Dark Matter, Dark Energy, and Elementary Particles and Forces

Thomas J. Buckholtz*

Abstract Patterns link properties of six quarks and three leptons, the set of fundamental forces, and possible properties of dark matter and dark energy.

Keywords Dark matter · Dark energy · Theory of everything · Fundamental forces · CPT symmetry · Uncertainty principle

1 Introduction

A formula approximates masses of three charged leptons and six quarks. The formula involves two integer variables and suggests a periodic table that includes the nine particles and three empty positions. Another formula provides the charges of the nine particles. (Section 2)

The pattern of empty positions in the periodic table pertaining to baryonic matter provides a basis for 23 other similar ensembles of proposed particles. Six ensembles become candidates for a super-ensemble that includes five dark matter ensembles and the baryonic matter ensemble. Three other super-ensembles become candidates for dark-energy material. Assuming that the ensembles share approximately equally the density of the universe, the numbers 1, 5, and 18 apportion the baryonic-matter, dark-matter, and dark-energy densities in a manner consistent with observations. (Section 3)

Expressions generalizing baryonic-matter formulas become candidate formulas for relating charges and masses within each of the 24 ensembles. At least one of the non-baryonic matter ensembles seems capable of having atom-like entities. (Section 4)

A constant that was estimated in the process of curve-fitting baryonic-matter masses and that appears in a formula for those masses can be computed based on the ratio, for an electron and a positron, of electromagnetic interaction to gravitational interaction. (Section 5)

For a space of interactions, consideration of a realm bounded by the electromagnetic interaction and the gravitational interaction leads to a relationship between graviton-mediated and photon-mediated interactions. Baryonic-matter excited states (such as the muon, bottom quark, and top quark) represent steps toward the electromagnetic end of the realm and away from the gravitational end of the realm (which features the electron, up quark, and down quark). (Section 6)

Various realms connect eight interactions. The eight interactions are a spin-4 interaction (s4), a spin-3 interaction (s3), gravity (gr, a spin-2 interaction), electromagnetism (em, a spin-1 interaction), the weak interaction (wk), the strong interaction (st), a basic mass interaction (bm), and a basic charge interaction (bc). The s4 interaction is universal to quark-like particles and to charged-lepton-like particles. The s4 interaction provides for the repulsion attributed to dark energy (as a force). Traversing realm s3/s4 leads to a four-fold "separation" based on the two

* Electronic mail: Thomas.J.Buckholtz@gmail.com

possible "charges" (matter and anti-matter) and two possible "parities" (left-handed and right-handed) associated with CPT symmetry. Baryonic matter falls into the "matter plus left-handed" branch. Baryonic matter does not easily detect "stuff" associated with the other three branches. Traversing realm s3/gr leads to a four-fold separation into the four super-ensembles. Baryonic matter falls into the "baryonic-matter plus dark-matter" super-ensemble. Baryonic matter does not easily detect gravitons from the other three super-ensembles. Traversing realm em/gr leads to a six-fold separation into ensembles. Baryonic matter does not easily detect photons from other ensembles. Traversing realm wk/em involves no split. Spin becomes a key property. Traversing realm st/wk, one finds that the strong interaction pertains to quarks but not to leptons. The basic mass interaction enables the s4 and gr interactions to scale from elementary particles to atoms to astrophysical objects. The basic charge interaction enables the s3 and em interactions to scale from elementary particles to atoms to astrophysical objects. (Section 7)

Considering the realm bounded by the weak and electromagnetic interactions points to both an approximation to the fine structure constant and an effect of the Z and W bosons' non-zero masses. (Section 8)

Considering the realm bounded by the strong and weak interactions leads to possible estimates regarding a ratio of strengths for the strong and weak interactions. (Section 9)

The interaction attributable to spin-4 bosons provides the repulsion popularly attributed to dark energy. (Section 10)

Analysis identifies six types of photons. This result supports an assertion that baryonic matter has difficulty detecting photons emitted by dark matter. Similar results point to a basis for difficulties each of the four super-ensembles has in detecting gravitation associated with the other three super-ensembles. (Section 11)

Each of a basic charge interaction and a basic mass interaction scales to include entity-pair interactions in which either interacting entity can be a fundamental particle, atom, or astrophysical object. The time-like dimension of space-time may be characterized as a collapse of three time-like dimensions into the observed time-like dimension. (Section 12)

This paper contains appendices. One appendix contains theory based on uncertainty, supports a key role of the number 4 in this paper, provides a generalization of the Dirac matrices, and develops a possible quantum mechanical parallel to the Einstein field equations. Some appendices point to relationships between, or rough bounds on, measured values for properties of baryonic matter. For example, a formula relates the mass of a proton to the mass of an electron. (Section 13)

Comments

This paper features attempts to identify and extrapolate patterns. Such patterns pertain mainly to domains of interactions and particle masses, not space-time and not energy-momentum space.

For each of some of the patterns, there is little observational evidence to support or refute the pattern. Readers can judge aspects of this work based on standards such as "proven beyond a reasonable doubt;" "supported by a preponderance of evidence;" "at least as good as any other known theories or conjectures;" "not contradicted by existing theory or observations, but not all that convincing;" "contradicts existing theory, but not observations;" and "contradicts accepted observation." The author hopes that little herein falls into the last category and that people will use this paper to guide subsequent observation, experimentation, and development of theory that help advance physics as well as help categorize or re-categorize (based on such standards) items in this paper.

To ease reading, this paper {a} provides key points via sections; {b} covers supporting material in appendices; and {c} structures each section around a summary (provided above in this Section 1), a core set of material, and comments. A comments sub-section provides commentary about the section in which it appears and may preview later sections.

2 Properties of Six Quarks and Three Charged Leptons

Table 1 presents a candidate for a periodic table of charged leptons and quarks. There are two integer indices, n_2 and n_1 .

Table 1 Each of the nine particles has a charge (relative to the charge of a positron), a generation number (gen) traditionally assigned to the particle, and a name. For each of the three empty positions in the table, no particle has been observed.

n_2	Particles and Properties		
	$n_1 = 2$	$n_1 = 1$	$n_1 = 0$
0	charge = -1 gen = 1 electron	charge = +2/3 gen = 1 up	charge = -1/3 gen = 1 down
1		charge = -1/3 gen = 2 strange	charge = +2/3 gen = 2 charm
2	charge = -1 gen = 2 muon	charge = -1/3 gen = 3 bottom	charge = +2/3 gen = 3 top
3	charge = -1 gen = 3 tauon		

As shown in Table 2, (1) approximates the masses of the particles in Table 1.

$$\frac{m(n_2,n_1)}{m_e} \approx exp\left(b_{gr} n_2 + b_{gr}{}^2 \frac{(1+n_2)(2-n_1)}{6} + \delta_{gr}(n_2) \right), \qquad (1)$$

m_e is the mass of an electron ($\approx 0.510998910(13)\ MeV/c^2$) [1],

n_2 is an integer, with $0 \le n_2 \le 3$, (2)

n_1 is an integer, with $0 \le n_1 \le 2$, (3)

$b_{gr} \approx 2.717993261$, (4)

$$\delta_{gr}(n_2) = \log\left(1 + d_{gr}\frac{\sin\left(\frac{2\pi}{3}n_2\right)}{\sin\left(\frac{2\pi}{3}\right)}\right), \quad (5)$$

$$d_{gr} \approx 0.099124099. \quad (6)$$

Formula (7), in which q_{do} is the charge of a down quark ($q_{do} = q_e/3 < 0$, with q_e being the charge of an electron), provides the charges q for particles in Table 1.

$$\frac{q(n_2,n_1)}{q_{do}} = b_{em}(-1)^{n_1}(1+n_1),\ if\ n_2 = 0\ or\ if\ n_1 = 2$$

$$\frac{q(n_2,n_1)}{q_{do}} = b_{em}(-1)^{1+n_1}(1+n_1^*),\ if\ 1 \le n_2 \le 2\ and\ 0 \le n_1 \le 1 \quad (7)$$

$$and\ in\ which\ 0 \le n_1^* \le 1\ and\ n_1^* \ne n_1$$

$$b_{em} = 1. \quad (8)$$

Table 2 For each particle, the upper value characterizes experimentally determined results [2] and the lower value is calculated per (1).

n_2	Masses (MeV/c^2) - Observed and Calculated		
	$n_1 = 2$	$n_1 = 1$	$n_1 = 0$
0	$0.510998910 \pm 0.00000013$ 0.510998910	1.7 to 3.3 1.750	4.1 to 5.8 5.996
1		101^{+29}_{-21} 99.845	1270^{+70}_{-90} 1171.6
2	105.658367 ± 0.000004 105.658367	4190^{+180}_{-60} 4246.9	$172000 \pm 900 \pm 1300$ 170706
3	1776.82 ± 0.16 1776.84		

Comments

Table 1 differs from traditional similar tables in the following ways. This table has 4 rows, not 3. In this table, only the top row presents all three relevant particles from one generation. In this table, the quarks in the second and third rows are in non-traditional positions. (The traditional table has, for example, only charge $= +2/3$ particles in the second column.) This table has empty cells, whereas the traditional table has no voids. This table does not include a column for zero-mass (or small-mass) neutrinos.

b_{gr} and d_{gr} were estimated by fitting the $n_1 = 2$ experimental data and the $n_2 = 2$ experimental data.

Neither (1) nor (7) applies to the (n_2, n_1) pairs for which there are no known particles.

Appendix 1 discusses possibilities that index integers take on four consecutive values. In subsequent sections, this paper explores circumstances in which $n_1 = 3$ applies.

The term δ_{gr} is zero for $n_2 = 0 \; or \; 3$. This term exemplifies adjustment terms that are found in similar equations in this paper. The term δ_{gr} is "relatively small" compared to the other two terms in the sum in (1). As estimated in (5), the term δ_{gr} does not depend on n_1.

The nine masses span a range of more than five orders of magnitude. Equation (1) approximately fits experimental data by using two integer variables (n_2 and n_1) and no more than four (m_e, b_{gr}, $b_{gr}{}^2/6$, and d_{gr}) non-integer constants. Perhaps one can reduce the number of observation-based constants by one by not counting $b_{gr}{}^2/6$. Perhaps one can reduce the number of observation-based constants by considering d_{gr} to be the following analytic number (in which $e \equiv \exp(1)$). Appendix 1 hints at the relevance of $3\log(3)$ as a factor. Appendix 5 further explores implications of using (9).

$$d_{gr} = 3log(3) / \left(\frac{9}{2}e^2\right) \approx 0.099120679 \,. \tag{9}$$

Appendix 2 provides an example of modifying δ_{gr}, by adding dependence on n_1, to try to better fit masses. (See (99) and Table 17.)

Equations like (10) become a means to look from a measured value ($q(0,n_1)$, in the case of (10)) toward a base-state value (q_{do}, or equivalently $q(n_2 = 0, n_1 = 0)$, in the case of (10)). The factor $1/(1+n_1)$ is key.

$$|q_{do}| = |q(0,0)| = \left|\frac{q(0,n_1)}{(1+\;n_1)}\right| . \tag{10}$$

Appendix 2 provides Table 16, which shows twenty "masses" (including ones for $n_1 = 3$ and $n_1 = -1$) calculated via (1).

The following equations define Δ_{gr} and show how closely b_{gr} approximates e.

$$b_{gr} = e(1 + \Delta_{gr}) \,, \tag{11}$$

$$\Delta_{gr} \approx -1.0616 \times 10^{-4} \,. \tag{12}$$

Aspects of this paper generalize on the pattern of particles and voids in Table 1 and Table 2. While this paper continues to use the term *void*, there may be possibilities that some such "*voids*" may actually represent particles that occur rarely or are hard to detect.

3 Possible Attributes of Dark Matter and Dark Energy

The second column of Table 3 provides observed data relevant to the total energy density of the universe.

Table 3 This table shows three observed densities [2]. Other densities - the pressureless matter density of the universe, the CMB radiation density of the universe, and the neutrino density of the universe - are not shown. Multiplying proposed numbers of ensembles by a multiplier produces estimated densities.

Type of Density	Observed Density	Number of Ensembles	Multiplier	Estimated Density
Baryon density of the universe	0.044(4)	1	$1/24$	0.042
Dark matter density of the universe	0.21(2)	5	$1/24$	0.21
Dark energy density of the ΛCDM universe	0.74(3)	18	$1/24$	0.75

The remaining columns show results of assuming that {a} there is one ensemble of baryonic matter (as partly described in Table 1 and Table 2); {b} there are five ensembles of dark matter, with each ensemble having some similarity to the baryonic matter ensemble; {c} baryonic-plus-dark matter is one super-ensemble; {d} there are three dark energy super-ensembles, each comprised of six ensembles; and {e} each of the ensembles contributes a somewhat similar share of the total density. Each calculated estimated density matches the corresponding observed density.

Patterns P describe 24 ensembles. Here, V_{DE} numbers the super-ensembles. Within any ensemble in a super-ensemble, {a}cells (in the ensemble's analogy to Table 1) having $n_1 = V_{DE}$ are void, {b} N_{LE} denotes the value of n_1 for which particles can be considered to be *leptons*, {c} other particles can be considered to be *quarks*, and {d} V_{LE} denotes the value of n_2 for which there is a void in the lepton column.

$$P(N_{DE}, N_{LE}, V_{LE}) \,, \tag{13}$$

$$0 \le V_{DE} \le 3 \,, \tag{14}$$

$$0 \le N_{LE} \le 3, \; N_{LE} \ne V_{DE} \,, \tag{15}$$

$$1 \le V_{LE} \le 2 \,. \tag{16}$$

Comments

These results do not depend on a choice among the following assumptions about fourth generation ($n_2 = 2$) quarks: {a} No fourth generation quarks exist. {b} Fourth generation quarks exist and (referring to Table 1) the pattern of charges mimics that for $n_2 = 0$. Fourth generation quarks exits and the pattern of charges mimics that for $n_2 = 1 \; or \; 2$.

Experiments (using baryonic matter and baryonic-matter equipment) have yet to detect photons attributed to sources other than baryonic matter [3]. Section 11 discusses the difficulty of one ensemble's detecting photons generated by another ensemble.

The baryonic matter ensemble is symbolized by $P(3,2,1)$.

Within a super-ensemble, there are two ensembles for each choice of N_{LE}.

For the five dark-matter ensembles (which share the relationship $V_{DE}=3$, but exclude the baryonic-matter ensemble $P(3,2,1)$), no experiments {a} contradict the ensembles' possible existences or {b} provide evidence as to the values for the corresponding masses or charges [3].

Within each of the 24 ensembles, the three $n_2=0$ particles are assumed to be *ground states* and all $n_2>0$ particles are considered to be *excited states*.

Every ensemble has an $n_2=3$ lepton.

(16) correlates with rows in which, for a forthcoming generalization (equation (22)) of (5), $\delta_{gr}(n_2)$ is non-zero.

4 Relationships between Particle Properties

The following equations could extend (7) and (1) to all 24 ensembles.

$$0\le n_2\le 3, \tag{17}$$

$$0\le n_1\le 3, \tag{18}$$

$$\frac{q(n_2,n_1)}{q(n_2=0,n_1=0)}=b_{em}(-1)^{n_1}(1+n_1),\ if\ n_2=0\ or\ if\ n_1=N_{LE}$$

$$\frac{q(n_2,n_1)}{q(n_2=0,n_1=0)}=b_{em}(-1)^{1+n_1}(1+n_1^{*}),\ if\ 1\le n_2\le 2\ and\ n_1\ne V_{DE}\ and\ N_{LE} \tag{19}$$

$$and\ in\ which\ 0\le n_1^{*}\le 3\ and\ n_1^{*}\ne V_{DE},N_{LE},and\ n_1$$

$$\frac{m(n_2,n_1)}{m(n_2=0,n_1=0)}\approx exp\left(b_{gr}n_2-b_{gr}{}^{2}\frac{(1+n_2)n_1}{f_{gr,1}(V_{DE},N_{LE},V_{LE})}+\delta_{gr}(n_2)\right), \tag{20}$$

$$f_{gr,1}(3,2,1)=6 \tag{21}$$

$$\delta_{gr}(n_2)=\log\left(1+d_{gr}\frac{\sin\left(\frac{2\pi}{3}n_2\right)}{\sin\left(\frac{2\pi}{3}\right)}\right), \tag{22}$$

$$d_{gr}=3log(3)/f_{gr,2}(V_{DE},N_{LE},V_{LE}), \tag{23}$$

$$f_{gr,2}(3,2,1)=\left(\frac{9}{2}e^{2}\right). \tag{24}$$

In the corresponding 4-by-4 periodic tables, voids occur for {a} the four $n_1 = V_{DE}$ positions, {b} the one $n_2 = V_{LE}, n_1 = N_{LE}$ position, and {c} the two positions for which $n_2 = 3 \; and \; V_{DE} \neq n_1 \neq N_{LE}$.

For (19), the following apply. $b_{em} = 1$. The exponential terms ensure that the sign of $q(n_2 = 0, n_1)$ alternates as n_1 changes by 1. The term involving $1 + n_1$ (or $1 + n_1^*$) scales the charge. Within an ensemble, particles having $n_1 = N_{LE}$ have the same charge. Within an ensemble, for a column specified by $V_{DE} \neq n_1 \neq N_{LE}$, two excited-state quarks have one charge and the ground-state quark has another charge. The expression pertaining to $1 \leq n_2 \leq 2$ provides a reversal of columns that generalizes (7). This paper does not speculate about possible $n_2 = 3$ quarks.

For example, for $N_{LE} = 3$, the values of $q(n_2 = 0, n_1)/q(n_2 = 0, n_1 = 0)$ are (in order, from $n_1 = 3$ downward) are $-1, 3/4, -1/2, \; and \; 1/4$. Within this example, if $V_{DE} = 0$, lepton-based measuring system would detect ground-state charges of (respectively) of $\mp 1, \pm 3/4, \mp 1/2,$ and void. The quarks would have the fractional charges, but the denominator is 4, not 3.

For (20), the least "mass" is for the particle or void specified by $n_2 = 0, n_1 = 3$. For the baryonic matter ensemble (which has $V_{DE} = 3$) there are no $n_1 = 3$ particles. In (20), the left-side denominator is not the mass of an electron and the right-side term has, in effect, a factor $-n_1$ in the spot in which (1) has $2 - n_1$. Also, the factor 6 has been re-expressed as $f_{gr,1}(V_{DE}, N_{LE}, V_{LE})$.

The stated expression for (20) does not point out voids. (22) is the same as (5). (23) and (24) replace (6) with an analytic value suitable for the baryonic matter ensemble. The factor $3 log(3)$ matches a number in Table 15 (in Appendix 1). Work leading to the number $3 log(3)$ seems not to depend on the choice of ensemble.

Comments

Perhaps, $f_{gr,1}(V_{DE}, N_{LE}, V_{LE}) = 2(1 + N_{LE})$. There is no known trend allowing estimating $f_{gr,2}(V_{DE}, N_{LE}, V_{LE})$ for ensembles other than $P(3,2,1)$.

(25) provides a form of (20) that looks more like (1) than does (20).

$$\frac{m(n_2, n_1)}{m(n_2 = 0, n_1 = N_{LE})} \approx exp\left(b_{gr} n_2 + b_{gr}{}^2 \frac{(1 + n_2)(N_{LE} - n_1)}{f_{gr,1}(V_{DE}, N_{LE}, V_{LE})} + \delta_{gr}(n_2) \right). \qquad (25)$$

There is no evidence to suggest the extent to which the $q(n_2 = 0, n_1 = 0)$ or $m(n_2 = 0, n_1 = 0)$ vary from ensemble to ensemble.

Section 11 points to possible variation, by super-ensemble, of b_{gr}. Equations (60) (with $n_4 = V_{DE}$) and (61) provide an example of what such variation might be. For that example, for $V_{DE} = 0 \; and \; N_{LE} = 2$, $m(tauon - like - lepton)/m(electron - like - lepton) > \exp(3e)$.

Assuming such base-state constants are roughly the same for each ensemble, (20) suggests that people need not consider the mass of an electron ($n_2 = 0, n_1 = 2$) to be a minimal mass for charged leptons. The mass of $n_2 = 0, n_1 = 3$ particles would be less than the mass of the $n_2 = 0, n_1 = 2$ particles. In the 18 ensembles for which $V_{DE} \neq 3$, the $n_2 = 0, n_1 = 3$ particle would have the lowest mass. In six such ensembles, $N_{LE} = 3$ and this particle would be a lepton.

Within the collection of 24 ensembles, for the baryonic matter ensemble ($P(3,2,1)$) {a} V_{DE} is at the maximum allowed value; {b} within the previous caveat, N_{LE} is at the maximum allowed value; {c} V_{LE} is at its minimum allowed value.

For the $P(3,2,2)$ ensemble, it seems likely that quarks can form nucleon-like particles and that atom-like entities can exist.

For the two $V_{DE} = 2$ *and* $N_{LE} = 3$ ensembles, the three charged leptons have the same sign for their charges as would have any proton-analogs. Atom-like entities with three-quark nucleons would need to {a} have nucleons that include excited-state quarks, or {b} involve anti-leptons. For the latter alternative, an electrically neutral atom could have an integer multiple of four proton-analogs and, for each four proton-analogs, three anti-leptons.

For each of the eight ensembles having $|V_{DE} - N_{LE}| = 2$, the two ground-state quarks have the same sign for their charges and the charged leptons have the opposite sign for their charges.

5 A Relationship between Electromagnetic and Gravitational Interaction Strengths

For an electron and a positron in a flat space-time, $R_{em/gr}$ is the relative strength of the electrical force to the gravitational force and does not depend on the distance separating the particles.

$$R_{em/gr} \equiv \frac{q_e^{\,2} / 4\pi\varepsilon_0}{G_N m_e^{\,2}} \approx 4.16562 \times 10^{42} \ , \tag{26}$$

$q_e \approx -1.602176487(40) \times 10^{-19}\, C$ is the charge of an electron [1],

$\varepsilon_0 \approx 8.854187817 \times 10^{-12}\, F\, m^{-1}$ is the permittivity of free space [1],

$G_N \approx 6.67428(67) \times 10^{-11}\, m^3\, kg^{-1}\, s^{-1}$ is the gravitational constant [1],

$m_e \approx 9.10938215(45) \times 10^{-31}\, kg$ is the mass of an electron [1].

(27) yields a number, $B_{em,gr}$, that is similar to b_{gr}.

$$B_{em,gr} \equiv \frac{1}{36} log\left(\frac{3}{4} R_{em/gr}\right) \approx 2.717993261 \ , \tag{27}$$

$$B_{em,gr} \approx b_{gr} \ . \tag{28}$$

Comments

This paper assumes $B_{em,gr} = b_{gr}$.

Appendix 3 indicates roughly the latitude inherent in the two numbers $R_{em/gr}$ and $B_{em,gr}$, given current experimental accuracies.

Appendix 4 provides an example of how improving the accuracy of some quantities linked by (28), (27), (26), and (1) could lead to more accurate results for other quantities. This example suggests that increasing the measurement accuracy of either the mass of a tauon or the baryonic-matter gravitational constant could lead to a better theoretic estimate for the other of those two quantities.

6 Electromagnetic, Intermediary, and Gravitational Interactions

The following equation restates (27).

$$R_{em/gr} \approx (4/3)exp(36B_{em,gr}) . \tag{29}$$

Assume there is an *interaction space* for which one realm, realm em/gr, is bounded by the electromagnetic and gravitational interactions. Aside from the $n_2 = 0$ particles (such as the electron), the various leptons and quarks in Table 1 correspond to non-zero steps in a transition in inter-particle interactions from spin-2 (gravitons) to spin-1 (photons). For example, as noted in Table 4, a realm-em/gr interaction of a muon with an electron is an interaction that is two steps along, out of thirty-six steps, from a purely graviton-mediated interaction between two electrons to a purely photon-mediated interaction between two electrons.

Table 4 These examples illustrate steps, in realm em/gr of interaction space, away from an electron-electron gravitational interaction toward an electron-electron electromagnetic interaction.

Pair of Interacting Particles	n_2 for the First Particle	n_2 for the Second Particle	Steps (n_2(first particle) + n_2(second particle))
Electron - Electron	0	0	0
Electron - Muon	0	2	2
Muon - Tauon	2	3	5

Staying with this family of lepton-lepton interactions yields the following possible interpretation of 36 steps. Start by considering an electron-electron graviton-mediated interaction. For the first particle, n_2(first particle) might seem to stop at 3, for which the first particle is a tauon. But, as yet there is no consideration regarding the five dark-matter ensembles. To get across the realm, the number of transitions for the first particle needs to get to 18. So does the number of transitions for the second particle. Thus, for a full journey from one end of the realm to the other, the following expressions pertain.

$$transitions(\text{first particle}) = transitions(\text{second particle}) = 18\,, \tag{30}$$
$$transitions(\text{first particle}) + transitions(\text{second particle}) = 36\,. \tag{31}$$

The same number, 18, applies when a particle is a ground-state ($n_2 = 0$) quark.

Given the assumption that all ensemble ground-state particles exist, (31) applies also to all non-void n_1 columns in all 24 ensembles.

(32) establishes notation. (33) reprises results from above.

$$\begin{aligned} &E_{\vartheta_1,\vartheta_2} \equiv \\ &(transitions\ per\ particle\ pair) \\ &\times \left(\frac{number\ of\ ensembles\ similarly\ treated\ at\ the\ \vartheta_2\ end\ of\ the\ realm}{number\ of\ ensembles\ similarly\ treated\ at\ the\ \vartheta_1\ end\ of\ the\ realm} \right) \end{aligned} \tag{32}$$

$$E_{em,gr} = 6 \times \frac{6}{1} = 36\,. \tag{33}$$

Comments

Assuming there are no fourth-generation quarks, each of the two occurrences of the number 18 in (30) includes 6 instances of transitions that would "land on" voids.

7 Interactions and Realms

Table 5 defines notation for various possible interactions and particle properties. The table also shows results developed above (such as for b_{gr}) or to be developed below (such as for b_{wk}).

Table 5 The notation standardizes subscripting conventions. Some of the symbols may not be useful. Numeric results apply for the $V_{DE} = 3$ super-ensemble. Some numbers may vary for other super-ensembles.

Interaction	Force constant	Particle property		Leading constant	Electron property
Basic charge	G_{bc}		m_{bc}	b_{bc}	
Basic mass	G_{bm}		m_{bm}	b_{bm}	
Strong	G_{st}		m_{st}	$b_{st} = 0$	$m_{st,e} = 0$
Weak	$G_{wk}(V_{DE} = 3) \equiv 4c/\hbar$	spin	$m_{wk} = s$	$b_{wk} = 0$	$m_{wk,e} \equiv \hbar/2$
Electromagnetism	$G_{em}(V_{DE} = 3) \equiv 1/4\pi\varepsilon_0$	charge	$m_{em} = q$	$b_{em} = 1$	$m_{em,e} \equiv q_e$
Gravitation	$G_{gr}(V_{DE} = 3) \equiv G_N$	mass	$m_{gr} = m$	$b_{gr} \approx 2.718$	$m_{gr,e} \equiv m_e$
Spin 3	G_{s3}	C&P	n_3	$b_{s3} \approx 4.482$	$n_{3,e} = 2$
Spin 4	G_{s4}	DE	n_4	$b_{s4} \approx 6.255$	$n_{4,e} = 3$

The notation C&P corresponds to the C and P in CPT symmetry. C denotes charge in the sense of matter and antimatter. P denotes handedness in the sense of left-handedness and right-handedness, as pertaining to the weak interaction.

The notation DE denotes dark energy.

n_3 and n_4 are integer variables, similar in some respects to n_1 and n_2. For example, $0 \le n_3 \le 3$ and $0 \le n_4 \le 3$.

Except for b_{s3} and b_{s4} , the numbers stated for the b_ϑ come from direct uses of data. Per Appendix 9, b_{s3} and b_{s4} are consistent with less direct uses of data. The stated values of b_ϑ are consistent with a mathematical series discussed in Appendix 6. Section 11 discusses possible variation, by super-ensemble, of b_{gr} and $B_{em,gr}$. Equations (60) and (61) provide an example of what such variation might be. Possibly, G_N varies by super-ensemble.

Table 6 defines notation for various possible realms. The table also shows results developed above (such as for $B_{em,gr}$) or to be developed below (such as for $B_{wk,em}$). Table 7 shows results developed above (such as for $R_{em/gr}$) or to be developed below (such as for $R_{wk/em}$).

Table 6 The notation standardizes subscripting conventions. Some of the symbols may not be useful.

Realm $\vartheta_1 / \vartheta_2$	Force strength ratio	Channel ratio	$E_{\vartheta_1,\vartheta_2}$	Key constant
bc/bm	$R_{bc/bm}$	$C_{bc,bm}$	$E_{bc,bm}$	$B_{bc,bm}$
bm/st	$R_{bm/st}$	$C_{bm,st}$	$E_{bm,st}$	$B_{bm,st}$
st/wk	$R_{st/wk}$	$C_{st,wk} \approx 21.8$ *or* ≈ 87.1?	$E_{st,wk}$	$B_{st,wk} = b_{wk}$
wk/em	$R_{wk/em}$	$C_{wk,em} \approx 2.94$	$E_{wk,em} = 6$	$B_{wk,em} = -b_{em}$
em/gr	$R_{em/gr}$	$C_{em,gr} = 4/3$	$E_{em.gr} = 36$	$B_{em,gr} = b_{gr}$
gr/s3	$R_{gr/s3}$	$C_{gr,s3} = 3/2$	$E_{gr,s3} = 24$	$B_{gr,s3} = -b_{s3}$
s3/s4	$R_{s3/s4}$	$C_{s3,s4} = 2/1$	$E_{s3,s4} = 24$	$B_{s3,s4} = b_{s4}$

Table 7 The force strength ratios are developed in various sections of this paper.

Realm $\vartheta_1 / \vartheta_2$	Force strength ratio	Force strength ratio	Ratio of realm-boundary forces for an electron and a positron
bc/bm	$R_{bc/bm}$		
bm/st	$R_{bm/st}$		
st/wk	$R_{st/wk}$	$C_{st,wk}$	$\frac{G_{st} m_{st,e}{}^2}{G_{wk} m_{wk,e}{}^2} = \frac{0}{\hbar c} = 0$
wk/em	$R_{wk/em}$	$\approx 7.297 \times 10^{-3}$	$R_{wk/em} \equiv \frac{G_{wk} m_{wk,e}{}^2}{G_{em} m_{em,e}{}^2} = \frac{\hbar c}{\frac{1}{4\pi\varepsilon_0} q_e{}^2}$
em/gr	$R_{em/gr}$	$\approx 4.1656 \times 10^{42}$	$R_{el/gr} \equiv \frac{G_{em} m_{em,e}{}^2}{G_{gr} m_{gr,e}{}^2} = \frac{\frac{1}{4\pi\varepsilon_0} q_e{}^2}{G_N m_e{}^2}$
gr/s3	$R_{gr/s3}$	$\sim 3 \times 10^{-47}$	
s3/s4	$R_{s3/s4}$	$\sim 3 \times 10^{65}$	

The value for $C_{em,gr}$ was developed in Section 5 and Section 6. Other channel ratios are developed in subsequent sections in this paper. Appendix 7 discusses the channel ratios for realms em/gr, gr/s3, and s3/s4 and provides representations, based on 5-by-5 matrices, for realms and realm boundaries.

Appendix 8 discusses possible approximations for the masses of the of the Z and W bosons. The number 5, which is the number of positions on the diagonal of each realm-boundary matrix, appears as a factor. This appendix also indicates a possible instance (that would parallel $C_{wk,em} \neq 3$) of a channel-ratio reduction related to bosons with non-zero mass.

Appendix 9 discusses possible approximations for the masses of the proton and some Δ particles. The appendix notes further appearances of 5 as a factor. It also suggests the values reported in Table 5 for n_3 and n_4 for the electron (and hence for baryonic matter). The value for n_4 may be based on $V_{DE} = 3$.

Appendix 5 calculates implications of using the possible analytic expression, (9), for d_{gr}. For example, it shows a possibly improved value for G_N , the gravitational constant for the $V_{DE} = 3$ super-ensemble. (106) provides a value for G_N. (107) provides a value for Δ_{gr}. Table 20 provides recalculated values for the masses of the six quarks and three charged leptons.

Table 8 notes characteristics for the spin-3 interaction and the spin-4 interaction, as developed in Section 10. Here, {a} r denotes the distance between (the centers of mass or charge of) two entities (denoted respectively by $\vartheta = 1 \textit{ and } \vartheta = 2$) in a flat space-time, {b} $q_\#$ is an arbitrary constant with dimensions of charge, {c} $m_\#$ is an arbitrary constant with dimensions of mass, {d} $r_\#$ is an arbitrary constant with dimensions of distance, {e} Q denotes charge, {f} M

Thomas.J.Buckholtz@gmail.com 1.650.854.7552 http://ThomasJBuckholtz.wordpress.com

denotes mass, {g} M_{boson} denotes the mass of a boson that facilitates the interaction, and {h} $a_{1,boson}$ and $a_{2,boson} > 0$ are scale-factor constants.

$$Q(\vartheta) \approx \frac{Q_{\vartheta}}{q_{\#}} \times \frac{r_{\#}}{r}, \ for\, 1 \le \vartheta \le 2, \tag{34}$$

$$M(\vartheta) \approx \frac{M_{\vartheta}}{m_{\#}} \times \frac{r_{\#}}{r}, \ for\, 1 \le \vartheta \le 2, \tag{35}$$

$$R_M(boson) \approx a_{1,boson} \frac{r_{\#}}{r} \times \exp\left(-a_{2,boson} \frac{M_{boson}}{m_{\#}} \times \frac{r}{r_{\#}} \right), \ for\, M_{boson} > 0, \tag{36}$$

$$R_0(boson) \approx 1, \ for\, M_{boson} = 0. \tag{37}$$

Table 8 The large-distance behavior for interactions mediated by bosons is based on r, the separation of the centers of charge (or mass) for two entities in a flat space-time. Either entity can be, for example, an elementary particle, atom, or astrophysical object.

Interaction	Factor (in large-distance behavior) from the roles of entities 1 and 2	Factor (in large-distance behavior) from the role of the boson	Interaction between two electrons	Interaction between an electron and a positron
Basic charge	$(Q(1)Q(2))^{-1}$	1		
Basic mass	$(M(1)M(2))^{-1}$	1		
Strong	$1 = (Q(1)Q(2))^{0}$	1		
Weak	$1 = (M(1)M(2))^{0}$	$R_M(boson)$		
Electromagnetism	$(Q(1)Q(2))^{1}$	1	Repulsive	Attractive
Gravitation	$(M(1)M(2))^{1}$	1	Attractive	Attractive
Spin 3	$(Q(1)Q(2))^{2}$	1	Attractive	Repulsive
Spin 4	$(M(1)M(2))^{2}$	1	Repulsive	Repulsive

Section 10 discusses implications of the second-power factors ($(Q(1)Q(2))^2$ and $(M(1)M(2))^2$) in the lowermost two rows of Table 8. Section 12 discusses the uppermost two rows of Table 5, Table 6, Table 7, and Table 8.

Comments

This paper assumes $DE = V_{DE}$.

The term "dark energy" may be a misnomer. This paper distinguishes {a} the "stuff" of dark energy, which features super-ensembles that include fermion particles; and {b} the expansion of the universe attributed to dark energy, which features an interaction based on a spin-4 boson.

Table 9 shows four combinations of stuff and interactions. People measure physics properties by using the matter plus left-handed weak interaction combination. Per remarks above, that combination corresponds to $n_3 = 2$.

Table 9 To achieve CPT symmetry, various combinations of stuff and interactions correspond to various (relative) signs of T.

n_3	Stuff	Weak interaction	Relative sign of T
(?)	antimatter	right-handed	+
(?)	antimatter	left-handed	-
2	matter	left-handed	+
(?)	matter	right-handed	-

The four values of n_3 correspond, in some not-yet-fully-determined order, to four combinations of stuff and interactions. The four combinations are {a} matter and left-handed weak interactions, {b} antimatter and left-handed weak interactions, {c} matter and right-handed weak interactions, and {d} antimatter and right-handed weak interactions.

8 Weak, Intermediate, and Electromagnetic Interactions

Table 10 presents a candidate for a periodic table for realm wk/em, which is bounded by the weak and electromagnetic interactions. There are two integer indices, n_1 and n_0.

Table 10 Each of the nine baryonic ensemble particles has a charge (relative to the charge of a positron) and a name.

n_1	Particles and Properties			
	$n_0 = 3$	$n_0 = 2$	$n_0 = 1$	$n_0 = 0$
0		charge = -1/3 bottom	charge = -1/3 strange	charge = -1/3 down
1		charge = +2/3 top	charge = +2/3 charm	charge = +2/3 up
2	charge = -1 tauon	charge = -1 muon		charge = -1 electron
3				

The following equations fit properties of the particles in Table 10. Here, s denotes spin and $\hbar$ is Planck's constant.

$$\frac{s(n_1,n_0)}{s(n_1=0,n_0=0)}=b_{wk}n_0+\delta_{wk}(n_1,n_0)\,, \quad (38)$$

$$s(n_1=0,n_0=0)=\frac{\hbar}{2}\,, \quad (39)$$

$b_{wk}=0$, (40)

$\delta_{wk}=1$, (41)

$\hbar=1.054571628(53)\times10^{-34}$ J s [2], (42)

$$\frac{q(n_1,n_0)}{q(n_1=0,n_0=0)}=b_{em}(-1)^{n_1}(1+n_1)\,. \quad (43)$$

The following equation provides an analogy to (29).

$$R_{wk/em}\approx C_{wk,em}exp(E_{wk,em}B_{wk,em})\,. \quad (44)$$

$B_{wk,em}=-b_{em}<0$ because the weak interaction is weaker than the electromagnetic interaction. $E_{wk,em}=6$ because there is no ensemble consolidation across the region. The following result fits the fine structure constant, α.

$$\alpha\equiv\frac{q_e^{\;2}}{(4\pi\varepsilon_0)\hbar c}=\frac{q_e^{\;2}/4\pi\varepsilon_0}{(\hbar/2)^2\,\frac{4c}{\hbar}}=7.2973525376(50)\times10^{-3}\ \ [2], \quad (45)$$

$$\alpha\approx R_{wk,el}\ \text{, for} \quad (46)$$

$$C_{wk,em}\approx 2.943962=3(1+\Delta_{wk,em})\,, \quad (47)$$

$$\Delta_{wk,em}\approx-0.01868 \quad (48)$$

Comments

$C_{wk,em}\sim3$ because the weak interaction has three carriers (the W^+, W^-, and Z bosons), while the electromagnetic interaction has one carrier (the photon). Appendix 8 points to other possible "partial blockage" of channels involving the W and Z bosons. The phenomenon may correlate with the bosons having non-zero-mass.

That $E_{wk,em}=6$ reinforces the concept of counting transitions (away from ground states) that land on or start from voids.

9 Strong and Weak Interactions

(49) extends trends and includes the assumption $B_{st,wk}=b_{wk}$. The logarithmic behavior in (51) extends the progression from exponential behavior in (20) to linear behavior in (43). (52)

follows from (51) because charged leptons do not partake in the strong interaction. Appendix 6 provides $b_{st} = 0$ and (53) follows.

$$R_{st/wk} \approx C_{st,wk} exp(E_{st,wk} B_{st,wk}) = C_{st,wk} \tag{49}$$

$$\frac{s(n_0, n_{-1})}{s(n_0 = 0, n_{-1} = 0)} = 1 \,, \tag{50}$$

$$\frac{m_{st}(n_0, n_{-1})}{m_{st}(n_0 = 0, n_{-1} = 0)} = \log(b_{st} n_{-1} + \delta_{st}(n_0, n_{-1})) \tag{51}$$

$$b_{st} n_{-1} + \delta_{st}(n_0, n_{-1}) = 1, \; for\, n_{-1} = N_{LE} = 2 \,, \tag{52}$$

$$\frac{m_{st}(n_0, n_{-1})}{m_{st}(n_0 = 0, n_{-1} = 0)} = \log(\delta_{st}(n_0, n_{-1})) \tag{53}$$

Comments

Assuming (54), there would be 64 or 256 gluons, with the larger number recognizing 4 quark colors and one void quark color state. The former number could be consistent with there being no fourth-generation quarks. The latter number could be consistent with the key role of voids in this paper. Based on there being three weak-interaction bosons, (55) follows.

$$number\ of\ gluons = 2^{number\ of\ relevant\ quarks} \tag{54}$$

$$\begin{aligned} R_{st/wk} &\approx C_{st,wk} \approx \frac{64}{2.94} \approx 21.8,\ if\ there\ are\ 3\ relevant\ quarks \\ R_{st/wk} &\approx C_{st,wk} \approx \frac{256}{2.94} \approx 87.1,\ if\ there\ are\ 4\ relevant\ quarks \end{aligned} \,. \tag{55}$$

This paper does not compare (55) to possible experimentally reported numbers. Possibly, literature does not provide a known-to-be suitable observed value for $R_{st/wk}$.

Table 11 presents a candidate for a relevant periodic table. There are two integer indices, n_0 and n_{-1}. The weak interaction applies throughout. The strong interaction applies to the rightmost two columns and not to the $n_{-1} = 2$ column.

Table 11 Each of the nine baryonic ensemble particles has a charge (relative to the charge of a positron) and a name.

n_0	Particles and Properties			
	$n_{-1}=3$	$n_{-1}=2$	$n_{-1}=1$	$n_{-1}=0$
0		charge = -1 electron	charge = +2/3 up	charge = -1/3 down
1			charge = +2/3 charm	charge = -1/3 strange
2		charge = -1 muon	charge = +2/3 top	charge = -1/3 bottom
3		charge = -1 tauon		

10 Gravitational, Spin-3, and Spin-4 Interactions

There are two more known sets of properties. One involves two considerations, {a} the charge (C) of the "matter" form of the electron and {b} the handedness (P) of the weak interaction. The other involves the four super-ensembles.

Assuming the trend of alternating signs in the rightmost column of Table 6, $B_{gr,s3}$ is negative, $R_{gr/s3}$ is less than one, $B_{s3,s4}$ is positive, and $R_{s3/s4}$ is greater than one.

$$R_{gr/s3} = C_{gr,s3}\exp(24B_{gr,s3}) = (3/2)\exp(24B_{gr,s3}) \sim 3\times10^{-47}, \quad (56)$$
$$R_{s3/s4} = C_{s3,s4}\exp(24B_{s3,s4}) = (2/1)\exp(24B_{s3,s4}) \sim 3\times10^{65}, \quad (57)$$
$$R_{em/s3} \equiv R_{em/gr}R_{gr/s3} \sim 1\times10^{-4} \quad (58)$$
$$R_{gr/s4} \equiv R_{gr/s3}R_{s3/s4} \sim 1\times10^{19} \quad (59)$$

(58) suggests that C&P repulsion is, for an electron and any one of its three counterparts, stronger than the electromagnetic interaction. In a laboratory, such does not apply for a positron as a counterpart. This paper assumes that the spin-3 and spin-4 forces decrease - in a flat space-time - in proportion to r^{-4}, unlike the r^{-2} behavior for electromagnetism and gravity.

Comments

The spin-4 interaction leads to the observed continuing expansion of the universe. (59) implies that, for an electron and a positron, the spin-4 force is weaker than gravity. And, the s4 force decreases, with distance, more rapidly than does gravity. However, the s4 repulsion accumulates as one considers ever-larger entities. Table 12 illustrates the scaling shown in Table 8. For a relatively uniformly populated universe of objects of increasingly large size, the s4 interaction can overtake the gravitational interaction at some distance and maintain its dominance at yet larger distances.

Table 12 The illustrated scaling assumes that entities 3 and 4 have double the scale-length of entities 1 and 2. For example, entity 3 has twice the radius of entity 1. And, entities 3 and 4 are twice as far apart as entities 1 and 2.

Property	Entity 1 or 2	Entities 1 and 2	Entity 3 or 4	Entities 3 and 4
relative mass	1		8	
relative separation		1		2
ratio of gravitational effects		1		$16 = \frac{8 \times 8}{2 \times 2}$
ratio of spin-4 effects		1		$256 = \left(\frac{8 \times 8}{2 \times 2}\right)^2$

Possibly, in processes related to the big bang, the spin-3 force repels and separates from each other the four combinations of stuff and interactions. A possible interpretation of Table 9 features our universe being associated with one or four 4-line vertex interactions in a super-universe. The possibility for four such interactions corresponds to the four values of DE. Each interaction would have one diagram-line corresponding to each of the rows in Table 9.

After the big bang, within an ensemble, overall large-scale effects of the s3 interaction average to negligible, just as do overall effects of the electromagnetic interaction.

11 Photons and Gravitons

Standard treatment of harmonic oscillators indicates an energy of $E_n = \hbar\omega(n + (1/2))$ for the $n-th$ state, with $\hbar\omega$ stating a unit of energy and $n \geq 0$.

For a hypothetical one-dimensional space component of space-time, the energy of a photon state is $E_n = \hbar\omega(n + (1/2))$.

For a three-dimensional space, with $<\mathrm{x}, \mathrm{y}, \mathrm{z}>$ denoting the state for each of the three spatial components of the vector potential, the ground state is a combination of states like $<0,0,-1>$. Here the energy is $\hbar\omega(1/2)$, not the $\hbar\omega(3/2)$ associated with $<0,0,0>$. The state $<k,0,-1>$ represents a $k-th$ x-linear-polarization excitation of a photon mode for photons moving parallel to the z direction. For $<0,0,-1>$, one has the potential for photon motion in the z-direction and excitation in the x and y directions. Within the realm of photonics, this explains the lack of longitudinal polarization. While the x and y states can be excited, the $\sqrt{1+n} = \sqrt{1-1} = \sqrt{0}$ factor attached to the raising operator (when applied to the -1 state) prohibits excitation of z states by a photon.

$<0,0,-1>$ can be excited to $<0,-1,1>$ by a graviton. For example, one can envision an interaction in which z absorbs a graviton, y absorbs a photon, and y sheds a graviton. The result is $< 0, -1, 1 >$ and the photon is now traveling parallel to the y axis. This type of interaction provides for the bending of light via the presence of gravity.

There is a yet-lower energy ground state available, assuming nature behaves (in the following regard) as if there are four dimensions, say $< \mathrm{w}, \mathrm{x}, \mathrm{y}, \mathrm{z} >$. States like $< -1, 0, 0, -1 >$ have energy of zero. The number of ways to distribute the two 0's and two -1's between four positions is six. Only one of those six has possibilities for both x-excitations and y-excitations via photons. Two combinations admit x-excitations but not y-excitations. Two combinations admit y-excitations but not x-excitations. One combination admits neither x-excitations nor y-excitations.

These considerations provide a basis for the matter in an ensemble (such as, for example, the baryonic matter ensemble) not being able to detect readily photons generated by other ensembles (such as, for the same example, the five dark matter ensembles) in the same super-ensemble. Each of the six ensembles emits and absorbs its own unique one of the six types of photons. For baryonic matter to observe a photon from another ensemble, involvement of gravitons (or potentially s3 or s4 bosons) is needed.

$< 0, 0, 0, -2 >$ states also have energy of zero. Paralleling discussion above, there are four ways to distribute three 0's and one 1 among four positions. This provides a basis for each of the four super-ensembles not being able to detect gravitational effects of the other three super-ensembles. Borrowing from the choice made above for baryonic matter, $< 0, 0, -2 >$ has the potential for graviton motion in the z-direction and excitation in the x and y directions. Thus, gravitons have no longitudinal polarization and have two possible transverse (or two possible circular) polarization states.

There seems to be no requirement that each super-ensemble has the same values as other super-ensembles for b_{gr} and $B_{em,gr}$. For example, perhaps the following equations pertain. If so, an average across super-ensembles of the b_{gr} (or $B_{em,gr}$) could be e.

$$B_{em,gr}(n_4) = b_{gr}(n_4) \approx \left(1 + \frac{2n_4 - 3}{3} \Delta_{gr}(3)\right) e,\ 0 \le n_4 \le 3, \tag{60}$$

$$\Delta_{gr}(3) \equiv \Delta_{gr} \text{ per (11) and (12).} \tag{61}$$

Comments

The number of available transverse dimensions for photon excitation correlates with (in a perhaps more sophisticated, but for the purposes of this paper not necessarily more revealing, spherically symmetric treatment) the number of available circular polarization states.

These considerations couple realms wk/em and gr/s3. For realm wk/em weak interactions, the s3 interaction implies that only one handedness is (relatively) easily observable. The other handedness becomes (much) harder to detect.

In three or more dimensions, the function $(1/r^2)\exp(-ar^2)$ (with $r \ge 0$ being a radial coordinate and with any $a > 0$) integrates to a finite value. Photon-related states discussed in this section can be represented by wave functions. The lowest-energy states have wave-functions that have radial dependence $(1/r)\exp(-ar^2)$ and normalization integrals that involve $(1/r^2)\exp(-ar^2)$.

The above remarks do not include discussions of excitations via the s3 and s4 interactions.

These considerations might suggest that neutrinos need not have mass in order to interact with gravity.

This paper does not address the extent to which, assuming b_{gr} and $B_{em,gr}$ vary by super-ensemble, various analog-factors vary by super-ensemble. Referring to (26), (27), (28), and Table 5, such analog-factors could include $V_{DE} \neq 3$ analogs to q_e, $1/4\pi\varepsilon_0$, G_N, and/or m_e.

This paper does not further address the evident opportunity to consider that "any" harmonic oscillator (with states, say, $< x >$) could or should be considered to be paired with another harmonic oscillator (with states, say, $< y >$) such that the states of x are characterized by $< x, y >=< k, -1 >$, with $k \geq 0$.

12 The Basic Charge and Basic Mass Interactions

Assume that the basic charge interaction and basic mass interaction (Section 7) exist. Based on {a} equation (35), {b} the second row in Table 8, and {c} standard considerations about harmonic oscillators, the following pertain regarding the basic mass interaction.

$$A(M(1)M(2), n) = n + \frac{1}{2}\text{, for some integers } n \geq 0\text{, where} \tag{62}$$

$$A(M(1)M(2), n)\text{, is the "applicability" of the factor } M(1)M(2). \tag{63}$$

Similar results pertain for the basic charge interaction. In practicality, each applicability factor should be able to take on values of zero and one. Paralleling work in Section 11, the following four-element state description pertains. Here the focus is on interactions, bc denotes the quantum number for the charge-related applicability factor, and bm denotes the quantum number for the mass-related applicability factor. There is a complementary quantum number for each factor.

$$\lfloor bc, bm, \overline{bc}, \overline{bm} \rfloor. \tag{64}$$

The interaction $\lfloor 1,1,-1,-1 \rfloor$ pertains to interactions between entities having non-zero charge and non-zero mass. Much of this paper has discussed fermion particles that match this description. This interaction also applies to entities such as atoms and astrophysical objects, even if the net charge of an entity is zero.

That these considerations cover interactions in which the entities can be elementary particles or astrophysical objects suggests (regarding (36) and (37)) that {a} (37) pertains, or {b} any mass for carrier bosons would likely need to be an effect of a combination of {i} an interaction between interaction space and space-time and {ii} the structure of curved space-time.

Comments

The interaction $\lfloor 0,0,0,0 \rfloor$ applies to unoccupied physics-allowed states of particles or other entities.

Perhaps, interactions between an entity and a photon or an ordinary (or "zero or low-mass") neutrino correspond to interactions like $\lfloor 0,0,1,-1 \rfloor$ and $\lfloor 0,0,-1,1 \rfloor$. Perhaps one combination of these corresponds to interactions with photons and an orthogonal combination corresponds to interactions with neutrinos. Such would seem to suggest that ordinary neutrinos have masses of zero. Per Table 8, neutrinos need not have mass in order to interact via the weak interaction.

Possibly, photons correspond to symmetric combinations and neutrinos correspond to anti-symmetric ones. Assume so. Then, $(1/\sqrt{2})(\lfloor k,0,1,-1 \rfloor + \lfloor k,0,-1,1 \rfloor)$ interactions correspond to $k-times$ excited photon states. And, $(1/\sqrt{2})(\lfloor k,k,1,-1 \rfloor + \lfloor k,k,-1,1 \rfloor)$ might pertain to, for example, an alpha-particle laser.

Drawing on work in Appendix 1 and paralleling work in Section 11, work in the present section suggests that the time-like dimension of space-time should be treated as being a collapse of three time-like dimensions. Such a view fulfills (92). For such a view, Appendix 1 provides a form of gamma matrices.

Table 8 shows a pattern of four pairs of "charge-related" and "mass-related" interactions. The four charge-related interactions are basic charge, strong, electromagnetism, and spin-3. The four mass-related interactions are basic mass, weak, gravitation, and spin-4. This paper does not speculate regarding the extent to which the four pairs correspond to another application of the concept of four consecutive values of an integer index, such as developed in Appendix 1. Nor does this paper speculate about the extent to which either of the interaction quadruples (the charge-related one or the mass-related one) corresponds to an application of the same concept.

This paper does not speculate about possibilities for interactions corresponding to rows that would lie above or below the eight rows shown in Table 8.

Perhaps there is a correspondence between magnetic monopoles and terms such as $M(1)Q(2)$. (See (34), (35), and Table 8.) If so, a lack of one implies a lack of the other.

13 Appendices

We develop theoretic underpinnings for integer-index ranges and limits, such as that n_2 ranges over and is limited to 4 consecutive integer values (as in (2)). We provide a basis for emphasizing integer-multiples of $6 = 2 \times 3$ transitions for transiting a realm. We provide a basis for the appearance of $3\log(3)$. We develop generalized gamma matrices. We link the Einstein field equations to a form of uncertainty. (Appendix 1)

We extend Table 2 to show "masses" for voids in that table and for a column with $n_1 = -1$. We explore the possibility of adjusting (1), to better fit the masses of low-mass quarks, by inserting a new term. (Appendix 2)

We explore the ranges of acceptable $R_{em/gr}$ and $B_{em,gr}$ for baryonic matter. (Appendix 3)

We explore a relationship between the strength or electromagnetism, the strength of gravity, and the masses of leptons. In particular, for baryonic matter, we estimate bounds on the gravitational constant, based on the accuracy of the mass of a tauon. (Appendix 4)

We explore the impact, on estimating the gravitational constant for baryonic matter, of using an analytical version of the constant $\delta_{gr}(n_2)$ in (1). (Appendix 5)

We discuss a family of functions for which zeros correspond to values of various b_ϑ. (Appendix 6)

We provide graphical depictions symbolizing some interaction-space realms. (Appendix 7)

We explore possible approximate expressions for the masses of Z and W bosons. Results support a previously used notion that these bosons correspond to the diagonal in a 5-by-5 matrix in interaction space. Results also support the concept of "partial blockage" for channels associated with bosons of non-zero mass. Another expression may link the masses of pi mesons to the mass of an electron. (Appendix 8)

We exhibit an expression linking the mass of a proton to the mass of an electron. This equation suggests that the integer index, n_3, related to the s3-interaction satisfies $n_3 = 2$ for baryonic matter. Another expression links the mass of Δ particles to the mass of an electron and indicates that $n_4 = 3$ for baryonic matter. (Appendix 9)

Comments

Some wording in appendices favors "we …" over "this paper …"

Appendix 1 Systems, subsystems, uncertainty operators, and integer-index ranges

Systems, subsystems, and uncertainty operators

We discuss concepts such as systems, interacting subsystems, and uncertainty operators.

We distinguish three concepts - a universe, a primary subsystem, and a complementary subsystem. We assume the complementary subsystem interacts with the primary subsystem. One set of operators acts on the primary subsystem. We use the notation $\hat{\vartheta}$ to apply to an aspect (generically ϑ) associated with a complementary subsystem. We use the term system to denote the combination of the primary subsystem and complementary subsystem. The universe contains the system and possibly more.

We denote the amplitude for the primary subsystem by Ψ. We assume the amplitude can be normalized. That is, …

$$0 < \langle 1 \rangle \equiv \langle \Psi^* | 1 | \Psi \rangle < \infty . \qquad (65)$$

We define, for an operator o, an uncertainty operator $U(o)$ by the following equation. In (66), the two appearances of $\langle 1 \rangle^2$ compensate for the two appearances of $\langle o \rangle$ and allow for computations in which one might not want to normalize Ψ. I denotes the identity operator.

$$\langle 1 \rangle^2 U(o) \equiv \langle 1 \rangle^2 o^2 - \langle o \rangle^2 I . \qquad (66)$$

For $\langle 1 \rangle = 1$, the expected values of terms in (66) yield the following statement of uncertainty. For the remainder of this paper, we assume $\langle 1 \rangle = 1$.

$$\langle U(o)\rangle = \langle o^2\rangle - \langle o\rangle^2 . \tag{67}$$

Eventually, we will use various sets of indices. Such indices label operators and are not continuous coordinates. The following are examples of such sets.

$$C(4n) \equiv \ 0,1,2,3 \ \ , \tag{68}$$
$$C(4a) \equiv \ t,x,y,z \ \ ,$$
$$C(6a) \equiv \ u,v,w,x,y,z \ \ .$$

We use the symbol $O \equiv \{o_j | j\epsilon C\}$ to denote a set of primary-subsystem operators o_j indexed by a set C. For example, for the index set $C(4n)$, the following are the elements of a set we denote by $O(4\sigma)$. The elements are the Pauli matrices.

$$\sigma_0 \equiv \begin{pmatrix} +1 & 0 \\ 0 & +1 \end{pmatrix}, \tag{69}$$
$$\sigma_1 \equiv \begin{pmatrix} 0 & +1 \\ +1 & 0 \end{pmatrix},$$
$$\sigma_2 \equiv \begin{pmatrix} 0 & -i \\ +i & 0 \end{pmatrix},$$
$$\sigma_3 \equiv \begin{pmatrix} +1 & 0 \\ 0 & -1 \end{pmatrix}.$$

Eventually, we will use various coordinate domains. For finite sets of discreet coordinates, we define the following notation for domains.

$$D(discreet, n_{min}, n_{max}) \equiv \{\vartheta | \vartheta \, is \, an \, integer, n_{min}, \leq \vartheta \leq, n_{max}\} .$$

We assume that the complementary subsystem interacts with the primary subsystem and that $\hat{O}$ is a set of operators for the complementary subsystem.

$$\hat{O} = \{\hat{g}_{jk} \mid j\epsilon C, k\epsilon C\} .$$

We define an uncertainty operator $U(\hat{O},O)$ as follows. This operator operates in the space $\{\hat{\Psi}\} \otimes \{\Psi\}$.

$$U(\hat{O},O) \equiv \sum_{j\epsilon C, k\epsilon C} o_j \hat{g}_{jk} o_k - \langle o_j\rangle \hat{g}_{jk} \langle o_k\rangle I \ \ . \tag{70}$$

An example of uncertainty

We explore consequences of (70) for a 2-state primary subsystem and a 2-state complementary subsystem. In particular, we look at effects of axis reversals on uncertainty. We discover a candidate quantum number. We discuss symmetries within a subsystem and identify raising and lowering operators.

For this example, the primary subsystem features $C(4n)$, $D(discreet,1,2)$, and $O(4\sigma)$. The complementary subsystem involves a set of amplitudes $\hat{\Psi}$ over a new instance of $D(discreet,1,2)$ and a use, $\hat{g}$, of operators paralleling those of $O(4\sigma)$.

$$\hat{O} = \{\hat{g}_{jk} \equiv \hat{\sigma}_j \hat{\sigma}_k \mid j \epsilon \hat{C}(4n) \equiv C(4n), k \epsilon \hat{C}(4n)\}. \tag{71}$$

Table 13 notes various terms in (70) and shows the contribution to $\langle U(\hat{O},O) \rangle$ of the sum of the terms in the respective rows. Here, $U(\hat{O},O)$ features a computation involving both $\hat{\Psi}$ and Ψ. While the $j=0, k\neq 0$ row and the $j\neq 0, k=0$ row contribute nothing to $\langle U(\hat{O},O) \rangle$, the corresponding six terms (as operators) are non-zero. For example, pertaining to $j=0, k\neq 0$, we note that $\sigma_k - \langle \sigma_k \rangle I \neq 0$. Generally, depending on $\hat{\Psi}$ and Ψ, $0 \leq \langle U(\hat{O},O)(\hat{\Psi},\ \Psi) \rangle \leq 4$. Table 14 provides examples.

Table 13 The right-most column shows the sum of contributions, from the corresponding terms to, $\langle U(\hat{O},O) \rangle$. Only the last row varies as a function of $\hat{\Psi}$ or Ψ.

j	k	Terms	Contribution to the expected value, from the sum of the terms
0	0	$\hat{\sigma}_0\hat{\sigma}_0\sigma_0\sigma_0 - \hat{\sigma}_0\hat{\sigma}_0 \langle \sigma_0 \rangle \langle \sigma_0 \rangle$ $= 0$	0
0	$\neq 0$	$\hat{\sigma}_0\hat{\sigma}_k\sigma_0\sigma_k - \hat{\sigma}_0\hat{\sigma}_k \langle \sigma_0 \rangle \langle \sigma_k \rangle$	0
$\neq 0$	0	$\hat{\sigma}_j\hat{\sigma}_0\sigma_j\sigma_0 - \hat{\sigma}_j\hat{\sigma}_0 \langle \sigma_j \rangle \langle \sigma_0 \rangle$	0
$\neq 0$	$= j$	$\hat{\sigma}_j\hat{\sigma}_j\sigma_j\sigma_j - \hat{\sigma}_j\hat{\sigma}_j \langle \sigma_j \rangle \langle \sigma_j \rangle$ $= \sigma_j\sigma_j - \langle \sigma_j \rangle \langle \sigma_j \rangle$	2
$\neq 0$	$\neq j$	$\hat{\sigma}_j\hat{\sigma}_k\sigma_j\sigma_k - \hat{\sigma}_j\hat{\sigma}_k \langle \sigma_j \rangle \langle \sigma_k \rangle$ $= i\varepsilon_{jkl}\hat{\sigma}_l \;\; i\varepsilon_{jkl}\sigma_l - \langle \sigma_j \rangle \langle \sigma_k \rangle$	$-2 \leq contribution \leq +2$

Table 14 The right-most column notes contributions values, which depend on the choice of $\hat{\Psi}$ and Ψ.

$\hat{\Psi}$	Ψ	$\left\langle U(\hat{O},O)(\hat{\Psi},\Psi)\right\rangle$
$\begin{pmatrix}1\\0\end{pmatrix}$	$\begin{pmatrix}1\\0\end{pmatrix}$	0
$\begin{pmatrix}0\\1\end{pmatrix}$	$\begin{pmatrix}1\\0\end{pmatrix}$	4
$\frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix}$	$\begin{pmatrix}1\\0\end{pmatrix}$	2

We consider symmetries under flip transformations defined by $\sigma_j \leftarrow -\sigma_j$, for $j\epsilon C(4n)$. For $\sigma_0 \leftarrow -\sigma_0$, the following pertain.

$$\begin{gathered}\sigma_0 \leftarrow -\sigma_0, \\ \sigma_k \leftarrow +\sigma_k, \, for\, k \neq 0, \\ \left\langle U(\hat{O},O)(\hat{\Psi},\Psi)\right\rangle \leftarrow +\left\langle U(\hat{O},O)(\hat{\Psi},\Psi)\right\rangle.\end{gathered} \tag{72}$$

For $j=1,2,or\,3$, considering the last row in Table 13, we find that for $\sigma_j \leftarrow -\sigma_j$ that the following pertain.

$$\begin{gathered}\sigma_j \leftarrow -\sigma_j, \\ \sigma_k \leftarrow +\sigma_k, \, for\, k \neq j, \\ \left\langle U(\hat{O},O)(\hat{\Psi},\Psi)\right\rangle \leftarrow 4-\left\langle U(\hat{O},O)(\hat{\Psi},\Psi)\right\rangle.\end{gathered} \tag{73}$$

In effect, we can describe two parity-like phenomena. One corresponds to the appearances of the ε_{jkl} in Table 13. The other corresponds to an ε_{jklm} that includes 0 as an index value. The former corresponds to aspects of parity/opposite-parity. The latter corresponds to aspects of particle/anti-particle. The latter links parity/opposite-parity to particle/anti-particle.

For a specific complementary subsystem, we can envision perhaps as many as $16=2^4$ primary-subsystem choices of axes, based on 4 choices of sign: $\pm\sigma_0, \pm\sigma_1, \pm\sigma_2,$ and $\pm\sigma_3$. Each of (72) and (73) halves the number of potentially indistinguishable isomers.

We assume that there is (without loss of generality) a starting configuration in which each primary subsystem axis can be considered to be aligned with the corresponding complementary subsystem axis. We adopt a quantum number n_f to denote the number of operators σ_j in the set $\{\sigma_1,\sigma_2,\sigma_3\}$ for which, after a transition, the sign is the opposite of the sign for the corresponding $\hat{\sigma}_j$. The four possible values for n_f are as follows. Table 15 summarizes the number of unique configurations that achieve the various values of n_f.

$$n_f = 0,1,2,3 . \qquad (74)$$

Table 15 The third column shows the number of configurations that can achieve the result indicated in the first column.

Final n_f	Transition to the Final n_f	$c(n_f) =$ Number of Configurations	$c(n_f) log(c(n_f))$
0	Flip no axes.	1	0
1	Flip once any 1 of 3 axes.	3	$3log(3)$
2	Flip once any 2 of 3 axes.	3	$3log(3)$
3	Flip once all 3 axes.	1	0

If the set $\{\hat{\Psi}\}$ is limited to the single element having $\hat{\Psi}(1) = 1$ and $\hat{\Psi}(2) = 0$, the set $\{\hat{\Psi}\}$ determines a preferred 3-axis for the primary subsystem. Borrowing from notation sometimes used with harmonic oscillators, we define the following raising operator (a_+) and lowering operator (a_-) for the primary subsystem.

$$a_+ = \frac{1}{2}(\sigma_1 + i\sigma_2) , \qquad (75)$$
$$a_- = \frac{1}{2}(\sigma_1 - i\sigma_2) .$$

We calculate the following.

$$\sigma_1 = a_+ + a_- , \qquad (76)$$
$$\sigma_2 = -i(a_+ - a_-) ,$$
$$\{a_+ , a_-\} \equiv a_+ \ a_- + a_- a_+$$
$$\sigma_1{}^2 = a_+{}^2 + \{a_+ , a_-\} + a_-{}^2 = 0 + \{a_+ , a_-\} + 0 = \{a_+ , a_-\} ,$$
$$\sigma_2{}^2 = -(a_+{}^2 - \{a_+ , a_-\} + a_-{}^2) = 0 + \{a_+ , a_-\} + 0 = \{a_+ , a_-\} .$$

Pulling together various items above yields the following.

$$U(\hat{O}, O) = 2\{a_+ , a_-\} . \qquad (77)$$

We can observe that (77) calls attention to an interpretation of $U(\hat{O}, O)$ as measuring an ability for the primary subsystem to interact (such as via mutual flips) with a complementary system. If we carry concepts and notation regarding harmonic oscillators farther, we can associate σ_1 (or σ_2) with a dimensionless momentum and σ_2 (or, respectively, σ_1) with a dimensionless position. A Hamiltonian H can be represented as the following.

$$H \equiv \left(\frac{1}{2}\sigma_1^{\,2} + \frac{1}{2}\sigma_2^{\,2} \right) = \{a_+, a_-\} \,. \tag{78}$$

We can consider that the following pertain.

$$\pm\sigma_1, \pm\sigma_2, \pm\sigma_3, \pm\sqrt{\{a_+, a_-\}} \,\epsilon\, \{o|o^2 = H\}, \tag{79}$$

$$\sqrt{H} = \pm\sigma_3 = \pm\frac{1}{2}(\sigma_1\sigma_2 - \sigma_2\sigma_1) \,. \tag{80}$$

If, instead, such a set $\{\hat{\Psi}\}$ includes all possible amplitudes $\hat{\Psi}$, the complementary system does not specify a preferred axis. We can consider that there is an orthogonal basis for equivalents to O that consists of one of each of the following.

$$\begin{gathered} \pm I = \sqrt{I} \;, \\ \pm \textit{ a dimensionless momentum}, \\ \pm \textit{ a dimensionless position}, \\ \pm \textit{ a dimensionless square root of energy}. \end{gathered} \tag{81}$$

There are potentially $16 = 2^4$ such sets equivalent to O. We have found the beginning of an isomer-counting path similar to that described above and also leading to a result of four. (See discussion about and after (72) and (73).) Also, (82) pertains.

$$\left\langle U(\hat{O}, O) \right\rangle = 2 \,. \tag{82}$$

A similar example, with a scenario featuring measurement

We explore consequences of (70) for a 2-state primary subsystem and a complementary subsystem that supplies metrics. We discuss an example involving a system in which physical constants can apply.

We consider 2-state amplitudes Ψ over $D(discreet,1,2)$. When the complementary subsystem measures the primary subsystem, the measurement, in effect, selects an axis. Without loss of generality for the purposes of this example, we assume the selected axis is the 3-axis. (For space-time measurements in traditional space-time, measurements involve scale factors, such as $\hbar / 2$ for spin, or coordinates, or derivates with respect to coordinates, but here we have no such factors or coordinates with which to work.) We note the following.

$$\begin{gathered} \sigma_0 = 1, \\ -1 \le \, < \sigma_3 > \, \le +1. \end{gathered} \tag{83}$$

The complementary subsystem can detect the following 2 items.

$$\textit{Existence}: \sigma_0 \,, \tag{84}$$

$$Property : \sigma_3 .$$

The above is invariant to $\sigma_j \leftarrow -\sigma_j$, for $j = 1 or 2$. Based on Table 13, the contributions from $j \neq 0, k \neq j$ terms to sum to zero.

$$\left\langle U(\hat{O},O) \right\rangle = 2 . \tag{85}$$

For systems that admit constants as parameters, the following provides a parallel derivation. It is natural to ask, "For such a system, what invariants exist and how can we construct them?" For this example, we adopt the following metric in which $\hat{\vartheta}$ and $\hat{s}$ are real numbers.

$$\begin{gathered} \hat{g}_{jk} = \hat{\vartheta}^2, for\ j = k = 0 , \\ \hat{g}_{jk} = \hat{s}^2, for\ j = k \neq 0 , \\ \hat{g}_{jk} = 0, for\ j \neq k . \end{gathered} \tag{86}$$

We compute the following.

$$\begin{gathered} U(\hat{O},O) = \sum_{j\epsilon C, k\epsilon C} (\sigma_j \hat{g}_{jk} \sigma_k - \left\langle \sigma_j \right\rangle \hat{g}_{jk} \left\langle \sigma_k \right\rangle I) , \\ = \hat{\vartheta}^2 (\sigma_0\sigma_0 - I) + \hat{s}^2 (\sigma_1\sigma_1 + \sigma_2\sigma_2 + \sigma_3\sigma_3 - I) = \hat{s}^2 (2I) , \\ \left\langle U(\hat{O},O) \right\rangle = 2\hat{s}^2 . \end{gathered} \tag{87}$$

Each of $\hat{\vartheta}$ and $\hat{s}$ is imposed by the metric. $U(\hat{O},O)$ is invariant with respect to choice of $\hat{\vartheta}$. $U(\hat{O},O)$ is invariant with respect to choices of Ψ. An interpretation of $\hat{s}$, say as a unit of angular momentum, involves considerations regarding the complementary subsystem. Units of measure also are associated with the complementary subsystem.

For the case, in a space-time, of measuring spin of a spin-J system, with $J = \hbar / 2$, the following pertain.

$$J(J+1) - J^2 = \left(\frac{1}{2} \times \frac{3}{2} - \left(\frac{1}{2} \right)^2 \right) \hbar^2 = 2 \left(\frac{\hbar}{2} \right)^2 = \left\langle U(\hat{O},O) \right\rangle . \tag{88}$$

Spin-1/2 fermions

We construct, from a previous example, operators for spin-1/2 fermions. We observe that doing so points to the existence of six axes.

We start with the system defined in discussion above regarding (71). Herein, we call that the original system and consider the possibility of a new complementary subsystem that interacts with a new primary subsystem consisting of the original system. The new 4-state primary subsystem (with domain $D(discreet,1,4)$ and amplitudes denoted by $_4\Psi$) combines various Ψ

from the original 2-state primary subsystem and various $\hat{\Psi}$ from the original 2-state complementary subsystem. The following operators can be associated with new primary subsystem.

$$\rho_{jk} = \hat{\sigma}_j \sigma_k \,,\ for\ j \epsilon C(4n)\ and\ k \epsilon C(4n)\,. \tag{89}$$

We define an index set $C(16n) \equiv \{(lm) | l \epsilon C(4n)\,, m \epsilon C(4n)\}$ and construct a set $O \equiv \{\sigma_{lm} | (lm) \epsilon C(16n)\}$ of operators by embedding 2-by-2 σ_m matrices inside of 2-by-2 σ_l matrices. Each $\sigma_{(lm)}$ is a square root of $I = \sigma_{(00)}$.

$$\sigma_{(lm)} \equiv \begin{pmatrix} \sigma_l(1,1)\sigma_m & \sigma_l(1,2)\sigma_m \\ \sigma_l(2,1)\sigma_m & \sigma_l(2,2)\sigma_m \end{pmatrix}, \tag{90}$$
$$for \begin{pmatrix} \sigma_l(1,1) & \sigma_l(1,2) \\ \sigma_l(2,1) & \sigma_l(2,2) \end{pmatrix} \equiv \sigma_l\,.$$

The Dirac gamma matrices (indexed by $C(4n)$) can be defined as follows.

$$\gamma_0 \equiv \sigma_{(30)} = \begin{pmatrix} +1 & 0 & 0 & 0 \\ 0 & +1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}, \tag{91}$$
$$\gamma_1 \equiv i\sigma_{(21)} = \begin{pmatrix} 0 & 0 & 0 & +1 \\ 0 & 0 & +1 & 0 \\ 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix},$$
$$\gamma_2 \equiv i\sigma_{(22)} = \begin{pmatrix} 0 & 0 & 0 & -i \\ 0 & 0 & +i & 0 \\ 0 & +i & 0 & 0 \\ -i & 0 & 0 & 0 \end{pmatrix},$$
$$\gamma_3 \equiv i\sigma_{(23)} = \begin{pmatrix} 0 & 0 & +1 & 0 \\ 0 & 0 & 0 & -1 \\ -1 & 0 & 0 & 0 \\ 0 & +1 & 0 & 0 \end{pmatrix}.$$

For 4x4 matrices operating on a set $\{{}_4\Psi\}$, the number of raising-operator/lowering-operator pairs is 6. For the new system, which includes a new primary subsystem (equal to the original system) involving spin-1/2 fermions described via $\{{}_4\Psi\}$, the parallel to (82) becomes the following.

$$\left\langle U(\hat{O},O)\right\rangle = 12 . \tag{92}$$

Work related to (85) might seem to imply uncertainty scales by a factor of 2 per space-time dimension. For a 4-dimensional space-time, we might expect a result of 8. From (92), we find a possible need for considering six axes.

We assume that the time-like dimension encompasses a collapse of three dimensions into one dimension. Such would be understood in terms of a consolidation $u,v,w \leftarrow t$ in a "collapse" $C(6a) \leftarrow C(4a)$ in (68).

A treatment similar to work related to (85) involves five operators. We select the four non-identity operators specified in the following set of five operators.

$$\hat{O} = \{\sigma_{(00)}, \sigma_{(30)}\ , \sigma_{(21)}, \sigma_{(22)}, \sigma_{(23)}\} . \tag{93}$$

To parallel (86), we assume the following for $j \epsilon C(16n)$ and $k \epsilon C(16n)$.

$$\begin{gathered} \hat{g}_{jk} = \hat{\vartheta}^2, \textit{ for } j = k = (00) \textit{ and for any number } \hat{\vartheta} , \\ \hat{g}_{jk} = \hat{s}^2, \textit{ for } j = k \neq (00) , \\ \hat{g}_{jk} = 0, \textit{ for } j \neq k . \end{gathered} \tag{94}$$

To illustrate (92), one can use the following Ψ as an example and obtain (96).

$$\Psi = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \tag{95}$$

$$\begin{gathered} U(\hat{O},O) = 12\hat{s}^2 I , \\ \left\langle U(\hat{O},O)\right\rangle = 12\hat{s}^2 . \end{gathered} \tag{96}$$

Analogously to the process symbolized in (90), we define the following seven 8-by-8 gamma-like matrices ${}_8\gamma_{\vartheta}$. Here, we associate respective elements of $C(4n)$ and $C(4a)$. For example, $0 \leftrightarrow t$ and $1 \leftrightarrow x$.

$$\begin{gathered} {}_8\gamma_j \equiv \gamma_{(jt)}, \textit{ for all } j \epsilon C(4a) , \\ {}_8\gamma_u \equiv \gamma_{(tx)} , \\ {}_8\gamma_v \equiv \gamma_{(ty)} , \end{gathered} \tag{97}$$

$${}_8\gamma_w \equiv \gamma_{(tz)} .$$

The set $\{{}_8\gamma_j | j \epsilon C(4a)\}$ can be used, for considerations regarding traditional physics, in manners similar to uses of the traditional 4-by-4 gamma matrices. The set $\{{}_8\gamma_j | j \epsilon C(6a)\}$ can be used for the possibility that the time-dimension of space-time should be treated as being a collapse of three time-like dimensions into one dimension. We characterize t, u, v, and w as temporal and x, y, and z as spatial. We note that, for either set, the following is satisfied for all relevant choices of j and k.

$$\{{}_8\gamma_j, {}_8\gamma_k\} = {}_8\gamma_j\, {}_8\gamma_k + {}_8\gamma_k\, {}_8\gamma_j = 2g_{jk} I , \tag{98}$$
$$g_{jj} = 1, \textit{ for temporal } j ,$$
$$g_{jj} = -1, \textit{ for spatial } j ,$$
$$g_{jk} = 0, \textit{otherwise} .$$

Comments

Considerations related to discussion starting near (75) lead to the acceptability of considering photon states with negative populations, as used in Section 11 and Section 12.

Considering physics within space-time and referring to (70), we consider {a} $C = C(4a)$; {b} $\hat{g}_{jk}$ is the space-time metric; and {c} the o_j are momentum operators of the general form symbolized by $-i\partial_j + qA_j$ in which $i = \sqrt{-1}$, ∂_j represents a derivative with respect to a space-time coordinate, q is the charge of a particle, A is the vector potential, units are chosen so that the speed of light is unity, and so forth. The 16 components of the sum in (70) become 16 components of a possible quantum-mechanical analog of the Einstein field equations expressed as $g_{jk}\Lambda = (8\pi G_N / c^4)T_{jk} - G_{jk}$, in which g_{jk} is the space-time metric, Λ is the cosmological constant, T_{jk} is the stress-energy tensor, and G_{jk} is the Einstein tensor. The uncertainty term in (70) aligns with the cosmological constant term. The first term on the right-hand side of (70) aligns with the stress-energy term. The second term on the right-hand side of (70) aligns with the Einstein tensor term. In the application of (70), the stress-energy-like term contributes derivatives of the space-time metric.

Appendix 2 Twenty "Masses"

Table 16 shows twenty baryonic-matter "masses" calculated via (1). As noted above, each of as many as nine of these "particles" may not exist. For the $n_1 = 3$ column, an extension of (7) provides $q = 4|q_e|/3$ for $n_2 = 0$ and $q = |q_e|/3$ for $n_2 = 1 \textit{ or } 2$.

Table 16 For each position in this table, the "mass" is calculated per (1) and (5).

n_2	Masses (MeV/c^2) - Calculated				
	$n_1=3$	$n_1=2$	$n_1=1$	$n_1=0$	$n_1=-1$
0	1.49×10^{-1}	5.11×10^{-1}	1.75	6.00	2.05×10^{1}
1	7.25×10^{-1}	8.51	9.98×10^{1}	1.17×10^{3}	1.37×10^{4}
2	2.63	1.06×10^{2}	4.25×10^{3}	1.71×10^{5}	6.86×10^{6}
3	1.29×10^{1}	1.78×10^{3}	2.45×10^{5}	3.37×10^{7}	4.64×10^{9}

While experimental data is not accurate enough to try to fit results much more precisely, we can demonstrate one (of possibly many) ways to bring the calculated masses of the two $n_2=0$ baryon ensemble quarks to within experimental limits. We try replacing (5) with the (99). Table 17 shows results. Each of the two $n_2=0$ quark masses is within its experimentally determined range shown in Table 2.

$$\delta_{gr}(n_2,n_1)=A+B, \tag{99}$$

$$A=\log\left(1+d_{gr}\frac{\sin\left\{\frac{2\pi}{3}n_2\right\}}{\sin\left\{\frac{2\pi}{3}\right\}}\right),$$

$$B=\log\left(1+d_{gr}\frac{\sin\left\{\frac{2\pi}{3}(2-n_1)\right\}}{\sin\left\{\frac{2\pi}{3}\right\}}\times\frac{\cos\left\{\frac{2\pi}{3}n_2\right\}-\cos\left\{\frac{2\pi}{3}\right\}}{\cos\{0\}-\cos\left\{\frac{2\pi}{3}\right\}}\right).$$

Table 17 For each position in this table, the "mass" is calculated per (1) and (99).

n_2	Masses (MeV/c^2) - Calculated				
	$n_1=3$	$n_1=2$	$n_1=1$	$n_1=0$	$n_1=-1$
0	1.49×10^{-1}	5.11×10^{-1}	1.92	5.40	2.05×10^{1}
1	7.25×10^{-1}	8.51	9.98×10^{1}	1.17×10^{3}	1.37×10^{4}
2	2.63	1.06×10^{2}	4.25×10^{3}	1.71×10^{5}	6.86×10^{6}
3	1.29×10^{1}	1.78×10^{3}	2.69×10^{5}	3.03×10^{7}	4.64×10^{9}

Comments

Regarding the possibilities for 4th generation quarks ($n_2=3$, $with\, n_1=1\, or\, 0$), [1] indicates some lower mass-limits (with confidences of $>95\%$). For a b' quark, minimums for a mass fall in the range $46\, to\, 199\, GeV/c^2$. For $n_2=3$, $n_1=1$, the result in Table 16 is not inconsistent with such minimums. For a t', a likely minimum mass is $256 GeV/c^2$. For $n_2 = 3$, $n_1 = 0$, the result in Table 16 is not inconsistent with such a minimum.

[1] indicates possibilities for a stable heavy neutral lepton with minimum mass of at least $39 GeV/c^2$ and a neutral heavy lepton with minimum mass of at least $80 GeV/c^2$. Such would seem not to be inconsistent with (1) for $n_1 = -1$ and $n_2 = 2$ and/or 3.

Appendix 3 Latitude for two numbers

Table 18 indicates roughly the latitude inherent in the numbers $R_{em/gr}$ and $B_{em,gr}$ for the super-ensemble that includes baryonic matter, given experimental accuracies noted above.

Table 18 This table indicates roughly a range for each of $R_{em/gr}$ and $B_{em,gr}$. The nominal-ratio numbers are based on the nominal numbers in (26). For the upper-ratio numbers, the parenthesized inaccuracy in q_e is added to the absolute-value of the nominal number and the parenthesized inaccuracies in G_N and m_e are subtracted, respectively, from the nominal numbers. For the lower-ratio numbers, the parenthesized inaccuracy in q_e is subtracted from the absolute-value of the nominal number and the parenthesized inaccuracies in G_N and m_e are added, respectively, to the nominal numbers.

	$R_{em/gr}$	$B_{em,gr}$
Upper ratio	$4.166040731 \times 10^{42}$	2.717996054
Nominal ratio	$4.165621902 \times 10^{42}$	2.717993261
Lower ratio	$4.165203157 \times 10^{42}$	2.717990469

Appendix 4 Links among electromagnetism, gravity, and masses

For the baryonic-matter ensemble, the observed ratio of the mass of a tauon to the mass of an electron is consistent with the result $exp(3b_{gr})$ provided by (1). The uncertainty in the electron mass is no more than 50 parts per billion. The uncertainty in the electron charge is no more than 50 parts per billion. The value for ε_0 is assumed to be exact. G_N is known to about 1 part in 10^4 [1]. The accuracy of tauon results from (1) is, therefore, limited by the accuracy of G_N.

Approximation (100) provides a basis for estimating the sensitivity of the mass of leptons, as calculated via (1), to inaccuracies in G_N. Assume a variation $G_N \leftarrow G_N(1+d)$.

$$
\begin{aligned}
& exp\left(n_2\left(\frac{1}{36}log\left(\frac{3}{4}R_{em/gr}\right)\right)\right) \\
& \leftarrow exp\left(n_2\left(\frac{1}{36}log\left(\frac{3R_{em/gr}}{4}\times\frac{1}{1+d}\right)\right)\right) \\
& \approx exp\left(n_2\left(\frac{1}{36}log\left(\frac{3R_{em/gr}}{4}\times(1-d)\right)\right)\right) \\
& = exp\left(n_2\left(\frac{1}{36}\left(log\left(\frac{3R_{em/gr}}{4}\right)+log(1-d)\right)\right)\right) \\
& \approx exp\left(n_2\left(\frac{1}{36}\left(log\left(\frac{3R_{em/gr}}{4}\right)-d\right)\right)\right) \\
& = exp\left(n_2\left(\frac{1}{36}\left(log\left(\frac{3R_{em/gr}}{4}\right)\right)\right)\right)exp\left(n_2\left(\frac{1}{36}(-d)\right)\right) \\
& \approx exp\left(n_2\left(\frac{1}{36}log\left(\frac{3}{4}R_{em/gr}\right)\right)\right)\times\left(1-\frac{n_2}{36}d\right) .
\end{aligned}
\tag{100}
$$

For a tauon $(n_2 = 3)$, the calculated mass is, as noted in (101), an order of magnitude more accurate than is G_N. Table 19 illustrates this relationship.

$$m(tauon) \approx 1776.84\,MeV / c^2 \text{, with an accuracy of about } 1 \text{ part in } 10^5 . \tag{101}$$

Table 19 These two values for the gravitational constant are consistent with the current experimental range for that constant and illustrate a possible relationship between the baryonic-matter gravitational constant and the mass of a tauon.

G_N $(m^3\,kg^{-1}\,s^{-1})$	$m(tauon)$ (MeV / c^2)
6.67495×10^{-11}	1776.825
6.67361×10^{-11}	1776.855

Appendix 5 A possible improvement to the value of the gravitational constant

This work is motivated by the right-hand column of Table 15 and the observation that the following number is approximately equal to the value used for d_{gr} in (6).

$$3log(3) / \left(\frac{9}{2}e^2\right) \approx 0.099120679 . \tag{102}$$

We assume (103) and (104). With those new values, nominal values for $R_{em/gr}$ and G_N for baryonic matter become as indicated below and the masses calculated via (1) become as shown in Table 20. The values of $R_{em/gr}$ and b_{gr} are within the ranges specified in Table 18. The value for G_N is consistent with the range shown in (26). The value of Δ_{gr} below contrasts with that in (12). Based on $c(\vartheta)$ as defined in Table 15, (108) restates (5).

$$d_{gr} = 3log(3)/\left(\frac{9}{2}e^2\right) \approx 0.099120679\,, \tag{103}$$

$$b_{gr} \approx 2.717991359\,, \tag{104}$$

$$R_{em/gr} \approx 4.165337 \times 10^{42}\,, \tag{105}$$

$$G_N \approx 6.6747372 \times 10^{-11}\, m^3\, kg^{-1}\, s^{-1}\,, \tag{106}$$

$$\Delta_{gr} \approx -1.0686 \times 10^{-4}\,, \text{ for } b_{gr} \text{ as in (104)}\,, \tag{107}$$

$$\delta_{gr}(n_2) = \log\left(1-(-1)^{n_2} c(n_2)\log(c(n_2)) \times \left(\frac{2}{(3e)^2}\right)\right). \tag{108}$$

Table 20 For each observed particle, the upper value characterizes experimentally determined results [2] and the lower value is calculated per (1), but using d_{gr} from (103) and b_{gr} from (104).

n_2	Masses (MeV/c^2) - Observed and Calculated		
	$n_1 = 2$	$n_1 = 1$	$n_1 = 0$
0	$0.510998910 \pm 0.00000013$ 0.510998910	1.7 to 3.3 1.750	4.1 to 5.8 5.996
1		101^{+29}_{-21} 99.844	1270^{+70}_{-90} 1171.6
2	105.658367 ± 0.000004 105.658367	4190^{+180}_{-60} 4246.9	$172000 \pm 900 \pm 1300$ 170704
3	1776.82 ± 0.16 1776.83		

Comments

This work uses the δ_{gr} expression from (5), not (99).

Per the right-most column in Table 15, the $3log(3)$ in (103) might correspond to an entropy related to the number of configurations.

We have a candidate value for a more precise G_N, though we do not estimate the accuracy.

Appendix 6 A family of functions

Summary

We define the following functions of a variable x, for ϑ an integer.

$$f(+,\vartheta,x) = \frac{1}{\vartheta!}x^{\vartheta} \text{ , } for\ \vartheta \geq 0 \text{ ,} \tag{109}$$
$$f(+,\vartheta,x) = 0 \text{ , } for\ \vartheta < 0 \text{ .}$$

In general, $f(+,\vartheta-1,x)$ is the derivative with respect to x of $f(+,\vartheta,x)$. We also consider integrations and start with a function with a negative exponent, say x^{-1}. Ignoring constants that might arise from indefinite integrations, we can consider a new series that includes the following functions.

$$f(-,4,x) = \frac{1}{288}x^4(12log(x)-25) \text{ ,} \tag{110}$$
$$f(-,3,x) = \frac{1}{36}x^3(6log(x)-11) \text{ ,}$$
$$f(-,2,x) = \frac{1}{4}x^2(2log(x)-3) \text{ ,}$$
$$f(-,1,x) = x(log(x)-1) \text{ ,}$$
$$f(-,0,x) = log(x) \text{ ,}$$
$$f(-,-1,x) = x^{-1} \text{ .}$$

Formula (1) contains one $x = b_{gr}$ instance of each of $f(+,1,x)$ and $f(+,2,x)$, assuming that a denominator in (1) can be characterized by $6 = 2\times 3$.

We assume that b_{gr} for baryonic matter can be considered to be an approximate solution to $f(-,1,x) = 0$.

$$b_{gr} = e(1+\ \Delta_{gr}) \text{ ,} \tag{111}$$
$$\Delta_{gr} \approx -1.0616\times 10^{-4} \text{, for (4) above,} \tag{112}$$
$$f(-,1,b_{gr}) \approx -2.9\times 10^{-4} \text{ .} \tag{113}$$

$b_{em} = 1$ solves $f(-,0,x) = log(x) = 0$. $f(-,-1,x) = x^{-1}$ has no zero solution. But, $b_{wk} = 0$ solves any of the expressions in (109). We project that $b_{st} = 0$. We project that b_{s3} approximately solves $f(-,2,x) = 0$. We project that b_{s4} approximately solves $f(-,3,x) = 0$.

$$b_{s3} \approx exp(3/2) \approx 4.4817 \text{ ,} \tag{114}$$
$$b_{s4} \approx exp(11/6) \approx 6.2547 \text{ .} \tag{115}$$

Appendix 7 Representations for realms em/gr, gr/s3, and s3/s4

Fig. 1 symbolizes realm em/gr. With the 5-by-5 matrix, there are $4/3$ as many photon channels available as graviton channels. This choice corresponds to the factor $4/3$ in (29) and to the entry $C_{em,gr} = 4/3$ in Table 6. At one end of realm em/gr, there are 4 pairs of spin-1 (or photon) raising and lowering operators (symbolized by em). At the other end of realm em/gr, there are 3 pairs of spin-2 (or graviton) raising and lowering operators (symbolized by gr).

$$\begin{pmatrix} & em & & & \\ em & & em & & \\ & em & & em & \\ & & em & & em \\ & & & em & \end{pmatrix} \leftrightarrow \begin{pmatrix} & & gr & & \\ & & & gr & \\ gr & & & & gr \\ & gr & & & \\ & & gr & & \end{pmatrix}$$

Fig. 1 This figure provides a representation of the electromagnetism-gravity range (realm em/gr) in an interaction space. The double arrow denotes the possibility of interactions that are, in some sense, between the two ends of the realm.

Realm gr/s3 is symbolized by Fig. 2. Interactions corresponding to one edge of the realm feature gravitons. Interactions corresponding to the other edge of the realm feature a zero-mass spin-3 boson. Appearances of $s3$ in the table symbolize two pairs of raising and lowering operators. This figure illustrates the origin of the entry $C_{gr,s3} = 3/2$ in Table 6.

$$\begin{pmatrix} & & gr & & \\ & & & gr & \\ gr & & & & gr \\ & gr & & & \\ & & gr & & \end{pmatrix} \leftrightarrow \begin{pmatrix} & & & s3 & \\ & & & & s3 \\ & & & & \\ s3 & & & & \\ & s3 & & & \end{pmatrix}$$

Fig. 2 This figure provides a representation of realm gr/s3 in an interaction space.

Realm s3/s4 is symbolized by Fig. 3. Interactions corresponding to one edge of the realm feature the spin-3 boson. Interactions corresponding to the other edge of the realm feature a zero-mass spin-4 boson. Appearances of $s4$ in the table symbolize one pair of raising and lowering operators and one channel. This figure illustrates the origin of the entry $C_{s3,s4} = 2/1$ in Table 6.

$$
\begin{pmatrix} & & s3 & \\ & & & s3 \\ s3 & & & \\ & s3 & & \end{pmatrix} \leftrightarrow \begin{pmatrix} & & & s4 \\ & & & \\ & & & \\ s4 & & & \end{pmatrix}
$$

Fig. 3 This figure provides a representation of realm s3/s4 in an interaction space.

Appendix 8 Possible approximations related to realm wk/em

For baryonic matter, we explore topics related to realm wk/em. Results may be seen as fortuitous or as indications of possible symmetries and theories not addressed in this paper.

The experimental value for the mass of a Z boson is approximately $91.19 GeV/c^2$ [2]. We note the following approximation.

$$
m(Z) \sim 5 \times \frac{2}{3} \times m_e \, exp(4b_{gr}) \approx 89.73 GeV/c^2 . \tag{116}
$$

The Z boson might correspond to the diagonal 5 elements in arrays such as the two in Fig. 1 (in Appendix 7). Hence, the factor 5 in (116).

The experimental value for mass of a W boson is approximately $80.40 GeV/c^2$ [2]. We note the following approximation.

$$
m(W) \sim 5 \times \frac{3}{5} \times m_e \, exp(4b_{gr}) \approx 80.76 GeV/c^2 . \tag{117}
$$

The following conjectural statements seem to support not dismissing outright the above two approximations.

In realm wk/em, the number of flips each of two interacting charged fermions can undergo in the interaction is two - between $n_1 = 0$ and $n_1 = 2$. (See Table 10.) Hence, the factor $exp(2 \times 2b_{gr})$ in each of (116) and (117).

Both Z bosons and photons are spin-1 particles and potentially might carry helicities of +1, 0, or -1. Photons have zero mass and (in a vacuum) carry only non-zero effective helicity. Z bosons have mass and should be able to interact via 0-helicity states. Perhaps a notion of two fully effective helicities, not three, correlates with the factor $2/3$ in (116). If so, perhaps a correction such as ϑ below for some, but not full, participation by the helicity-0 channel is appropriate.

$$
\left(\frac{2+\vartheta}{3}\right) / \left(\frac{2}{3}\right) \approx \frac{91.19}{89.73} , \tag{118}
$$
$$
\vartheta \approx 0.0325 .
$$

To the extent W-boson-mediated interactions cannot use two (perhaps the uppermost and lowermost) channels corresponding to the diagonal in the array symbolizing interactions (for example, because of a need to transfer a unit of charge), there is a possible explanation for the factor of $3/5$ in (117).

Looking elsewhere than at Z and W bosons, possibly a 1-step transition for each of two interacting particles has meaning. If so, a key mass could be, for some factor ϑ, the following.

$$m \sim \vartheta \times m_e exp(2b_{gr}) \approx \vartheta \times 117.28\, MeV / c^2 . \tag{119}$$

The following ϑ would fit the masses of pions. Perhaps the amounts $\vartheta - 1$ correspond to a form of a δ_{**} function (such as that in (5)).

$$\begin{gathered} \vartheta \approx 1.190,\ for\ charged\ pions\,, \\ \vartheta \approx 1.151,\ for\ the\ neutral\ pion\,. \end{gathered} \tag{120}$$

Appendix 9 Approximations possibly related to baryon masses

We might expect that, for baryonic matter, each of realms s3/gr and s4/s3 present opportunities regarding fermions other than leptons and quarks. For example, we note the following, which might provide a pointer toward theory linking the mass of a proton ([2]) to the mass of an electron.

$$\frac{m_p}{m_e} \approx exp\left(5 \frac{b_{s3}}{3} (1 + \Delta_{(3p)}) \right), \tag{121}$$

$$\Delta_{(3p)} \approx 0.00615 . \tag{122}$$

(114), (121), and discussion related to (10) support $n_3 = 3 - 1 = 2$ for baryonic matter.

Table 21 shows results for approximating the masses of baryon-octet particles ([2]) via (123) or (124) and n_p as specified by (125). As with the periodic table for quarks and charged leptons, there is not a one-to-one correspondence between used and available values of integer variables. For (123), there is no $n_p = 1$ particle. For (124), there are no $n_\Sigma = 1$ particles for $S = 0$ or $S = 2$.

$$m_{calc} = m_p\, exp(\frac{n_p}{12}) \,, \tag{123}$$

$$m_{calc} = m_p\, exp(\frac{S}{6} + \frac{n_\Sigma}{12}) \,, \tag{124}$$

$$\begin{aligned} & n_p = 2S + n_\Sigma\,, \\ & n_\Sigma = 1 \text{ for a } \Sigma \text{ particle}\,, \\ & n_\Sigma = 0 \text{ otherwise}\,. \end{aligned} \tag{125}$$

Table 21 For each of the particles in the baryon octet, the mass m_{calc} calculated via (123) or (124) approximates the observed mass m_{obs} [2]. For five of the particles n_p equals twice the strangeness, *S*.

Particle	m_{obs} (MeV/c^2)	S	n_Σ	n_p	m_{calc} (MeV/c^2)	$\frac{m_{obs}-m_{calc}}{m_{calc}}$
p	938.272	0	0	0	938.272	0.0000
n	939.565	0	0	0	938.272	0.0014
Λ	1115.68	1	0	2	1108.437	0.0065
Σ^+	1189.37	1	1	3	1204.765	-0.0128
Σ^0	1192.64	1	1	3	1204.765	-0.0101
Σ^-	1197.45	1	1	3	1204.765	-0.0061
Ξ^0	1314.86	2	0	4	1309.464	0.0041
Ξ^-	1321.71	2	0	4	1309.464	0.0093

For the baryon decuplet, the masses of Δ particles are approximately $1232 MeV/c^2$ [2]. Equations (126) and (127) pertain for that mass.

$$\frac{m}{m_e} \approx exp\left(5\frac{b_{s4}}{4}(1+\Delta_{(s4)})\right), \tag{126}$$

$$\Delta_{(s4)} \approx -0.004\,. \tag{127}$$

(115), (126), and discussion related to (10) support $n_4 = 4-1 = 3$ for baryonic matter.

References

1. Particle Data Group, July 2008 Particle Physics Booklet, Lawrence Berkeley National Laboratory (2008), http://pdg.lbl.gov/2009/download/rpp-2008-booklet.pdf. Extracted from C. Amsler, *et al.*, Physics Letters **B 667**, 1 (2008).
2. K. Nakamura *et al.* (Particle Data Group), J. Phys. G **37**, 075021 (2010), http://pdg.lbl.gov/2010/reviews/contents_sports.html .
3. B. Cabrera, "What Makes Up the Dark Matter of the Universe?", 2011 Segre Lecture, University of California, Berkeley, http://physics.berkeley.edu/index.php?Itemid=223&id=37&option=com_content&task=view.