group element for paths in the algebraic variety.

One key result is that pure motives using numerical equivalence are equivalent with the category of representations of an algebraic group called motivic Galois group which has Lie algebra and is thus looks like a continuous group.

1. Lie algebra structure for something apparently discrete indeed makes sense for profinite groups (synonymous to Stone spaces). Spaces with p-adic topology are basic examples of this kind of spaces. For instance, 2-adic integers is a Stone space obtained as the set of all bit sequences allowed to contain infinite number of non-vanishing digits. This implies that real discreteness transforms to p-adic continuity and the notion of Lie algebra makes sense. For polynomials this would correspond to polynomials with strictly infinite degree unless one considers the absolute Galois group associated with the algebraic extension of rationals associated with an ordinary polynomial. For infinite primes this would correspond to many-fermion states containing infinite number of fermions kicked out from the Dirac sea and from the point of view of physics would look like an idealization.

2. Motivic Galois group does not obviously correspond to the Galois groups as they are introduced above. Absolute Galois group for the extension of say rationals however emerges if one performs the lift to the algebraic completion and this might be how one ends up with motivic Galois group and also with p-adic physics. One can perhaos say that the Galois groups as introduced above make sense in the intersection of real and p-adic worlds.

3. The choice of algebraic extension might be encountered also in the construction of roots for the polynomials associated with infinite primes and since this choice is not unique it seems that one must use quantum superposition of the different choices and must introduce the action of an appropriate absolute Galois group. This group would be absolute Galois group for algebraic extension of polynomials of n variables at n:th level and ordinary Galois group at the lowest level of hierarchy which should be or less the same as the Galois group introduced above. This could bring in additional spin like degrees of freedom in which the absoltey Galois group acts.

   The fascinating question is whether one could regard not only the degrees of freedom associated with the finite Galois groups but even those associated with the absolute Galois group as physical. Physically the analogs of color quantum numbers whose net values vanish for confined states would be in question. To sum up, it seems that number theory could contain implicitly an incredible rich spectrum of physics.

5 Motives and twistor approach applied to TGD

Motivic cohomology has turned out to pop up in the calculations of the twistorial amplitudes using Grassmannian approach [3, 10]. The amplitudes reduce to multiple residue integrals over smooth projective sub-varieties of projective spaces. Therefore they represent the simplest kind of algebraic geometry for which cohomology theory exists. Also in Grothendieck's vision about motivic cohomology [54] projective spaces are fundamental as spaces to which more general spaces can be mapped in the construction of the cohomology groups (factorization).

5.1 Number theoretic universality, residue integrals, and symplectic symmetry

A key challenge in the realization of the number theoretic universality is the definition of p-adic definite integral. In twistor approach integration reduces to the calculation of multiple residue integrals over closed varieties. These could exist also for p-adic number fields. Even more general integrals identifiable as integrals of forms can be defined in terms of motivic cohomology.

Yangian symmetry [42], [11] is the symmetry behind the successes of twistor Grassmannian approach [7] and has a very natural realization in zero energy ontology [21]. Also the basic prerequisites for twistorialization are satisfied. Even more, it is possible to have massive states as bound states of massless ones and one can circumvent the IR difficulties of massless gauge theories. Even UV divergences are tamed since virtual particles consist of massless wormhole throats without bound state condition on masses. Space-like momentum exchanges correspond to pairs of throats with opposite sign of energy.

Algebraic universality could be realized if the calculation of the scattering amplitudes reduces to multiple residue integrals just as in twistor Grassmannian approach. This is because also p-adic integrals could be defined as residue integrals. For rational functions with rational coefficients field the outcome would be an algebraic number apart from power of 2π, which in p-adic framework is a nuisance unless it is possible to get rid of it by a proper normalization or unless one can accepts the infinite-dimensional transcendental extension defined by 2π. It could also happen that physical predictions do not contain the power of 2π.

Motivic cohomology defines much more general approach allowing to calculate analogs of integrals of forms over closed varieties for arbitrary number fields. In motivic integration [51] - to be discussed below - the basic idea is to replace integrals as real numbers with elements of so called scissor group whose elements are geometric objects. In the recent case one could consider the possibility that $(2\pi)^n$ is interpreted as torus $(S^1)^n$ regarded as an element of scissor group which is free group formed by formal sums of varieties modulo certain natural relations meaning.

Motivic cohomology allows to realize integrals of forms over cycles also in p-adic context. Symplectic transformations are transformation leaving areas invariant. Symplectic form and its exterior powers define natural volume measures as elements of cohomology and p-adic variant of integrals over closed and even surfaces with boundary might make sense. In TGD framework symplectic transformations indeed define a fundamental symmetry and quantum fluctuating degrees of freedom reduce to a symplectic group assignable to $\delta M^4 \pm \times CP_2$ in well-defined sense [4]. One might hope that they could allow to define scissor group with very simple canonical representatives- perhaps even polygons- so that integrals could be defined purely algebraically using elementary area (volume) formulas and allowing continuation to real and p-adic number fields. The basic argument could be that varieties with rational symplectic volumes form a dense set of all varieties involved.

5.2 How to define the p-adic variant for the exponent of Kähler action?

The exponent of Kähler function defined by the Kähler action (integral of Maxwell action for induced Kähler form) is central for quantum at least in the real sector of WCW. The question is whether this exponent could have p-adic counterpart and if so, how it should be defined.

In the real context the replacement of the exponent with power of p changes nothing but in the p-adic context the interpretation is affected in a dramatic manner. Physical intuition provided by p-adic thermodynamics [12] suggest that the exponent of Kähler function is analogous to Bolzmann weight replaced in the p-adic context with non-negative power of p in order to achieve convergence of the series defining the partition function not possible for the exponent function in p-adic context.

1. The quantization of Kähler function as $K = rlog(m/n)$, where r is integer, $m > n$ is divisible by a positive power of p and n is indivisible by a power of p, implies that the exponent of Kähler function is of form $(m/n)^r$ and therefore exists also p-adically. This would guarantee the p-adic

existence of the vacuum functional for any prime dividing m and for a given prime p would select a restricted set of p-adic space-time sheets (or partonic 2-surfaces) in the intersection of real and p-adic worlds. It would be possible to assign several p-adic primes to a given space-time sheet (or partonic 2-surface). In elementary particle physics a possible interpretation is that elementary particle can correspond to several p-adic mass scales differing by a power of two [14]. One could also consider a more general quantization of Kähler action as sum $K = K_1 + K_2$ where $K_1 = rlog(m/n)$ and $K_2 = n$, with n divisible by p since $exp(n)$ exists in this case and one has $exp(K) = (m/n)^r \times exp(n)$. Also transcendental extensions of p-adic numbers involving $p + n - 2$ powers of $e^{1/n}$ can be considered.

2. The natural continuation to p-adic sector would be the replacement of integer coefficient r with a p-adic integer. For p-adic integers not reducing to finite integers the p-adic norm of the vacuum functional would however vanish and their contribution to the transition amplitude vanish unless the number of these space-time sheets increases with an exponential rate making the net contribution proportional to a finite positive power of p. This situation would correspond to a critical situation analogous to that encounted in string models as the temperature approaches Hagedorn temperature [6] and the number states with given energy increases as fast as the Boltzmann weight. Hagedorn temperature is essentially due to the extended nature of particles identified as strings. Therefore this kind of non-perturbative situation might be encountered also now.

3. Rational numbers m/n with n not divisible by p are also infinite as real integers. They are somewhat problematic. Does it make sense to speak about algebraic extensions of p-adic numbers generated by $p^{1/n}$ and giving $n-1$ fractional powers of p in the extension or does this extension reduce to something equivalent with the original p-adic number field when one redefines the p-adic norm as $|x|_p \rightarrow |x|^{1/n}$? Physically this kind of extension could have a well defined meaning. If this does not make sense, it seems that one must treat p-adic rationals as infinite real integers so that the exponent would vanish p-adically.

4. If one wants that Kähler action exists p-adically a transcendental extension of rational numbers allowing all powers of $log(p)$ and $log(k)$, where $k < p$ is primitive $p-1$:th root of unity in $G(p)$. A weaker condition would be an extension to a ring with containing only $log(p)$ and $log(k)$ but not their powers. That only single $k < p$ is needed is clear from the identity $log(k^r) = rlog(k)$, from primitive root property, and from the possibility to expand $log(k^r + pn)$, where n is p-adic integer, to powers series with respect to p. If the exponent of Kähler function is the quantity coding for physics and naturally required to be ordinary p-adic number, one could allow $log(p)$ and $log(k)$ to exists only in symbolic sense or in the extension of p-adic numbers to a ring with minimal dimension.

   Remark: One can get rid of the extension by $log(p)$ and $log(k)$ if one accepts the definition of p-adic logarithm as $log(x) = log(p^{-k}x/x_0)$ for $x = p^k(x_0 + py)$, $|y|_p < 1$. To me this definition looks somewhat artificial since this function is not strictly speaking the inverse of exponent function but might have a deeper justification.

5. What happens in the real sector? The quantization of Kähler action cannot take place for all real surfaces since a discrete value set for Kähler function would mean that WCW metric is not defined. Hence the most natural interpretation is that the quantization takes place only in the intersection of real and p-adic worlds, that is for surfaces which are algebraic surfaces in some sense. What this actually means is not quite clear. Are partonic 2-surfaces and their tangent space data algebraic in some preferred coordinates? Can one find a universal identification for the preferred coordinates- say as subset of imbedding space coordinates selected by isometries?

If this picture inspired by p-adic thermodynamics holds true, p-adic integration at the level of WCW would give analog of partition function with Boltzman weight replaced by a power of p reducing a sum over contributions corresponding to different powers of p with WCW integra.l over space-time sheets with this value of Kähler action defining the analog for the degeneracy of states with a given value of energy. The integral over space-time sheets corresponding to fixed value of Kähler action should allow definition in terms of a symplectic form defined in the p-adic variant of WCW. In finite-dimensional case one could worry about odd dimension of this sub-manifold but in infinite-dimensional case this need not be a problem. Kähler function could defines one particular zero mode of WCW Kähler metric possessing an infinite number of zero modes.

One should also give a meaning to the p-adic integral of Kähler action over space-time surface assumed to be quantized as multiples of $log(m/n)$.

1. The key observation is that Kähler action for preferred extrememals reduces to 3-D Chern-Simons form by the weak form of electric-magnetic duality. Therefore the reduction to cohomology takes place and the existing p-adic cohomology gives excellent hopes about the existence of the p-adic variant of Kähler action. Therefore the reduction of TGD to almost topological QFT would be an essential aspect of number theoretical universality.

2. This integral should have a clear meaning also in the intersection of real and p-adic world. Why the integrals in the intersection would be quantized as multiple of $log(m/n)$, m/n divisible by a positive power of p? Could $log(m/n)$ relate to the integral of $\int_1^p dx/x$, which brings in mind $\oint dz/z$ in residue calculus. Could the integration range $[1, m/n]$ be analogous to the integration range $[0, 2\pi]$. Both multiples of 2π and logarithms of rationals indeed emerge from definite integrals of rational functions with rational coefficients and allowing rational valued limits and in both cases $1/z$ is the rational function responsible for this.

3. $log(m/n)$ would play a role similar to 2π in the approach based on motivic integration where integral has geometric objects as its values. In the case of 2π the value would be circle. In the case of $log(m/n)$ the value could be the arc between the points $r = m/n > 1$ and $r = 1$ with r identified the radial coordinate of light-cone boundary with conformally invariant length measures dr/r. One can also consider the idea that $log(m/n)$ is the hyperbolic angle analogous to 2π so that these two integrals could correspond to hyper-complex and complex residue calculus respectively.

4. TGD as almost topological QFT means that for preferred extremals the Kähler action reduces to 3-D Chern-Simons action, which is indeed 3-form as cohomology interpretation requires, and one could consider the possibility that the integration giving $log(m/n)$ factor to Kähler action is associated with the integral of Chern-Simons action density in time direction along light-like 3-surface and that the integral over the transversal degrees of freedom could be reduced to the flux of the induced CP_2 Kähler form. The logarithmic quantization of the effective distance between the braid end points the in metric defined by modified gamma matrices has been proposed earlier [8].

Since p-adic objects do not possess boundaries, one could argue that only the integrals over closed varieties make sense. Hence the basic premise of cohomology would fail when one has p-adic integral over braid strand since it does not represent closed curve. The question is whether one could identify the end points of braid in some sense so that one would have a closed curve effectively or alternatively relative cohomology. Periodic boundary conditions is certainly one prerequisite for this kind of identification.

1. In one of the many cohomologies known as quantum cohomology [28] one indeed assumes that the intersection of varieties is fuzzy in the sense that two surfaces for which points are connected by what is called pseudo-holomorphic curve can be said to intersect at these points. As a special case pseudo-holomorphic curve reduce to holomorphic curve defined by a holomorphic map of 2-D Kähler manifold to complex manifold with Kähler structure. The question arises what "pseudoholomorphic curve connects points" really means. In the recent case a natural analog would be 2-D string world sheets or partonic 2-surfaces so that complex numbers are replaced by hyper-complex numbers effectively. The boundaries of string world sheets would be 1-D braid strands at wormhole throats and at the end of space-time sheet at boundaries of CD. In spirit of algebraic geometry one could also call the 1-D braid strands holomorphic curves connecting points of the partonic 2-surfaces at the two light-like boundaries of CD. In the similar manner space-like braid strands would connect points of partonic 2-surface at same end of CD.

2. In the construction of the solutions of the modified Dirac equation one assumes periodic boundary conditions so that in physical sense these points are identified [8]. This assumption actually

reduces the locus of solutions of the modified Dirac equation to a union of braids at light-like 3-surfaces so that finite measurement resolution for which discretization defines space-time correlates becomes an inherent property of the dynamics. The coordinate varying along the braid strands is light-like so that the distance in the induced metric vanishes between its end points (unlike the distance in the effective metric defined by the modified gamma matrices): therefore also in metric sense the end points represent intersection point. Also the effective 2-dimensionality means are effectively one and same point.

3. The effective metric 2-dimensionality of the light-like 2-surfaces implies the counterpart of conformal invariance with the light-like coordinate varying along braid strands so that it might make sense to say that braid strands are pseudo-holomorphic curves. Note also that the end points of a braid along light-like 3-surface are not causally independent: this is why M-matrix in zero energy ontology is non-trivial. Maybe the causal dependence together with periodic boundary conditions, light-likeness, and pseudo-holomorphy could imply a variant of quantum cohomology and justify the p-adic integration over the braid strands.

5.3 Motivic integration

While doing web searches related to motivic cohomology I encountered also the notion of motivic measure [51] proposed first by Kontsevich. Motivic integration is a purely algebraic procedure in the sense that assigns to the symbol defining the variety for which one wants to calculate measure. The measure is not real valued but takes values in so called scissor group, which is a free group with group operation defined by a formal sum of varieties subject to relations. Motivic measure is number theoretical universal in the sense that it is independent of number field but can be given a value in particular number field via a homomorphism of motivic group to the number field with respect to sum operation.

Some examples are in order.

1. A simple example about scissor group is scissor group consisting operations needed in the algorithm transforming plane polygon to a rectangle with unit edge. Polygon is triangulated; triangles are transformed to rectangle using scissors; long rectangles are folded in one half; rectangles are rescaled to give an unit edge (say in horizontal direction); finally the resulting rectangles with unit edge are stacked over each other so that the height of the stack gives the area of the polygon. Polygons which can be transformed to each other using the basic area preserving building bricks of this algorithm are said to be congruent.

   The basic object is the free abelian group of polygons subject to two relations analogous to second homology group. If P is polygon which can be cut to two polygons P_1 and P_2 one has $[P] = [P_1] + [P_2]$. If P and P' are congruent polygons, one has $[P] = [P']$. For plane polygons the scissor group turns out to be the group of real numbers and the area of polygon is the area of the resulting rectangle. The value of the integral is obtained by mapping the element of scissor group to a real number by group homomorphism.

2. One can also consider symplectic transformations leaving areas invariant as allowed congruences besides the slicing to pieces as congruences appearing as parts of the algorithm leading to a standard representation. In this framework polygons would be replaced by a much larger space of varieties so that the outcome of the integral is variety and integration means finding a simple representative for this variety using the relations of the scissor group. One might hope that a symplectic transformations singular at the vertices of polygon combined with with scissor transformations could reduce arbitrary area bounded by a curve into polygon.

3. One can identify also for discrete sets the analog of scissor group. In this case the integral could be simply the number of points. Even more abstractly: one can consider algebraic formulas defining algebraic varieties and define scissor operations defining scissor congruences and scissor group as sums of the formulas modulo scissor relations. This would obviously abstract the analytic calculation algorithm for integral. Integration would mean that transformation of the formula to a formula stating the outcome of the integral. Free group for formulas with disjunction of formulas is the additive operation [54]. Congruence must correspond to equivalence of some

kind. For finite fields it could be bijection between solutions of the formulas. The outcome of the integration is the scissor group element associated with the formula defining the variety.

4. For residue integrals the free group would be generated as formal sums of even-dimensional complex integration contours. Two contours would be equivalent if they can be deformed to each other without going through poles. The standard form of variety consists of arbitrary small circles surrounding the poles of the integrand multiplied by the residues which are algebraic numbers for rational functions. This generalizes to rational functions with both real and p-adic coefficients if one accepts the identification of integral as a variety modulo the described equivalence so that $(2\pi)^n$ corresponds to torus $(S^1)^n$. One can replace torus with 2π if one accepts an infinite-dimensional algebraic extension of p-adic numbers by powers of 2π. A weaker condition is that one allows ring containing only the positive powers of 2π.

5. The Grassmannian twistor approach for two-loop hexagon Wilson gives dilogarithm functions $L_k(s)$ [10]. General polylogarithm is defined by obey the recursion formula:

$$Li_{s+1}(z) = \int_0^z Li_s(t)\frac{dt}{t} \quad .$$

   Ordinary logarithm $Li_1(z) = -log(1-z)$ exists p-adically and generates a hierarchy containing dilogarithm, trilogarithm, and so on, which each exist p-adically for $|x| < 1$as is easy to see. If one accepts the general definition of logariths one finds that the entire function series exists p-adically for integer values of s. An interesting question is how strong constraints p-adic existence gives to the thetwistor loop integrals and to the underlying QFT.

6. The ring having p-adic numbers as coefficients and spanned by transcendentals $log(k)$ and $log(p)$, where k is primitive root of unity in $G(p)$ emerges in the proposed p-adicization of vacuum functional as exponent of Kähler action. The action for the preferred extremals reducing to 3-D Chern-Simons action for space-time surfaces in the intersection of real and p-adic worlds would be expressible p-adically as a linear combination of $log(p)$ and $log(k)$. $log(m/n)$ expressible in this manner p-adically would be the symbolic outcome of p-adic integral $\int dx/x$ between rational points. x could be identified as a preferred coordinate along braid strand. A possible identification for x earlier would be as the length in the effective metric defined by modified gamma matrices appearing in the modified Dirac equation [8] .

5.4 How could one calculate p-adic integrals numerically?

Riemann sum gives the simplest numerical approach to the calculation of real integrals. Also p-adic integrals should allow a numerical approach and very probably such approaches already exist and "motivic integration" presumably is the proper word to google. The attempts of an average physicist to dig out this kind of wisdom from the vastness of mathematical literature however lead to a depression and deep feeling of inferiority. The only manner to avoid the painful question "To whom should I blame for ever imagining that I could become a real mathematical physicist some day?" is a humble attempt to extrapolate real common sense to p-adic realm. One must believe that the almost trivial Riemann integral must have an almost trivial p-adic generalization although this looks far from obvious.

5.4.1 A proposal for p-adic numerical integration

The physical picture provided by quantum TGD gives strong constraints on the notion of p-adic integral.

1. The most important integrals should be over partonic 2-surfaces. Also p-adic variants of 3-surfaces and 4-surfaces can be considered. The p-adic variant of Kähler action would be an especially interesting integral and reduces to Chern-Simons terms over 3-surfaces for preferred extremals. One should use this definition also in the p-adic context since the reduction of a total divergence to boundary term is not expected to take place in numerical approach if one begins from a 4-dimensional Kähler action since in p-adic context topological boundaries do not

exist. The reduction to Chern-Simons term means also a reduction to cohomology and p-adic cohomology indeed exists.

At the first step one could restrict the consideration to algebraic varieties - in other words zero loci for a set of polynomials $P_i(x)$ at the boundary of causal diamond consisting of pieces of $\delta M^4_\pm \times CP_2$. 5 equations are needed. The simplest integral would be the p-adic volume of the partonic 2-surface.

2. The numerics must somehow rely on the p-adic topology meaning that very large powers p^n are very small in p-adic sense. In the p-adic context Riemann sum makes no sense since the sum never has p-adic norm larger than the maximum p-adic norm for summands so that the limit would give just zero. Finite measurement resolution suggests that the analog for the limit $\Delta x \to 0$ is pinary cutoff $O(p^n) = 0$, $n \to \infty$, for the function f to be integrated. In the spirit of algebraic geometry one must asume at least power series expansion if not even the representability as a polynomial or rational function with rational or p-adic coefficients.

3. Number theoretic approach suggests that the calculation of the volume $vol(V)$ of a p-adic algebraic variety V as integral should reduce to the counting of numbers for the solutions for the equations $f_i(x) = 0$ defining the variety. Together with the finite pinary cutoff this would mean counting of numbers for the solutions of equations $f_i(x)\ mod\ p^n = 0$. The p-adic volume $Vol(V,n)$ of the variety in the measurement resolution $O(p^n) = 0$ would be simply the number of p-adic solutions to the equations $f_i(x)\ mod\ p^n = 0$. Although this number is expected to become infinite as a real number at the limit $n \to \infty$, its p-adic norm is never larger than one. In the case that the limit is a well-defined as p-adic integer, one can say that the variety has a well-defined p-adic valued volume at the limit of infinite measurement resolution. The volume $Vol(V,n)$ could behave like n_p^n and exist as a well defined p-adic number only if n_p is divisible by p.

4. The generalization of the formula for the volume to an integral of a function over the volume is straightforward. Let f be the function to be integrated. One considers solutions to the conditions $f(x) = y$, where y is p-adic number in resolution $O(p^n) = 0$, and therefore has only a finite number of values. The condition $f(x) - y = 0$ defines a codimension 1 sub-variety V_y of the original variety and the integral is defined as the weighted sum $\sum_y y \times vol(V_y)$, where y denotes the point in the finite set of allowed values of $f(x)$ so that calculation reduces to the calculation of volumes also now.

5.4.2 General coordinate invariance

From the point of view of physics general coordinate invariance of the volume integral and more general integrals is of utmost importance.

1. The general coordinate invariance with respect to the internal coordinates of surface is achieved by using a subset of imbedding space-cooordinates as preferred coordinates for the surface. This is of also required if one works in algebraic geometric setting. In the case of projective spaces and similar standard imbedding spaces of algebraic varieties natural preferred coordinates exist. In TGD framework the isometries of $M^4 \times CP_2$ define natural preferred coordinate systems.

2. The question whether the formula can give rise to a something proportional to the volume in the induced metric in the intersection of real and rational worlds interesting. One could argue that one must include the square root of the determinant of the induced metric to the definition of volume in preferred coordinates but this might not be necessary. In fact, p-adic integration is genuine summation whereas the determinant of metric corresponds density of volume and need not make no sense in p-adic context. Could the fact that the preferred coordinates transform in simple manner under isometries of the imbedding space (linearly under maximal subgoup) alone guarantee that the information about the imbedding space metric is conveyed to the formula?

3. Indeed, since the volume is defined as the number of p-adic points, the proposed formula should be invariant at least under coordinate transformations mediated by bijections of the preferred coordinates expressible in terms of rational functions. In fact, even more general bijections mapping p-adic numbers to p-adic numbers could be allowed since they effectively mean the

introduction of new summation indices. Since the determinant of metric changes in coordinate transformations this requires that the metric determinant is not present at all. Thus summation is what allows to achieve the p-adic variant of general coordinate invariance.

4. This definition of volume and more general integrals amounts to solving the remaining coordinates of imbedding space as (in general) many-valued functions of these coordinates. In the integral those branches contribute to the integral for which the solution is p-adic number or belongs to the extension of p-adic numbers in question. By p-adic continuity the number of p-adic value solutions is locally constant. In the case that one integrates function over the surface one obtains effectively many-valued function of the preferred coordinates and can perform separate integrals over the branches.

5.4.3 Numerical iteration procedure

A convenient iteration procedure is based on the representation of integrand f as sum $\sum_k f_k$ of functions associated with different p-adic valued branches $z_k = z_k(x)$ for the surface in the coordinates chosen and identified as a subset of preferred imbedding space coordinates. The number of branches z_k contributing is by p-adic continuity locally constant.

The function f_k -call it g for simplicity - can in turn be decomposed into a sum of piecewise constant functions by introducing first the piecewise constant pinary cutoffs $g_n(x)$ obtained in the approximation $O(p^{n+1}) = 0$. One can write g as

$$g(x) = \sum h_n(x) \quad , \quad h_0(x) = g_0(x) \quad , \quad h_n = g_n(x) - g_{n-1}(x) \ \text{ for } n > 0 \quad .$$

Note that $h_n(x)$ is of form $g_n(x) = a_n(x)p^n$, $a_n(x) \in \{0, p-1\}$ so that the representation for integral as a sum of integrals for piecewise constant functions h_n converge rapidly. The technical problem is the determination of the boundaries of the regions inside which these functions contribute.

The integral reduces to the calculation of the number of points for given value of $h_n(x)$ and by the local constancy for the number of p-adic valued roots $z_k(x)$ the number of points for $N_0 \sum_{k\geq 0} p^k = N_0/(1-p)$, where N_0 is the number of points x with the property that not all points $y = x(1+O(p))$ represent p-adic points $z(x)$. Hence a finite number of calculational steps is enough to determine completely the contribution of given value to the integral and the only approximation comes from the cutoff in n for $h_n(x)$.

5.4.4 Number theoretical universality

This picture looks nice but it is far from clear whether the resulting integral is that what physicist wants. It is not clear whether the limit $Vol(V, n)$, $n \rightarrow \infty$, exists or even should exist always.

1. In TGD Universe a rather natural condition is algebraic universality requiring that the p-adic integral is proportional to a real integral in the intersection of real and p-adic worlds defined by varieties identified as loci of polynomials with integer/rational coefficients. Number theoretical universality would require that the value of the p-adic integral is p-adic rational (or algebraic number for extensions of p-adic numbers) equal to the value of the real integral and in algebraic sense independent of the number field. In the eyes of physicist this condition looks highly non-trivial. For a mathematician it should be extremely easy to show that this condition cannot hold true. If true the equality would represent extremely profound number theoretic truth.

   The basic idea of the motivic approach to integration is to generalize integral formulas so that the same formula applies in any number field: the specialization of the formula to given number field would give the integral in that particular number field. This is of course nothing but number theoretical universality. Note that the existence of this kind of formula requires that in the intersection of the real and p-adic worlds real and p-adic integrals reduce to same rational or transcendentals (such as $log(1+x)$ and polylogarithms).

2. If number theoretical universality holds true one can imagine that one just takes the real integral, expresses it as a function of the rational number valued parameters (continuable to real numbers) characterizing the integrand and the variety and algebraically continues this expression to p-adic number fields. This would give the universal formula which can be specified to any number

field. But it is not at all clear whether this definition is consistent with the proposed numerical definition.

3. There is also an intuitive expectation in an apparent conflict with the number theoretic universality. The existence of the limit for a finite number p-adic primes could be interpreted as mathematical realization of the physical intuition suggesting that one can assign to a given partonic 2-surface only a finite number of p-adic primes [8]. Indeed, quantum classical correspondence combined with the p-adic mass calculations suggests that the partonic 2-surfaces assignable to a given elementary particle in the intersection of real and p-adic worlds corresponds to a finite number of p-adic primes somehow coded by the geometry of the partonic 2-surface.

   One way out of the difficulty is that the functions - say polynomials - defining the surface have as coefficients powers of e^n. For given prime p only the powers of e^p exist p-adically so that only the primes p dividing n would be allowed. The transcendentals of form $log(1+px)$ and their polylogarithmic generalizations resulting from integrals in the intersection of real and p-adic worlds would have the same effect. Second way out of the difficulty would be based on the condition that the functional integral over WCW ("world of classical worlds") converges. There is a good argument stating that the exponent of Kähler action reduces to an exponent of integer n and since all powers of n appear the convergence is achieved only for p-adic primes dividing n.

5.4.5 Can number theoretical universality be consistent with the proposed numerical definition of the p-adic integral?

The equivalence of the proposed numerical integral with the algebraic definition of p-adic integral motivated by the algebraic formula in the real context expressed in terms of various parameters defining the variety and the integrand and continued to all number fields would be such a number theoretical miracle that it deserves italics around it:

For algebraic surfaces the real volume of the variety equals apart from constant C to the number of p-adic points of the variety in the case that the volume is expressible as p-adic integer.

The proportionality constant C can depend on p-adic number field , and the previous numerical argument suggests that the constant could be simply the factor $1/(1-p)$ resulting from the sum of p-adic points in p-adic scales so short that the number of the p-adic branches $z_k(x)$ is locally constant. This constant is indeed needed: without it the real integrals in the intersection of real and p-adic worlds giving integer valued result $I = m$ would correspond to functions for which the number of p-adic valued points is finite.

The statement generalizes also to the integrals of rational and perhaps even more general functions. The equivalence should be considered in a weak form by allowing the transcendentals contained by the formulas have different meanings in real and p-adic number fields. Already the integrals of rational functions contain this kind of transcendentals.

The basic objection that number of p-adic points without cannot give something proportional to real volume with an appropriate interpretation cannot hold true since real integral contains the determinant of the induced metric. As already noticed the preferred coordinates for the imbedding space are fixed by the isometries of the imbedding space and therefore the information about metric is actually present. For constant function the correspondence holds true and since the recipe for performing of the integral reduce to that for an infinite sum of constant functions, it might be that the miracle indeed happens.

The proposal can be tested in a very simple manner. The simplest possible algebraic variety is unit circle defined by the condition $x^2 + y^2 = 1$.

1. In the real context the circumference is 2π and p-adic transcendental requiring an infinite-dimensional algebraic extension defined in terms of powers of 2π. Does this mean that the number of p-adic points of circle at the limit $n \to \infty$ for the pinary cutoff $O(p^n) = 0$ is ill-defined? Should one define 2π as this integral and say that the motivic integral calculus based on manipulation of formulas reduces the integrals to a combination of p-adically existing numbers and 2π? In motivic integration the outcome of the integration is indeed formula rather than number and only a specialization gives it a value in a particular number field. Does 2π have a

specialization to the original p-adic number field or should one introduce it via transcendental extension?

2. The rational points $(x, y) = (k/m, l/m)$ of the p-adic unit circle would correspond to Pythagorean triangles satisfying $k^2 + l^2 = m^2$ with the general solution $k = r^2 - s^2$, $l = 2rs$, $m = r^2 + s^2$. Besides this there is an infinite number of p-adic points satisfying the same equation: some of the integers k, l, m would be however infinite as real integers. These points can be solved by starting from $O(p) = 0$ approximation $(k, l, m) \rightarrow (k, l, m)\ mod\ p \equiv (k_0, l_0, m_0)$. One must assume that the equations are satisfied only modulo p so that Pythagorean triangles modulo p are the basic objects. Pythagorean triangles can be also degenerate modulo p so that either k_0, l_0 or even m_0 vanishes. Note that for surfaces $x^n + y^n = z^n$ no non-trivial solutions exists for $x^n, y^n, z^n < p$ for $n > 2$ and all p-adic points are infinite as real integers.

   The Pythagorean condition would give a constraint between higher powers in the expressions for k, l and m. The challenge would be to calculate the number of this kind of points. If one can choose the integers $k - (k\ mod\ p)$ and $l - (l\ mod\ p)$ freely and solve $m - (m\ mod\ p)$ from the quadratic equations uniquely, the number of points of the unit circle consisting of p-adic integers must be of form $N_0/(1 - p)$. At the limit $n \rightarrow \infty$ the p-adic length of the unit circle would be in p-adic topology equal to the number of modulo p Pythagorean triangles (r, s). The p-adic counterpart of 2π would be ordinary p-adic number depending on p. This definition of the length of unit circle as number of its modulo p Pythagorean points also Pythagoras would have agreed with since in the Pythagorean world view only rational triangles were accepted.

3. One can look the situation also directly solving y as $y = \pm\sqrt{1 - x^2}$. The p-adic square root exists always for $x = O(p^n)$, $n > 0$. The number of these points x is $2/(1 - p)$. For $x = O(p^0)$ the square root exist for roughly one half of the integers $n \in \{0, p - 1\}$. The number of integers $(x^2)_0$ is therefore roughly $(p-1)/2$. The study of $p = 5$ cae suggests that the number of integers $(1 - (x^2)_0)_0 \in \{0, p - 1\}$ which are squares is about $(p - 1)/4$. Taking into account the $\pm$ sign the number of these points by $N_0 \simeq (p - 1)/2$. In this case the higher $O(p)$ contribution to x is arbitrary and one obtains total contribution $N_0/(1 - p)$. Altogether one would have $(N_0 + 2)/(1 - p)$ so that eliminating the proportionality factor the estimate for the p-adic counterpart of 2π would be $(p + 3)/2$.

4. One could also try a trick. Express the points of circle as $(x, y) = (cos(t), sin(t))$ such that t is any p-adic number with norm smaller than one in p-adic case. This unit circle is definitely not the same object as the one defined as algebraic variety in plane. One can however calculate the number of p-adic points at the limit $n \rightarrow \infty$. Besides $t = 0$, all p-adic numbers with norm larger than p^{-n} and smaller than 1 are acceptable and one obtains as a result $N(n) = 1 + p^{n-1}$, where "1" comes from overall important point $t = 0$. One has $N(n) \rightarrow 1$ in p-adic sense. If $t = 0$ is not allowed the length vanishes p-adically. The circumference of circle in p-adic context would have length equal to 1 in p-adic topology so that no problems would be encountered (numbers $exp(i2\pi/n)$ would require algebraic extension of p-adic numbers and would not exist as power series).

   The replacement of the coordinates (x, y) with coordinate t does not respect the rules of algebraic geometry since trigonometric functions are not algebraic functions. Should one allow also exponential and trigonometric functions and their inverses besides rational functions and define circle also in terms of these. Note that these functions are exceptional in that corresponding transcendental extensions -say that containing e and its powers- are finite-dimensional?

5. To make things more complicated, one could allow algebraic extensions of p-adic numbers containing roots $U_n = exp(i2\pi/n)$ of unity. This would affect the count too but give a well-defined answer if one accepts that the points of unit circle correspond to the Pythagorean points multiplied by the roots of unity.

5.4.6 p-Adic thermodynamics for measurement resolution?

The proposed definition is rather attractive number theoretically since everything would reduce to the counting of p-adic points of algebraic varieties. The approach generalizes also to algebraic extensions of p-adic numbers. Mathematicians and also physicists love partition functions, and one can indeed assign to the volume integral a partition function as p-adic valued power series in powers $Z(t) = \sum v_n t^n$ with the coefficients v_n giving the volume in $O(p^n) = 0$ cutoff. One can also define partition functions $Z_f(t) = \sum f_n t^n$, with f_n giving the integral of f in the same approximation.

Could this kind of partition functions have a physical interpretation as averages over physical measurements over different pinary cutoffs? p-Adic temperature can be identified as $t = p^{1/T}$, $T = 1/k$. For p-adically small temperatures the lowest terms corresponding to the worst measurement resolution dominate. At first this sounds counter-intuitive since usually low temperatures are thought to make possible good measurement resolution. One can however argue that one must excite p-adic short range degrees of freedom to get information about them. These degrees of freedom correspond to the higher pinary digits by p-adic length scale hypothesis and high energies by Uncertainty Principle. Hence high p-adic temperatures are needed. Also measurement resolution would be subject to p-adic thermodynamics rather than being freely fixed by the experimentalist.

5.5 Infinite rationals and multiple residue integrals as Galois invariants

In TGD framework one could consider also another kind of cohomological interpretation. The basic structures are braids at light-like 3-surfaces and space-like 3-surfaces at the ends of space-time surfaces. Braids intersects have common ends points at the partonic 2-surfaces at the light-like boundaries of a causal diamond. String world sheets define braid cobordism and in more general case 2-knot [10]. Strong form of holography with finite measurement resolution would suggest that physics is coded by the data associated with the discrete set of points at partonic 2-surfaces. Cohomological interpretation would in turn would suggest that these points could be identified as intersections of string world sheets and partonic 2-surface defining dual descriptions of physics and would represent intersection form for string world sheets and partonic 2-surfaces.

Infinite rationals define rational functions and one can assign to them residue integrals if the variables x_n are interpreted as complex variables. These rational functions could be replaced with a hierarchy of sub-varieties defined by their poles of various dimensions. Just as the zeros allow realization as braids or braids also poles would allow a realization as braids of braids. Hence the n-fold residue integral could have a representation in terms of braids. Given level of the braid hierarchy with n levels would correspond to a level in the hierarchy of complex varieties with decreasing complex dimension.

One can assign also to the poles (zeros of polynomial in the denominator of rational function) Galois group and obtains a hierarchy of Galois groups in this manner. Also the braid representation would exists for these Galois groups and define even cohomology and homology if they do so for the zeros. The intersections of braids with of the partonic 2-surfaces would represent the poles in the preferred coordinates and various residue integrals would have representation in terms of products of complex points of partonic 2-surface in preferred coordinates. The interpretation would be in terms of quantum classical correspondence.

Galois groups transform the poles to each other and one can ask how much information they give about the residue integral. One would expect that the n-fold residue integral as a sum over residues expressible in terms of the poles is invariant under Galois group. This is the case for the simplest integrals in plane with n poles and probably quite generally. Physically the invariance under the hierarchy of Galois group would mean that Galois groups act as the symmetry group of quantum physics. This conforms with the number theoretic vision and one could justify the formula for the residue integral also as a *definition* motivated by the condition of Galois invariance. Of course, all symmetric functions of roots would be Galois invariants and would be expected to appear in the expressions for scattering amplitudes.

The Galois groups associated with zeros and poles of the infinite rational seem to have a clear physical significance. This can be understood in zero energy ontology if positive (negative) physical states are indeed identifiable as infinite integers and if zero energy states can be mapped to infinite rationals which as real numbers reduce to real units. The positive/negative energy part of the zero energy state would correspond to zeros/poles in this correspondence. An interesting question is how strong correlations the real unit property poses on the two Galois groups hierarchies. The asymmetry between positive and negative energy states would have interpretation in terms of the thermodynamic arrow of geometric time [1] implied by the condition that either positive or negative energy states correspond to state function reduced/prepared states with well defined particle numbers and minimum amount of entanglement.

5.6 Twistors, hyperbolic 3-manifolds, and zero energy ontology

While performing web searches for twistors and motives I have begun to realize that Russian mathematicians have been building the mathematics needed by quantum TGD for decades by realizing the vision of Grothendieck. One of the findings was the article Volumes of hyperbolic manifolds and mixed Tate motives [50] by Goncharov- one of the great Russian mathematicians involved with the drama- about volumes of hyperbolic n-manifolds and motivic integrals.

Hyperbolic n-manifolds [19] are n-manifolds equipped with complete Riemann metric having constant sectional curvature equal to -1 (with suitable choice of length unit) and therefore obeying Einstein's equations with cosmological constant. They are obtained as coset spaces on proper-time constant hyperboloids of n+1-dimensional Minkowski space by dividing by the action of discrete subgroup of SO(n,1), whose action defines a lattice like structure on the hyperboloid. What is remarkable is that the volumes of these closed spaces are homotopy invariants in a well-define sense.

What is even more remarkable that hyperbolic 3-manifolds [18] are completely exceptional in that there are very many of them. The complements of knots and links in 3-sphere are often cusped hyperbolic 3-manifolds (having therefore tori as boundaries). Also Haken manifolds are hyperbolic. According to Thurston's geometrization conjecture, proved by Perelman (whom we all know!), any closed, irreducible, atoroidal 3-manifold with infinite fundamental group is hyperbolic. There is an analogous statement for 3-manifolds with boundary. One can perhaps say that very many 3-manifolds are hyperbolic.

The geometrization conjecture of Thurston [14] allows to see hyperbolic 3-manifolds in a wider framework. The theorem states that compact 3-manifolds can be decomposed canonically into submanifolds that have geometric structures. It was Perelman who sketched the proof of the conjecture. The prime decomposition with respect to connected sum reduces the problem to the classification of prime 3-manifolds and geometrization conjecture states that closed 3-manifold can be cut along tori such that the interior of each piece has a geometric structure with finite volume serving as a topological invariant. There are 8 possible geometric structures in dimension three and they are characterized by the isometry group of the geometry and the isotropy group of point.

Important is also the behavior under Ricci flow [31] $\partial_t g_{ij} = -2R_{ij}$: here t is not space-time coordinate but a parameter of homotopy. If I have understood correctly, Ricci flow is a dissipative flow gradually polishing the metric for a particular region of 3-manifold to one of the 8 highly symmetric metrics defining topological invariants. This conforms with the general vision about dissipation as the source of maximal symmetries. For compact n-manifolds the normalized Ricci flow $\partial_t g_{ij} = -2R_{ij} + (2/n)Rg_{ij}$ preserving the volume makes sense. Interestingly, for $n = 4$ the right hand side is Einstein tensor so that the solutions of vacuum Einstein's equations in dimension four are fixed points of normalized Ricci flow. Ricci flow expands the negatively curved regions and contracts the positively curved regions of space-time time. Hyperbolic geometries represent one these 8 geometries and for the Ricci flow is expanding. The outcome is amazingly simple and gives also support for the idea that the preferred extremals of Kähler action could represent maximally symmetries 4-geometries defining topological or algebraic geometric invariants: the preferred extremals would be maximally symmetric representatives - kind of archetypes- for a given topology or algebraic geometry.

The volume spectrum for hyperbolic 3-manifolds forms a countable set which is however not discrete: some reader might understand what the statement that one can assign to them ordinal ω^ω could possibly mean for the man of the street. What comes into my simple mind is that p-adic integers and more generally, profinite spaces which are not finite, are something similar: one can enumerate them by infinitely long sequences of pinary digits so that they are countable (I do not know whether also infinite p-adic primes must be allowed). They are totally disconnected in real sense but do not form a discrete set since since can connect any two points by a p-adically continuous curve.

What makes twistor people excited is that the polylogarithms emerging from twistor integrals and making sense also p-adically seems to be expressible in terms of the volumes of hyperbolic manifolds. What fascinates me is that the moduli spaces for causal diamonds or rather for the double light-cones associated with their M^4 projections with second tip fixed are naturally lattices of the3-dimensional hyperbolic space defined by all positions of the second tip and 3-dimensional hyperbolici spaces are the most intersting ones! At least in the intersection of the real and p-adic worlds number theoretic discretization requires discretization and volume could be quantized in discrete manner.

For $n = 3$ the group defining the lattice is a discrete subgroup of the group of SO(3,1) which equals to $PSL(2,C)$ obtained by identifying $SL(2,C)$ matrices with opposite sign. The divisor group defining the lattice and hyperbolic spaces as its lattice cell is therefore a subgroup of $PSL(2, Z_c)$, where Z_c denotes complex integers. Recall that $PSL(2, Z_c)$ acts also in complex plane (and therefore on partonic 2-surfaces) as discrete Möbius transformations wheras $PSL(2, Z)$ correspond to 3-braid group. Reader is perhaps familiar with fractal like orbits of points under iterated Möbius transformations. The lattice cell of this lattice obtained by identifying symmetry related points defines hyperbolic 3-manifolds. Therefore zero energy ontology realizes directly the hyperboliic manifolds whose volumes should somehow represent the polylogarithms.

The volumes, which are topological invariants, are said to be highly transcendental. In the intersection of real and p-adic worlds only algebraic volumes are possible unless one allows extension by say finite number of roots of e (e^p is p-adic number). The p-adic existence of polylogarithms suggests that also p-adic variants of hyperbolic spaces make sense and that one can assign to them volume as topological invariance although the notion of ordinary volume integral is problematic. In fact, hyperbolic spaces are symmetric spaces and general arguments allow to imagine what the p-adic variants of real symmetric spaces could be.

6 Floer homology and TGD

Floer homology [11] has provided considerable understanding of symplectic manifolds using physics based approach relying on 2-D variational principle called symplectic action. One variant of Floer theory has been applied also to deduce topological invariants of 3-manifolds in terms of SU(2) Chern-Simons action. The basics of Floer homology without recourse to quantum field theoretic approach are described at technical level in the lectures of Dietmar Salamon [55]. The notion of quantum cohomology closely related to Floer homology and related approaches and involving also supersymmetry is described by Alexander Givental in [28].

The quantum fluctuating degrees of freedom of TGD Universe are parameterized by symplectic group acting as isometries of WCW, which can be regarded as a union of symmetric spaces assignable to the symplectic group. Hence the optimistic hunch is that Floer homology might provide new insights about quantum TGD - in particular about the problem of understanding the preferred extremals of Kähler action. Especially interesting is the relationship of Floer homology to the proposed vision about braided Galois homology. The following considerations encourage this optimism. In particular, completely new insights about the role of Minkowskian and Euclidian regions emerge.

6.1 Trying to understand the basic ideas of Floer homology

I do not have competence to describe Floer's homology as a mathematician. Instead, I try just to outline the basic ideas as I have (possibly mis-)understood them as a physicist by reading the basic introduction to the theory [11]. The motivation for the symplectic Floer homology came from Arnold's conjecture stating that for a closed symplectic manifold the number of fixed points for non-degenerate (isolated critical points) symplecto-morphisms has the sum of the Betti numbers as a lower bound. The equivalence of Floer's symplectic homology for closed symplectic manifolds with singular homology proves this conjecture. This means that symplectic Floer homology as such is not interesting from TGD view point of view.

6.1.1 Morse function in the loop space of the symplectic manifold

Recall that Morse function is a monotonically increasing real valued function in n-manifold for which critical points are isolated. Its level surfaces induce the slicing of the manifold $n-1$-dimensional surfaces. At the extrema the topology of the slice changes as is clear from a simple example provided by torus (standing on tangent plane orthogonal to the plane defined by the torus with Morse function identified as the height function defined by the coordinate orthogonal to the plane). There is minimum and maximum and two saddle points. Quite generally, the signature of the matrix defined by the second derivatives of the Morse function -Hessian- characterizes the properties of the critical point. Hessian allows to deduce information about the topology of the manifold and Morse theorem states that the number of critical points has a lower limit given by the sum of the Betti numbers defining the dimensions of various homology groups of the manifolds in singular homology.

Floer generalizes Morse theory from the level of symplectic manifold M with a Morse function defined by Hamiltonian to the level of the free loop space LM of M. This Morse function depends on preferred Hamiltonian and its cyclic time variation defining a loop in LM. Salamon represents the approach without recourse to the methods of topological quantum field theories [55]. A very schematic representation -even more schematic than that in [28] - using referring to quantum about what one does is attempted in following.

1. 2-dimensional action for an orbit of string in M replaces Morse function. The extrema of the action analogous to critical points of Morse function are crucial for calculating path integral in QFT approach using saddle point approximation. In topological QFTs path integral reduces to a well-defined finite dimensional integrals over moduli spaces. One constructs action principle in the form

$$S = \int_{-\infty}^{\infty}(||\partial_u m||^2 + ||\nabla f||^2)du \tag{6.1}$$

   where u can be seen regarded as a coordinate parallel to cylinder axes defined by the orbit of the loop of M and t could be regarded as an angle coordinate of the loop. f denotes the symplectic action functional of the loop defined by time dependent Hamiltonian H_t. ∇f is the functional gradient of f with respect to coordinates of m regarded as analogous to fields $S^1 \times R$. $||...||^2$ defines inner product in the space of maps $S^1 \to M$ involving integral over the circle parameterized by coordinate t. Note that this action introduces preferred parameterization of the cylinder meaning breaking of at least manifest general coordinate invariance.

2. Schematically the field equations read as

$$\partial_u^2 m = \nabla^2 f \ , \tag{6.2}$$

   where ∇^2 is functional d'Alembertian reducing to its analog at the level of M but depending on preferred Hamilton H_t. This condition states that the cylinder represents a harmonic map $S^1 \times R \to M$ with respect to the almost Kähler metric of M.

3. Assuming the analog of $\mathcal{N} = 2$ supersymmetry for the solution the above equation reduces to

$$\partial_u m = \pm\nabla f \ . \tag{6.3}$$

   This condition is just the condition saying that one has a wave packed moving to right or left and state the hyper-complex variant of holomorphy. These left and right moving solutions are in key role in string model. In Euclidian metric of $S^1 \times R$ the conditions have interpretation as the generalization of Cauchy-Riemann conditions stating that the map $S^1 \times R \to M$ commutes with complex conjugation: in other worlds the multiplication by imaginary unit in $S^1 \times R$ is equivalent with the tensor multiplication defined by the almost Kähler form in M. The tangent space of image is complex sub-space of tangent space of M. Depending on the sign on the right hand side one has pseudo-holomorphy or anti-pseudo-holomorphy.

4. The solutions with finite action become asymptotically independent of u so that one has $\nabla f = 0$. This states that the loop represents a cyclic solution of Hamilton's equations for Hamilton H. Hamilton could also depend on time in periodic manner so that for $t = 0$ and $t = 2\pi$ one has $H_t = H$.

5. One can consider also solutions which are independent of u and t asymptotically so that the circles reduce to critical points asymptotically. One can also consider solutions representing spheres with more than two critical points as marked points. Also solutions with higher genus can be considered These solutions are relate closely to the definition of Gromov-Witten invariants in quantum cohomology.

This approach generalizes also to Chern-Simons action by replacing f with Chern-Simons action for the 3-manifold X^3 and $R \times S^1$ with $R \times X^3$ to get space-time. The symplectic manifold is replaced with the space of Yang-Mills gauge potentials. In this case field equations from the variational principle are YM equations and instanton and anti-instanton equations are obtained in the super-symmetric case. Time independent solutions correspond asymptotically to static solutions describing magnetic monopoles. In this case the critical points of Morse function can be seen as points at which the topology of the slice of field space defined by the Morse function changes its topology. A good intuitive guideline is Morse function for torus.

6.1.2 About Witten's approach to Floer homology

Using the ideas discussed for the first time in Witten's classic work revealing a connection between supersymmetry and Morse theory [52], one can extend M to a super-manifold. Witten defines $\mathcal{N} = 2$ SUSY algebra by introducing a parameter dependent deformation of the exterior algebra via $d_t = exp(-th)dexp(th)$ and its conjugate $d_t^* = exp(th)dexp(-th)$: for $t = 0$ one has $d_t = d_t^*$. h takes the role the role of Morse function. $Q_1 = d_t + d_t^*$ and $Q_2 = i(d_t - d_t^*)$ obey standard supersymmetry algebra $Q_1Q_2 + Q_2Q_1 = 0$ and $Q_1^2 = Q_2^2 \equiv H_t$. The solutions of $d_t\Psi = 0$ are differential forms of various degrees and correspond to zero energy solutions for which the supersymmetry is not broken. The deformed cohomology is equivalent with the original cohomology by $\Psi \to exp(th)\Psi$. This gives a direct connection between cohomology and supersymmetry whose existence is to be expected from the basic properties of exterior algebra.

The motivation for the deformation is that for degree p closed forms are localized around critical points of h with Hessian having p negative eigenvalues so that the correspondence between homology generators and critical points becomes manifest. There is indeed a natural mapping from de Rham cohomology to the critical points such that the degree of the form correspond to the number of negative eigenvalues of the Hessian.

Later Witten managed to expand his ideas about supersymmetric Morse theory so that it could be applied to Floer homology (1+1 case) and to the calculation of Donaldson invariants of 4-manifold (1+3 case). Recently Witten has been working with the applications to knot theory (1+2 case) for ordinary knots and for 2-knots and cobordisms of 1-knots (1+3 case) [57, 39, 58].

6.1.3 Representation of loops with fixed based in terms of Hamiltonians with cyclic time dependence

As already noticed Floer - whose work preceded Witten's work - considered instead of the symplectic manifold M its free loop space LM. One begins with symplectic action identified as the sum of the symplectic area of the loop expressible as the value of the one-form defining the symplectic form over the loop and integral of the Hamiltonian H around the loop. The natural choice of the loop parameter is as the canonical conjugate of the symplectic potential so that the integrated quantity is analogous to the minimal substitution $p - eA$ of familiar from elementary quantum mechanics. The variational equations for the symplectic action are Hamiltonian equations of motion in the force field defined by the Hamiltonian H and one considers periodic orbits (recall that there is conserved energy associated with the orbits defined by the Hamiltonian). The counterparts of critical points are loops which correspond to the extrema of symplectic action.

One can also consider time dependent Hamiltonians H_t for which the initial and final value of the Hamiltonian is the same preferred Hamiltonian. This kind of Hamiltonians define via their time evolutions loops in the loop space LG of the symplectic group. At the level of LM the resulting map of M to itself is symplecto-morphism. Now however energy is not in general conserved. By periodicity the critical points of the Hamiltonian H correspond to cyclic orbits of periodically time varying Hamiltonian so that the homotopies of LM with base point defined by H are mapped to a collection of homotopies of M defined by the critical points of the Hamiltonian. For constant Hamiltonian $H_t = H$ the critical orbits reduce to a point and the need to obtain non-trivial elements of homotopy group of M explains why one needs Hamiltonians with cyclic time dependence. The homotopy group of LM is mapped to that of M by homomorphism.

One could consider also higher homotopy groups of the loop space. The first homotopy group would correspond to loops in loop space mapped to tori associated with the fixed points of the Hamiltonian. In this manner one would obtain analogs of homotopy groups defined by mappings from $(S^1)^n$ to loop space to M and also of homotopy groups. By taking the initial loop to be trivial so that initial Hamiltonian is constant Hamiltonian, one obtains the symplectic analogs of ordinary homotopy groups defined as a map from S^n to loop space to M. Also the condition that loops are contracted to points asymptotically gives rise to homotopy groups.

6.1.4 Representation of non-closed paths of LM as paths connecting critical points of M

In Floer homology one considers also paths of LM and M,which are not closed. These paths form the first homotopy groupoid of LM. Since the elements of $\pi_0(LM)$ (loops not deformable to each other) represented by Hamiltonians with cyclic time dependence are mapped to those of $\pi_1(M)$ at critical points, a good guess is that the elements of homotopy group $\pi_1(LM)$ can be mapped to elements of $\pi_2(M)$ connecting critical points of H. If the loops at the ends of cylinder reduce to points the images of $\pi_1(LM)$ are indeed elements of $\pi_2(M)$ containing two critical points. As noticed, the number critical points can be also higher.

To achieve the representation of first homotopy group one considers a path of LM parameterized by a parameter u defining a cylinder in M which should connect the critical points. This requires that the deformation becomes at the limit $u \rightarrow \pm\infty$ independent of u so that one obtains a cyclic deformation of H. The partial differential equations state that one has gradient flow defined by symplectic action in loop space. The equations (resulting from supersymmetry in QFT approach) pseudo-holomorphy or generalized Cauchy-Riemann conditions as

$$\partial_u m \pm L_{H_t}(m) = 0 \ ,$$

where $L_{H_t}(m) = 0$ denotes Hamiltonian equations for the coordinates m of M so that $L_{H_t}m$ is indeed the functional gradient of symplectic action. At the asymptotic limit $\partial_u m \rightarrow 0$ boundary conditions give just Hamiltonian equations.

As already found, one can assign to to these equations a supersymmetric action functional defined in terms of the almost Kähler metric defining the analog of energy. As a matter fact, the existence of almost complex structure in M is enough (transitions functions between coordinate patches need not be holomorphic in this case). The condition that the energy is finite requires asymptotic u-independence and super-symmetry condition since energy density is the sum of kinetic energy densities associated with the motion in u direction and of the square of the vector $L_{H_t}m$. Since the time evolution with respect to u is not energy conserving, the cylinders can connect different critical points of H. This motivates the term "connecting cylinder". From the point of view of physicist the role of the field equations is to perform a "gauge choice" selecting particular representative for homotopy.

6.1.5 The orbit of the loop as a pseudo-holomorphic surface

The cylinder defined by the loop defines a pseudo-holomorphic surface. The sub-spaces connected by pseudo-holomorphic surfaces intersect in quantum cohomology and Gromow-Witten invariant counts for the number of the pseudo-holomorphic surfaces connecting/intersecting given n surfaces. Stringy interpretation for the pseudo-holomorphic curves (holomorphic for Kähler manifolds) would be as string world sheets. There is an obvious connection with the vision about branes connected by string world sheets. If the asymptotic images of S^1 contract to points, they correspond to critical points (marked points). One can consider also more general solutions of field with n asymptotic circles containing n critical points as marked points.

The statement of quantum cohomology that two surfaces intersect in fuzzy sense when they are connected by pseudo-holomorphic curve would mean that that two surfaces intersect when they both have points common with the pseudo-holomorphic curve. The 2-dimensional mapping cylinders can be filled to 3-D objects by adding the 2-dimensional pseudo-holomorphic surface. From this the connection with Chern-Simons action and possibility to apply analogous construction to 3-D manifold topology becomes obvious. Chern-Simons action in turn implies connection to 4-D manifold topology.

6.1.6 The correspondence with the singular homology

Symplectic Floer homology for closed symplectic manifolds is equivalent with singular homology. This means that one has one-to-one map of the space spanned by the critical points to the singular homology. Critical points are classified by the signature of the Hessian of Hamiltonian so that there is natural ordering of the critical points, which should correspond to the ordering of the homology groups since signature varies from n (maximum of Morse function) to zero (minimum of Morse function). The study of the homology of torus defined in terms of critical points of height function h serves as a guide-line when one tries to guess the idea behind the correspondence.

To each critical point one can assign a tangent plane defined as the plane of negative signature of the Hessian of h. Its value equals to 0,1,1,2 for the critical points of h. The critical manifolds assigned with the negative signature tangent space at critical points can be identified as point, first homologically non-trivial circle, second homologically non-trivial circle, and the entire torus and correspond to the generators of the homology. In Floer homology the correspondence need not be as simple as this but one expect similar correspondence so that the value of grading of homology corresponds to the signature of the critical point. One must allow only the connections going to the direction of smaller energy and by a proper choices of signs the dynamics defined by the action defined gradient flow is indeed dissipative so that this condition is satisfied.

6.1.7 Quantum cup product and pseudo-holomorphic surfaces

As the analog of intersection product in ordinary cohomology homology, the cohomology associated with the symplectic Floer homology corresponds to the so called pair of pants product of quantum cohomology [28] which is a deformed cup product having fuzzy intersection as its dual at the level of homology.

Ordinary cup product for two forms of degree n_1 and n_2 is a form which is characterized by its values for the elements of homology with co-dimension n_1+n_2 so that $d-n_1-n_2$ is the dimension of the intersection of the corresponding surfaces. The product is characterized by a coefficients $W(\alpha,\beta,\gamma)$ where the arguments represent homology equivalence classes identifiable as Gromov-Witten invariants assignable to sphere with three punctures. One can say that three representatives α,β,γ of homology give rise to a non-vanishing coefficient $W(\alpha,\beta,\gamma)$ if there is a pair of pants having non-empty intersections with α,β,γ. The coefficient $W(\alpha,\beta,\gamma)$ is analogous to a coupling constant associated with vertex with α,β,γ representing the particles entering to the vertex.

The factors of the cup product of quantum cohomology are associated with the two legs of the pants and the outcome of the product to the "waist". More abstractly, by conformal transformations the legs and "waist" can be reduced to 3 marked points and the number of marked points can be arbitrary and represent the intersection points for n manifolds connected by a pseudo-holomorphic surface with n marked points. One can indeed generalize the variational principle to allow besides cylinders also pseudo-holomorphic surfaces with arbitrary number holes whose boundaries are associated with loops containing critical point so that critical points would indeed represent marked points of a sphere with holes. When H_t reduces to H, loops and marked spheres reduce to point a so that ordinary cup product results.

6.2 Could Floer homology teach something new about Quantum TGD?

The understanding of both quantum TGD and its classical counterpart is still far from from comprehensive. For instance, skeptic could argue that the understanding of the preferred extremals of Kähler action is still just a bundle of ideas without a coherent overview. Also the physical roles of Kähler actions for Euclidian and Minkowskian space-time regions is far from clear. Do they provide dual descriptions as suggested or are both needed? Kähler action for preferred extremal in Euclidian regions defines naturally Kähler function. Could Kähler action in Minkowskian regions- naturally imaginary by negative sign of metric determinant- give an imaginary contribution to the vacuum functional and define Morse function so that both Kähler and Morse would find a prominent role in the world order of TGD? One might hope that the mathematical insights from Floer homology combined with the physical picture and constraints from quantum classical correspondence could provide additional insights about the construction preferred extremals of Kähler action.

6.2.1 Basic picture about preferred extremals of Kähler action

It is useful to gather some basic ideas about construction of preferred extremals before the discussion of ideas inspired by Floer homology.

1. For the preferred extremals Kähler action reduces to Chern-Simons term at the light-like surfaces defining orbits of partonic 2-surfaces and space-like 3-surfaces the ends of the space-time sheets. These 3-surfaces are extremals of Chern-Simons action subject to the constraint force defined by the weak form of electric-magnetic duality implying that TGD does not reduce to a mere topological QFT. One has clearly two dynamics: one along light-like 3-surfaces and one along space-like 3-surfaces and their internal consistency is a powerful constraint.

2. The Chern-Simons contributions from Minkowskian region is imaginary and corresponds to almost topological QFT aspect of TGD. The argument reducing the action to Chern-Simons term has been discussed in detail only in Minkowskian regions and involves in an essential manner the notions of local polarization and light-like momentum direction: the latter one does not make sense in Euclidian regions. Note however that Laplace equation makes sense and local polarization and momentum directions are replaced by those for color quantum numbers. It will be found that internal consistency requires holography both in Minkowskian and Euclidian regions. In any case, the Euclidian contribution would give rise to the exponent of Kähler function and Minkowskian contribution to a phase factor appearing usually in path integral defining topological QFT. Exponent of Kähler function would guarantee that integration over WCW is mathematically well-defined.

3. How could one extend the 3-surfaces to 4-surfaces using strong form of holography. One could think of having for each time=constant collection of 2-D slices of the light-like 3-surfaces a space-like Chern-Simons dynamics connecting them to each other. One would have two dynamics-one time-like and one space-like as effective 2-dimensionality required by the strong form of holography requires. These dynamics should be mutually consistent and this should give consistency conditions. The time parameters for these two dynamics would correspond to the two coordinates of string world sheets involved.

4. The idea that one could assign Hamiltonians to the marked points of the partonic 2-surfaces as carriers is physically compelling. The Hamiltonians of $\delta M^4_{\pm} \times CP_2$ inducing Hamiltonians of WCW play essential role in quantum theory. Also the Hamiltonians at ends of braid strands should have classical counterparts at space-time level. Could braid strand obey Hamiltonian dynamics defined by Hamiltonian attached to it? This would give a constraint to the wormhole throat making itself visible also a properties of the space-time sheet. If so then braid strands would define a kind of the skeleton for the space-time sheet. This idea could be generalized so that one would have a skeleton of space-time consisting of string world sheets and finite measurement resolution would mean the restriction of consideration to this skeleton. Also the braid strands carrying fermion number (other than right handed neutrino number) should obey their own dynamics.

6.2.2 Braided Galois homology as counterpart of Floer homology?

The picture suggested by braided Galois homology seems to have natural correspondences with that provided by Floer homology.

1. The quantum fluctuating degrees of freedom correspond to the symplectic group of $\delta M^4_{\pm} \times CP_2$. Finite measurement resolution leads to the discretization. One considers the subgroup G of symplectic group of $\delta M^4_{\pm} \times CP_2$ permuting a given set of n points of the partonic 2-surface defining the end points of braids. Subgroup of S_n having interpretation as Galois group is in question. The normal subgroup H of symplecto-morphisms leaving these points invariant and the factor group G/H is the target of primary interest and expected to be discrete group. The braiding of this group is intuitively equivalent with the replacement of symplectic transformations with flows and the points can be interpreted as critical points of infinite number of Hamiltonian belonging to H. In Floer's theory one makes a gauge choice selecting a generic non-degenerate Hamiltonian. This choice -or a generalization of it- should have a definite physical meaning in TGD framework in terms of classical correlates for the quantum numbers of the zero energy state.

2. Preferred Hamiltonian acting and its time dependent deformation play a key role in Floer homology and represent homotopy in symplectic group. In the recent case braided Galois homology

assigns to preferred extremals subgroup of symplectic flow in Minkowskian space-time regions and the braid points are invariant under its normal subgroup. The flow defined by time dependent deformation a Hamiltonian of subgroup defines a candidate for the flow defined by preferred Hamiltonian. The connecting flows in turn would correspond to the Galois group. The condition that the flow lines of the Hamilton along 3-surfaces poses a strong condition on the choice of Hamiltonian on one hand and on the preferred extremal on the other hand. The time evolution of Hamiltonian could be realized by the slicing of imbedding space by light-cone boundaries parallel to the lower or upper boundary of CD.

3. For braided Galois homology the generators d_i representing boundary homomorphisms whose square maps to commutator subgroup and to zero after abelianization define candidates for the algebra of SUSY generators. Parameter dependent deformation of these generators would make sense also now and give rise a homology analogous to that of Witten. The generators of the cohomology would correspond to supersymmetric ground states and one would expect that cohomology is non-trivial for the critical points of Morse function. This super-symmetry, which need not have anything to do with the standard notion of supersymmetry, would be assigned to Minkowskian regions of space-time. One cannot of course exclude purely fermionic representations of braided Galois homology and number theoretic quantization of fermions would pose a powerful constraint on the spectrum of fermionic modes.

6.2.3 Kähler function as Kähler action in Euclidian regions and Morse function as Kähler action in Minkowskian regions?

The role of Kähler action in the Floer like aspects of TGD has been already briefly discussed.

1. Symplectic Floer homology for imbedding space gives just the homology groups of $S^2 \times CP_2$. This homology is crucial for the interpretation of TGD but much more detailed information is required. The analog of Floer homology must be associated with WCW for which quantum fluctuating degrees of freedom are parametrized by symplectic group of $\delta M^4_\pm \times CP_2$ or symmetric space associated with it. In finite measurement resolution one would have discrete subgroup defined as a factor group of subgroup permuting braid points and normal subgroup leaving them invariant identifiable in terms of a hierarchy of Galois groups. Flows must be considered in order to have braiding. The flows could also correspond to parameter dependent Hamiltonians with the parameter varying along light-like wormhole throat or space-like 3-surface at the end of CD.

2. In the case of Chern-Simons action the critical points correspond to flat connections and define the generators of the homology for the space of connections. For YM action instanton solutions play similar role. In the recent case the space of 3-surfaces associated with given CD seems to be natural object of study.

   Kähler function -to be distinguished from Kähler action whose value for the preferred extremal defines Kähler function - would be the first guess for the Morse function in WCW and the analog of Floer homology would be formally defined by the sums of the 3-surfaces which correspond to the extrema of Kähler function. This idea fails. Kähler metric must be positive definite. Therefore the Hessian of the Kähler function in holomorphic quantum fluctuating degrees of freedom characterized by complex coordinates of WCW should have only non-negative or non-positive eigen values.

   One could try to circumvent the difficulty by assuming that the allowed extrema with varying signature of Hessian are associated with the zero modes. Therefore the analog of Floer homology based on Kähler function would not however tell anything about symplectic degrees of freedom -at least those assignable to the Euclidian regions.

   Remark: One can wonder how the Kähler function can escape the implications of Morse theorem. In the case of CP_2 the degeneracy of Kähler function -meaning that it depends on single $U(2)$ invariant CP_2 coordinate only - takes care of the problem. Also now infinite-dimensional symmetries of WCW are expected to allow to circumvent the Morse theorem.

3. The only manner to save this idea is that the Euclidian regions defined by the generalized Feynman graphs define Kähler function and Minkowskian regions the analog of the action defining

path integral. The earlier proposed duality states that the formulation TGD is possible either as a functional integral or a path integral. If duality holds true, its effect would be analogous to that of Wick rotation. The alternative approach would assign physical significance to both contributions. The Kähler action in Minkowskian regions could serve as Morse function. This identification is rather natural since the determinant of the induced metric appearing in the action indeed gives imaginary unit in Minkowskian regions. If this were the case interference effects would result already at the level of action and the connection with quantum field theories would be much tighter than previously thought.

Euclidian regions would guarantee the convergence of the functional integral and one would have a mathematically well-defined theory. The analog of Floer homology would represent quantum superpositions of critical points identifiable a ground states defined by the extrema of Kähler action for Minkowskian regions. Perturbative approach to quantum TGD would rely on functional integrals around the extrema of Kähler function.

4. Should one assume that the reduction to Chern-Simons terms occurs for the preferred extremals in *both* Minkowskian and Euclidian regions or only in Minkowskian regions?

   (a) All arguments for this have been represented for Minkowskian regions [8] involve local light-like momentum direction which does not make sense in the Euclidian regions. This does not however kill the argument: one can have non-trivial solutions of Laplacian equation in the region of CP_2 bounded by wormhole throats: for CP_2 itself only covariantly constant right-handed neutrino represents this kind of solution and at the same time supersymmetry. In the general case solutions of Laplacian represent broken super-symmetries and should be in one-one correspondences with the solutions of the modified Dirac equation. The interpretation for the counterparts of momentum and polarization would be in terms of classical representation of color quantum numbers.

   If the reduction occurs in Euclidian regions, it gives in the case of CP_2 two 3-D terms corresponding to two 3-D gluing regions for three coordinate patches needed to define coordinates and spinor connection for CP_2 so that one would have two Chern-Simons terms. Without any other contributions the first term would be identical with that from Minkowskian region apart from imaginary unit. Second Chern-Simons term would be however independent of this. For wormhole contacts the two terms could be assigned with opposite wormhole throats and would be identical with their Minkowskian cousins from imaginary unit. This looks a little bit strange.

   (b) There is however a very delicate issue involved. Quantum classical correspondence requires that the quantum numbers of partonic states must be coded to the space-time geometry, and this is achieved by adding to the action a measurement interaction term which reduces to what is almost a gauge term present only in Chern-Simons-Dirac equation but not at space-time interior [8]. This term would represent a coupling to Poincare quantum numbers at the Minkowskian side and to color and electro-weak quantum numbers at CP_2 side. Therefore the net Chern-Simons contributions and would be different.

   (c) There is also a very beautiful argument stating that Dirac determinant for Chern-Simons-Dirac action equals to Kähler function, which would be lost if Euclidian regions would not obey holography. The argument obviously generalizes and applies to both Morse and Kähler function.

5. The preferred extremal of Kähler action itself would connect 3-surfaces at the opposite boundaries of CD just as the action for Floer theory connects two loops assignable to critical points. In zero energy ontology the unions of 3-surfaces at the ends of CD is the basic unit and correspond to the critical points of Morse function. The question is whether objects can be mapped to a set of critical points of the preferred Hamiltonian in a natural manner. Braided Galois homology with preferred Hamiltonian defining the braids as its flow lines gives hopes about this.

6. In Floer theory the homology of LM is mapped to homology of M. The homology of the WCW cannot be mapped to that of the imbedding space. The hierarchy of Planck constants [7] assigned to the multivalued correspondence between canonical momentum densities of Kähler action and time derivatives of imbedding space coordinates leads to the introduction of singular covering

spaces of the imbedding space with the number of sheets of covering depending on space-time region. The homology of WCW might be mapped homomorphically to the homology of this space.

In the case of loop space $H_0(LM)$ is mapped to $H_1(M)$. Something similar should take place now since all odd homology groups of WCW must vanish if it is Kähler manifold whereas zeroth homology could be non-trivial. In zero energy ontology 3-surfaces having disjoint components at the ends of CD indeed correspond naturally to paths of connected 3-surface so that this condition might be realized.

On basis of these arguments it seems that the general structure of Floer homology fits rather nicely the structure of quantum TGD.

6.2.4 TGD counterparts for pseudo-holomorphic surfaces

If the Morse function exists as Kähler action for preferred extremal in the Minkowskian regions of the space-time, there are good hopes of obtaining the analog of Floer homology in TGD framework. Consider first pseudo-holomorphic surfaces.

1. The analogy with Floer homology would suggest that the analogs of pseudo-holomorphic surfaces assignable to the critical points of Morse function correspond to 3-surfaces at the ends of CD are 3-surface defined by the simultaneous vanishing of two holomorphic rational functions of the complex coordinates of $S^2 \subset \delta M^4_\pm$ and of CP_2 depending parametrically on the light-like radial coordinate of $\delta M^\pm$ giving $7-4=3$ conditions. The effective metric 2-dimensionality implied by the strong form of holography is expected to pose conditions on the radial dependence of these functions.

2. Pseudo-holomorphic closed string world sheets with punctures provide a beautiful geometric realization of quantum cohomology. If positive and negative energy parts of zero energy states can be regarded as elements of homology, space-time sheets could take a similar role. In finite measurement resolution string world sheets would perform the same function so that closed strings would be replaced with open ones as connectors in TGD based quantum cohomology. Signature is not a problem: in string theories the hypercomplex variant of holomorphy is allowed. String world sheets would connect partonic two surfaces at the given end of partonic CD and also at different ends of CD. String world sheets could branch but the mechanism would be the decay of open string creating new partonic 2-surfaces meeting at TGD counterpart of Feynman vertex. Note that also in Witten's approach to Floer theory and Donaldson theory the signature of string world sheets is Minkowskian.

   Remarks:

   (a) One can imagine an extremely simple definition for the intersection for partonic 2-surfaces at opposite boundaries of CD proposed actually earlier. One could identify the opposite boundaries of CD given by pieces $\delta M^4_\pm \times CP_2$ by identifying δM^4_+ and δM^4_- in an obvious manner. This definition is however a natural dynamical counterpart for intersection in classical sense obtained by identifying the boundaries of CD.

   (b) So called massless extremals represent one example about the analogs of right and left moving solutions in TGD framework [2]. They distinguish sharply between classical TGD and Maxwell's hydrodynamics. There are arguments suggesting that quite generally the preferred extremals in Minkowskian regions representable as graphs of maps $M^4 \times CP_2$ decompose to regions characterized by local directions of momentum and polarization representing propagation of massless waves. This would be the classical space-time correlate for the decomposition of radiation to massless quanta.

3. Partonic 2-surfaces with particles at the ends of braid strands would define basic objects and would naturally correspond to holomorphic surfaces for the critical points of Morse function defined by the contribution of Minkowskian regions to Kähler action. The hyper-complex string world sheets and hyper-quaternionicity are however necessary for the $M^4 \times CP_2 - M^8$ correspondence suggested by physics as generalized number theory vision. The finite dimensions of

the moduli spaces would not be a problem since holomorphy would characterize only the critical points. The connection between super-symmetry and cohomology plays a key role in TQFT and pseudo-holomorphy is an excellent candidate for the geometric correlate of supersymmetry of some kind.

The natural question is whether pseudo-holomorphy could generalize in 4-D context to its quaternionic analog.

1. One of the basic conjectures of TGD is that preferred extremals of Kähler action can be regarded as hyper-quaterionic sub-manifolds. The tangent spaces of space-time surfaces would define hyper-quaternionic sub-spaces of complexified octonions with imaginary units of quaternions would be multiplied by commuting imaginary unit.

2. The tangent spaces of space-time surface would also contain a preferred hyper-complex plane or more generally, a hyper-complex plane which depends on position so that these planes integrate to string world sheet. This would allow to regard space-time surfaces either as surfaces in $M^4 \times CP_2$ or in hyper-octonionic subspace M^8 [19]. Integrable distributions of the hyper-complex sub-manifolds would define string world sheets analogous with hypercomplex sub-manifolds. The physical interpretation would be in terms of local preferred planes of un-physical polarizations. The philosophical motivation of hyper-quaternionicity would be that associativity for space-time surfaces and commutativity for string world sheets could define a number theoretical variational principle.

3. The role of pseudo-holomorphy suggests that hyper-quaternionicity could characterize the critical points of Morse function defined by Kähler action in Minkowskian regions of space-time. If all preferred extremals are hyper-quaternionic, this property cannot imply holomorphy of the partonic surfaces.

4. It was already mentioned that finite measurement resolution defines a skeleton of space-time surface realized in terms of string world sheets. This skeleton would generalize a curve of complex plane at which holomorphic function defining a complex coordinate is real to hyper-complex sub-manifold of hyper-quaternionic space-time surface. Given this skeleton, the construction of space-time surface would be analogous to an analytic continuation from hyper-complex realm to hyper-quaternionic realm.

6.2.5 Hierarchy of Planck constants, singular coverings of the imbedding space, and homology of WCW

1. As already noticed, the homology groups of imbedding space are certainly too simple to be of interest from the point of physics and quantum TGD. Physically interesting analogs of homology groups could be associated with the space-time surface itself or with the singular covering of imbedding space allowing to describe the many-valued correspondence between canonical momentum densities and time derivatives of imbedding space coordinates. This would allow to interpret the resulting non-trivial homology as a property of either space-time surface or of effective imbedding space. In any case, one should add to the homology the constraint that the elements of homology are representable as sub-varieties for the preferred extremals of Kähler action. This might allow to code physics using the formalism of homology theory. Floer like theory would also define a homomorphism mapping the homology $H_n(WCW)$ to the homology group H_{m+1} of the singular covering of the imbedding space.

2. The recent interpretation for the effective hierarchy of Planck constants coming as integer multiples of ordinary Planck constants has interpretation in terms of effective coverings of space-time surface implied by the 1-to-many character of the map assigning to canonical momentum densities of Kähler action time derivatives of imbedding space coordinates. The strange sounding proposal is that at partonic two surfaces branching occurs in the sense that the various branchings of the many-valued function involved with this correspondence co-incide. Branching would however occur both in the direction of the light-like 3-surface and space-like 3-surface at the end of CD. Branching could occur at both ends of given CD or only at single end if the branching is taken as a space-time correlate for dissipation and arrow of time, and perhaps even for quantum superposition as will be discussed below.

3. This branching brings in mind the emergence of homologically non-trivial curves from the critical points in Floer cohomology and possibility of several curves connecting two critical points (torus serves as a good illustration also now). The analogy would be more convincing if one could assign to the branches a sign factor analogous to the sign of the eigenvalue of Hessian as physical signature. One possibility is that the sign factor tells whether the line is incoming or outgoing. Also the sign of energy in the case of virtual particles could appear in the sign factor.

6.2.6 How detailed quantum classical correspondence can be?

The gradient dynamics is quite essential for the super-symmetric solutions of Floer theory and typically gradient dynamics is dissipative leading to fixed points of the function function involved. Dissipative dynamics allows to order critical points in terms of the energy defined by Hamilton and also connect different critical points. Physicist would obviously ask whether this aspect of the dynamics is only an artifact of the model or whether it has a much deeper physical significance. If it does not, the following considerations can be taken only as a proposal for how the quantum correlates could be represented at space-time level and how detailed they can be.

Can the dynamics defined by preferred extremals of Kähler action be dissipative in some sense? The generation of the arrow of time has a nice realization in zero energy ontology as a choice of well-defined particle numbers and other quantum numbers at the "lower" end of CD. By quantum classical correspondence this should have a space-time correlate. Gradient dynamics is a highly phenomenological realization of the dissipative dynamics and one must try to identify a microscopic variant of dissipation in terms of entropy growth of some kind. If the arrow of time and dissipation has space-time correlate, there are hopes about the identification of this kind of correlate.

Quantum classical correspondence has been perhaps the most useful guiding principle in the construction of quantum TGD. What is says that not only quantum numbers but also quantum jump sequences should have space-time correlates: about this the failure of strict determinism of Kähler action gives good hopes. Even the quantum superposition- at least for certain situations -might have space-time correlates.

1. Measurement interaction term in the modified Dirac action at the upper end of CD indeed defines a coupling to the classical dynamics [8] in a very delicate manner. This kind of measurement interaction is indeed basic element of quantum TGD. Also the color and charges and angular momentum associated with the Hamiltonians at point of braids could couple to the dynamics via the boundary conditions.

2. The braid strand with a given Hamiltonian could obey Hamiltonian equations of motion: this would give rise to a skeleton of space-time defined by braid strands possibly continued to string world sheets and would provided different realization of quantum classical correspondence.

3. Quantum TGD can be regarded as a square root of thermodynamics in well-defined sense. Could it be possible to couple the Hermitian square root of density matrix appearing in M-matrix and characterizing zero energy state thermally to the geometry of space-time sheets by coupling it to the classical dynamical via boundary conditions depending on its eigenvalues? The necessity to choose single eigenvalue spoils the attempt and one obtains only a representation for single measurement outcome. It seems that one can achieve only a representation of the ensemble at space-time level consisting of space-time sheets representing various outcomes of measurement. This ensemble would be realized as ensemble of sub-CDs for a given CD.

4. One can pose even more ambigous question: could quantum superposition of WCW spinor fields have a space-time correlate in the sense that all space-time surfaces in the superposition would carry information about the superposition itself? Obviously this would mean self-referentiality via quantum-classical feedback.

The following discussion concentrates on possible space-time correlates for the quantum superposition of WCW spinor fields and for the arrow of time.

1. It seems difficult to imagine space-time correlate for the quantum superposition of final states with varying quantum numbers since these states correspond to quantum superpositions of different space-time surfaces. How could one code information about quantum superposition of

space-time surfaces to the space-time surfaces appearing in the superposition? This kind of self-referentiality seems to be necessary if one requires that various quantum numbers characterizing the superposition (say momentum) couple via boundary conditions to the space-time dynamics.

2. The failure of non-determinism of quantum dynamics is behind dissipation and strict determinism fails for Kähler action. This gives hopes that the dynamics induces also arrow of time. Energy non-conservation is of course excluded and one should be able to identify a measure of entropy and the analog of second law of thermodynamics telling what happens at for preferred extremals when the situation becomes non-deterministic. The vertices of generalized Feynman graphs are natural places were non-determinism emerges as are also sub-CDs. Naive physical intuition would suggest that dissipation means generation of entropy: the vertices would favor decay of particles rather than their spontaneous assembly. The analog of blackhole entropy assignable to partonic 2-surfaces might allow to characterize this quantatively. The symplectic area of partonic 2-surface could be a symplectic invariant of this kind.

3. Could the mysterious branching of partonic 2-surfaces -obviously analogous to even more mysterious branching of quantum state in many worlds interpretation of quantum mechanics- assigned to the multivalued character of the correspondence between canonical momentum densities and time derivatives of H coordinates allow to understand how the arrow of time is represented at space-time level?

    (a) This branching would effectively replace CD with its singular covering with number of branches depending on space-time region. The relative homology with respect to the upper boundary of CD (so that the branches of the trees would effectively meet there) could define the analog of Floer homology with various paths defined by the orbits of partonic 2-surfaces along lines of generalize Feynman diagram defining the first homology group. Typically tree like structures would be involved with the ends of the tree at the upper boundary of CD effectively identified.

    (b) This branching could serve as a representation for the branching of quantum state to a superposition of eigenstates of measured quantum observables. If this is the case, the various branches to which partonic 2-surface decays at partonic 2-surface would more or less relate to quantum superposition of final states in particle reaction. The number of branches would be finite by finite measurement resolution. For a given choice of the arrow of geometric time the partonic surface would not fuse back at the upper end of CD.

    (c) Rather paradoxically, the space-time correlate for the dissipation would reduce the dissipation by increasing the effective value of $\hbar$: the interpretation would be however in terms of dark matter identified in terms of large $\hbar$ phase. In the same manner dissipation would be accompanied by evolution since the increase of $\hbar$ naturally implies formation of macroscopically quantum coherent states. The space-time representation of dissipation would compensate the increase of entropy at the ensemble level.

    (d) The geometric representation of quantum superposition might take place only in the intersection of real and p-adic worlds and have interpretation in terms of cognitive representations. In the intersection one can also have a generalization of second law [13] in which the generation of genuine negentropy in some space-time regions via the build up of cognitive representation compensated by the generation of entropy at other space-time regions. The entropy generating behavior of living matter conforms with this modification of the second law. The negentropy measure in question relies on the replacement of logarithms of probabilities with logarithms of their p-adic norms and works for rational probabilities and also their algebraic variants for finite-dimensional algebraic extensions of rationals.

    (e) Each state in the superposition of WCW quantum states would contain this representation as its space-time correlate realizing self-referentiality at quantum level in the intersection of real and p-adic worlds. Also the state function reduced members of ensemble could contain this cognitive representation at space-time level. Essentially quantum memory making possible self-referential linguistic representation of quantum state in terms of space-time geometry and topology would be in question. The formulas written by mathematicians would define similar map from quantum level to the space-time level making possible to "see" one's thoughts.

7 Could Gromov-Witten invariants and braided Galois homology together allow to construct WCW spinor fields?

The challenge of TGD is to understand the structure of WCW spinor fields both in the zero modes which correspond to symplectically invariant degrees of freedom not contributing to the WCW Kähler metric and in quantum fluctuating degrees of freedom parametrized by the symplectic group of $\delta M^4_{\pm} \times CP_2$. The following arguments suggest that an appropriate generalization of Gromov-Witten invariants to covariants combined with braid Galois homology could allow do construct WCW spinor fields and at the same time M-matrices defining the rows of the unitary U-matrix between zero energy states.

7.1 Gromov-Witten invariants

Gromov-Witten invariants [15] are rational numbers $GW^{X,A}_{g,n}$, which in a loose sense count the number of pseudo-holomorphic curves of genus g and n marked points and homology equivalence class A in symplectic space X meeting n surfaces of X with given homology equivalence classes. These invariants can distinguish between different symplectic manifolds. Since also the proposed generalized homology groups would define symplectic invariants if the realization of braided Galois groups as symplectic flows works, the attempt to understand the relation of Gromov-Witten invariants of TGD is well-motivated.

Let X be a symplectic manifold with almost complex structure J (the transition functions are not holomorphic) and C be an algebraic variety in X of genus g and with complex structure j having n marked points $x_1,...x_n$, which are points of X. Pseudo-homolomorphic maps of C to X are by definition maps, whose Jacobian map commutes with the multiplication of the tangent space vectors with the antisymmetric tensor representing imaginary unit $J \circ df = df \circ j$. If the symplectic manifold allows Kähler structure, one can say that pseudohomolomorphic maps commute with the multiplication by imaginary unit so that tangent plane of complex 2-manifold is mapped to a complex tangent plane of X.

The moduli space $M_{g,n}(X)$ of the pseudoholomorphic maps is finite-dimensional. One considers also its subspaces $M_{g,n}(X,A)$ of $M_{g,n}(X)$, where A represents a fixed homology equivalence class A for the image of C in X. The so called evaluation map from $M_{g,n}(X,A)$ to $M_{g,n}(X)) \times X^n$ defined by $(C, x_1, x_2, ...x_n, f) \rightarrow (st(C, x_1, x_2, ...x_n); f(x_1), ..., f(x_n))$. Here $st(C, x_1, x_2, ...x_n)$ denotes so called stabilization of $(C, x_1, ...x_n)$ defined in the following manner. A smooth component of Riemann surface is said to be stable if the number of automorphisms (conformal transformations) leaving the marked and nodal (double) points invariant is finite. Stabilization is obtained by dropping away the unstable components from the domain of C.

The image of the fundamental class of the moduli space $M_{g,n}(X)$ defines a homology class in $M_{g,n}(X)) \times X^n$. Since the homology groups of $M_{g,n}(X)) \times X^n$ are by Künneth theorem expressible as convolutions of homology groups of $M_{g,n}(X)$ and n copies of X, this homology class can be expressed as a sum

$$\sum_{\beta,\alpha_i} GW^{X,A}_{g,n} \beta \times \alpha_1 ... \times \alpha_n \ .$$

The coefficients, which in the general case are rational valued, define Gromov-Witten invariants. One can roughly say that these rational numbers count the number of surfaces C intersecting the n homology classes α_i of X. n surfaces intersect when there is a surface of genus g with n marked points intersection the surfaces at marked points and Gromov-Witten invariant counts the number of homologically non-equivalent pseudo-holomorphic 2-surfaces of this kind [28].

Branes connected by closed strings would represent a basic example about quantum intersections. Also in Floer homology [55] and quantum cohomology [28] this kind of fuzzy intersection is encountered. The fundamental Gromov-coeffiecients $W(\alpha, \beta, \gamma)$ are for three homology generators α, β, γ and connecting surface correspond to pseudo-holomorphic spheres (or higher genus surfaces) with three marked points obtained by contracting the outgoing three strings of stringy trouser vertex to point.

7.2 Gromov-Witten invariants and topological string theory of type A

Gromow-Witten invariants appear in topological string theory of type A [37] for which the scattering amplitudes depend on Kähler structure of X only. The target space X of this theory is 6-dimensional symplectic manifold. X can correspond to 6-dimensional Calabi-Yau manifold. Twistor space is one particular example of this kind of manifold and one can indeed relate twistor amplitudes to those of topological string theory in twistor space.

Type A topological string theory contains both fundamental string orbits, which are 2-surfaces wrapping over 2-real-D homorphic curves in X and D2 branes, whose 3-D "orbits" in X wrap over Lagrangian manifolds having by definition a vanishing induced symplectic form. There are also strings connecting the branes. C corresponds now to the world sheet of string with n marked points representing emitted particles. Gromov-Witten invariants are defined as integrals over the moduli spaces $M_{g,n}(X)$ and provide a rigorous definition for path integral and the free energy at given genus g is the generating function for Gromov-Witten invariants.

Witten introduced the formulation of the topological string theories in terms of topological sigma models [36]. The formulation involves the analog of BRST symmetry encountered in gauge fixing meaning that one replaces target space with super-space by assigning to target space-coordinates anticommutating partners which do not however represent genuine fermionic degrees of freedom. One also replaces string world sheet with a super-manifold $\mathcal{N} = (2,2)$ SUSY and spinors are world sheet spinors and Lorentz transformations act on string world sheet. Topological string models are characterized by continuous R-symmetries and the mixing of rotational and R-symmetries takes place. The R-symmetry associated with 2-D world sheet Lorentz transformation compensates for the spin rotation so that one indeed obtains a BRST charge Q (for elementary introduction to BRST symmetry see [13]), which is scalar and the condition $Q^2 = 0$ is satisfied identically so that cohomology is obtained.

7.3 Gromov-Witten invariants and WCW spinor fields in zero mode degrees of freedom

One can ask whether Gromow-Witten invariants of something more general could emerge naturally in TGD framework.

1. Gromov-Witten invariants modified so that closed string orbits are replaced by open string world sheets with boundaries identifiable as braid strands relate to the braided Galois homology. Both the geometric interpretation these invariants in terms of fuzzy quantum intersection induced by connecting string world sheets and the discussion of the Floer homology like aspects of quantum TGD support this idea.

2. Another interpretation is that Gromov-Witten invariants or their generalizations emerge in the construction of WCW spinor fields in zero mode degrees of freedom, which do not contribute to the line element of WCW Kähler metric. Contrary to the first hopes there is no convincing support for this view.

7.3.1 Comparison of the basic geometric frameworks

The basic geometric frameworks are sufficiently similar to encourage the idea that Gromov-Witten type invariants might make sense in TGD framework.

1. In the standard formulation of TGD the 6-dimensional symplectic manifold is replaced with the metrically 6-dimensional manifold $\delta M^4_\pm \times CP_2$ having degenerate symplectic and Kähler structure and reducing effectively (metrically) to the symplectic manifold $S^2 \times CP_2$. Partonic 2-surfaces at the light-like boundaries of CD identifiable as wormhole throats define the counterparts of fundamental string like object of topological string theory of type A. The n marked points of Gromov-Witten theory could correspond to the ends of braid strands carrying purely bosonic quantum numbers characterized by the attached $\delta M^4_\pm \times CP_2$ Hamiltonians with well defined angular momentum and color quantum numbers. One must distinguish these braid strands from the braid strands carrying fermion quantum numbers.

2. There are also differences. One assigns 3-D surfaces to the boundaries of CD and partonic 2-surfaces at CD are connected with are interpreted as strings so that partonic 2-surfaces have also brane like character. One can identify 3-D surfaces for which induced Kähler forms of CP_2 and $\delta M^4_\pm$ vanish (any surface with 1-D projection to $\delta M^4_\pm$ and 2-D CP_2 projection with Lagrangian

manifold would define counterpart of brane) but it is not natural to raise these objects to a special role.

3. I have proposed that quantum TGD is analogous to a physical analog of Turing machine in the sense that the inclusions of HFFs could allow to emulate any QFT with almost gauge group assignable to the included algebra [7]. The representation of these gauge groups as subgroups of symplectic transformations leaving the marked points of the partonic 2-surfaces invariant gives hopes of realizing this idea mathematically. Symplectic groups are indeed completely exceptional because of their representative power [35] and used already in classical mechanics and field theory to represent symmetries. An interesting question is whether the symplectic group associated with $\delta M^4_\pm \times CP_2$ could be universal in the sense that any gauge group of this kind allows a faithful homomorphism to this group.

One should understand what pseudo-holomorphy means in TGD framework. One must consider both the identification of pseudo-holomorphic surfaces as string world sheets or as partonic 2-surfaces. Consider first the interpretation of pseudo-holomorphic 2-surfaces as string world sheets assignable to the space-time sheets.

1. String world sheets would not represent closed strings and their ends would define braid strands at light-like 3-surfaces and at the space-like 3-surfaces defining the ends of space-time. This is not a problem: also the standard picture about pseudo-holomorphic surfaces as spheres with punctures is obtained by idealizing the holes of closed string with punctures [55]. Open string world sheet be seen as a string containing holes defined by the boundary braid strands. Disjoint partonic two surfaces at the ends of braid strands would intersect in quantum sense. The interpretation for the fuzzy intersection would be in terms of causal dependence of the quantum state at the ends of CD so that the assignment of Gromov-Witten invariants to them would be natural.

2. This option looks very natural from TGD point of view since the moduli space is expected to be finite-dimensional and have interpretation in terms of the preferred extremal property. For a given partonic 2-surfaces and tangent space data at them the moduli would be fixed more or less uniquely and the variation of the tangent space data would vary the moduli.

Also the identification of pseudo-holomorphic surfaces as partonic 2-surfaces can be considered. It would apparently conform with the canonical identification of pseudo-holomorphic surfaces but the interpretation as connectors in fuzzy cup product can be challenged.

1. Since the moduli space of pseudo-holomorphic surfaces is finite-dimensional, only a very restricted set of partonic 2-surfaces satisfies pseudo-holomorphy condition. The induced metric of the partonic 2-surface defines a unique complex structure. Pseudo-holomorphy states that Jacobian takes the complex tangent place of partonic 2-surface to a comlex plane of the tangent space of $\delta M^4_\pm \times CP_2$. Pseudo-holomorphy is implied by holomorphy stating that both CP_2 coordinates and S^2 coordinates as functions of the complex coordinate of the partonic 2-surface are holomorphic functions implying that the induced metric as the standard $ds^2 = g_{z\overline{z}} dz d\overline{z}$. Holomorphy is also implied if one can express as a variety using functions which are holomorphic functions of $\delta M^4_\pm$ and CP_2 complex coordinates and analytic functions of the radial coordinate r. These surfaces are characterized by the homology-equivalence classes of their projections in $\delta M^4_\pm$ (3-D Euclidian space with puncture at origin) and in CP_2. Both are characterized by integer. These surfaces obviously define a subset of partonic 2-surfaces and one can actually assign to the string-like objects as cartesian products of string world sheets satisfying minimal surface equations and of 2-D complex sub-manifolds of CP_2.

2. The first objection is that partonic two-surfaces do not represent time-evolution so punctures associated with them cannot be regarded as causally dependent. From physics point of view it does not make sense to speak about fuzzy intersection except in terms of finite measurement resolution implying that second quantized induced spinor fields have finite number of modes so that they do not anticommute at partonic 2-surfaces anymore.

3. Second objection is that there is nothing physically interesting that partonic 2-surfaces could connect!

4. The third counter argument is that pseudo-holomorphy condition allows only finite-dimensional moduli space whereas the space of partonic 2-surfaces is infinite-dimensional. Two explanations suggest itself.

   (a) The finite-measurement resolution might imply an effective reduction of the space of partonic 2-surfaces to this moduli space? Finite measurement resolution could be understood also as a kind of gauge invariance when realized in terms of inclusion of hyper-finite factors of type II_1 (HFFs) with the action of sub-factor having no effect on its observable properties. Holomorphy would serve as a gauge fixing condition.

   (b) If TGD as almost topological QFT can be formulated as an analog of Floer's theory relying on action principle, the natural proposal is that holomorphic partonic 2-surfaces correspond to critical values for the Kähler action assignable to the Minkowskian regions of the preferred extremal.

It seems relatively safe to conclude that only the string world sheets have a natural interpretation as connectors the deformed interwection product in TGD framework.

7.3.2 Could an analog of topological string theory make sense in TGD framework

The observations of previous paragraphs motivate the question whether an analog of type A topological string theory could emerge in the construction of WCW spinor fields. The basic problem is to understand how the WCW spinor fields depend on symplectic invariants, which however need not correspond to zero modes which should be expressible in terms of symplectic fluxes alone. One might hope that topological string theory of some kind could allow to construct this kind of symplectic invariants.

1. The encouraging symptom is that the n-point functions of both A and B type topological string theories are non-trivial only in dimension $D = 6$, which is the metric dimension of $\delta M^4_\pm \times CP_2$. Since the n-point functions of type A topological string theory depend only on the Kähler structure associated now by CP_2 and $\delta M^4_\pm$ Kähler forms they could code for the physics associated with the zero modes representing non-quantum fluctuating degrees of freedom. Since type B topological string theory requires vanishing of the first Chern class implying Calabi-Yau property, this theory is not possible in the standard formulation of TGD.

   The emergence of the topological string theory of type A seems to be in conflict with what twistorialization suggests. Witten suggested in his classic article [17] boosting the twistor revolution, that the Fourier transforms of the sattering amplitudes from momentum space to twistor space scattering amplitudes for perturbative $\mathcal{N} = 4$ SUSY could be interpreted in terms of D-instanton expansion of topological string theory of type B defined in twistor space CP_3. Twistorial considerations however led to a proposal [21] that TGD allows formulation also in terms of 6-dimensional surfaces in $CP_3 \times CP_3$, which are sphere bundles. CP_3 is a Calabi-Yau manifold and the natural question is whether the analog of topological string theory of type B might emerge in this formulation. The counterpart of the mirror symmetry relating A and B type models for different Calabi-Yau models would relate the two formulations of quantum TGD.

2. One can identify the marked points as the end points of both space-like and time-like braids but it is not natural to assign them fermionic quantum numbers except those of covariantly constant right-handed neutrino spinor with the points of symplectic triangulation. This is well-motivated since symplectic algebra extends to super-symplectic algebra with covariantly constant right handed neutrino spinor defining the super-symmetry. One can assign the values of Hamiltonians of $\delta M^4_\pm \times CP_2$ to the marked points belonging to the irreducible representations of rotation group and color group such that the total quantum numbers vanish by the symplectic invariance. n-point functions would be correlation functions for Hamiltonians. In a well-defined sense one would have color and angular momentum confinement in WCW degrees of freedom.

   The vanishing of net quantum numbers need not hold true for single connected partonic 2-surface. Also it could hold true only for a collection of partonic 2-surfaces associated with same 3-surface at either end of CD. The most general condition would be that the total color and spin numbers of positive and negative energy parts of the state sum up to zero in symplectic degrees of freedom.

3. The generating function for Gromov-Witten invariants is defined for a connected pseudo-holomorphic 2-surface with a fixed genus g as such is not general enough if one allows partonic 2-surfaces with several components. The generalization would provide information about the preferred extremal of Kähler action and about the topology of space-time surface. The generalization of the Gromov-Witten partition function would define as its inverse the normalization factor for zero energy state identifiable as M-matrix defined as a positive diagonal square root of density matrix multiplied by S-matrix for which initial partons possess fixed genus and which contains superposition over braids with arbitrary number of strands. The intuition from ordinary thermodynamics suggests that this partitition function is in a reasonable approximation expressible as convolution for n-points functions for individual partonic 2-surfaces allowing the set of marked points to carry net $\delta M^4_\pm$ angular momentum and color quantum numbers.

7.3.3 Description of super-symmetries in TGD framework

It is interesting to see whether the formulation of super-symmetries in the framework of topological sigma models giving rise to Gromov-Witten invariants [36] has any reasonable relation to TGD where the notion of super-space does not look natural as a fundamental notion although it might be very useful as a formal tool in the formulation of SUSY QFT limit [9] and even quantum TGD itself.

1. Almost topological QFT property means that Kähler action for the preferred extremals reduces to Chern-Simons action assuming the weak form of electric magnetic duality. In the fermionic sector one must use modified gamma matrices defined as contractions of the canonical momentum densities for Kähler action (Kähler-Chern-Simon action) with imbedding space gamma matrices in the counterpart of Dirac action in the interior of space-time sheet and at 3-D wormhole throats. The modified gamma matrices define effective metric quadratic in canonical momentum densities which is typically highly degenerate. It contains information about the induced metric. Therefore one cannot expect that topological sigma model approach could work as such in TGD framework.

2. In TGD framework supersymmetries are generated by right-handed covariantly constant neutrinos and antineutrinos with both spin directions. These spinors are imbedding space spinors rather than world sheet spinors but one can say that the induction of the spinor structure makes them world sheet spinors. Since the momentum of the spinors is vanishing, one can assign all possible spin directions to the neutrinos.

3. Covariantly constant right-handed neutrino and antineutrino can have all possible spin directions and for fixed choice of quantization axes two spin directions are possible. Therefore one could say that rotation group acts as non-Abelian group of R-symmetries. TGD formulation need not be based on sigma model so that it is not all clear whether a twisted Lorenz transformations are needed. If so, the most obvious guess is that space-time rotations are accompanied by R-symmetry rotation of right-handed neutrino spinors compensating the ordinary rotation it as in the case of topological sigma model originally introduced by Witten.

It is interesting to look the situation also from the point of view of the breaking of SUSY for supergravity defined in dimension 8 by using the table listing super-gravities in various dimensions [2].

1. One can assign to the causal diamond a fixed direction as a WCW correlate for the fixing of spin quantization axis and this direction corresponds to a particular modulus. The preferred time directiond defined by the line connecting the tips of CD and this direction define a plane of non-physical polarizations having in number theoretical approach as a preferred hypercomplex plane of hyper-octonions [19]. Hence it would seem that by the symmetry breaking by the choice of quantization axes allows only two spin directions the right handed neutrino and antineutrino and that different choices of the quantization axes correspond to different values for the moduli space of CDs.

2. Since imbedding space spinors are involved, the sugra counterpart of TGD is $\mathcal{N} = 2$ super gravity in dimension 8 for which super charges are Dirac spinors and their hermitian conjugates with $U(2)$ acting as R-symmetries. Note that the supersymmetry does not require Majorana spinors unlike $N = 1$ supersymmetry does in string model and fixes the target space dimension to $D = 10$ or $D = 11$. Just like $D = 11$ of M-theory is the unique maximal dimension if one requires fundamental Majorana spinors (for which there is no empirical support), $D = 8$ of TGD is the unique maximal dimension if one allows only Dirac spinors.

3. In dimensional reduction to $D = 6$, which is the metric dimension of the boundary of δCD a breaking of $\mathcal{N} = 8$ sugra $\mathcal{N} = (2, 2)$ sugra occurs, and one obtains decomposition into pseudoreal representations with supercharges in representations (4,0) and (0,4) of $R = Sp(2) \times Sp(2)$ ($Sp(2) = Sl(2, R)$ corresponds to 2-D symplectic transformations identifiable also as Lorentz group SO(1,2)). (4,0) and (0,4) could correspond to left and right handed neutrinos with both directions of helicities and thus potentially massive. CP_2 geometry breaks this supersymmetry.

4. The reduction to the level of right handed neutrinos requires a further symmetry breaking and $D = 5$ sugra indeed contains supercharges Q and their conjugates in 4-D pseudoreal representation of $R = Sp(4)$. Note that this group corresponds to 2×2 quaternionic matrices. A possible interpretation would be as a reduction in CP_2 degrees freedom to $U(2) \times U(1)$ invariant sphere.

5. The R-symmetries mixing neutrinos and antineutrinos are pysically questionable so that a breaking of R-symmetry to $Sp(2)\times Sp(2)$ to $SU(2)\times SU(2)$ or even $SU(2)$ should take place. A further reduction to homologically non-trivial geodesic sphere of CP_2 might reduce the action of $CP_2(2)$ holonomies to those generated by electric charge and weak isospin and thus leaving right-handed neutrinos invariant. Fixing the quantization axis of spin would reduce R-symmetry to $U(1)$. The inverse imaged of this geodesic sphere is identified as string world sheet [10].

7.3.4 How braided Galois homology and Gromov-Witten type homology and WCW spinor fields could relate?

One can distinguish between WCW "orbital" degrees of freedom and fermionic degrees of freedom and in the case of WCW degrees of freedom also between zero modes expressible in terms of Kähler fluxes and quantum fluctuating degrees of freedom expressible using wave functions in symplectic group.

1. Quantum fluctuating degrees of freedom

As far as quantum number are considered, quantum fluctuating degrees of freedom correspond to the symplectic algebra in the basis defined by Hamiltonians belonging to the irreps of rotation group and color group.

1. At the level of partonic 2-surfaces finite measurement resolution leads to discretization in terms of braid ends and symplectic triangulation. At the level of WCW discretization replaces symplectic group with its discrete subgroup. This discrete subgroup must result as a coset space defined by the subgroup of symplectic group acting as Galois group in the set of braid points and its normal subgroup leaving them invariant. The group algebra of this discrete subgroup of symplectic group would have interpretation in terms of braided Galois cohomology. This picture provides an elegant realization for finite measurement resolutions and there is also a connection with the realization of finite measurement resolution using categorification [43], [3].

2. The proposed generalized homology theory involving braided Galois group and symplectic group of $\delta M^4_\pm \times CP_2$ would realize the "almost" in TGD as almost topological QFT in finite measurement resolution replacing symplectic group with its discretized version. This algebra would relate to the quantum fluctuating degrees of freedom. The braids would carry only fermion number and there would be no Hamiltonians attached with them. The braided Galois homology could define in the more general situation invariants of symplectic isotopies.

3. The generalization of Gromov-Witten invariants to n-point functions defined by Hamiltonians of $\delta M^4_\pm \times CP_2$ are symplectic invariants if net $\delta M^4_\pm \times CP_2$ quantum numbers vanish. As As a special case one obtains Gromove-Witten invariants. The most general definition assumes that the vanishing of quantum numbers occurs only for zero energy states having disjoint unions

of partonic 2-surfaces at the boundaries of CDs as geometric correlate. Since Hamiltonians correspond to quantum fluctuating degrees of freedom the interpretation in terms of zero modes is not not possible. The comparison of Floer homology with quantum TGD encourages to think that the generalizations of Gromov-Witten invariants can be assigned to the braided Galois homology.

4. One should also add four-momenta and twistors to this picture. The separation of dynamical fermionic and sup-symplectic degrees of freedom suggesets that the Fourier transforms for amplitudes containing the fermionic braid end points as arguments define twistorial amplitudes. The representations of light-like momenta using twistors would lead to a generalization of the twistor formalism. At zero momentum limit one would obtain symplectic QFT with states characterized by collections of Hamiltonians and their super-counterparts.

2. Zero modes

WCW spinor field depends also on zero modes and the challenge is to identify the appropriate variables coding for this information in accordance with quantum classical correspondence. The best that one could achieve would be a basis for the parts of WCW spinor fields in these degrees of freedom. Zero modes correspond essentially to the non-local symplectic invariants assignable to the projections of the $\delta M^4_{\pm}$ and CP_2 symplectic forms to the space-time surface and expressible in terms of symplectic fluxes only. The appropriate symplectic fluxes should be determined by the information about the quantum state in quantum fluctuating degrees of freedom by quantum classical correspondence.

1. The exponent of Kähler action for preferred extremal- by above proposal real in Euclidian regions and imaginary in Minkowskian regions and reducing to Chern-Simons action at both sides - contains also information about zero modes and would code implicitly the vacuum functional in zero modes. What would be needed is an explicit representation for this part of vacuum functional. The identification of zero modes as classical variables requires entanglement between zero modes and quantum fluctuating degrees of freedom and one-one correspondence analogous to that between the states of the measurement apparatus and the outcome of quantum measurement is expected. This duality would express quantum holography and quantum classical correspondence crucial for quantum measurement theory.

2. Could the generating function for appropriately generalized Gromow-Witten invariants define a candidate for what might be regarded as a vacuum functional in zero modes separating into a factor in WCW spinor field? The first thing to notice is that symplectic invariance is not equivalent with zero mode property. In Floer homology there is a preferred Hamiltonian interpreted in TGD framework in terms of the braiding defining braided Galois homology. Neither Floer homology, Gromov-Witten invariants nor braided Galois homology do depend on the details of the Hamiltonian. Does this mean that the TGD counterparts of Gromov-Witten invariants might could be interpreted as zero modes and generating function for these invariants as vacuum functional in zero modes? Or does the fact that Hamiltonian flow is involved mean that information about quantum fluctuating degrees of freedom is present?

Symplectic QFT [3] provides a more promising approach to the description of zero modes in terms of symplectic fluxes.

1. The earlier proposal [3] for symplectic QFT defined as a generalization of conformal QFT coding for these degrees of freedom assigns to the partonic 2-surface collections of marked points defining its division to 2-polygons carrying Kähler magnetic flux together with the signed area defined by R^3_+ symplectic form (essentially solid angle assignable to partonic 2-surface or its portion with respect to the tip of light-cone). A given assignment of marked points defines symplectic fusion algebra and these algebras integrate to an operad with a product defined by the product of fusion algebras.

2. Symplectic triangulation would define symplectic invariants. The nodes of the symplectic triangulation could be identified as the ends of braid strands assignable to string world sheets. If the information about quantum state can be used to fix the edges of the triangulation, the phases defined by the fluxes associated with the triangles define physically interesting symplectic

invariants. If one assumes that each Hamiltonian assignable to the partonic 2-surface defines its own symplectic triangulation, the Hamiltonian equations associated with the Hamiltonian would naturally define the edges of the triangulation. Symplectic triangulation would characterize a Bose-Einstein condensate like state assignable to single Hamiltonian. The total magnetic flux for the triangulation would characterize the Hamiltonian. If only single Hamiltonian is involved the orbit should be a closed orbit connecting the node to itself and also now could assign to it a symplectic area.

3. Symplectic triangulation would add additional pieces to the proposed skeleton of the space-time surface. If the symplectic triangulation can be continued from partonic 2-surfaces to the interior of space-time in both time and spatial direction it would provide space-time with a web string world sheets connected by sheets assignable to the edges of the symplectic triangulation.

8 K-theory, branes, and TGD

K-theory is an essential part of the motivic cohomology. Unfortunately, this theory is very abstract and the articles written by mathematicians are usually incomprehensible for a physicist. Hence the best manner to learn K-theory is to learn about its physics applications. The most important applications are brane classification in super string models and M-theory. The excellent lectures by Harah Evslin with title *What doesn't K-theory classify?* [9] make it possible to learn the basic motivations for the classification, what kind of classifications are possible, and what are the failures. Also the Wikipedia article [1] gives a bird's eye of view about problems. As a by-product one learns something about the basic ideas of K-theory.

In the sequel I will discuss critically the basic assumptions of brane world scenario, sum up my understanding about the problems related to the topological classification of branes and also to the notion itself, ask what goes wrong with branes and demonstrate how the problems are avoided in TGD framework, and conclude with a proposal for a natural generalization of K-theory to include also the division of bundles inspired by the generalization of Feynman diagrammatics in quantum TGD, by zero energy ontology, and by the notion of finite measurement resolution.

8.1 Brane world scenario

The brane world scenario looks attractive from the mathematical point of view ine one is able to get accustomed with the idea that basic geometric objects have varying dimensions. Even accepting the varying dimensions, the basic physical assumptions behind this scenario are vulnerable to criticism.

1. Branes are geometric objects of varying dimension in the 10-/11-dimensional space-time -call it M- of superstring theory/M-theory. In M-theory the fundamental strings are replaced with M-branes, which are 2-D membranes with 3-dimensional orbit having as its magnetic dual 6-D M5-brane. Branes are thought to emerge non-perturbatively from fundamental 2-branes but what this really means is not understood. One has D-p-branes with Dirichlet boundary conditions fixing a $p+1$-dimensional surface of M as brane orbit: one of the dimensions corresponds to time. Also S-branes localized in time have been proposed.

2. In the description of the classical limit branes interact with the classical fields of the target space by the generalization of the minimal coupling of charged point-like particle to electromagnetic gauge potential. The coupling is simply the integral of the gauge potential over the world-line - the value of 1-form for the wordline. Point like particle represents 0-brane and in the case of p-brane the generalization is obtained by replacing the gauge potential represented by a 1-from with $p+1$-form. The exterior derivative of this $p+1$-form is $p+2$-form representing the analog of electromagnetic field. Complete dimensional democracy strongly suggests that string world sheets should be regarded as 1-branes.

3. From TGD point of view the introduction of branes looks a rather ad hoc trick. By generalizing the coupling of electromagnetic gauge potential to the word line of point like particle one could introduce extended objects of various dimensions also in the ordinary 4-D Maxwell theory but they would be always interpreted as idealizations for the carriers of 4- currents. Therefore the

crucial step leading to branes involves classical idealization in conflict with Uncertainty Principle and the genuine quantal description in terms of fields coupled to gauge potentials.

My view is that the most natural interpretation for what is behind branes is in terms of currents in D=10 or D= 11 space-time. In this scheme branes have role only as semi-classical idealizations making sense only above some scale. Both the reduction of string theories to quantum field theories by holography and the dynamical character of the metric of the target space conforms with super-gravity interpretation. Internal consistency requires also the identification of strings as branes so that superstring theories and M-theory would reduce to an idealization to 10-/11-dimensional quantum gravity.

In this framework the brave brane world episode would have been a very useful Odysseia. The possibility to interpret various geometric objects physically has proved to be an extremely powerful tool for building provable conjectures and has produced lots of immensely beautiful mathematics. As a fundamental theory this kind of approach does not look convincing to me.

8.2 The basic challenge: classify the conserved brane charges associated with branes

One can of course forget these critical arguments and look whether this general picture works. The first thing that one can do is to classify the branes topologically. I made the same question about 32 years ago in TGD framework: I thought that cobordism for 3-manifolds might give highly interesting topological conservation laws. I was disappointed. The results of Thom's classical article about manifold cobordism demonstrated that there is no hope for really interesting conservation laws. The assumption of Lorentz cobordism meaning the existence of global time-like vector field would make the situation more interesting but this condition looked too strong and I could not see a real justification for it. In generalized Feynman diagrammatics there is no need for this kind of condition.

There are many alternative approaches to the classification problem. One can use homotopy, homology, cohomology and their relative and other variants, topological or algebraic K-theory, twisted K-theory, and variants of K-theory not yet existing but to be proposed within next years. The list is probably endless unless something like motivic cohomology brings in enlightment.

1. First of all one must decide whether one classifies p-dimensional time=constant sections of p-branes or their $p+1$-dimensional orbits. Both approaches have been applied although the first one is natural in the standard view about spontaneous compactification. For the first option topological invariants could be seen as conserved charges: homotopy invariants and homological and cohomological characteristics of branes provide this kind of invariants. For the latter option the invariants would be analogous to instanton number characterizing the change of magnetic charge.

2. Purely topological invariants come first in mind. Homotopy groups of the brane are invariants inherent to the brane (the brane topology can however change). Homological and cohomological characteristics of branes in singular homology characterize the imbedding to the target space. There are also more delicate differential topological invariants such as de Rham cohomology defining invariants analogous to magnetic charges. Dolbeault cohomology emerges naturally for even-dimensional branes with complex structure.

3. Gauge theories - both abelian and non-Abelian - define a standard approach to the construction of brane charges for the bundle structures assigned with branes. Chern-Simons classes are fundamental invariants of this kind. Also more delicate invariants associated with gauge potentials can be considered. Chern-Simons theory with vanishing field strengths for solutions of field equations provides a basic example about this. For intance, SU(2) Chern-Simons theory provides 3-D topological invariants and knot invariants.

4. More refined approaches involve K-theory -closely related to motivic cohomology - and its twisted version. The idea is to reduce the classification of branes to the classification of the bundle structures associated with them. This approach has had remarkable successes but has also its short-comings.

The challenge is to find the mathematical classification which suits best the physical intuitions (, which might be fatally wrong as already proposed) but is universal at the same time. This challenge has turned out to be tough. The Ramond-Ramond (RR) p-form fields of type II superstring theory are rather delicate objects and a source of most of the problems. The difficulties emerge also by the presence of Neveu-Schwartz 3-form $H = dB$ defining classical background field.

K-theory has emerged as a good candidate for the classification of branes. It leaves the confines of homology and uses bundle structures associated with branes and classifies these. There are many K-theories. In topological K-theory bundles form an algebraic structure with sum, difference, and multiplication. Sum is simply the direct sum for the fibers of the bundle with common base space. Product reduces to a tensor product for the fibers. The difference of bundles represents a more abstract notion. It is obtained by replacing bundles with pairs in much the same way as rationals can be thought of as pairs of integers with equivalence $(m, n) = (km, kn)$, k integer. Pairs $(n, 1)$ representing integers and pairs $(1, n)$ their inverses. In the recent case one replaces multiplication with sum and regards bundle pairs and (E, F) and $(E + G, F + G)$ equivalent. Although the pair as such remains a formal notion, each pair must have also a real world representativs. Therefore the sign for the bundle must have meaning and corresponds to the sign of the charges assigned to the bundle. The charges are analogous to winding of the brane and one can call brane with negative winding antibrane. The interpretation in terms of orientation looks rather natural. Later a TGD inspired concrete interpretation for the bundle sum, difference, product and also division will be proposed.

8.3 Problems

The classification of brane structures has some problems and some of them could be argued to be not only technical but reflect the fact that the physical picture is wrong.

8.3.1 Problems related to the existence of spinor structure

Many problems in the classification of brane charges relate to the existence of spinor structure. The existence of spinor structure is a problem already in general general relativity since ordinary spinor structure exists only if the second Stiefel-Whitney class [34] of the manifold is non-vanishing: if the third Stiefel-Whitney class vanishes one can introduce so called spin^c structure. This kind of problems are encountered already in lattice QCD, where periodic boundary conditions imply non-uniqueness having interpretation in terms of 16 different spinor structures with no obvious physical interpretation. One the strengths of TGD is that the notion of induced spinor structure eliminates all problems of this kind completely. One can therefore find direct support for TGD based notion of spinor structure from the basic inconsistency of QCD lattice calculations!

1. Freed-Witten anomaly [5] appearing in type II string theories represents one of the problems. Freed and Witten show that in the case of 2-branes for which the generalized gauge potential is 3-form so called spin^c structure is needed and exists if the third Stiefel-Whitney class w_3 related to second Stiefel Whitney class whose vanishing guarantees the existence of ordinary spin structure (in TGD framework spin^c structure for CP_2 is absolutely essential for obtaining standard model symmetries).

   It can however happen that w_3 is non-vanishing. In this case it is possible to modify the spin^c structure if the condition $w_3 + [H] = 0$ holds true. It can however happen that there is an obstruction for having this structure - in other words $w_3 + [H]$ does not vanish - known as Freed-Witten anomaly. In this case K-theory classification fails. Witten and Freed argue that physically the wrapping of cycle with non-vanishing $w_3 + [H]$ by a Dp-brane requires the presence of $D(p-2)$ brane cancelling the anomaly. If $D(p-2)$ brane ends to anti-Dp in which case charge conservation is lost. If there is not place for it to end one has semi-infinite brane with infinite mass, which is also problematic physically. Witten calls these branes baryons: these physically very dubious objects are not classified by K-theory.

2. The non-vanishing of $w_3 + [H] = 0$ forces to generalize K-theory to twisted K-theory [38]. This means a modification of the exterior derivative to get twisted de Rham cohomology and twisted K-theory and the condition of closedness in this cohomology for certain form becomes the condition guaranteeing the existence of the modified spin^c structure. D-branes act as sources of

these fields and the coupling is completely analogous to that in electrodynamics. In the presence of classical Neveu-Schwartz (NS-NS) 3-form field H associated with the back-ground geometry the field strength $G^{p+1} = dC_p$ is not gauge invariant anymore. One must replace the exterior derivative with its twisted version to get twisted de Rham cohomology:

$$d \rightarrow d + H \wedge \quad .$$

There is a coupling between p- and p+2-forms together and gauge symmetries must be modified accordingly. The fluxes of twisted field strengths are not quantized but one can return to original p-forms which are quantized. The coupling to external sources also becomes more complicated and in the case of magnetic charges one obtains magnetically charged Dp-branes. Dp-brane serves as a source for $D(p-2)$- branes.

This kind of twisted cohomology is known by mathematicians as Deligne cohomology. At the level of homology this means that if branes with dimension of p are presented then also branes with dimension $p+2$ are there and serve as source of Dp-branes emanating from them or perhaps identifiable as their sub-manifolds. Ordinary homology fails in this kind of situation and the proposal is that so called twisted K-theory could allow to classify the brane charges.

3. A Lagrangian formulation of brane dynamics based on the notion of p-brane democracy [15] due to Peter Townsend has been developed by various authors.

Ashoke Sen has proposed a grand vision for understanding the brane classification in terms of tachyon condensation in absence of NS-NS field H [14]. The basic observation is that stacks of space-filling D- and anti D-branes are unstable against process called tachyon condensation which however means fusion of $p+1$-D brane orbits rather than p-dimensional time slicse of branes. These branes are however accompanied by lower-dimensional branes and the decay process cannot destroy these. Therefore the idea arises that suitable stacks of D9 branes and anti-D9-branes could code for all lower-dimensional brane configurations as the end products of the decay process.

This leads to a creation of lower-dimensional branes. All decay products of branes resulting in the decay cascade would be by definition equivalent. The basic step of the decay process is the fusion of D-branes in stack to single brane. In bundle theoretic language one can say that the D-branes and anti-D branes in the stack fuse together to single brane with bundle fiber which is direct sum of the fibers on the stack. This fusion process for the branes of stack would correspond in topological K-theory. The fusion of D-branes and anti-D branes would give rise to nothing since the fibers would have opposite sign. The classification would reduce to that for stacks of D9-branes and anti D9-branes.

8.3.2 Problems with Hodge duality and S-duality

The K-theory classification is plagued by problems all of which need not be only technical.

1. R-R fields are self dual and since metric is involved with the mapping taking forms to their duals one encounters a problem. Chern characters appearing in K-theory are rational valued but the presence of metric implies that the Chern characters for the duals need not be rational valued. Hence K-theory must be replaced with something less demanding.

   The geometric quantization inspired proposal of Diaconescu, Moore and Witten [4] is based on the polarization using only one half of the forms to get rid of the proboem. This is like thinking the 10-D space-time as phase space and reducing it effectively to 5-D space: this brings strongly in mind the identification of space-time surfaces as hyper-quaternionic (associative) sub-manifolds of imbedding space with octonionic structure and one can ask whether the basic objects also in M-theory should be taken 5-dimensional if this line of thought is taken seriously. An alternative approach uses K-theory to classify the intersections of branes with 9-D space-time slice as has been porposed by Maldacena, Moore and Seiberg [12].

2. There another problem related to classification of the brane charges. Witten, Moore and Diaconescu [4] have shown that there are also homology cycles which are unstable against decay and this means that twisted K-theory is inconsistent with the S-duality of type IIB string theory. Also these cycles should be eliminated in an improved classification if one takes charge conservation as the basic condition and an hitherto un-known modification of cohomology theory is needed.

3. There is also the problem that K-theory for time slices classifies only the R-R field strengths. Also R-R gauge potentials carry information just as ordinary gauge potentials and this information is crucial in Chern-Simons type topological QFTs. K-theory for entire target space classifies D-branes as $p+1$-dimensional objects but in this case the classification of R-R field strengths is lost.

8.3.3 The existence of non-representable 7-D homology classes for targent space dimension $D > 9$

There is a further nasty problem which destroys the hopes that twisted K-theory could provide a satisfactory classification. Even worse, something might be wrong with the superstring theory itself. The problem is that not all homology classes allow a representation as non-singular manifolds. The first dimension in which this happens is $D = 10$, the dimension of super-string models! Situation is of course the same in M-theory. The existence of the non-representables was demonstrated by Thom - the creator of catastrophe theory and of cobordism theory for manifolds- for a long time ago.

What happens is that there can exist 7-D cycles which allow only singular imbeddings. A good example would be the imbedding of twistor space CP_3, whose orbit would have conical singularity for which CP_3 would contract to a point at the "moment of big bang". Therefore homological classification not only allows but demands branes which are orbifolds. Should orbifolds be excluded as unphysical? If so then homology gives too many branes and the singular branes must be excluded by replacing the homology with something else. Could twisted K-theory exclude non-representable branes as unstable ones by having non-vanishing $w_3 + [H]$? The answer to the question is negative: D6-branes with $w_3 + [H] = 0$ exist for which K-theory charges can be both vanishing or non-vanishing.

One can argue that non-representability is not a problem in superstring models (M-theory) since spontaneous compactification leads to $M \times X_6$ ($M \times X_7$). On the other hand, Cartesian product topology is an approximation which is expected to fail in high enough length scale resolution and near big bang so that one could encounter the problem. Most importantly, if M-theory is theory of everything it cannot contain this kind of beauty spots.

8.4 What could go wrong with super string theory and how TGD circumvents the problems?

As a proponent of TGD I cannot avoid the temptation to suggest that at least two things could go wrong in the fundamental physical assumptions of superstrings and M-theory.

1. The basic failure would be the construction of quantum theory starting from semiclassical approximation assuming localization of currents of 10 - or 11-dimensional theory to lower-dimensional sub-manifolds. What should have been a generalization of QFT by replacing pointlike particles with higher-dimensional objects would reduce to an approximation of 10- or 11-dimensional supergravity.

   This argument does not bite in TGD. 4-D space-time surfaces are indeed fundamental objects in TGD as also partonic 2-surfaces and braids. This role emerges purely number theoretically inspiring the conjecture that space-time surfaces are associative sub-manifolds of octonionic imbedding spaces, from the requirement of extended conformal invariance, and from the non-dynamical character of the imbedding space.

2. The condition that all homology equivalence classes are representable as manifolds excludes all dimensions $D > 9$ and thus super-strings and M-theory as a physical theory. This would be the case since branes are unavoidable in M-theory as is also the landscape of compactifications. In semiclassical supergravity interpretation this would not be catastrophe but if branes are fundamental objects this shortcoming is serious. If the condition of homological representability is accepted then target space must have dimension $D < 10$ and the arguments sequence leading to D=8 and TGD is rather short. The number theoretical vision provides the mathematical justification for TGD as the unique outcome.

3. The existence of spin structure is clearly the source of many problems related to R-R form. In TGD framework the induction of spin^c structure of the imbedding space resolves all problems

associated with sub-manifold spin structures. For some reason the notion of induced spinor structure has not gained attention in super string approach.

4. Conservative experimental physicist might criticize the emergence of branes of various dimensions as something rather weird. In TGD framework electric-magnetic duality can be understood in terms of general coordinate invariance and holography and branes and their duals have dimension 2, 3, and 4 organize to sub-manifolds of space-time sheets. The TGD counterpart for the fundamental M-2-brane is light-like 3-surface. Its magnetic dual has dimension given by the general formula $p_{dual} = D - p - 4$, where D is the dimension of the target space [8]. In TGD one has $D = 8$ giving $p_{dual} = 2$. The first interpretation is in terms of self-duality. A more plausible interpretation relies on the identification of the duals of light-like 3-surfaces as spacelike-3-surfaces at the light-like boundaries of CD. General Coordinate Invariance in strong sense implies this duality. For partonic 2-surface one would have $p = 1$ and $p_{dual} = 3$. The identification of the dual would be as space-time surface. The crucial distinction to M-theory would be that branes of different dimension would be sub-manifolds of space-time surface.

5. For $p = 0$ one would have $p_{dual} = 4$ assigning five-dimensional surface to orbits of point-like particles identifiable most naturally as braid strands. One cannot assign to it any direct physical meaning in TGD framework and gauge invariance for the analogs of brane gauge potentials indeed excludes even-dimensional branes in TGD since corresponding forms are proportional to Kähler gauge potential (so that they would be analogous to odd-dimensional branes allowed by type II_B superstrings).

   4-branes could be however mathematically useful by allowing to define Morse theory for the critical points of the Minkowskian part of Kähler action. While writing this I learned that Witten has proposed a 4-D gauge theory approach with $\mathcal{N} = 4$ SUSY to the classification of knots. Witten also ends up with a Morse theory using 5-D space-times in the category-theoretical formulation of the theory [45]. For some time ago I also proposed that TGD as almost topological QFT defines a theory of knots, knot braidings, and of 2-knots in terms of string world sheets [10]. Maybe the 4-branes could be useful for understanding of the extrema of TGD of the Minkowskian part of Kähler action which would take take the same role as Hamiltonian in Floer homology: the extrema of 5-D brane action would connect these extrema.

6. Light-like 3-surfaces could be seen as the analogs von Neuman branes for which the boundary conditions state that the ends of space-like 3-brane defined by the partonic 2-surfaces move with light-velocity. The interpretation of partonic 2-surfaces as space-like branes at the ends of CD would in turn make them D-branes so that one would have a duality between D-branes and N-brane interpretations. T-duality exchanges von Neumann and Dirichlet boundary conditions so that strong from of general coordinate invariance would correspond to both electric-magnetic and T-duality in TGD framework. Note that T-duality exchanges type II_A and type II_B superstrings with each other.

7. What about causal diamonds and their 7-D lightlike boundaries? Could one regard the light-like boundaries of CDs as analogs of 6-branes with light-like direction defining time-like direction so that space-time surfaces would be seen as 3-branes connecting them? This brane would not have magnetic dual since the formula for the dimensions of brane and its magnetic dual allows positive brane dimension p only in the range (1,3).

8.5 Can one identify the counterparts of R-R and NS-NS fields in TGD?

R-R and NS-NS 3-forms are clearly in fundamental role in M-theory. Since in TGD partonic 2-surfaces define the analogs of fundamental M-2-branes, one can wonder whether these 3-forms could have TGD counterparts.

1. In TGD framework the 3-forms $G_{3,A} = dC_{2,A}$ defined as the exterior derivatives of the two-forms $C_{2,A}$ identified as products $C_{2,A} = H_A J$ of Hamiltonians H_A of $\delta M^4_\pm \times CP_2$ with Kähler forms of factors of $\delta M^4_\pm \times CP_2$ define an infinite family of closed 3-forms belonging to various irreducible representations of rotation group and color group. One can consider also the algebra generated by products $H_A A$, $H_A J$, $H_A A \wedge J$, $H_A J \wedge J$, where A *resp.* J denotes the Kähler gauge potential

resp. Kähler form or either $\delta M^4_\pm$ or CP_2. A *resp.* Also the sum of Kähler potentials *resp.* forms of $\delta M^4_\pm$ and CP_2 can be considered.

2. One can define the counterparts of the fluxes $\int Adx$ as fluxes of $H_A A$ over braid strands, $H_A J$ over partonic 2-surfaces and string world sheets, $H_A A \wedge J$ over 3-surfaces, and $H_A J \wedge J$ over space-time sheets.Gauge invariance however suggests that for non-constant Hamiltonians one must exclude the fluxes assigned to odd dimensional surfaces so that only odd-dimensional branes would be allowed. This would exclude 0-branes and the problematic 4-branes. These fluxes should be quantized for the critical values of the Minkowskian contributions and for the maxima with respect to zero modes for the Euclidian contributions to Kähler action. The interpretation would be in terms of Morse function and Kähler function if the proposed conjecture holds true. One could even hope that the charges in Cartan algebra are quantized for all preferred extremals and define charges in these irreducible representations for the isometry algebra of WCW. The quantization of electric fluxes for string world sheets would give rise to the familiar quantization of the rotation $\int E \cdot dl$ of electric field over a loop in time direction taking place in superconductivity.

3. Should one interpret these fluxes as the analogs of NS-NS-fluxes or R-R fluxes? The exterior derivatives of the forms G_3 vanish which is the analog for the vanishing of magnetic charge densities (it is however possible to have the analogs of homological magnetic charge). The self-duality of Ramond p-forms could be posed formally ($G_p =^* G_{8-p}$) but does not have any implications for $p < 4$ since the space-time projections vanish in this case identically for $p > 3$. For $p = 4$ the dual of the instanton density $J \wedge J$ is proportional to volume form if M^4 and is not of topological interest. The approach of Witten eliminating one half of self dual R-R-fluxes would mean that only the above discussed series of fluxes need to be considered so that one would have no troubles with non-rational values of the fluxes nor with the lack of higher dimensional objects assignable to them. An interesting question is whether the fluxes could define some kind of K-theory invariants.

4. In TGD imbedding space is non-dynamical and there seems to be no counterpart for the NS 3-form field $H = dB$. The only natural candidate would correspond to Hamiltonian $B = J$ giving $H = dB = 0$. At quantum level this might be understood in terms of bosonic emergence [16] meaning that only Ramond representations for fermions are needed in the theory since bosons correspond to wormhole contacts with fermion and anti-fermions at opposite throats. Therefore twisted cohomology is not needed and there is no need to introduce the analogy of brane democracy and 4-D space-time surfaces containing the analogs of lower-dimensional brains as sub-manifolds are enough. The fluxes of these forms over partonic 2-surfaces and string world sheets defined non-abelian analogs of ordinary gauge fluxes reducing to rotations of vector potentials and suggested be crucial for understanding braidings of knots and 2-knots in TGD framework. [10]. Note also that the unique dimension D=4 for space-time makes 4-D space-time surfaces homologically self-dual so that only they are needed.

8.6 What about counterparts of S and U dualities in TGD framework?

The natural question is what could be the TGD counterparts of $S-$, $T-$ and U-dualities. If one accepts the identification of U-duality as product $U = ST$ and the proposed counterpart of T duality as a strong form of general coordinate invariance, it remains to understand the TGD counterpart of S-duality - in other words electric-magnetic duality - relating the theories with gauge couplings g and $1/g$. Quantum criticality selects the preferred value of g_K: Kähler coupling strength is very near to fine structure constant at electron length scale and can be equal to it. Since there is no coupling constant evolution associated with α_K, it does not make sense to say that g_K becomes strong and is replaced with its inverse at some point. One should be able to formulate the counterpart of S-duality as an identity following from the weak form of electric-magnetic duality and the reduction of TGD to almost topological QFT. This seems to be the case.

1. For preferred extremals the interior parts of Kähler action reduces to a boundary term because the term $j^\mu A_\mu$ vanishes. The weak form of electric-magnetic duality requires that Kähler electric charge is proportional to Kähler magnetic charge, which implies reduction to abelian

Chern-Simons term: the Kähler coupling strength does not appear at all in Chern-Simons term. The proportionality constant beween the electric and magnetic parts J_E and J_B of Kähler form however enters into the dynamics through the boundary conditions stating the weak form of electric-magnetic duality. At the Minkowskian side the proportionality constant must be proportional to g_K^2 to guarantee a correct value for the unit of Kähler electric charge - equal to that for electric charge in electron length scale- from the assumption that electric charge is proportional to the topologically quantized magnetic charge. It has been assumed that

$$J_E = \alpha_K J_B$$

holds true at *both sides* of the wormhole throat but this is an un-necessarily strong assumption at the Euclidian side. In fact, the self-duality of CP_2 Kähler form stating

$$J_E = J_B$$

favours this boundary condition at the Euclidian side of the wormhole throat. Also the fact that one cannot distinguish between electric and magnetic charges in Euclidian region since all charges are magnetic can be used to argue in favor of this form. The same constraint arises from the condition that the action for CP_2 type vacuum extremal has the value required by the argument leading to a prediction for gravitational constant in terms of the square of CP_2 radius and α_K the effective replacement $g_K^2 \to 1$ would spoil the argument.

2. Minkowskian and Euclidian regions should correspond to a strongly/weakly interacting phase in which Kähler magnetic/electric charges provide the proper description. In Euclidian regions associated with CP_2 type extremals there is a natural interpretation of interactions between magnetic monopoles associated with the light-like throats: for CP_2 type vacuum extremal itself magnetic and electric charges are actually identical and cannot be distinguished from each other. Therefore the duality between strong and weak coupling phases seems to be trivially true in Euclidian regions if one has $J_B = J_E$ at Euclidian side of the wormhole throat. This is however an un-necessarily strong condition as the following argument shows.

3. In Minkowskian regions the interaction is via Kähler electric charges and elementary particles have vanishing total Kähler magnetic charge consisting of pairs of Kähler magnetic monopoles so that one has confinement characteristic for strongly interacting phase. Therefore Minkowskian regions naturally correspond to a weakly interacting phase for Kähler electric charges. One can write the action density at the Minkowskian side of the wormhole throat as

$$\frac{(J_E^2 - J_B^2)}{\alpha_K} = \alpha_K J_B^2 - \frac{J_B^2}{\alpha_K} \quad .$$

The exchange $J_E \leftrightarrow J_B$ accompanied by $\alpha_K \to -1/\alpha_K$ leaves the action density invariant. Since only the behavior of the vacuum functional infinitesimally near to the wormhole throat matters by almost topological QFT property, the duality is realized. Note that the argument goes through also in Euclidian regions so that it does not allow to decide which is the correct form of weak form of electric-magnetic duality.

4. S-duality could correspond geometrically to the duality between partonic 2-surfaces responsible for magnetic fluxes and string worlds sheets responsible for electric fluxes as rotations of Kähler gauge potentials around them and would be very closely related with the counterpart of T-duality implied by the strong form of general coordinate invariance and saying that space-like 3-surfaces at the ends of space-time sheets are equivalent with light-like 3-surfaces connecting them.

The boundary condition $J_E = J_B$ at the Euclidian side of the wormhole throat inspires the question whether all Euclidian regions could be self-dual so that the density of Kähler action would be just the instanton density. Self-duality follows if the deformation of the metric induced by the deformation of the canonically imbedded CP_2 is such that in CP_2 coordinates for the Euclidian region the tensor $(g^{\alpha\beta}g^{\mu\nu} - g^{\alpha\nu}g^{\mu\beta})/\sqrt{g}$ remains invariant. This is certainly the case for CP_2 type vacuum extremals since by the light-likeness of M^4 projection the metric remains invariant. Also conformal scalings of the induced metric would satisfy this condition. Conformal scaling is not consistent with the degeneracy of the 4-metric at the wormhole throat. Self-duality is indeed an un-necessarily strong condition.

8.6.1 Comparison with standard view about dualities

One can compare the proposed realization of T, S and U to the more general dualities defined by the modular group $SL(2,Z)$, which in QFT framework can hold true for the path integral over all possible gauge field configurations. In the resent case the dualities hold true for every preferred extremal separately and the functional integral is only over the space-time projections of fixed Kähler form of CP_2. Modular invariance for Maxwell action was discussed by E. Verlinde for Maxwell action with θ term for a general 4-D compact manifold with Euclidian signature of metric in [16]. In this case one has path integral giving sum over infinite number of extrema characterized by the cohomological equivalence class of the Maxwell field the action exponential to a high degree. Modular invariance is broken for CP_2: one obtains invariance only for $\tau \rightarrow \tau + 2$ whereas S induces a phase factor to the path integral.

1. In the recent case these homology equivalence classes would correspond to homology equivalence classes of holomorphic partonic 2-surfaces associated with the critical points of Kähler function with respect to zero modes.

2. In the case that the Euclidian contribution to the Kähler action is expressible solely in terms of wormhole throat Chern-Simons terms, and one can neglect the measurement interaction terms, the exponent of Kähler action can be expressed in terms of Chern-Simons action density as

$$\begin{aligned} L &= \tau L_{C-S} \ , \\ L_{C-S} &= J \wedge A \ , \\ \tau &= \frac{1}{g_K^2} + i\frac{k}{4\pi} \ , \quad k = 1 \ . \end{aligned} \tag{8.1}$$

Here the parameter τ transforms under full $SL(2,Z)$ group as

$$\tau \rightarrow \frac{a\tau + b}{c\tau + d} \ . \tag{8.2}$$

The generators of $SL(2,Z)$ transformations are $T : \tau \rightarrow \tau + 1$, $S : \tau \rightarrow -1/\tau$. The imaginary part in the exponents corresponds to Kac-Moody central extension $k = 1$.

This form corresponds also to the general form of Maxwell action with CP breaking θ term given by

$$L = \frac{1}{g_K^2} J \wedge^* J + i\frac{\theta}{8\pi^2} J \wedge J \ , \quad \theta = 2\pi \ . \tag{8.3}$$

Hence the Minkowskian part mimicks the θ term but with a value of θ for which the term does not give rise to CP breaking in the case that the action is full action for CP_2 type vacuum extremal so that the phase equals to 2π and phase factor case is trivial. It would seem that the deviation from the full action for CP_2 due to the presence of wormhole throats reducing the value of the full Kähler action for CP_2 type vacuum extremal could give rise to CP breaking. One can visualize the excluded volume as homologically non-trivial geodesic spheres with some thickness in two transverse dimensions. At the limit of infinitely thin geodesic spheres CP breaking would vanish. The effect is exponentially sensitive to the volume deficit.

8.6.2 CP breaking and ground state degeneracy

Ground state degeneracy due to the possibility of having both signs for Minkowskian contribution to the exponent of vacuum functional provides a general view about the description of CP breaking in TGD framework.

1. In TGD framework path integral is replaced by inner product involving integral over WCV. The vacuum functional and its conjugate are associated with the states in the inner product so that the phases of vacuum functionals cancel if only one sign for the phase is allowed. Minkowskian contribution would have no physical significance. This of course cannot be the case. The ground state is actually degenerate corresponding to the phase factor and its complex conjugate since $\sqrt{g}$ can have two signs in Minkowskian regions. Therefore the inner products between states associated with the two ground states define 2×2 matrix and non-diagonal elements contain interference terms due to the presence of the phase factor. At the limit of full CP_2 type vacuum extremal the two ground states would reduce to each other and the determinant of the matrix would vanish.

2. A small mixing of the two ground states would give rise to CP breaking and the first principle description of CP breaking in systems like $K - \overline{K}$ and of CKM matrix should reduce to this mixing. K^0 mesons would be CP even and odd states in the first approximation and correspond to the sum and difference of the ground states. Small mixing would be present having exponential sensitivity to the actions of CP_2 type extremals representing wormhole throats. This might allow to understand qualitatively why the mixing is about 50 times larger than expected for B^0 mesons.

3. There is a strong temptation to assign the two ground states with two possible arrows of geometric time. At the level of M-matrix the two arrows would correspond to state preparation at either upper or lower boundary of CD. Do long- and shortlived neutral K mesons correspond to almost fifty-fifty orthogonal superpositions for the two arrow of geometric time or almost completely to a fixed arrow of time induced by environment? Is the dominant part of the arrow same for both or is it opposite for long and short-lived neutral measons? Different lifetimes would suggest that the arrow must be the same and apart from small leakage that induced by environment. CP breaking would be induced by the fact that CP is performed only K^0 but not for the environment in the construction of states. One can probably imagine also alternative interpretations.

Remark: The proportionality of Minkowskian and Euclidian contributions to the same Chern-Simons term implies that the critical points with respect to zero modes appear for both the phase and modulus of vacuum functional. The Kähler function property does not allow extrema for vacuum functional as a function of complex coordinates of WCW since this would mean Kähler metric with non-Euclidian signature. If this were not the case. the stationary values of phase factor and extrema of modulus of the vacuum functional would correspond to different configurations.

8.7 Could one divide bundles?

TGD differs from string models in one important aspects: stringy diagrams do not have interpretation as analogs of vertices of Feynman diagrams: the stringy decay of partonic 2-surface to two pieces does not represent particle decay but a propagation along different paths for incoming particle. Particle reactions in turn are described by the vertices of generalized Feynman diagrams in which the ends of incoming and outgoing particles meet along partonic 2-surface. This suggests a generalization of K-theory for bundles assignable to the partonic 2-surfaces. It is good to start with a guess for the concrete geometric realization of the sum and product of bundles in TGD framework.

1. The analogs of string diagrams could represent the analog for direct sum. Difference between bundles could be defined geometrically in terms of trouser vertex $A + B \to C$. B would by definition represent $C - A$. Direct sum could make sense for single particle states and have as space-time correlate the conservation of braid strands.

2. A possible concretization in TGD framework for the tensor product is in terms of the vertices of generalized Feynman diagrams at which incoming light-like 3-D orbits of partons meet along

their ends. The tensor product of incoming state spaces defined by fermionic oscillator algebras is naturally formed. Tensor product would have also now as a space-time correlate conservation of braid strands. This does not mean that the number of braid strands is conserved in reactions if also particular exchanges can carry the braid strands of particles coming to the vertex.

Why not define also division of bundles in terms of the division for tensor product? In terms of the 3-vertex for generalized Feynman diagrams $A \otimes B = C$ representing tensor product B would be by definition C/A. Therefore TGD would extend the K-theory algebra by introducing also division as a natural operation necessitated by the presence of the join along ends vertices not present in string theory. I would be surprised if some mathematician would not have published the idea in some exotic journal. Below I represent an argument that this notion could be also applied in the mathematical description of finite measurement resolution in TGD framework using inclusions of hyper-finite factor. Division could make possible a rigorous definition for for non-commutative quantum spaces.

Tensor division could have also other natural applications in TGD framework.

1. One could assign bundles M_+ and M_- to the upper and lower light-like boundaries of CD. The bundle M_+/M_- would be obtained by formally identifying the upper and lower light-like boundaries. More generally, one could assign to the boundaries of CD positive and negative energy parts of WCW spinor fields and corresponding bundle structures in "half WCW". Zero energy states could be seen as sections of the unit bundle just like infinite rationals reducing to real units as real numbers would represent zero energy states.

2. Finite measurement resolution would encourage tensor division since finite measurement resolution means essentially the loss of information about everything below measurement resolution represented as a tensor product factor. The notion of coset space formed by hyper-finite factor and included factor could be understood in terms of tensor division and give rise to quantum group like space with fractional quantum dimension in the case of Jones inclusions [20]. Finite measurement resolution would therefore define infinite hierarchy of finite dimensional non-commutative spaces characterized by fractional quantum dimension. In this case the notion of tensor product would be somewhat more delicate since complex numbers are effectively replaced by the included algebra whose action creates states not distinguishable from each other [20]. The action of algebra elements to the state $|B\rangle$ in the inner product $\langle A|B\rangle$ must be equivalent with the action of its hermitian conjugate to the state $\langle A|$. Note that zero energy states are in question so that the included algebra generates always modifications of states which keep it as a zero energy state.

9 A connection between cognition, number theory, algebraic geometry, topology, and quantum physics

I have had some discussions with Stephen King and Hitoshi Kitada in a closed discussion group about the idea that the duality between Boolean algebras and Stone spaces could be important for the understanding of consciousness, at least cognition. In this vision Boolean algebras would represent conscious mind and Stone spaces would represent the matter: space-time would emerge.

I am personally somewhat skeptic because I see consciousness and matter as totally different levels of existence. Consciousness (and information) is about something, matter just is. Consciousness involves always a change as we no from basic laws about perception. There is of course also the experience of free will and the associated non-determinism. Boolean algebra is a model for logic, not for conscious logical reasoning. There are also many other aspects of consciousness making it very difficult to take this kind of duality seriously.

I am also skeptic about the emergence of space-time say in the extremely foggy form as it used in entropic gravity arguments. Recent day physics poses really strong constraints on our view about space-time and one must take them very seriously.

This does not however mean that Stone spaces could not serve as geometrical correlates for Boolean consciousness. In fact, p-adic integers can be seen as a Stone space naturally assignable to Boolean algebra with infinite number of bits.

9.1 Innocent questions

I ende up with the innocent questions, as I was asked to act as some kind of mathematical consultant and explain what Stone spaces actually are and whether they could have a connection to p-adic numbers. Anyone can of course go to Wikipedia and read the article Stone's representation theorem for Boolean algebras. For a layman this article does not however tell too much.

Intuitively the content of the representation theorem looks rather obvious, at least at the first sight. As a matter fact, the connection looks so obvious that physicists often identify the Boolean algebra and its geometric representation without even realizing that two different things are in question. The subsets of given space- say Euclidian 3-space- with union and intersection as basic algebraic operations and inclusion of sets as ordering relation defined a Boolean algebra for the purposes of physicist. One can assign to each point of space a bit. The points for which the value of bit equals to one define the subset. Union of subsets corresponds to logical OR and intersection to AND. Logical implication B→ A corresponds to A contains B.

When one goes to details problems begin to appear. One would like to have some non-trivial form of continuity.

1. For instance, if the sets are form open sets in real topology their complements representing negations of statements are closed, not open. This breaks the symmetry between statement and it negation unless the topology is such that closed sets are open. Stone's view about Boolean algebra assumes this. This would lead to discrete topology for which all sets would be open sets and one would lose connection with physics where continuity and differential structure are in key role.

2. Could one dare to disagree with Stone and allow both closed and open sets of E^3 in real topology and thus give up clopen assumption? Or could one tolerate the asymmetry between statements and their negations and give some special meaning for open or closet sets- say as kind of axiomatic statements holding true automatically. If so, one an also consider algebraic varieties of lower dimension as collections of bits which are equal to one. In Zariski topology used in algebraic geometry these sets are closed. Again the complements would be open. Could one regard the lower dimensional varieties as identically true statements so that the set of identically true statements would be rather scarce as compared to falsities? If one tolerates some quantum TGD, one could ask whether the 4-D quaternionic/associative varieties defining classical space-times and thus classical physics could be identified as the axiomatic truths. Associativity would be the basic truth inducing the identically true collections of bits.

9.2 Stone theorem and Stone spaces

For reasons which should be clear it is perhaps a good idea to consider in more detail what Stone duality says. Stone theorem states that Boolean algebras are dual with their Stone spaces. Logic and certain kind of geometry are dual. More precisely, any Boolean algebra is isomorphic to closed open subsets of some Stone space and vice versa. Stone theorem respects category theory. The homomorphisms between Boolean algebras A and B corresponds to homomorphism between Stone spaces $S(B)$ and $S(A)$: one has contravariant functor between categories of Boolean algebras and Stone spaces. In the following set theoretic realization of Boolean algebra provides the intuitive guidelines but one can of course forget the set theoretic picture altogether and consider just abstract Boolean algebra.

1. Stone space is defined as the space of homomorphisms from Boolean algebra to 2-element Boolean algebra. More general spaces are spaces of homomorphisms between two Boolean algebras. The analogy in the category of linear spaces would be the space of linear maps between two linear spaces. Homomorphism is in this case truth preserving map: $h(A\ AND\ B) = h(a)\ AND\ h(B)$, $h(\ OR\ B) = h(a)\ OR\ h(B)$ and so on.

2. For any Boolean algebra Stone space is compact, totally disconnected Hausdorff space. Conversely, for any topological space, the subsets, which are both closed and open define Boolean algebra. Note that for a real line this would give 2-element Boolean algebra. Set is closed and open simultaneously only if its boundary is empty and in p-adic context there are no boundaries. Therefore for p-adic numbers closed sets are open and the sets of p-adic numbers with p-adic

norm above some lower bound and having some set of fixed pinary digits, define closed-open subsets.

3. Stone space dual to the Boolean algebra does not conform with the physicist's ideas about space-time. Stone space is a compact totally disconnected Hausdorff space. Disconnected space is representable as a union of two or more disjoint open sets. For totally disconnected space this is true for every subset. Path connectedness is stronger notion than connected and says that two points of the space can be always connected by a curve defined as a mapping of *real* unit interval to the space. Our physical space-time seems to be however connected in this sense.

4. The points of the Stone space $S(B)$ can be identified ultrafilters. Ultrafilter defines homomorphism of B to 2-element of Boolean algebra Boolean algebra. Set theoretic realization allows to understand what this means. Ultrafilter is a set of subsets with the property that intersections belong to it and if set belongs to it also sets containing it belong to it: this corresponds to the fact that set inclusion $A \supset B$ corresponds to logical implication. Either set or its complement belongs to the ultrafilter (either statement or its negation is true). Empty set does not. Ultrafilter obviously corresponds to a collection of statements which are simultaneously true without contradictions. The sets of ultrafilter correspond to the statements interpreted as collections of bits for which each bit equals to 1.

5. The subsets of B containing a fixed point b of Boolean algebra define an ultrafilter and imbedding of b to the Stone space by assigning to it this particular principal ultrafilter. b represents a statement which is always true, kind of axiom for this principal ultrafilter and ultrafilter is the set of all statements consistent with b.

   Actually any finite set in the Boolean algebra consisting of a collection of fixed bits b_i defines an ultrafilter as the set all subsets of Boolean algebra containing this subset. Therefore the space of all ultra-filters is in one-one correspondence with the space of subsets of Boolean statements. This set corresponds to the set of statements consistent with the truthness of b_i analogous to axioms.

9.3 2-adic integers and 2-adic numbers as Stone spaces

I was surprised to find that p-adic numbers are regarded as a totally disconnected space. The intuitive notion of connected is that one can have a continuous curve connecting two points and this is certainly true for p-adic numbers with curve parameter which is p-adic number but not for curves with real parameter which became obvious when I started to work with p-adic numbers and invented the notion of p-adic fractal. In other words, p-adic integers form a continuum in p-adic but not in real sense. This example shows how careful one must be with definitions. In any case, to my opinion the notion of path based on p-adic parameter is much more natural in p-adic case. For given p-adic integers one can find p-adic integers arbitrary near to it since at the limit $n \to \infty$ the p-adic norm of p^n approaches zero. Note also that most p-adic integers are infinite as real integers.

Disconnectedness in real sense means that 2-adic integers define an excellent candidate for a Stone space and the inverse of the Stone theorem allows indeed to realize this expectation. Also 2-adic numbers define this kind of candidate since 2-adic numbers with norm smaller than 2^n for any n can be mapped to 2-adic integers. One would have union of Boolean algebras labelled by the 2-adic norm of the 2-adic number. p-Adic integers for a general prime p define obviously a generalization of Stone space making sense for effectively p-valued logic: the interpretation will be discussed below.

Consider now a Boolean algebra consisting of all possible infinitely long bit sequences. This algebra corresponds naturally to 2-adic integers. The generating Boolean statements correspond to sequences with single non-vanishing bit: by taking the unions of these points one obtains all sets. The natural topology is that for which the lowest bits are the most significant. 2-adic topology realizes this idea since n:th bit has norm 2^{-n}. 2-adic integers as an p-adic integers are as spaces totally disconnected.

That 2-adic integers and more generally, 2-adic variants of n-dimensional p-adic manifolds would define Stone bases assignable to Boolean algebras is consistent with the identification of p-adic space-time sheets as correlates of cognition. Each point of 2-adic space-time sheet would represent 8 bits as a point of 8-D imbedding space. In TGD framework WCW ("world of classical worlds") spinors correspond to Fock space for fermions and fermionic Fock space has natural identification as a Boolean algebra. Fermion present/not present in given mode would correspond to true/false. Spinors decompose to a tensor product of 2-spinors so that the labels for Boolean statements form a Boolean algebra too in this case. A possible interpretation is as statements about statements.

In TGD Universe life and thus cognition reside in the intersection of real and p-adic worlds. Therefore the intersections of real and p-adic partonic 2-surfaces represent the intersection of real and p-adic worlds, those Boolean statements which are expected to be accessible for conscious cognition. They correspond to rational numbers or possibly numbers in n algebraic estension of rationals. For rationals pinary expansion starts to repeat itself so that the number of bits is finite. This intersection is also always discrete and for finite real space-time regions finite so that the identification looks a very natural since our cognitive abilities seem to be rather limited. In TGD inspired physics magnetic bodies are the key players and have much larger size than the biological body so that their intersection with their p-adic counterparts can contain much more bits. This conforms with the interpretation that the evolution of cognition means the emergence of increasingly longer time scales. Dark matter hierarchy realized in terms of hierarchy of Planck constants realizes this.

9.4 What about p-adic integers with $p > 2$?

The natural generalization of Stone space would be to a geometric counterpart of p-adic logic which I discussed for some years ago. The representation of the statements of p-valued logic as sequences of pinary digits makes the correspondence trivial if one accepts the above represented arguments. The generalization of Stone space would consist of p-adic integers and imbedding of a p-valued analog of Boolean algebra would map the number with only n:th digit equal to $1, ..., p-1$ to corresponding p-adic number.

One should however understand what p-valued statements mean and why p-adic numbers near powers of 2 are important. What is clear that p-valued logic is too romantic to survive. At least our every-day cognition is firmly anchored to a reality where everything is experience to be true or false.

1. The most natural explanation for $p > 2$ adic logic is that all Boolean statements do not allow a physical representation and that this forces reduction of 2^n valued logic to $p < 2^n$-valued one. For instance, empty set in the set theoretical representation of Boolean logic has no physical representation. In the same manner, the state containing no fermions fails to represent anything physically. One can represent physically at most $2^n - 1$ one statements of n-bit Boolean algebra and one must be happy with $n - 1$ completely represented digits. The remaining statements containing at least one non-vanishing digit would have some meaning, perhaps the last digit allowed could serve as a kind of parity check.

2. If this is accepted then p-adic primes near to power 2^n of 2 but below it and larger than the previous power 2^{n-1} can be accepted and provide a natural topology for the Boolean statements grouping the binary digits to p-valued digit which represents the allowed statements in 2^n valued Boolean algebra. Bit sequence as a unit would be represented as a sequence of physically realizable bits. This would represent evolution of cognition in which simple yes or not statements are replaced with sequences of this kind of statements just as working computer programs are fused as modules to give larger computer programs. Note that also for computers similar evolution is taking place: the earliest processors used byte length 8 and now 32, 64 and maybe even 128 are used.

3. Mersenne primes $M_n = 2^n - 1$ would be ideal for logic purposes and they indeed play a key role in quantum TGD. Mersenne primes define p-adic length scales characterize many elementary particles and also hadron physics. There is also evidence for p-adically scaled up variants of hadron physics (also leptohadron physics allowed by the TGD based notion of color predicting colored excitations of leptons). LHC will certainly show whether M_{89} hadron physics at TeV energy scale is realized and whether also leptons might have scaled up variants.

4. For instance, M_{127} assignable to electron secondary p-adic time scale is .1 seconds, the fundamental time scale of sensory perception. Thus cognition in .1 second time scale single pinary statement would contain 126 digits as I have proposed in the model of memetic code. Memetic codons would correspond to 126 digit patterns with duration of .1 seconds giving 126 bits of information about percept.

If this picture is correct, the interpretation of p-adic space-time sheets- or rather their intersections with real ones- would represent space-time correlates for Boolean algebra represented at quantum level by fermionic many particle states. In quantum TGD one assigns with these intersections braids- or number theoretic braids- and this would give a connection with topological quantum field theories (TGD can be regarded as almost topological quantum field theory).

9.5 One more road to TGD

The following arguments suggests one more manner to end up with TGD by requiring that fermionic Fock states identified as a Boolean algebra have their Stone space as space-time correlate required by quantum classical correspondence. Second idea is that space-time surfaces define the collections of binary digits which can be equal to one: kind of eternal truths. In number theoretical vision associativity condition in some sense would define these divine truths. Standard model symmetries are a must- at least as their p-adic variants -and simple arguments forces the completion of discrete lattice counterpart of M^4 to a continuum.

1. If one wants Poincare symmetries at least in p-adic sense then a 4-D lattice in M^4 with $SL(2, Z)\times T^4$, where T^4 is discrete translation group is a natural choice. $SL(2, Z)$ acts in discrete Minkowski space T^4 which is lattice. Poincare invariance would be discretized. Angles and relative velocities would be discretized, etc..

2. The p-adic variant of this group is obtained by replacing Z and T^4 by their p-adic counterparts: in other words Z is replaced with the group Z_p of p-adic integers. This group is p-adically continuous group and acts continuously in T^4 defining a p-adic variant of Minkowski space consisting of all bit sequences consisting of 4-tuples of bits. Only in real sense one would have discreteness: note also that most points would be at infinity in real sense. Therefore it is possible to speak about analytic functions, differential calculus, and to write partial differential equations and to solve them. One can construct group representations and talk about angular momentum, spin and 4-momentum as labels of quantum states.

3. If one wants standard model symmetries p-adically one must replace T^4 with $T^4 \times CP_2$. CP_2 would be now discrete version of CP_2 obtained from discrete complex space C^3 by identifying points different by a scaling by complex integer. Discrete versions of color and electroweak groups would be obtained.

The next step is to ask what are the laws of physics. TGD fan would answer immediately: they are of course logical statements which can be true identified as subsets of $T^4 \times CP_2$ just as subset in Boolean algebra of sets corresponds to bits which are true.

1. The collections of 8-bit sequences consisting of only 1:s would define define 4-D surfaces in discrete $T^4 \times CP_2$. Number theoretic vision would suggest that they are quaternionic surfaces so that one associativity be the physical law at geometric level. The conjecture is that preferred extremals of Kähler action are associative surfaces using the definition of associativity as that assignable to a 4-plane defined by modified gamma matrices at given point of space-time surface.

2. Induced gauge field and metric make sense for p-adic integers. p-Adically the field equations for Kähler action make also sense. These p-adic surfaces would represent the analog of Boolean algebra. They would be however something more general than Stone assumes since they are not closed-open in the 8-D p-adic topology.

One however encounters a problem.

1. Although the field equations associated with Kähler action make sense, Kähler action itself does not exists as integral nor does the genuine minimization make sense since p-adically numbers are not well ordered and one cannot in general say which of two numbers is the larger one. This is a real problem and suggests that p-adic field equations are not enough and must be accompanied by real ones. Of course, also the metric properties of p-adic space-time are in complete conflict with what we believe about them.

2. One could argue that for preferred extremals the integral defining Kähler action is expressible as an integral of 4-form whose value could be well-defined since integrals of forms for closed algebraic surfaces make sense in p-adic cohomology theory pioneered by Grothendieck. The idea would be to use the definition of Kähler action making sense for preferred extremals as its definition in p-adic context. I have indeed proposed that space-time surfaces define representatives for homology with inspiration coming from TGD as almost topological QFT. This would give powerful constraints on the theory in accordance with the interpretation as a generalized Bohr orbit.

3. This argument together with what we know about the topology of space-time on basis of everyday experience however more or less forces the conclusion that also real variant of $M^4 \times CP_2$ is there and defines the proper variational principle. The finite points (on real sense) of $T^4 \times CP_2$ (in discrete sense) would represent points common to real and p-adic worlds and the identification in terms of braid points makes sense if one accepts holography and restricts the consideration to partonic 2-surfaces at boundaries of causal diamond. These discrete common points would represent the intersection of cognition and matter and living systems and provide a representation for Boolean cognition.

4. Finite measurement resolution enters into the picture naturally. The proper time distance between the tips would be quantized in multiples of CP_2 length. There would be several choices for the discretized imbedding space corresponding to different distance between lattice points: the interpretation is in terms of finite measurement resolution.

It should be added that discretized variant of Minkowski space and its p-adic variant emerge in TGD also in different manner in zero energy ontology.

1. The discrete space $SL(2,Z) \times T^4$ would have also interpretation as acting in the moduli space for causal diamonds identified as intersections of future and past directed light-cones. T^4 would represent lattice for possible positions of the lower tip of CD and and $SL(2,Z)$ leaving lower tip invariant would act on hyperboloid defined by the position of the upper tip obtained by discrete Lorentz transformations. This leads to cosmological predictions (quantization of red shifts). CP_2 length defines a fundamental time scale and the number theoretically motivated assumption is that the proper time distances between the tips of CDs come as integer multiples of this distance.

2. The stronger condition explaining p-adic length scale hypothesis would be that only octaves of the basic scale are allowed. This option is not consistent with zero energy ontology. The reason is that for more general hypothesis the M-matrices of the theory for Kac-Moody type algebra with finite-dimensional Lie algebra replaced with an infinite-dimensional algebra representing hermitian square roots of density matrices and powers of the phase factor replaced with powers of S-matrix. All integer powers must be allowed to obtain generalized Kac-Moody structure, not only those which are powers of 2 and correspond naturally to integer valued proper time distance between the tips of CD. Zero energy states would define the symmetry Lie-algebra of S-matrix with generalized Yangian structure.

3. p-Adic length scale hypothesis would be an outcome of physics and it would not be surprising that primes near power of two are favored because they are optimal for Boolean cognition.

The outcome is TGD. Reader can of course imagine alternatives but remember the potential difficulties due to the fact that minimization in p-adic sense does not make sense and action defined as integral does not exist p-adically. Also the standard model symmetries and quantum classical correspondence are to my opinion "must":s.

9.6 A connection between cognition and algebraic geometry

Stone space is synonym for profinite space. The Galois groups associated with algebraic extensions of number fields represent an extremely general class of profinite group [27]. Every profinite group appears in Galois theory of some field K. The most most interesting ones are inverse limits of $Gal(F_1/K)$ where F_1 varies over all intermediate fields. Profinite groups appear also as fundamental groups in algebraic geometry. In algebraic topology fundamental groups are in general not profinite. Profiniteness means that p-adic representations are especially natural for profinite groups.

There is a fascinating connection between infinite primes and algebraic geometry discussed above leads to the proposal that Galois groups - or rather their projective variants- can be represented as braid groups acting on 2-dimensional surfaces. These findings suggest a deep connection between space-time correlates of Boolean cognition, number theory, algebraic geometry, and quantum physics and TGD based vision about representations of Galois groups as groups lifted to braiding groups acting on the intersection of real and p-adci variants of partonic 2-surface conforms with this.

Fermat theorem serves as a good illustration between the connection between cognitive representations and algebraic geometry. A very general problem of algebraic geometry is to find rational points of an algebraic surface. These can be identified as common rational points of the real and p-adic variant of the surface. The interpretation in terms of consciousness theory would be as points defining cognitive representation as rational points common to real partonic 2-surface and and its p-adic variants. The mapping to polynomials given by their representation in terms of infinite primes to braids of braids of braids.... at partonic 2-surfaces would provide the mapping of n-dimensional problem to 2-dimensional one.

One considers the question whether there are integer solutions to the equation $x^n + y^n + z^n = 1$. This equation defines 2-surfaces in both real and p-adic spaces. In p-adic context it is easy to construct solutions but they usually represent infinite integers in real sense. Only if the expansion in powers of p contains finite number of powers of p, one obtains real solution as finite integers.

The question is whether there are any real solutions at all. If they exist they correspond to the intersections of the real and p-adic variants of these surfaces. In other words p-adic surface contains cognitively representable points. For $n > 2$ Fermat's theorem says that only single point $x = y = z = 0$ exists so that only single p-adic multi-bit sequence $(0, 0, 0, ...)$ would be cognitively representable.

This relates directly to our mathematical cognition. Linear and quadratic equations we can solve and in these cases the number in the intersection of p-adic and real surfaces is indeed very large. We learn the recipes already in school! For $n > 2$ difficulties begin and there are no general recipes and it requires mathematician to discover the special cases: a direct reflection of the fact that the number of intersection points for real and p-adic surfaces involved contains very few points.

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