The New Prime theorems (1291)一（1340）

Chun-Xuan Jiang
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Abstract

Using Jiang function we are able to prove almost all prime problems in prime distribution. This is the Book proof. No great mathematicians study prime problems and prove Riemann hypothesis in AIM, CLAYMA, IAS, THES, MPIM, MSRI. In this paper using Jiang function $J_2(\omega)$ we prove that the new prime theorems (1291)-(1340) contain infinitely many prime solutions and no prime solutions. From (6) we are able to find the smallest solution $\pi_k(N_0,2) \geq 1$. This is the Book theorem.

It will be another million years at least, before we understand the primes.

Paul Erdos (1913-1996)

TATEMENT OF INTENT

If elected. I am willing to serve the IMU and the international mathematical community as president of the IMU. I am willing to take on the duties and responsibilities of this function.

These include (but are not restricted to) working with the IMU's Executive Committee on policy matters and its tasks related to organizing the 2014 ICM, fostering the development of mathematics, in particular in developing countries and among young people worldwide, representing the interests of our community in contacts with other international scientific bodies, and helping the IMU committees in their function.

--IMU president Ingrid Daubechies—

Satellite conference to ICM 2010

Analytic and combinatorial number theory (August 29-September 3, ICM2010) is a conjecture. The sieve methods and circle method are outdated methods which cannot prove twin prime conjecture and Goldbach's conjecture. The papers of Goldston-Pintz-Yildirim and Gree-Tao are based on the Hardy-Littlewood prime k-tuple conjecture (1923). But the Hardy-Littlewood prime k-tuple conjecture is false:

(http://www.wbabin.net/math/xuan77.pdf)

(http://vixra.org/pdf/1003.0234v1.pdf)

The world mathematicians read Jiang's book and papers. In 1998 Jiang disproved Riemann hypothesis. In 1996 Jiang prove Goldabch conjecture and twin prime conjecture. Using a new analytical tool Jiang invented the Jiang function. Jiang prove almost all prime problems in prime distribution. Jiang epoch-making works in ICM2002 which was a failure congress. China considers Jiang epoch-making works to be pseudoscience. Jiang negated ICM2006 Fields medal (Green and Tao theorem is false) to see.

(http://www.wbabin.net/math/xuan39e.pdf)

(http://www.vixra.org/pdf/0904.00001v1.pdf)

There are no Jiang's epoch-making works in ICM2010. It cannot represent the modern epoch-making works. For fostering the development of Jiang prime theory IMU is willing to take on the duty and responsibility of this function to see [new prime k-tuple theorems (1)-(20)] and [the new prime theorems (1)-(1340)]: (http//www.wbabin.net/xuan.htm#chun-xuan)

(http://vixra.org/numth/)

The New Prime Theorem (1291)

$$p, jp^{2522} + k - j(j = 1, 2, \cdots, k-1)$$

Chun-Xuan Jiang

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Abstract

Using Jiang function we prove that $jp^{2522} + k - j$ contain infinitely many prime solutions and no prime solutions.

Theorem. Let k be a given odd prime.

$$p, jp^{2522} + k - j(j = 1, \cdots, k-1) \tag{1}$$

Contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega) = \prod_{P>2} [P - 1 - \chi(P)]. \tag{2}$$

where $\omega = \prod_P P$, $\chi(P)$ is the number of solutions of congruence

$$\prod_{j=1}^{k-1}(jq^{2522} + k - j) \equiv 0 \pmod{p}, q = 1, \cdots, p-1. \tag{3}$$

If $\chi(P) \le P-2$ then from (2) and (3) we have

$$J_2(\omega) \neq 0 \tag{4}$$

We prove that (1) contain infinitely many prime solutions that is for any k there are infinitely many primes P such that each of $jp^{2522} + k - j$ is a prime.

Using Fermat's little theorem from (3) we have $\chi(P) = P-1$. Substituting it into (2) we have

$$J_2(\omega) = 0 \tag{5}$$

We prove that (1) contain no prime solutions [1,2]

If $J_2(\omega) \neq 0$ then we have asymptotic formula [1,2]

$$\pi_k(N,2) = \left| \left\{ P \le N : jp^{2522} + k - j = \text{prime} \right\} \right| \sim \frac{J_2(\omega)\omega^{k-1}}{(2522)^{k-1}\phi^k(\omega)} \frac{N}{\log^k N}, \tag{6}$$

where $\phi(\omega) = \prod_P (P-1)$.

From (6) we are able to find the smallest solution $\pi_k(N_0, 2) \ge 1$.

Example 1. Let $k = 3$

From (2) and (3) we have

$$J_2(\omega) = 0. \tag{7}$$

We prove that for $k = 3$

（1）contain no prime solutions. 1 is not a prime.

Example 2. Let $k \neq 3$

From (2) and (3) we have

$$J_2(\omega) \neq 0. \tag{8}$$

We prove that for $k \neq 3$

(1) contain infinitely many prime solutions.

The New Prime Theorem (1292)

$$p, jp^{2524} + k - j(j = 1, 2, \cdots, k-1)$$

Chun-Xuan Jiang

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Abstract

Using Jiang function we prove that $jp^{2524} + k - j$ contain infinitely many prime solutions and no prime solutions.

Theorem. Let k be a given odd prime.

$$p, jp^{2524} + k - j(j = 1, \cdots, k-1) \tag{1}$$

Contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega) = \prod_{P>2} [P - 1 \chi \quad P]. \tag{2}$$

where $\omega = \prod_P P$, $\chi(P)$ is the number of solutions of congruence

$$\prod_{j=1}^{k-1}(jq^{2524} + k - j) \equiv 0 \pmod{p}, q = 1, \cdots, p-1. \tag{3}$$

If $\chi(P) \le P - 2$ then from (2) and (3) we have

$$J_2(\omega) \ne (\tag{4}$$

We prove that (1) contain infinitely many prime solutions that is for any k there are infinitely many primes P such that each of $jp^{2524} + k - j$ is a prime.

Using Fermat's little theorem from (3) we have $\chi(P) = P - 1$. Substituting it into (2) we have

$$J_2(\omega) = (\tag{5}$$

We prove that (1) contain no prime solutions [1,2]

If $J_2(\omega) \ne 0$ then we have asymptotic formula [1,2]

$$\pi_k(N,2) = \left| \{ P \le N : jp^{2524} + k - j = \text{prime} \} \right| \sim \frac{J_2(\omega)\omega^{k-1}}{(2524)^{k-1}\phi^k(\omega)} \frac{N}{\log^k N}, \tag{6}$$

where $\phi(\omega) = \prod_P(P-1)$.

From (6) we are able to find the smallest solution $\pi_k(N_0, 2) \ge 1$.

Example 1. Let $k = 3, 5$

From (2) and (3) we have

$$J_2(\omega) = (. \tag{7}$$

We prove that for $k = 3, 5$

（1）contain no prime solutions. 1 is not a prime.

Example 2. Let $k \ne 3, 5$

From (2) and (3) we have

$$J_2(\omega) \ne (. \tag{8}$$

We prove that for $k \ne 3, 5$

(1) contain infinitely many prime solutions.

The New Prime Theorem (1293)

$$p, jp^{2526} + k - j(j = 1, 2, \cdots, k-1)$$

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Abstract

Using Jiang function we prove that $jp^{2526} + k - j$ contain infinitely many prime solutions and no prime solutions.

Theorem. Let k be a given odd prime.

$$p, jp^{2526} + k - j(j = 1, \cdots, k-1) \tag{1}$$

Contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega) = \prod_{P>2} [P - 1 - \chi(P)]. \tag{2}$$

where $\omega = \prod_P P$, $\chi(P)$ is the number of solutions of congruence

$$\prod_{j=1}^{k-1}(jq^{2526} + k - j) \equiv 0 \pmod{p}, q = 1, \cdots, p-1. \tag{3}$$

If $\chi(P) \leq P - 2$ then from (2) and (3) we have

$$J_2(\omega) \neq 0 \tag{4}$$

We prove that (1) contain infinitely many prime solutions that is for any k there are infinitely many primes P such that each of $jp^{2526} + k - j$ is a prime.

Using Fermat's little theorem from (3) we have $\chi(P) = P - 1$. Substituting it into (2) we have

$$J_2(\omega) = 0 \tag{5}$$

We prove that (1) contain no prime solutions [1,2]

If $J_2(\omega) \neq 0$ then we have asymptotic formula [1,2]

$$\pi_k(N, 2) = \left| \{ P \leq N : jp^{2526} + k - j = \text{prime} \} \right| \sim \frac{J_2(\omega)\omega^{k-1}}{(2526)^{k-1}\phi^k(\omega)} \frac{N}{\log^k N}, \tag{6}$$

where $\phi(\omega) = \prod_P (P-1)$.

From (6) we are able to find the smallest solution $\pi_k(N_0, 2) \geq 1$.

Example 1. Let $k = 3$

From (2) and (3) we have

$$J_2(\omega) = 0. \tag{7}$$

We prove that for $k = 3$

（1）contain no prime solutions. 1 is not a prime.

Example 2. Let $k \neq 3$

From (2) and (3) we have

$$J_2(\omega) \neq 0. \tag{8}$$

We prove that for $k \neq 3$

(1) contain infinitely many prime solutions.

The New Prime Theorem (1294)

$$p, jp^{2528} + k - j(j = 1, 2, \cdots, k-1)$$

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Abstract

Using Jiang function we prove that $jp^{2528} + k - j$ contain infinitely many prime solutions and no prime solutions.

Theorem. Let k be a given odd prime.

$$p, jp^{2528} + k - j(j = 1, \cdots, k-1) \tag{1}$$

Contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega) = \prod_{P>2} [P - 1 - \chi(P)]. \tag{2}$$

where $\omega = \prod_P P$, $\chi(P)$ is the number of solutions of congruence

$$\prod_{j=1}^{k-1}(jq^{2528} + k - j) \equiv 0 \pmod{p}, q = 1, \cdots, p-1. \tag{3}$$

If $\chi(P) \leq P - 2$ then from (2) and (3) we have

$$J_2(\omega) \neq 0 \tag{4}$$

We prove that (1) contain infinitely many prime solutions that is for any k there are infinitely many primes P such that each of $jp^{2528} + k - j$ is a prime.

Using Fermat's little theorem from (3) we have $\chi(P) = P - 1$. Substituting it into (2) we have

$$J_2(\omega) = 0 \tag{5}$$

We prove that (1) contain no prime solutions [1,2]

If $J_2(\omega) \neq 0$ then we have asymptotic formula [1,2]

$$\pi_k(N,2) = \left| \left\{ P \leq N : jp^{2528} + k - j = \text{prime} \right\} \right| \sim \frac{J_2(\omega)\omega^{k-1}}{(2528)^{k-1}\phi^k(\omega)} \frac{N}{\log^k N}, \tag{6}$$

where $\phi(\omega) = \prod_P(P-1)$.

From (6) we are able to find the smallest solution $\pi_k(N_0, 2) \geq 1$.

Example 1. Let $k = 3, 5, 17, 317$

From (2) and (3) we have

$$J_2(\omega) = 0 \tag{7}$$

We prove that for $k = 3, 5, 17, 317$

（1）contain no prime solutions. 1 is not a prime.

Example 2. Let $k \neq 3, 5, 17, 317$

From (2) and (3) we have

$$J_2(\omega) \neq 0 \tag{8}$$

We prove that for $k \neq 3, 5, 17, 317$

(1) contain infinitely many prime solutions.

The New Prime Theorem (1295)

$$p, jp^{2530} + k - j(j = 1, 2, \cdots, k-1)$$

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Abstract

Using Jiang function we prove that $jp^{2530} + k - j$ contain infinitely many prime solutions and no prime solutions.

Theorem. Let k be a given odd prime.

$$p, jp^{2530} + k - j(j = 1, \cdots, k-1) \tag{1}$$

Contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega) = \prod_{P>2} [P - 1 - \chi(P)]. \tag{2}$$

where $\omega = \prod_P P$, $\chi(P)$ is the number of solutions of congruence

$$\prod_{j=1}^{k-1}(jq^{2530} + k - j) \equiv 0 \pmod{p}, q = 1, \cdots, p-1. \tag{3}$$

If $\chi(P) \le P - 2$ then from (2) and (3) we have

$$J_2(\omega) \ne 0 \tag{4}$$

We prove that (1) contain infinitely many prime solutions that is for any k there are infinitely many primes P such that each of $jp^{2530} + k - j$ is a prime.

Using Fermat's little theorem from (3) we have $\chi(P) = P - 1$. Substituting it into (2) we have

$$J_2(\omega) = 0 \tag{5}$$

We prove that (1) contain no prime solutions [1,2]

If $J_2(\omega) \ne 0$ then we have asymptotic formula [1,2]

$$\pi_k(N, 2) = \left| \left\{ P \le N : jp^{2530} + k - j = \text{prime} \right\} \right| \sim \frac{J_2(\omega)\omega^{k-1}}{(2530)^{k-1}\phi^k(\omega)} \frac{N}{\log^k N}, \tag{6}$$

where $\phi(\omega) = \prod_P (P-1)$.

From (6) we are able to find the smallest solution $\pi_k(N_0, 2) \ge 1$.

Example 1. Let $k = 3, 11, 23, 47, 2531$

From (2) and (3) we have

$$J_2(\omega) = 0 \tag{7}$$

We prove that for $k = 3, 11, 23, 47, 2531$

（1）contain no prime solutions. 1 is not a prime.

Example 2. Let $k \ne 3, 11, 23, 47, 2531$

From (2) and (3) we have

$$J_2(\omega) \ne 0 \tag{8}$$

We prove that for $k \ne 3, 11, 23, 47, 2531$

(1) contain infinitely many prime solutions.

The New Prime Theorem (1296)

$$p, jp^{2532} + k - j(j = 1, 2, \cdots, k-1)$$

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Abstract

Using Jiang function we prove that $jp^{2532} + k - j$ contain infinitely many prime solutions and no prime solutions.

Theorem. Let k be a given odd prime.

$$p, jp^{2532} + k - j(j = 1, \cdots, k-1) \tag{1}$$

Contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega) = \prod_{P>2} [P - 1 - \chi(P)] . \tag{2}$$

where $\omega = \prod_P P$, $\chi(P)$ is the number of solutions of congruence

$$\prod_{j=1}^{k-1}(jq^{2532} + k - j) \equiv 0 \pmod{p}, q = 1, \cdots, p-1. \tag{3}$$

If $\chi(P) \leq P - 2$ then from (2) and (3) we have

$$J_2(\omega) \neq 0 \tag{4}$$

We prove that (1) contain infinitely many prime solutions that is for any k there are infinitely many primes P such that each of $jp^{2532} + k - j$ is a prime.

Using Fermat's little theorem from (3) we have $\chi(P) = P-1$. Substituting it into (2) we have

$$J_2(\omega) = 0 \tag{5}$$

We prove that (1) contain no prime solutions [1,2]

If $J_2(\omega) \neq 0$ then we have asymptotic formula [1,2]

$$\pi_k(N,2) = \left| \left\{ P \leq N : jp^{2532} + k - j = \text{prime} \right\} \right| \sim \frac{J_2(\omega)\omega^{k-1}}{(2532)^{k-1}\phi^k(\omega)} \frac{N}{\log^k N}, \tag{6}$$

where $\phi(\omega) = \prod_P (P-1)$.

From (6) we are able to find the smallest solution $\pi_k(N_0, 2) \geq 1$.

Example 1. Let $k = 3, 5, 7, 13$

From (2) and (3) we have

$$J_2(\omega) = 0 \tag{7}$$

We prove that for $k = 3, 5, 7, 13$

（1）contain no prime solutions. 1 is not a prime.

Example 2. Let $k \neq 3, 5, 7, 13$

From (2) and (3) we have

$$J_2(\omega) \neq 0 \tag{8}$$

We prove that for $k \neq 3, 5, 7, 13$

(1) contain infinitely many prime solutions.

The New Prime Theorem (1297)

$$p, jp^{2534} + k - j(j = 1, 2, \cdots, k-1)$$

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Abstract

Using Jiang function we prove that $jp^{2534} + k - j$ contain infinitely many prime solutions and no prime solutions.

Theorem. Let k be a given odd prime.

$$p, jp^{2534} + k - j(j = 1, \cdots, k-1) \tag{1}$$

Contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega) = \prod_{P>2} [P - 1 - \chi(P)]. \tag{2}$$

where $\omega = \prod_P P$, $\chi(P)$ is the number of solutions of congruence

$$\prod_{j=1}^{k-1}(jq^{2534} + k - j) \equiv 0 \pmod{p}, q = 1, \cdots, p-1. \tag{3}$$

If $\chi(P) \le P - 2$ then from (2) and (3) we have

$$J_2(\omega) \ne 0 \tag{4}$$

We prove that (1) contain infinitely many prime solutions that is for any k there are infinitely many primes P such that each of $jp^{2534} + k - j$ is a prime.

Using Fermat's little theorem from (3) we have $\chi(P) = P - 1$. Substituting it into (2) we have

$$J_2(\omega) = 0 \tag{5}$$

We prove that (1) contain no prime solutions [1,2]

If $J_2(\omega) \ne 0$ then we have asymptotic formula [1,2]

$$\pi_k(N,2) = \left| \left\{ P \le N : jp^{2534} + k - j = \text{prime} \right\} \right| \sim \frac{J_2(\omega)\omega^{k-1}}{(2534)^{k-1}\phi^k(\omega)} \frac{N}{\log^k N}, \tag{6}$$

where $\phi(\omega) = \prod_P (P-1)$.

From (6) we are able to find the smallest solution $\pi_k(N_0, 2) \ge 1$.

Example 1. Let $k = 3$

From (2) and (3) we have

$$J_2(\omega) = 0 \tag{7}$$

We prove that for $k = 3$

（1）contain no prime solutions. 1 is not a prime.

Example 2. Let $k \ne 3$

From (2) and (3) we have

$$J_2(\omega) \ne 0 \tag{8}$$

We prove that for $k \ne 3$

(1) contain infinitely many prime solutions.

The New Prime Theorem (1298)

$$p, jp^{2536} + k - j(j = 1, 2, \cdots, k - 1)$$

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Abstract

Using Jiang function we prove that $jp^{2536} + k - j$ contain infinitely many prime solutions and no prime solutions.

Theorem. Let k be a given odd prime.

$$p, jp^{2536} + k - j(j = 1, \cdots, k - 1) \tag{1}$$

Contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega) = \prod_{P>2} [P - 1 - \chi(P)]. \tag{2}$$

where $\omega = \prod_P P$, $\chi(P)$ is the number of solutions of congruence

$$\prod_{j=1}^{k-1}(jq^{2536} + k - j) \equiv 0 \pmod{p}, q = 1, \cdots, p-1. \tag{3}$$

If $\chi(P) \leq P - 2$ then from (2) and (3) we have

$$J_2(\omega) \neq 0 \tag{4}$$

We prove that (1) contain infinitely many prime solutions that is for any k there are infinitely many primes P such that each of $jp^{2536} + k - j$ is a prime.

Using Fermat's little theorem from (3) we have $\chi(P) = P - 1$. Substituting it into (2) we have

$$J_2(\omega) = 0 \tag{5}$$

We prove that (1) contain no prime solutions [1,2]

If $J_2(\omega) \neq 0$ then we have asymptotic formula [1,2]

$$\pi_k(N, 2) = \left| \left\{ P \leq N : jp^{2536} + k - j = \text{prime} \right\} \right| \sim \frac{J_2(\omega)\omega^{k-1}}{(2536)^{k-1}\phi^k(\omega)} \frac{N}{\log^k N}, \tag{6}$$

where $\phi(\omega) = \prod_P (P-1)$.

From (6) we are able to find the smallest solution $\pi_k(N_0, 2) \geq 1$.

Example 1. Let $k = 3, 5$

From (2) and (3) we have

$$J_2(\omega) = 0 \tag{7}$$

We prove that for $k = 3, 5$

（1）contain no prime solutions. 1 is not a prime.

Example 2. Let $k \neq 3, 5$

From (2) and (3) we have

$$J_2(\omega) \neq 0 \tag{8}$$

We prove that for $k \neq 3, 5$

(1) contain infinitely many prime solutions.

The New Prime Theorem (1299)

$$p, jp^{2538} + k - j(j = 1, 2, \cdots, k-1)$$

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Abstract

Using Jiang function we prove that $jp^{2538} + k - j$ contain infinitely many prime solutions and no prime solutions.

Theorem. Let k be a given odd prime.

$$p, jp^{2538} + k - j(j = 1, \cdots, k-1) \tag{1}$$

Contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega) = \prod_{P>2} [P - 1 - \chi(P)] . \tag{2}$$

where $\omega = \prod_P P$, $\chi(P)$ is the number of solutions of congruence

$$\prod_{j=1}^{k-1}(jq^{2538} + k - j) \equiv 0 \pmod{p}, q = 1, \cdots, p-1. \tag{3}$$

If $\chi(P) \leq P - 2$ then from (2) and (3) we have

$$J_2(\omega) \neq 0 \tag{4}$$

We prove that (1) contain infinitely many prime solutions that is for any k there are infinitely many primes P such that each of $jp^{2538} + k - j$ is a prime.

Using Fermat's little theorem from (3) we have $\chi(P) = P - 1$. Substituting it into (2) we have

$$J_2(\omega) = 0 \tag{5}$$

We prove that (1) contain no prime solutions [1,2]

If $J_2(\omega) \neq 0$ then we have asymptotic formula [1,2]

$$\pi_k(N, 2) = \left| \left\{ P \leq N : jp^{2538} + k - j = \text{prime} \right\} \right| \sim \frac{J_2(\omega)\omega^{k-1}}{(2538)^{k-1}\phi^k(\omega)} \frac{N}{\log^k N}, \tag{6}$$

where $\phi(\omega) = \prod_P (P-1)$.

From (6) we are able to find the smallest solution $\pi_k(N_0, 2) \geq 1$.

Example 1. Let $k = 3, 7, 19, 283, 2539$

From (2) and (3) we have

$$J_2(\omega) = 0 \tag{7}$$

We prove that for $k = 3, 7, 19, 283, 2539$

（1）contain no prime solutions. 1 is not a prime.

Example 2. Let $k \neq 3, 7, 19, 283, 2539$

From (2) and (3) we have

$$J_2(\omega) \neq 0 \tag{8}$$

We prove that for $k \neq 3, 7, 19, 283, 2539$

(1) contain infinitely many prime solutions.

The New Prime Theorem (1300)

$$p, jp^{2540} + k - j(j = 1, 2, \cdots, k-1)$$

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Abstract

Using Jiang function we prove that $jp^{2540} + k - j$ contain infinitely many prime solutions and no prime solutions.

Theorem. Let k be a given odd prime.

$$p, jp^{2540} + k - j(j = 1, \cdots, k-1) \tag{1}$$

Contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega) = \prod_{P>2} [P - 1 - \chi(P)]. \tag{2}$$

where $\omega = \prod_P P$, $\chi(P)$ is the number of solutions of congruence

$$\prod_{j=1}^{k-1} (jq^{2540} + k - j) \equiv 0 \pmod{p}, q = 1, \cdots, p-1. \tag{3}$$

If $\chi(P) \leq P - 2$ then from (2) and (3) we have

$$J_2(\omega) \neq 0 \tag{4}$$

We prove that (1) contain infinitely many prime solutions that is for any k there are infinitely many primes P such that each of $jp^{2540} + k - j$ is a prime.

Using Fermat's little theorem from (3) we have $\chi(P) = P - 1$. Substituting it into (2) we have

$$J_2(\omega) = 0 \tag{5}$$

We prove that (1) contain no prime solutions [1,2]

If $J_2(\omega) \neq 0$ then we have asymptotic formula [1,2]

$$\pi_k(N, 2) = \left| \left\{ P \leq N : jp^{2540} + k - j = \text{prime} \right\} \right| \sim \frac{J_2(\omega)\omega^{k-1}}{(2540)^{k-1}\phi^k(\omega)} \frac{N}{\log^k N}, \tag{6}$$

where $\phi(\omega) = \prod_P (P-1)$.

From (6) we are able to find the smallest solution $\pi_k(N_0, 2) \geq 1$.

Example 1. Let $k = 3, 5, 11, 509$

From (2) and (3) we have

$$J_2(\omega) = 0. \tag{7}$$

We prove that for $k = 3, 5, 11, 509$

（1）contain no prime solutions. 1 is not a prime.

Example 2. Let $k \neq 3, 5, 11, 509$

From (2) and (3) we have

$$J_2(\omega) \neq 0. \tag{8}$$

We prove that for $k \neq 3, 5, 11, 509$

(1) contain infinitely many prime solutions.

The New Prime Theorem (1301)

$$p, jp^{2542} + k - j(j = 1, 2, \cdots, k-1)$$

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Abstract

Using Jiang function we prove that $jp^{2542} + k - j$ contain infinitely many prime solutions and no prime solutions.

Theorem. Let k be a given odd prime.

$$p, jp^{2542} + k - j(j = 1, \cdots, k-1) \tag{1}$$

Contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega) = \prod_{P>2} [P - 1 - \chi(P)]. \tag{2}$$

where $\omega = \prod_P P$, $\chi(P)$ is the number of solutions of congruence

$$\prod_{j=1}^{k-1}(jq^{2542} + k - j) \equiv 0 \pmod{p}, q = 1, \cdots, p-1. \tag{3}$$

If $\chi(P) \le P - 2$ then from (2) and (3) we have

$$J_2(\omega) \ne 0 \tag{4}$$

We prove that (1) contain infinitely many prime solutions that is for any k there are infinitely many primes P such that each of $jp^{2542} + k - j$ is a prime.

Using Fermat's little theorem from (3) we have $\chi(P) = P - 1$. Substituting it into (2) we have

$$J_2(\omega) = 0 \tag{5}$$

We prove that (1) contain no prime solutions [1,2]

If $J_2(\omega) \ne 0$ then we have asymptotic formula [1,2]

$$\pi_k(N,2) = \left| \left\{ P \le N : jp^{2542} + k - j = \text{prime} \right\} \right| \sim \frac{J_2(\omega)\omega^{k-1}}{(2542)^{k-1}\phi^k(\omega)} \frac{N}{\log^k N}, \tag{6}$$

where $\phi(\omega) = \prod_P (P-1)$.

From (6) we are able to find the smallest solution $\pi_k(N_0, 2) \ge 1$.

Example 1. Let $k = 3, 83, 2543$

From (2) and (3) we have

$$J_2(\omega) = 0 \tag{7}$$

We prove that for $k = 3, 83, 2543$

（1）contain no prime solutions. 1 is not a prime.

Example 2. Let $k \ne 3, 83, 2543$

From (2) and (3) we have

$$J_2(\omega) \ne 0 \tag{8}$$

We prove that for $k \ne 3, 83, 2543$

(1) contain infinitely many prime solutions.

The New Prime Theorem (1302)

$$p, jp^{2544} + k - j(j = 1, 2, \cdots, k-1)$$

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Abstract

Using Jiang function we prove that $jp^{2544} + k - j$ contain infinitely many prime solutions and no prime solutions.

Theorem. Let k be a given odd prime.

$$p, jp^{2544} + k - j(j = 1, \cdots, k-1) \tag{1}$$

Contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega) = \prod_{P>2} [P - 1 - \chi(P)]. \tag{2}$$

where $\omega = \prod_P P$, $\chi(P)$ is the number of solutions of congruence

$$\prod_{j=1}^{k-1}(jq^{2544} + k - j) \equiv 0 \pmod{p}, q = 1, \cdots, p-1. \tag{3}$$

If $\chi(P) \leq P - 2$ then from (2) and (3) we have

$$J_2(\omega) \neq 0 \tag{4}$$

We prove that (1) contain infinitely many prime solutions that is for any k there are infinitely many primes P such that each of $jp^{2544} + k - j$ is a prime.

Using Fermat's little theorem from (3) we have $\chi(P) = P - 1$. Substituting it into (2) we have

$$J_2(\omega) = 0 \tag{5}$$

We prove that (1) contain no prime solutions [1,2]

If $J_2(\omega) \neq 0$ then we have asymptotic formula [1,2]

$$\pi_k(N,2) = \left| \left\{ P \leq N : jp^{2544} + k - j = \text{prime} \right\} \right| \sim \frac{J_2(\omega)\omega^{k-1}}{(2544)^{k-1}\phi^k(\omega)} \frac{N}{\log^k N}, \tag{6}$$

where $\phi(\omega) = \prod_P (P-1)$.

From (6) we are able to find the smallest solution $\pi_k(N_0, 2) \geq 1$.

Example 1. Let $k = 3, 5, 7, 13, 17$

From (2) and (3) we have

$$J_2(\omega) = 0. \tag{7}$$

We prove that for $k = 3, 5, 7, 13, 17$

（1）contain no prime solutions. 1 is not a prime.

Example 2. Let $k \neq 3, 5, 7, 13, 17$

From (2) and (3) we have

$$J_2(\omega) \neq 0. \tag{8}$$

We prove that for $k \neq 3, 5, 7, 13, 17$

(1) contain infinitely many prime solutions.

The New Prime Theorem (1303)

$$p, jp^{2546} + k - j(j = 1, 2, \cdots, k-1)$$

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Abstract

Using Jiang function we prove that $jp^{2546} + k - j$ contain infinitely many prime solutions and no prime solutions.

Theorem. Let k be a given odd prime.

$$p, jp^{2546} + k - j(j = 1, \cdots, k-1) \tag{1}$$

Contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega) = \prod_{P>2} [P - 1 - \chi(P)] . \tag{2}$$

where $\omega = \prod_P P$, $\chi(P)$ is the number of solutions of congruence

$$\prod_{j=1}^{k-1}(jq^{2546} + k - j) \equiv 0 \pmod{p}, q = 1, \cdots, p-1. \tag{3}$$

If $\chi(P) \leq P - 2$ then from (2) and (3) we have

$$J_2(\omega) \neq 0 \tag{4}$$

We prove that (1) contain infinitely many prime solutions that is for any k there are infinitely many primes P such that each of $jp^{2546} + k - j$ is a prime.

Using Fermat's little theorem from (3) we have $\chi(P) = P - 1$. Substituting it into (2) we have

$$J_2(\omega) = 0 \tag{5}$$

We prove that (1) contain no prime solutions [1,2]

If $J_2(\omega) \neq 0$ then we have asymptotic formula [1,2]

$$\pi_k(N,2) = \left| \left\{ P \leq N : jp^{2546} + k - j = \text{prime} \right\} \right| \sim \frac{J_2(\omega)\omega^{k-1}}{(2546)^{k-1}\phi^k(\omega)} \frac{N}{\log^k N}, \tag{6}$$

where $\phi(\omega) = \prod_P (P-1)$.

From (6) we are able to find the smallest solution $\pi_k(N_0, 2) \geq 1$.

Example 1. Let $k = 3$

From (2) and (3) we have

$$J_2(\omega) = 0. \tag{7}$$

We prove that for $k = 3$

（1）contain no prime solutions. 1 is not a prime.

Example 2. Let $k \neq 3$

From (2) and (3) we have

$$J_2(\omega) \neq 0. \tag{8}$$

We prove that for $k \neq 3$

(1) contain infinitely many prime solutions.

The New Prime Theorem (1304)

$$p, jp^{2548} + k - j(j = 1, 2, \cdots, k-1)$$

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Abstract

Using Jiang function we prove that $jp^{2548} + k - j$ contain infinitely many prime solutions and no prime solutions.

Theorem. Let k be a given odd prime.

$$p, jp^{2548} + k - j(j = 1, \cdots, k-1) \tag{1}$$

Contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega) = \prod_{P>2} [P - 1 \chi \quad P. \tag{2}$$

where $\omega = \prod_P P$, $\chi(P)$ is the number of solutions of congruence

$$\prod_{j=1}^{k-1}(jq^{2548} + k - j) \equiv 0 \pmod{p}, q = 1, \cdots, p-1. \tag{3}$$

If $\chi(P) \leq P - 2$ then from (2) and (3) we have

$$J_2(\omega) \neq (\tag{4}$$

We prove that (1) contain infinitely many prime solutions that is for any k there are infinitely many primes P such that each of $jp^{2548} + k - j$ is a prime.

Using Fermat's little theorem from (3) we have $\chi(P) = P - 1$. Substituting it into (2) we have

$$J_2(\omega) = (\tag{5}$$

We prove that (1) contain no prime solutions [1,2]

If $J_2(\omega) \neq 0$ then we have asymptotic formula [1,2]

$$\pi_k(N,2) = \left| \left\{ P \leq N : jp^{2548} + k - j = \text{prime} \right\} \right| \sim \frac{J_2(\omega)\omega^{k-1}}{(2548)^{k-1}\phi^k(\omega)} \frac{N}{\log^k N}, \tag{6}$$

where $\phi(\omega) = \prod_P (P-1)$.

From (6) we are able to find the smallest solution $\pi_k(N_0, 2) \geq 1$.

Example 1. Let $k = 3, 5, 29, 197, 2549$

From (2) and (3) we have

$$J_2(\omega) = (\tag{7}$$

We prove that for $k = 3, 5, 29, 197, 2549$

(1) contain no prime solutions. 1 is not a prime.

Example 2. Let $k \neq 3, 5, 29, 197, 2549$

From (2) and (3) we have

$$J_2(\omega) \neq (\tag{8}$$

We prove that for $k \neq 3, 5, 29, 197, 2549$

(1) contain infinitely many prime solutions.

The New Prime Theorem (1305)

$$p, jp^{2550} + k - j(j = 1, 2, \cdots, k-1)$$

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Abstract

Using Jiang function we prove that $jp^{2550} + k - j$ contain infinitely many prime solutions and no prime solutions.

Theorem. Let k be a given odd prime.

$$p, jp^{2550} + k - j(j = 1, \cdots, k-1) \tag{1}$$

Contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega) = \prod_{P>2} [P - 1 - \chi(P)]. \tag{2}$$

where $\omega = \prod_P P$, $\chi(P)$ is the number of solutions of congruence

$$\prod_{j=1}^{k-1}(jq^{2550} + k - j) \equiv 0 \pmod{p}, q = 1, \cdots, p-1. \tag{3}$$

If $\chi(P) \le P - 2$ then from (2) and (3) we have

$$J_2(\omega) \ne 0 \tag{4}$$

We prove that (1) contain infinitely many prime solutions that is for any k there are infinitely many primes P such that each of $jp^{2550} + k - j$ is a prime.

Using Fermat's little theorem from (3) we have $\chi(P) = P - 1$. Substituting it into (2) we have

$$J_2(\omega) = 0 \tag{5}$$

We prove that (1) contain no prime solutions [1,2]

If $J_2(\omega) \ne 0$ then we have asymptotic formula [1,2]

$$\pi_k(N, 2) = \left| \left\{ P \le N : jp^{2550} + k - j = \text{prime} \right\} \right| \sim \frac{J_2(\omega)\omega^{k-1}}{(2550)^{k-1}\phi^k(\omega)} \frac{N}{\log^k N}, \tag{6}$$

where $\phi(\omega) = \prod_P (P-1)$.

From (6) we are able to find the smallest solution $\pi_k(N_0, 2) \ge 1$.

Example 1. Let $k = 3, 7, 31, 103, 151, 2551$

From (2) and (3) we have

$$J_2(\omega) = 0 \tag{7}$$

We prove that for $k = 3, 7, 31, 103, 151, 2551$

（1）contain no prime solutions. 1 is not a prime.

Example 2. Let $k \ne 3, 7, 31, 103, 151, 2551$

From (2) and (3) we have

$$J_2(\omega) \ne 0 \tag{8}$$

We prove that for $k \ne 3, 7, 31, 103, 151, 2551$

(1) contain infinitely many prime solutions.

The New Prime Theorem (1306)

$$p, jp^{2552} + k - j(j = 1, 2, \cdots, k-1)$$

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Abstract

Using Jiang function we prove that $jp^{2552} + k - j$ contain infinitely many prime solutions and no prime solutions.

Theorem. Let k be a given odd prime.

$$p, jp^{2552} + k - j(j = 1, \cdots, k-1) \tag{1}$$

Contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega) = \prod_{P>2} [P - 1 - \chi(P)]. \tag{2}$$

where $\omega = \prod_P P$, $\chi(P)$ is the number of solutions of congruence

$$\prod_{j=1}^{k-1} (jq^{2552} + k - j) \equiv 0 \pmod{p}, q = 1, \cdots, p-1. \tag{3}$$

If $\chi(P) \le P - 2$ then from (2) and (3) we have

$$J_2(\omega) \ne 0 \tag{4}$$

We prove that (1) contain infinitely many prime solutions that is for any k there are infinitely many primes P such that each of $jp^{2552} + k - j$ is a prime.

Using Fermat's little theorem from (3) we have $\chi(P) = P-1$. Substituting it into (2) we have

$$J_2(\omega) = 0 \tag{5}$$

We prove that (1) contain no prime solutions [1,2]

If $J_2(\omega) \ne 0$ then we have asymptotic formula [1,2]

$$\pi_k(N, 2) = \left| \left\{ P \le N : jp^{2552} + k - j = \text{prime} \right\} \right| \sim \frac{J_2(\omega)\omega^{k-1}}{(2552)^{k-1}\phi^k(\omega)} \frac{N}{\log^k N}, \tag{6}$$

where $\phi(\omega) = \prod_P (P-1)$.

From (6) we are able to find the smallest solution $\pi_k(N_0, 2) \ge 1$.

Example 1. Let $k = 3, 5, 23, 59, 89, 1277$

From (2) and (3) we have

$$J_2(\omega) = 0 \tag{7}$$

We prove that for $k = 3, 5, 23, 59, 89, 1277$

（1）contain no prime solutions. 1 is not a prime.

Example 2. Let $k \ne 3, 5, 23, 59, 89, 1277$

From (2) and (3) we have

$$J_2(\omega) \ne 0 \tag{8}$$

We prove that for $k \ne 3, 5, 23, 59, 89, 1277$

(1) contain infinitely many prime solutions.

The New Prime Theorem (1307)

$$p, jp^{2554} + k - j(j = 1, 2, \cdots, k-1)$$

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Abstract

Using Jiang function we prove that $jp^{2434} + k - j$ contain infinitely many prime solutions and no prime solutions.

Theorem. Let k be a given odd prime.

$$p, jp^{2554} + k - j(j = 1, \cdots, k-1) \tag{1}$$

Contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega) = \prod_{P>2} [P - 1 \chi \ P \ . \tag{2}$$

where $\omega = \prod_P P$, $\chi(P)$ is the number of solutions of congruence

$$\prod_{j=1}^{k-1} (jq^{2554} + k - j) \equiv 0 \pmod{p}, q = 1, \cdots, p-1. \tag{3}$$

If $\chi(P) \leq P - 2$ then from (2) and (3) we have

$$J_2(\omega) \neq (\tag{4}$$

We prove that (1) contain infinitely many prime solutions that is for any k there are infinitely many primes P such that each of $jp^{2554} + k - j$ is a prime.

Using Fermat's little theorem from (3) we have $\chi(P) = P - 1$. Substituting it into (2) we have

$$J_2(\omega) = (\tag{5}$$

We prove that (1) contain no prime solutions [1,2]

If $J_2(\omega) \neq 0$ then we have asymptotic formula [1,2]

$$\pi_k(N, 2) = \left| \left\{ P \leq N : jp^{2554} + k - j = \text{prime} \right\} \right| \sim \frac{J_2(\omega)\omega^{k-1}}{(2554)^{k-1}\phi^k(\omega)} \frac{N}{\log^k N}, \tag{6}$$

where $\phi(\omega) = \prod_P (P-1)$.

From (6) we are able to find the smallest solution $\pi_k(N_0, 2) \geq 1$.

Example 1. Let $k = 3$

From (2) and (3) we have

$$J_2(\omega) = (. \tag{7}$$

We prove that for $k = 3$

（1）contain no prime solutions. 1 is not a prime.

Example 2. Let $k \neq 3$

From (2) and (3) we have

$$J_2(\omega) \neq (. \tag{8}$$

We prove that for $k \neq 3$

(1) contain infinitely many prime solutions.

The New Prime Theorem (1308)

$$p, jp^{2556} + k - j(j = 1, 2, \cdots, k-1)$$

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Abstract

Using Jiang function we prove that $jp^{2556} + k - j$ contain infinitely many prime solutions and no prime solutions.

Theorem. Let k be a given odd prime.

$$p, jp^{2556} + k - j(j = 1, \cdots, k-1) \tag{1}$$

Contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega) = \prod_{P>2} [P - 1 - \chi(P)]. \tag{2}$$

where $\omega = \prod_P P$, $\chi(P)$ is the number of solutions of congruence

$$\prod_{j=1}^{k-1}(jq^{2556} + k - j) \equiv 0 \pmod{p}, q = 1, \cdots, p-1. \tag{3}$$

If $\chi(P) \leq P - 2$ then from (2) and (3) we have

$$J_2(\omega) \neq 0 \tag{4}$$

We prove that (1) contain infinitely many prime solutions that is for any k there are infinitely many primes P such that each of $jp^{2556} + k - j$ is a prime.

Using Fermat's little theorem from (3) we have $\chi(P) = P - 1$. Substituting it into (2) we have

$$J_2(\omega) = 0 \tag{5}$$

We prove that (1) contain no prime solutions [1,2]

If $J_2(\omega) \neq 0$ then we have asymptotic formula [1,2]

$$\pi_k(N,2) = \left| \left\{ P \leq N : jp^{2556} + k - j = \text{prime} \right\} \right| \sim \frac{J_2(\omega)\omega^{k-1}}{(2556)^{k-1}\phi^k(\omega)} \frac{N}{\log^k N}, \tag{6}$$

where $\phi(\omega) = \prod_P (P-1)$.

From (6) we are able to find the smallest solution $\pi_k(N_0, 2) \geq 1$.

Example 1. Let $k = 3, 5, 7, 13, 19, 37, 853, 1279, 2557$

From (2) and (3) we have

$$J_2(\omega) = 0 \tag{7}$$

We prove that for $k = 3, 5, 7, 13, 19, 37, 853, 1279, 2557$

（1）contain no prime solutions. 1 is not a prime.

Example 2. Let $k \neq 3, 5, 7, 13, 19, 37, 853, 1279, 2557$

From (2) and (3) we have

$$J_2(\omega) \neq 0 \tag{8}$$

We prove that for $k \neq 3, 5, 7, 13, 19, 37, 853, 1279, 2557$

(1) contain infinitely many prime solutions.

The New Prime Theorem (1309)

$$p, jp^{2558} + k - j(j = 1, 2, \cdots, k-1)$$

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Abstract

Using Jiang function we prove that $jp^{2558} + k - j$ contain infinitely many prime solutions and no prime solutions.

Theorem. Let k be a given odd prime.

$$p, jp^{2558} + k - j(j = 1, \cdots, k-1) \tag{1}$$

Contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega) = \prod_{P>2} [P - 1 - \chi(P)]. \tag{2}$$

where $\omega = \prod_P P$, $\chi(P)$ is the number of solutions of congruence

$$\prod_{j=1}^{k-1}(jq^{2558} + k - j) \equiv 0 \pmod{p}, q = 1, \cdots, p-1. \tag{3}$$

If $\chi(P) \leq P - 2$ then from (2) and (3) we have

$$J_2(\omega) \neq 0 \tag{4}$$

We prove that (1) contain infinitely many prime solutions that is for any k there are infinitely many primes P such that each of $jp^{2558} + k - j$ is a prime.

Using Fermat's little theorem from (3) we have $\chi(P) = P - 1$. Substituting it into (2) we have

$$J_2(\omega) = 0 \tag{5}$$

We prove that (1) contain no prime solutions [1,2]

If $J_2(\omega) \neq 0$ then we have asymptotic formula [1,2]

$$\pi_k(N, 2) = \left| \{ P \leq N : jp^{2558} + k - j = \text{prime} \} \right| \sim \frac{J_2(\omega)\omega^{k-1}}{(2558)^{k-1}\phi^k(\omega)} \frac{N}{\log^k N}, \tag{6}$$

where $\phi(\omega) = \prod_P (P-1)$.

From (6) we are able to find the smallest solution $\pi_k(N_0, 2) \geq 1$.

Example 1. Let $k = 3$

From (2) and (3) we have

$$J_2(\omega) = 0 \tag{7}$$

We prove that for $k = 3$

（1）contain no prime solutions. 1 is not a prime.

Example 2. Let $k \neq 3$

From (2) and (3) we have

$$J_2(\omega) \neq 0 \tag{8}$$

We prove that for $k \neq 3$

(1) contain infinitely many prime solutions.

The New Prime Theorem (1310)

$$p, jp^{2560} + k - j(j = 1, 2, \cdots, k-1)$$

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Abstract

Using Jiang function we prove that $jp^{2560} + k - j$ contain infinitely many prime solutions and no prime solutions.

Theorem. Let k be a given odd prime.

$$p, jp^{2560} + k - j(j = 1, \cdots, k-1) \tag{1}$$

Contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega) = \prod_{P>2} [P - 1 - \chi(P)]. \tag{2}$$

where $\omega = \prod_P P$, $\chi(P)$ is the number of solutions of congruence

$$\prod_{j=1}^{k-1}(jq^{2560} + k - j) \equiv 0 \pmod{p}, q = 1, \cdots, p-1. \tag{3}$$

If $\chi(P) \le P - 2$ then from (2) and (3) we have

$$J_2(\omega) \ne 0 \tag{4}$$

We prove that (1) contain infinitely many prime solutions that is for any k there are infinitely many primes P such that each of $jp^{2560} + k - j$ is a prime.

Using Fermat's little theorem from (3) we have $\chi(P) = P - 1$. Substituting it into (2) we have

$$J_2(\omega) = 0 \tag{5}$$

We prove that (1) contain no prime solutions [1,2]

If $J_2(\omega) \ne 0$ then we have asymptotic formula [1,2]

$$\pi_k(N,2) = \left|\left\{ P \le N : jp^{2560} + k - j = \text{prime} \right\}\right| \sim \frac{J_2(\omega)\omega^{k-1}}{(2560)^{k-1}\phi^k(\omega)} \frac{N}{\log^k N}, \tag{6}$$

where $\phi(\omega) = \prod_P (P-1)$.

From (6) we are able to find the smallest solution $\pi_k(N_0, 2) \ge 1$.

Example 1. Let $k = 3, 5, 11, 17, 41, 257, 641$

From (2) and (3) we have

$$J_2(\omega) = 0 \tag{7}$$

We prove that for $k = 3, 5, 11, 17, 41, 257, 641$

（1）contain no prime solutions. 1 is not a prime.

Example 2. Let $k \ne 3, 5, 11, 17, 41, 257, 641$

From (2) and (3) we have

$$J_2(\omega) \ne 0 \tag{8}$$

We prove that for $k \ne 3, 5, 11, 17, 41, 257, 641$

(1) contain infinitely many prime solutions.

The New Prime Theorem (1311)

$$p, jp^{2562} + k - j(j = 1, 2, \cdots, k-1)$$

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Abstract

Using Jiang function we prove that $jp^{2562} + k - j$ contain infinitely many prime solutions and no prime solutions.

Theorem. Let k be a given odd prime.

$$p, jp^{2562} + k - j(j = 1, \cdots, k-1) \tag{1}$$

Contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega) = \prod_{P>2} [P - 1 - \chi(P)]. \tag{2}$$

where $\omega = \prod_P P$, $\chi(P)$ is the number of solutions of congruence

$$\prod_{j=1}^{k-1}(jq^{2562} + k - j) \equiv 0 \pmod{p}, q = 1, \cdots, p-1. \tag{3}$$

If $\chi(P) \leq P - 2$ then from (2) and (3) we have

$$J_2(\omega) \neq 0 \tag{4}$$

We prove that (1) contain infinitely many prime solutions that is for any k there are infinitely many primes P such that each of $jp^{2562} + k - j$ is a prime.

Using Fermat's little theorem from (3) we have $\chi(P) = P - 1$. Substituting it into (2) we have

$$J_2(\omega) = 0 \tag{5}$$

We prove that (1) contain no prime solutions [1,2]

If $J_2(\omega) \neq 0$ then we have asymptotic formula [1,2]

$$\pi_k(N, 2) = \left| \left\{ P \leq N : jp^{2562} + k - j = \text{prime} \right\} \right| \sim \frac{J_2(\omega)\omega^{k-1}}{(2562)^{k-1}\phi^k(\omega)} \frac{N}{\log^k N}, \tag{6}$$

where $\phi(\omega) = \prod_P (P-1)$.

From (6) we are able to find the smallest solution $\pi_k(N_0, 2) \geq 1$.

Example 1. Let $k = 3, 7, 43, 367$

From (2) and (3) we have

$$J_2(\omega) = 0. \tag{7}$$

We prove that for $k = 3, 7, 43, 367$

（1）contain no prime solutions. 1 is not a prime.

Example 2. Let $k \neq 3, 7, 43, 367$

From (2) and (3) we have

$$J_2(\omega) \neq 0. \tag{8}$$

We prove that for $k \neq 3, 7, 43, 367$

(1) contain infinitely many prime solutions.

The New Prime Theorem (1312)

$$p, jp^{2564} + k - j(j = 1, 2, \cdots, k-1)$$

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Abstract

Using Jiang function we prove that $jp^{2564} + k - j$ contain infinitely many prime solutions and no prime solutions.

Theorem. Let k be a given odd prime.

$$p, jp^{2564} + k - j(j = 1, \cdots, k-1) \tag{1}$$

Contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega) = \prod_{P>2} [P - 1 - \chi(P)]. \tag{2}$$

where $\omega = \prod_P P$, $\chi(P)$ is the number of solutions of congruence

$$\prod_{j=1}^{k-1}(jq^{2564} + k - j) \equiv 0 \pmod{p}, q = 1, \cdots, p-1. \tag{3}$$

If $\chi(P) \le P-2$ then from (2) and (3) we have

$$J_2(\omega) \ne 0 \tag{4}$$

We prove that (1) contain infinitely many prime solutions that is for any k there are infinitely many primes P such that each of $jp^{2564} + k - j$ is a prime.

Using Fermat's little theorem from (3) we have $\chi(P) = P-1$. Substituting it into (2) we have

$$J_2(\omega) = 0 \tag{5}$$

We prove that (1) contain no prime solutions [1,2]

If $J_2(\omega) \ne 0$ then we have asymptotic formula [1,2]

$$\pi_k(N,2) = \left| \left\{ P \le N : jp^{2564} + k - j = \text{prime} \right\} \right| \sim \frac{J_2(\omega)\omega^{k-1}}{(2564)^{k-1}\phi^k(\omega)} \frac{N}{\log^k N}, \tag{6}$$

where $\phi(\omega) = \prod_P (P-1)$.

From (6) we are able to find the smallest solution $\pi_k(N_0, 2) \ge 1$.

Example 1. Let $k = 3, 5, 1283$

From (2) and (3) we have

$$J_2(\omega) = 0 \tag{7}$$

We prove that for $k = 3, 5, 1283$

（1）contain no prime solutions. 1 is not a prime.

Example 2. Let $k \ne 3, 5, 1283$

From (2) and (3) we have

$$J_2(\omega) \ne 0 \tag{8}$$

We prove that for $k \ne 3, 5, 1283$

(1) contain infinitely many prime solutions.

The New Prime Theorem (1313)

$$p, jp^{2566}+k-j(j=1,2,\cdots,k-1)$$

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Abstract

Using Jiang function we prove that $jp^{2566}+k-j$ contain infinitely many prime solutions and no prime solutions.

Theorem. Let k be a given odd prime.

$$p, jp^{2566}+k-j(j=1,\cdots,k-1) \tag{1}$$

Contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega)= \prod_{P>2} [P-1-\chi(P)]. \tag{2}$$

where $\omega=\prod_P P$, $\chi(P)$ is the number of solutions of congruence

$$\prod_{j=1}^{k-1}(jq^{2566}+k-j)\equiv 0 \pmod{p}, q=1,\cdots,p-1. \tag{3}$$

If $\chi(P)\le P-2$ then from (2) and (3) we have

$$J_2(\omega)\ne 0 \tag{4}$$

We prove that (1) contain infinitely many prime solutions that is for any k there are infinitely many primes P such that each of $jp^{2566}+k-j$ is a prime.

Using Fermat's little theorem from (3) we have $\chi(P)=P-1$. Substituting it into (2) we have

$$J_2(\omega)=0 \tag{5}$$

We prove that (1) contain no prime solutions [1,2]

If $J_2(\omega)\ne 0$ then we have asymptotic formula [1,2]

$$\pi_k(N,2)=\left|\left\{P\le N: jp^{2566}+k-j=\text{prime}\right\}\right|\sim \frac{J_2(\omega)\omega^{k-1}}{(2566)^{k-1}\phi^k(\omega)}\frac{N}{\log^k N}, \tag{6}$$

where $\phi(\omega)=\prod_P(P-1)$.

From (6) we are able to find the smallest solution $\pi_k(N_0,2)\ge 1$.

Example 1. Let $k=3$

From (2) and (3) we have

$$J_2(\omega)=0. \tag{7}$$

We prove that for $k=3$

（1）contain no prime solutions. 1 is not a prime.

Example 2. Let $k\ne 3$

From (2) and (3) we have

$$J_2(\omega)\ne 0. \tag{8}$$

We prove that for $k\ne 3$

(1) contain infinitely many prime solutions.

The New Prime Theorem (1314)

$$p, jp^{2568} + k - j(j = 1, 2, \cdots, k-1)$$

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Abstract

Using Jiang function we prove that $jp^{2568} + k - j$ contain infinitely many prime solutions and no prime solutions.

Theorem. Let k be a given odd prime.

$$p, jp^{2568} + k - j(j = 1, \cdots, k-1) \tag{1}$$

Contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega) = \prod_{P>2} [P - 1 - \chi(P)]. \tag{2}$$

where $\omega = \prod_P P$, $\chi(P)$ is the number of solutions of congruence

$$\prod_{j=1}^{k-1}(jq^{2568} + k - j) \equiv 0 \pmod{p}, q = 1, \cdots, p-1. \tag{3}$$

If $\chi(P) \leq P - 2$ then from (2) and (3) we have

$$J_2(\omega) \neq 0 \tag{4}$$

We prove that (1) contain infinitely many prime solutions that is for any k there are infinitely many primes P such that each of $jp^{2568} + k - j$ is a prime.

Using Fermat's little theorem from (3) we have $\chi(P) = P - 1$. Substituting it into (2) we have

$$J_2(\omega) = 0 \tag{5}$$

We prove that (1) contain no prime solutions [1,2]

If $J_2(\omega) \neq 0$ then we have asymptotic formula [1,2]

$$\pi_k(N,2) = \left|\left\{P \leq N : jp^{2568} + k - j = \text{prime}\right\}\right| \sim \frac{J_2(\omega)\omega^{k-1}}{(2568)^{k-1}\phi^k(\omega)} \frac{N}{\log^k N}, \tag{6}$$

where $\phi(\omega) = \prod_P (P-1)$.

From (6) we are able to find the smallest solution $\pi_k(N_0, 2) \geq 1$.

Example 1. Let $k = 3, 5, 7, 13, 643, 857$

From (2) and (3) we have

$$J_2(\omega) = 0 \tag{7}$$

We prove that for $k = 3, 5, 7, 13, 643, 857$

（1）contain no prime solutions. 1 is not a prime.

Example 2. Let $k \neq 3, 5, 7, 13, 643, 857$

From (2) and (3) we have

$$J_2(\omega) \neq 0 \tag{8}$$

We prove that for $k \neq 3, 5, 7, 13, 643, 857$

(1) contain infinitely many prime solutions.

The New Prime Theorem (1315)

$$p, jp^{2570} + k - j (j = 1, 2, \cdots, k - 1)$$

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Abstract

Using Jiang function we prove that $jp^{2570} + k - j$ contain infinitely many prime solutions and no prime solutions.

Theorem. Let k be a given odd prime.

$$p, jp^{2570} + k - j (j = 1, \cdots, k - 1) \tag{1}$$

Contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega) = \prod_{P>2} [P - 1 - \chi(P)]. \tag{2}$$

where $\omega = \prod_P P$, $\chi(P)$ is the number of solutions of congruence

$$\prod_{j=1}^{k-1}(jq^{2570} + k - j) \equiv 0 \pmod{p}, q = 1, \cdots, p - 1. \tag{3}$$

If $\chi(P) \leq P - 2$ then from (2) and (3) we have

$$J_2(\omega) \neq 0 \tag{4}$$

We prove that (1) contain infinitely many prime solutions that is for any k there are infinitely many primes P such that each of $jp^{2570} + k - j$ is a prime.

Using Fermat's little theorem from (3) we have $\chi(P) = P - 1$. Substituting it into (2) we have

$$J_2(\omega) = 0 \tag{5}$$

We prove that (1) contain no prime solutions [1,2]

If $J_2(\omega) \neq 0$ then we have asymptotic formula [1,2]

$$\pi_k(N,2) = \left| \left\{ P \leq N : jp^{2570} + k - j = \text{prime} \right\} \right| \sim \frac{J_2(\omega)\omega^{k-1}}{(2570)^{k-1}\phi^k(\omega)} \frac{N}{\log^k N}, \tag{6}$$

where $\phi(\omega) = \prod_P (P - 1)$.

From (6) we are able to find the smallest solution $\pi_k(N_0, 2) \geq 1$.

Example 1. Let $k = 3, 11$

From (2) and (3) we have

$$J_2(\omega) = 0. \tag{7}$$

We prove that for $k = 3, 11$

（1）contain no prime solutions. 1 is not a prime.

Example 2. Let $k \neq 3, 11$.

From (2) and (3) we have

$$J_2(\omega) \neq 0. \tag{8}$$

We prove that for $k \neq 3, 11$

(1) contain infinitely many prime solutions.

The New Prime Theorem (1316)

$$p, jp^{2572} + k - j(j = 1, 2, \cdots, k-1)$$

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Abstract

Using Jiang function we prove that $jp^{2572} + k - j$ contain infinitely many prime solutions and no prime solutions.

Theorem. Let k be a given odd prime.

$$p, jp^{2572} + k - j(j = 1, \cdots, k-1) \tag{1}$$

Contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega) = \prod_{P>2} [P - 1 - \chi(P)] . \tag{2}$$

where $\omega = \prod_P P$, $\chi(P)$ is the number of solutions of congruence

$$\prod_{j=1}^{k-1}(jq^{2572} + k - j) \equiv 0 \pmod{p}, q = 1, \cdots, p-1. \tag{3}$$

If $\chi(P) \leq P - 2$ then from (2) and (3) we have

$$J_2(\omega) \neq 0 \tag{4}$$

We prove that (1) contain infinitely many prime solutions that is for any k there are infinitely many primes P such that each of $jp^{2572} + k - j$ is a prime.

Using Fermat's little theorem from (3) we have $\chi(P) = P - 1$. Substituting it into (2) we have

$$J_2(\omega) = 0 \tag{5}$$

We prove that (1) contain no prime solutions [1,2]

If $J_2(\omega) \neq 0$ then we have asymptotic formula [1,2]

$$\pi_k(N, 2) = \left| \left\{ P \leq N : jp^{2572} + k - j = \text{prime} \right\} \right| \sim \frac{J_2(\omega)\omega^{k-1}}{(2572)^{k-1}\phi^k(\omega)} \frac{N}{\log^k N}, \tag{6}$$

where $\phi(\omega) = \prod_P (P-1)$.

From (6) we are able to find the smallest solution $\pi_k(N_0, 2) \geq 1$.

Example 1. Let $k = 3, 5$

From (2) and (3) we have

$$J_2(\omega) = 0 \tag{7}$$

We prove that for $k = 3, 5$

（1）contain no prime solutions. 1 is not a prime.

Example 2. Let $k \neq 3, 5$

From (2) and (3) we have

$$J_2(\omega) \neq 0 \tag{8}$$

We prove that for $k \neq 3, 5$

(1) contain infinitely many prime solutions.

The New Prime Theorem (1317)

$$p, jp^{2574} + k - j(j = 1, 2, \cdots, k-1)$$

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Abstract

Using Jiang function we prove that $jp^{2574} + k - j$ contain infinitely many prime solutions and no prime solutions.

Theorem. Let k be a given odd prime.

$$p, jp^{2574} + k - j(j = 1, \cdots, k-1) \tag{1}$$

Contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega) = \prod_{P>2} [P - 1 - \chi(P)]. \tag{2}$$

where $\omega = \prod_P P$, $\chi(P)$ is the number of solutions of congruence

$$\prod_{j=1}^{k-1}(jq^{2574} + k - j) \equiv 0 \pmod{p}, q = 1, \cdots, p-1. \tag{3}$$

If $\chi(P) \leq P - 2$ then from (2) and (3) we have

$$J_2(\omega) \neq 0 \tag{4}$$

We prove that (1) contain infinitely many prime solutions that is for any k there are infinitely many primes P such that each of $jp^{2574} + k - j$ is a prime.

Using Fermat's little theorem from (3) we have $\chi(P) = P - 1$. Substituting it into (2) we have

$$J_2(\omega) = 0 \tag{5}$$

We prove that (1) contain no prime solutions [1,2]

If $J_2(\omega) \neq 0$ then we have asymptotic formula [1,2]

$$\pi_k(N, 2) = \left| \left\{ P \leq N : jp^{2574} + k - j = \text{prime} \right\} \right| \sim \frac{J_2(\omega)\omega^{k-1}}{(2574)^{k-1}\phi^k(\omega)} \frac{N}{\log^k N}, \tag{6}$$

where $\phi(\omega) = \prod_P (P-1)$.

From (6) we are able to find the smallest solution $\pi_k(N_0, 2) \geq 1$.

Example 1. Let $k = 3, 7, 19, 23, 67, 79, 199, 859$

From (2) and (3) we have

$$J_2(\omega) = 0 \tag{7}$$

We prove that for $k = 3, 7, 19, 23, 67, 79, 199, 859$

（1）contain no prime solutions. 1 is not a prime.

Example 2. Let $k \neq 3, 7, 19, 23, 67, 79, 199, 859$

From (2) and (3) we have

$$J_2(\omega) \neq 0 \tag{8}$$

We prove that for $k \neq 3, 7, 19, 23, 67, 79, 199, 859$

(1) contain infinitely many prime solutions.

The New Prime Theorem (1318)

$$p, jp^{2576} + k - j(j = 1, 2, \cdots, k-1)$$

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Abstract

Using Jiang function we prove that $jp^{2576} + k - j$ contain infinitely many prime solutions and no prime solutions.

Theorem. Let k be a given odd prime.

$$p, jp^{2576} + k - j(j = 1, \cdots, k-1) \tag{1}$$

Contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega) = \prod_{P>2} [P - 1 - \chi(P)]. \tag{2}$$

where $\omega = \prod_P P$, $\chi(P)$ is the number of solutions of congruence

$$\prod_{j=1}^{k-1}(jq^{2576} + k - j) \equiv 0 \pmod{p}, q = 1, \cdots, p-1. \tag{3}$$

If $\chi(P) \leq P - 2$ then from (2) and (3) we have

$$J_2(\omega) \neq 0 \tag{4}$$

We prove that (1) contain infinitely many prime solutions that is for any k there are infinitely many primes P such that each of $jp^{2576} + k - j$ is a prime.

Using Fermat's little theorem from (3) we have $\chi(P) = P - 1$. Substituting it into (2) we have

$$J_2(\omega) = 0 \tag{5}$$

We prove that (1) contain no prime solutions [1,2]

If $J_2(\omega) \neq 0$ then we have asymptotic formula [1,2]

$$\pi_k(N,2) = \left|\left\{P \leq N : jp^{2576} + k - j = \text{prime}\right\}\right| \sim \frac{J_2(\omega)\omega^{k-1}}{(2576)^{k-1}\phi^k(\omega)} \frac{N}{\log^k N}, \tag{6}$$

where $\phi(\omega) = \prod_P(P-1)$.

From (6) we are able to find the smallest solution $\pi_k(N_0, 2) \geq 1$.

Example 1. Let $k = 3, 5, 17, 29, 47, 113, 1289$

From (2) and (3) we have

$$J_2(\omega) = 0 \tag{7}$$

We prove that for $k = 3, 5, 17, 29, 47, 113, 1289$

（1）contain no prime solutions. 1 is not a prime.

Example 2. Let $k \neq 3, 5, 17, 29, 47, 113, 1289$

From (2) and (3) we have

$$J_2(\omega) \neq 0 \tag{8}$$

We prove that for $k \neq 3, 5, 17, 29, 47, 113, 1289$

(1) contain infinitely many prime solutions.

The New Prime Theorem (1319)

$$p, jp^{2578} + k - j(j = 1, 2, \cdots, k-1)$$

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Abstract

Using Jiang function we prove that $jp^{2578} + k - j$ contain infinitely many prime solutions and no prime solutions.

Theorem. Let k be a given odd prime.

$$p, jp^{2578} + k - j(j = 1, \cdots, k-1) \tag{1}$$

Contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega) = \prod_{P>2} [P - 1 - \chi(P)]. \tag{2}$$

where $\omega = \prod_P P$, $\chi(P)$ is the number of solutions of congruence

$$\prod_{j=1}^{k-1}(jq^{2578} + k - j) \equiv 0 \pmod{p}, q = 1, \cdots, p-1. \tag{3}$$

If $\chi(P) \leq P-2$ then from (2) and (3) we have

$$J_2(\omega) \neq 0 \tag{4}$$

We prove that (1) contain infinitely many prime solutions that is for any k there are infinitely many primes P such that each of $jp^{2578} + k - j$ is a prime.

Using Fermat's little theorem from (3) we have $\chi(P) = P-1$. Substituting it into (2) we have

$$J_2(\omega) = 0 \tag{5}$$

We prove that (1) contain no prime solutions [1,2]

If $J_2(\omega) \neq 0$ then we have asymptotic formula [1,2]

$$\pi_k(N, 2) = \left| \left\{ P \leq N : jp^{2578} + k - j = \text{prime} \right\} \right| \sim \frac{J_2(\omega)\omega^{k-1}}{(2578)^{k-1}\phi^k(\omega)} \frac{N}{\log^k N}, \tag{6}$$

where $\phi(\omega) = \prod_P (P-1)$.

From (6) we are able to find the smallest solution $\pi_k(N_0, 2) \geq 1$.

Example 1. Let $k = 3, 2579$

From (2) and (3) we have

$$J_2(\omega) = 0. \tag{7}$$

We prove that for $k = 3, 2579$

（1）contain no prime solutions. 1 is not a prime.

Example 2. Let $k \neq 3, 2579$

From (2) and (3) we have

$$J_2(\omega) \neq 0. \tag{8}$$

We prove that for $k \neq 3, 2579$

(1) contain infinitely many prime solutions.

The New Prime Theorem (1320)

$$p, jp^{2580} + k - j(j = 1, 2, \cdots, k-1)$$

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Abstract

Using Jiang function we prove that $jp^{2580} + k - j$ contain infinitely many prime solutions and no prime solutions.

Theorem. Let k be a given odd prime.

$$p, jp^{2580} + k - j(j = 1, \cdots, k-1) \tag{1}$$

Contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega) = \prod_{P>2} [P - 1 - \chi(P)]. \tag{2}$$

where $\omega = \prod_P P$, $\chi(P)$ is the number of solutions of congruence

$$\prod_{j=1}^{k-1}(jq^{2580} + k - j) \equiv 0 \pmod{p}, q = 1, \cdots, p-1. \tag{3}$$

If $\chi(P) \leq P - 2$ then from (2) and (3) we have

$$J_2(\omega) \neq 0 \tag{4}$$

We prove that (1) contain infinitely many prime solutions that is for any k there are infinitely many primes P such that each of $jp^{2580} + k - j$ is a prime.

Using Fermat's little theorem from (3) we have $\chi(P) = P - 1$. Substituting it into (2) we have

$$J_2(\omega) = 0 \tag{5}$$

We prove that (1) contain no prime solutions [1,2]

If $J_2(\omega) \neq 0$ then we have asymptotic formula [1,2]

$$\pi_k(N, 2) = \left| \left\{ P \leq N : jp^{2580} + k - j = \text{prime} \right\} \right| \sim \frac{J_2(\omega)\omega^{k-1}}{(2580)^{k-1}\phi^k(\omega)} \frac{N}{\log^k N}, \tag{6}$$

where $\phi(\omega) = \prod_P (P-1)$.

From (6) we are able to find the smallest solution $\pi_k(N_0, 2) \geq 1$.

Example 1. Let $k = 3, 5, 7, 11, 13, 31, 61, 431, 1291$

From (2) and (3) we have

$$J_2(\omega) = 0 \tag{7}$$

We prove that for $k = 3, 5, 7, 11, 13, 31, 61, 431, 1291$

（1）contain no prime solutions. 1 is not a prime.

Example 2. Let $k \neq 3, 5, 7, 11, 13, 31, 61, 431, 1291$

From (2) and (3) we have

$$J_2(\omega) \neq 0 \tag{8}$$

We prove that for $k \neq 3, 5, 7, 11, 13, 31, 61, 431, 1291$

(1) contain infinitely many prime solutions.

The New Prime Theorem (1321)

$$p, jp^{2582} + k - j(j = 1, 2, \cdots, k-1)$$

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Abstract

Using Jiang function we prove that $jp^{2582} + k - j$ contain infinitely many prime solutions and no prime solutions.

Theorem. Let k be a given odd prime.

$$p, jp^{2582} + k - j(j = 1, \cdots, k-1) \tag{1}$$

Contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega) = \prod_{P>2} [P - 1 - \chi(P)]. \tag{2}$$

where $\omega = \prod_P P$, $\chi(P)$ is the number of solutions of congruence

$$\prod_{j=1}^{k-1}(jq^{2582} + k - j) \equiv 0 \pmod{p}, q = 1, \cdots, p-1. \tag{3}$$

If $\chi(P) \le P - 2$ then from (2) and (3) we have

$$J_2(\omega) \ne 0 \tag{4}$$

We prove that (1) contain infinitely many prime solutions that is for any k there are infinitely many primes P such that each of $jp^{2582} + k - j$ is a prime.

Using Fermat's little theorem from (3) we have $\chi(P) = P - 1$. Substituting it into (2) we have

$$J_2(\omega) = 0 \tag{5}$$

We prove that (1) contain no prime solutions [1,2]

If $J_2(\omega) \ne 0$ then we have asymptotic formula [1,2]

$$\pi_k(N, 2) = \left| \left\{ P \le N : jp^{2582} + k - j = \text{prime} \right\} \right| \sim \frac{J_2(\omega)\omega^{k-1}}{(2582)^{k-1}\phi^k(\omega)} \frac{N}{\log^k N}, \tag{6}$$

where $\phi(\omega) = \prod_P (P-1)$.

From (6) we are able to find the smallest solution $\pi_k(N_0, 2) \ge 1$.

Example 1. Let $k = 3$

From (2) and (3) we have

$$J_2(\omega) = 0. \tag{7}$$

We prove that for $k = 3$

（1）contain no prime solutions. 1 is not a prime.

Example 2. Let $k \ne 3$

From (2) and (3) we have

$$J_2(\omega) \ne 0. \tag{8}$$

We prove that for $k \ne 3$

(1) contain infinitely many prime solutions.

The New Prime Theorem (1322)

$$p, jp^{2584} + k - j(j = 1, 2, \cdots, k-1)$$

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Abstract

Using Jiang function we prove that $jp^{2584} + k - j$ contain infinitely many prime solutions and no prime solutions.

Theorem. Let k be a given odd prime.

$$p, jp^{2584} + k - j(j = 1, \cdots, k-1) \tag{1}$$

Contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega) = \prod_{P>2} [P - 1 - \chi(P)]. \tag{2}$$

where $\omega = \prod_P P$, $\chi(P)$ is the number of solutions of congruence

$$\prod_{j=1}^{k-1}(jq^{2584} + k - j) \equiv 0 \pmod{p}, q = 1, \cdots, p-1. \tag{3}$$

If $\chi(P) \leq P - 2$ then from (2) and (3) we have

$$J_2(\omega) \neq 0 \tag{4}$$

We prove that (1) contain infinitely many prime solutions that is for any k there are infinitely many primes P such that each of $jp^{2584} + k - j$ is a prime.

Using Fermat's little theorem from (3) we have $\chi(P) = P - 1$. Substituting it into (2) we have

$$J_2(\omega) = 0 \tag{5}$$

We prove that (1) contain no prime solutions [1,2]

If $J_2(\omega) \neq 0$ then we have asymptotic formula [1,2]

$$\pi_k(N, 2) = \left|\left\{ P \leq N : jp^{2584} + k - j = \text{prime} \right\}\right| \sim \frac{J_2(\omega)\omega^{k-1}}{(2584)^{k-1}\phi^k(\omega)} \frac{N}{\log^k N}, \tag{6}$$

where $\phi(\omega) = \prod_P (P-1)$.

From (6) we are able to find the smallest solution $\pi_k(N_0, 2) \geq 1$.

Example 1. Let $k = 3, 5, 137, 647$

From (2) and (3) we have

$$J_2(\omega) = 0. \tag{7}$$

We prove that for $k = 3, 5, 137, 647$

（1）contain no prime solutions. 1 is not a prime.

Example 2. Let $k \neq 3, 5, 137, 647$

From (2) and (3) we have

$$J_2(\omega) \neq 0. \tag{8}$$

We prove that for $k \neq 3, 5, 137, 647$

(1) contain infinitely many prime solutions.

The New Prime Theorem (1323)

$$p, jp^{2586} + k - j(j = 1, 2, \cdots, k-1)$$

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Abstract

Using Jiang function we prove that $jp^{2586} + k - j$ contain infinitely many prime solutions and no prime solutions.

Theorem. Let k be a given odd prime.

$$p, jp^{2586} + k - j(j = 1, \cdots, k-1) \tag{1}$$

Contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega) = \prod_{P>2} [P - 1 - \chi(P)]. \tag{2}$$

where $\omega = \prod_P P$, $\chi(P)$ is the number of solutions of congruence

$$\prod_{j=1}^{k-1}(jq^{2586} + k - j) \equiv 0 \pmod{p}, q = 1, \cdots, p-1. \tag{3}$$

If $\chi(P) \leq P - 2$ then from (2) and (3) we have

$$J_2(\omega) \neq 0 \tag{4}$$

We prove that (1) contain infinitely many prime solutions that is for any k there are infinitely many primes P such that each of $jp^{2586} + k - j$ is a prime.

Using Fermat's little theorem from (3) we have $\chi(P) = P - 1$. Substituting it into (2) we have

$$J_2(\omega) = 0 \tag{5}$$

We prove that (1) contain no prime solutions [1,2]

If $J_2(\omega) \neq 0$ then we have asymptotic formula [1,2]

$$\pi_k(N, 2) = \left| \left\{ P \leq N : jp^{2586} + k - j = \text{prime} \right\} \right| \sim \frac{J_2(\omega)\omega^{k-1}}{(2586)^{k-1}\phi^k(\omega)} \frac{N}{\log^k N}, \tag{6}$$

where $\phi(\omega) = \prod_P (P-1)$.

From (6) we are able to find the smallest solution $\pi_k(N_0, 2) \geq 1$.

Example 1. Let $k = 3, 7, 863$

From (2) and (3) we have

$$J_2(\omega) = 0. \tag{7}$$

We prove that for $k = 3, 7, 863$

（1）contain no prime solutions. 1 is not a prime.

Example 2. Let $k \neq 3, 7, 863$

From (2) and (3) we have

$$J_2(\omega) \neq 0. \tag{8}$$

We prove that for $k \neq 3, 7, 863$

(1) contain infinitely many prime solutions.

The New Prime Theorem (1324)

$$p, jp^{2588} + k - j(j = 1, 2, \cdots, k - 1)$$

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Abstract

Using Jiang function we prove that $jp^{2588} + k - j$ contain infinitely many prime solutions and no prime solutions.

Theorem. Let k be a given odd prime.

$$p, jp^{2588} + k - j(j = 1, \cdots, k - 1) \tag{1}$$

Contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega) = \prod_{P>2} [P - 1 - \chi(P)]. \tag{2}$$

where $\omega = \prod_P P$, $\chi(P)$ is the number of solutions of congruence

$$\prod_{j=1}^{k-1}(jq^{2588} + k - j) \equiv 0 \pmod{p}, q = 1, \cdots, p - 1. \tag{3}$$

If $\chi(P) \leq P - 2$ then from (2) and (3) we have

$$J_2(\omega) \neq 0 \tag{4}$$

We prove that (1) contain infinitely many prime solutions that is for any k there are infinitely many primes P such that each of $jp^{2588} + k - j$ is a prime.

Using Fermat's little theorem from (3) we have $\chi(P) = P - 1$. Substituting it into (2) we have

$$J_2(\omega) = 0 \tag{5}$$

We prove that (1) contain no prime solutions [1,2]

If $J_2(\omega) \neq 0$ then we have asymptotic formula [1,2]

$$\pi_k(N, 2) = \left| \left\{ P \leq N : jp^{2588} + k - j = \text{prime} \right\} \right| \sim \frac{J_2(\omega)\omega^{k-1}}{(2588)^{k-1}\phi^k(\omega)} \frac{N}{\log^k N}, \tag{6}$$

where $\phi(\omega) = \prod_P (P - 1)$.

From (6) we are able to find the smallest solution $\pi_k(N_0, 2) \geq 1$.

Example 1. Let $k = 3, 5$

From (2) and (3) we have

$$J_2(\omega) = 0 \tag{7}$$

We prove that for $k = 3, 5$

（1）contain no prime solutions. 1 is not a prime.

Example 2. Let $k \neq 3, 5$

From (2) and (3) we have

$$J_2(\omega) \neq 0 \tag{8}$$

We prove that for $k \neq 3, 5$

(1) contain infinitely many prime solutions.

The New Prime Theorem (1325)

$$p, jp^{2590} + k - j (j = 1, 2, \cdots, k-1)$$

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Abstract

Using Jiang function we prove that $jp^{2590} + k - j$ contain infinitely many prime solutions and no prime solutions.

Theorem. Let k be a given odd prime.

$$p, jp^{2590} + k - j (j = 1, \cdots, k-1) \tag{1}$$

Contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega) = \prod_{P>2} [P - 1 - \chi(P)]. \tag{2}$$

where $\omega = \prod_P P$, $\chi(P)$ is the number of solutions of congruence

$$\prod_{j=1}^{k-1}(jq^{2590} + k - j) \equiv 0 \pmod{p}, q = 1, \cdots, p-1. \tag{3}$$

If $\chi(P) \leq P - 2$ then from (2) and (3) we have

$$J_2(\omega) \neq 0 \tag{4}$$

We prove that (1) contain infinitely many prime solutions that is for any k there are infinitely many primes P such that each of $jp^{2590} + k - j$ is a prime.

Using Fermat's little theorem from (3) we have $\chi(P) = P - 1$. Substituting it into (2) we have

$$J_2(\omega) = 0 \tag{5}$$

We prove that (1) contain no prime solutions [1,2]

If $J_2(\omega) \neq 0$ then we have asymptotic formula [1,2]

$$\pi_k(N, 2) = \left| \left\{ P \leq N : jp^{2590} + k - j = \text{prime} \right\} \right| \sim \frac{J_2(\omega)\omega^{k-1}}{(2590)^{k-1}\phi^k(\omega)} \frac{N}{\log^k N}, \tag{6}$$

where $\phi(\omega) = \prod_P (P-1)$.

From (6) we are able to find the smallest solution $\pi_k(N_0, 2) \geq 1$.

Example 1. Let $k = 3, 11, 71, 2591$

From (2) and (3) we have

$$J_2(\omega) = 0 \tag{7}$$

We prove that for $k = 3, 11, 71, 2591$

（1）contain no prime solutions. 1 is not a prime.

Example 2. Let $k \neq 3, 11, 71, 2591$

From (2) and (3) we have

$$J_2(\omega) \neq 0 \tag{8}$$

We prove that for $k \neq 3, 11, 71, 2591$

(1) contain infinitely many prime solutions.

The New Prime Theorem (1326)

$$p, jp^{2592} + k - j(j = 1, 2, \cdots, k-1)$$

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Abstract

Using Jiang function we prove that $jp^{2592} + k - j$ contain infinitely many prime solutions and no prime solutions.

Theorem. Let k be a given odd prime.

$$p, jp^{2592} + k - j(j = 1, \cdots, k-1) \tag{1}$$

Contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega) = \prod_{P>2} [P - 1 - \chi(P)]. \tag{2}$$

where $\omega = \prod_P P$, $\chi(P)$ is the number of solutions of congruence

$$\prod_{j=1}^{k-1}(jq^{2592} + k - j) \equiv 0 \pmod{p}, q = 1, \cdots, p-1. \tag{3}$$

If $\chi(P) \le P - 2$ then from (2) and (3) we have

$$J_2(\omega) \neq 0 \tag{4}$$

We prove that (1) contain infinitely many prime solutions that is for any k there are infinitely many primes P such that each of $jp^{2592} + k - j$ is a prime.

Using Fermat's little theorem from (3) we have $\chi(P) = P - 1$. Substituting it into (2) we have

$$J_2(\omega) = 0 \tag{5}$$

We prove that (1) contain no prime solutions [1,2]

If $J_2(\omega) \neq 0$ then we have asymptotic formula [1,2]

$$\pi_k(N,2) = \left| \left\{ P \le N : jp^{2592} + k - j = \text{prime} \right\} \right| \sim \frac{J_2(\omega)\omega^{k-1}}{(2592)^{k-1}\phi^k(\omega)} \frac{N}{\log^k N}, \tag{6}$$

where $\phi(\omega) = \prod_P (P-1)$.

From (6) we are able to find the smallest solution $\pi_k(N_0, 2) \ge 1$.

Example 1. Let $k = 3, 5, 7, 13, 17, 19, 37, 73, 97, 109, 163, 433, 1297, 2593$

From (2) and (3) we have

$$J_2(\omega) = 0. \tag{7}$$

We prove that for $k = 3, 5, 7, 13, 17, 19, 37, 73, 97, 109, 163, 433, 1297, 2593$

（1）contain no prime solutions. 1 is not a prime.

Example 2. Let $k \neq 3, 5, 7, 13, 17, 19, 37, 73, 97, 109, 163, 433, 1297, 2593$

From (2) and (3) we have

$$J_2(\omega) \neq 0. \tag{8}$$

We prove that for $k \neq 3, 5, 7, 13, 17, 19, 37, 73, 97, 109, 163, 433, 1297, 2593$

(1) contain infinitely many prime solutions.

The New Prime Theorem (1327)

$$p, jp^{2594} + k - j(j = 1, 2, \cdots, k-1)$$

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Abstract

Using Jiang function we prove that $jp^{2594} + k - j$ contain infinitely many prime solutions and no prime solutions.

Theorem. Let k be a given odd prime.

$$p, jp^{2594} + k - j(j = 1, \cdots, k-1) \qquad (1)$$

Contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega) = \prod_{P>2} [P - 1 - \chi(P)]. \qquad (2)$$

where $\omega = \prod_P P$, $\chi(P)$ is the number of solutions of congruence

$$\prod_{j=1}^{k-1}(jq^{2594} + k - j) \equiv 0 \pmod{p}, q = 1, \cdots, p-1. \qquad (3)$$

If $\chi(P) \le P - 2$ then from (2) and (3) we have

$$J_2(\omega) \ne 0 \qquad (4)$$

We prove that (1) contain infinitely many prime solutions that is for any k there are infinitely many primes P such that each of $jp^{2594} + k - j$ is a prime.

Using Fermat's little theorem from (3) we have $\chi(P) = P - 1$. Substituting it into (2) we have

$$J_2(\omega) = 0 \qquad (5)$$

We prove that (1) contain no prime solutions [1,2]

If $J_2(\omega) \ne 0$ then we have asymptotic formula [1,2]

$$\pi_k(N,2) = \left|\left\{P \le N : jp^{2594} + k - j = \text{prime}\right\}\right| \sim \frac{J_2(\omega)\omega^{k-1}}{(2594)^{k-1}\phi^k(\omega)} \frac{N}{\log^k N}, \qquad (6)$$

where $\phi(\omega) = \prod_P (P-1)$.

From (6) we are able to find the smallest solution $\pi_k(N_0, 2) \ge 1$.

Example 1. Let $k = 3$

From (2) and (3) we have

$$J_2(\omega) = 0. \qquad (7)$$

We prove that for $k = 3$

（1）contain no prime solutions. 1 is not a prime.

Example 2. Let $k \ne 3$

From (2) and (3) we have

$$J_2(\omega) \ne 0. \qquad (8)$$

We prove that for $k \ne 3$

(1) contain infinitely many prime solutions.

The New Prime Theorem (1328)

$$p, jp^{2596} + k - j(j = 1, 2, \cdots, k-1)$$

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Abstract

Using Jiang function we prove that $jp^{2596} + k - j$ contain infinitely many prime solutions and no prime solutions.

Theorem. Let k be a given odd prime.

$$p, jp^{2596} + k - j(j = 1, \cdots, k-1) \tag{1}$$

Contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega) = \prod_{P>2} [P - 1 - \chi(P)]. \tag{2}$$

where $\omega = \prod_P P$, $\chi(P)$ is the number of solutions of congruence

$$\prod_{j=1}^{k-1} (jq^{2596} + k - j) \equiv 0 \pmod{p}, q = 1, \cdots, p-1. \tag{3}$$

If $\chi(P) \leq P - 2$ then from (2) and (3) we have

$$J_2(\omega) \neq 0 \tag{4}$$

We prove that (1) contain infinitely many prime solutions that is for any k there are infinitely many primes P such that each of $jp^{2596} + k - j$ is a prime.

Using Fermat's little theorem from (3) we have $\chi(P) = P - 1$. Substituting it into (2) we have

$$J_2(\omega) = 0 \tag{5}$$

We prove that (1) contain no prime solutions [1,2]

If $J_2(\omega) \neq 0$ then we have asymptotic formula [1,2]

$$\pi_k(N,2) = \left| \left\{ P \leq N : jp^{2596} + k - j = \text{prime} \right\} \right| \sim \frac{J_2(\omega)\omega^{k-1}}{(2596)^{k-1}\phi^k(\omega)} \frac{N}{\log^k N}, \tag{6}$$

where $\phi(\omega) = \prod_P (P-1)$.

From (6) we are able to find the smallest solution $\pi_k(N_0, 2) \geq 1$.

Example 1. Let $k = 3, 5, 23$

From (2) and (3) we have

$$J_2(\omega) = 0. \tag{7}$$

We prove that for $k = 3, 5, 23$

（1）contain no prime solutions. 1 is not a prime.

Example 2. Let $k \neq 3, 5, 23$

From (2) and (3) we have

$$J_2(\omega) \neq 0. \tag{8}$$

We prove that for $k \neq 3, 5, 23$

(1) contain infinitely many prime solutions.

The New Prime Theorem (1329)

$$p, jp^{2598}+k-j(j=1,2,\cdots,k-1)$$

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Abstract

Using Jiang function we prove that $jp^{2598}+k-j$ contain infinitely many prime solutions and no prime solutions.

Theorem. Let k be a given odd prime.

$$p, jp^{2598}+k-j(j=1,\cdots,k-1) \tag{1}$$

Contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega)=\prod_{P>2}[P-1-\chi(P)]. \tag{2}$$

where $\omega=\prod_P P$, $\chi(P)$ is the number of solutions of congruence

$$\prod_{j=1}^{k-1}(jq^{2598}+k-j)\equiv 0 \pmod{p}, q=1,\cdots,p-1. \tag{3}$$

If $\chi(P)\leq P-2$ then from (2) and (3) we have

$$J_2(\omega)\neq 0 \tag{4}$$

We prove that (1) contain infinitely many prime solutions that is for any k there are infinitely many primes P such that each of $jp^{2598}+k-j$ is a prime.

Using Fermat's little theorem from (3) we have $\chi(P)=P-1$. Substituting it into (2) we have

$$J_2(\omega)=0 \tag{5}$$

We prove that (1) contain no prime solutions [1,2]

If $J_2(\omega)\neq 0$ then we have asymptotic formula [1,2]

$$\pi_k(N,2)=\left|\left\{P\leq N: jp^{2598}+k-j=\text{prime}\right\}\right|\sim \frac{J_2(\omega)\omega^{k-1}}{(2598)^{k-1}\phi^k(\omega)}\frac{N}{\log^k N}, \tag{6}$$

where $\phi(\omega)=\prod_P(P-1)$.

From (6) we are able to find the smallest solution $\pi_k(N_0,2)\geq 1$.

Example 1. Let $k=3,7$

From (2) and (3) we have

$$J_2(\omega)=0. \tag{7}$$

We prove that for $k=3,7$

（1）contain no prime solutions. 1 is not a prime.

Example 2. Let $k\neq 3,7$

From (2) and (3) we have

$$J_2(\omega)\neq 0. \tag{8}$$

We prove that for $k\neq 3,7$

(1) contain infinitely many prime solutions.

The New Prime Theorem (1330)

$$p, jp^{2600} + k - j(j = 1, 2, \cdots, k-1)$$

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Abstract

Using Jiang function we prove that $jp^{2600} + k - j$ contain infinitely many prime solutions and no prime solutions.

Theorem. Let k be a given odd prime.

$$p, jp^{2600} + k - j(j = 1, \cdots, k-1) \tag{1}$$

Contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega) = \prod_{P>2} [P - 1 - \chi(P)]. \tag{2}$$

where $\omega = \prod_P P$, $\chi(P)$ is the number of solutions of congruence

$$\prod_{j=1}^{k-1}(jq^{2600} + k - j) \equiv 0 \pmod{p}, q = 1, \cdots, p-1. \tag{3}$$

If $\chi(P) \le P-2$ then from (2) and (3) we have

$$J_2(\omega) \ne 0 \tag{4}$$

We prove that (1) contain infinitely many prime solutions that is for any k there are infinitely many primes P such that each of $jp^{2600} + k - j$ is a prime.

Using Fermat's little theorem from (3) we have $\chi(P) = P-1$. Substituting it into (2) we have

$$J_2(\omega) = 0 \tag{5}$$

We prove that (1) contain no prime solutions [1,2]

If $J_2(\omega) \ne 0$ then we have asymptotic formula [1,2]

$$\pi_k(N,2) = \left|\left\{ P \le N : jp^{2600} + k - j = \text{prime} \right\}\right| \sim \frac{J_2(\omega)\omega^{k-1}}{(2600)^{k-1}\phi^k(\omega)} \frac{N}{\log^k N}, \tag{6}$$

where $\phi(\omega) = \prod_P (P-1)$.

From (6) we are able to find the smallest solution $\pi_k(N_0, 2) \ge 1$.

Example 1. Let $k = 3, 5, 11, 41, 53, 101, 521$

From (2) and (3) we have

$$J_2(\omega) = 0 \tag{7}$$

We prove that for $k = 3, 5, 11, 41, 53, 101, 521$

（1）contain no prime solutions. 1 is not a prime.

Example 2. Let $k \ne 3, 5, 11, 41, 53, 101, 521$

From (2) and (3) we have

$$J_2(\omega) \ne 0 \tag{8}$$

We prove that for $k \ne 3, 5, 11, 41, 53, 101, 521$

(1) contain infinitely many prime solutions.

The New Prime Theorem (1331)

$$p, jp^{2602} + k - j(j = 1, 2, \cdots, k-1)$$

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Abstract

Using Jiang function we prove that $jp^{2602} + k - j$ contain infinitely many prime solutions and no prime solutions.

Theorem. Let k be a given odd prime.

$$p, jp^{2602} + k - j(j = 1, \cdots, k-1) \tag{1}$$

Contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega) = \prod_{P>2} [P - 1 - \chi(P)]. \tag{2}$$

where $\omega = \prod_P P$, $\chi(P)$ is the number of solutions of congruence

$$\prod_{j=1}^{k-1}(jq^{2602} + k - j) \equiv 0 \pmod{p}, q = 1, \cdots, p-1. \tag{3}$$

If $\chi(P) \leq P - 2$ then from (2) and (3) we have

$$J_2(\omega) \neq 0 \tag{4}$$

We prove that (1) contain infinitely many prime solutions that is for any k there are infinitely many primes P such that each of $jp^{2602} + k - j$ is a prime.

Using Fermat's little theorem from (3) we have $\chi(P) = P - 1$. Substituting it into (2) we have

$$J_2(\omega) = 0 \tag{5}$$

We prove that (1) contain no prime solutions [1,2]

If $J_2(\omega) \neq 0$ then we have asymptotic formula [1,2]

$$\pi_k(N, 2) = \left|\left\{P \leq N : jp^{2602} + k - j = \text{prime}\right\}\right| \sim \frac{J_2(\omega)\omega^{k-1}}{(2602)^{k-1}\phi^k(\omega)} \frac{N}{\log^k N}, \tag{6}$$

where $\phi(\omega) = \prod_P (P-1)$.

From (6) we are able to find the smallest solution $\pi_k(N_0, 2) \geq 1$.

Example 1. Let $k = 3$

From (2) and (3) we have

$$J_2(\omega) = 0 \tag{7}$$

We prove that for $k = 3$

（1）contain no prime solutions. 1 is not a prime.

Example 2. Let $k \neq 3$

From (2) and (3) we have

$$J_2(\omega) \neq 0 \tag{8}$$

We prove that for $k \neq 3$

(1) contain infinitely many prime solutions.

The New Prime Theorem (1332)

$$p, jp^{2604} + k - j(j = 1, 2, \cdots, k - 1)$$

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Abstract

Using Jiang function we prove that $jp^{2604} + k - j$ contain infinitely many prime solutions and no prime solutions.

Theorem. Let k be a given odd prime.

$$p, jp^{2604} + k - j(j = 1, \cdots, k - 1) \tag{1}$$

Contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega) = \prod_{P > 2} [P - 1 - \chi(P)]. \tag{2}$$

where $\omega = \prod_P P$, $\chi(P)$ is the number of solutions of congruence

$$\prod_{j=1}^{k-1}(jq^{2604} + k - j) \equiv 0 \pmod{p}, q = 1, \cdots, p - 1. \tag{3}$$

If $\chi(P) \le P - 2$ then from (2) and (3) we have

$$J_2(\omega) \ne 0 \tag{4}$$

We prove that (1) contain infinitely many prime solutions that is for any k there are infinitely many primes P such that each of $jp^{2604} + k - j$ is a prime.

Using Fermat's little theorem from (3) we have $\chi(P) = P - 1$. Substituting it into (2) we have

$$J_2(\omega) = 0 \tag{5}$$

We prove that (1) contain no prime solutions [1,2]

If $J_2(\omega) \ne 0$ then we have asymptotic formula [1,2]

$$\pi_k(N, 2) = \left| \left\{ P \le N : jp^{2604} + k - j = \text{prime} \right\} \right| \sim \frac{J_2(\omega)\omega^{k-1}}{(2604)^{k-1}\phi^k(\omega)} \frac{N}{\log^k N}, \tag{6}$$

where $\phi(\omega) = \prod_P (P - 1)$.

From (6) we are able to find the smallest solution $\pi_k(N_0, 2) \ge 1$.

Example 1. Let $k = 3, 5, 7, 13, 29, 43, 373$

From (2) and (3) we have

$$J_2(\omega) = 0 \tag{7}$$

We prove that for $k = 3, 5, 7, 13, 29, 43, 373$

（1）contain no prime solutions. 1 is not a prime.

Example 2. Let $k \ne 3, 5, 7, 13, 29, 43, 373$

From (2) and (3) we have

$$J_2(\omega) \ne 0 \tag{8}$$

We prove that for $k \ne 3, 5, 7, 13, 29, 43, 373$

(1) contain infinitely many prime solutions.

The New Prime Theorem (1333)

$$p, jp^{2606} + k - j(j = 1, 2, \cdots, k-1)$$

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Abstract

Using Jiang function we prove that $jp^{2606} + k - j$ contain infinitely many prime solutions and no prime solutions.

Theorem. Let k be a given odd prime.

$$p, jp^{2606} + k - j(j = 1, \cdots, k-1) \tag{1}$$

Contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega) = \prod_{P>2} [P - 1 - \chi(P)]. \tag{2}$$

where $\omega = \prod_P P$, $\chi(P)$ is the number of solutions of congruence

$$\prod_{j=1}^{k-1}(jq^{2606} + k - j) \equiv 0 \pmod{p}, q = 1, \cdots, p-1. \tag{3}$$

If $\chi(P) \leq P - 2$ then from (2) and (3) we have

$$J_2(\omega) \neq 0 \tag{4}$$

We prove that (1) contain infinitely many prime solutions that is for any k there are infinitely many primes P such that each of $jp^{2606} + k - j$ is a prime.

Using Fermat's little theorem from (3) we have $\chi(P) = P - 1$. Substituting it into (2) we have

$$J_2(\omega) = 0 \tag{5}$$

We prove that (1) contain no prime solutions [1,2]

If $J_2(\omega) \neq 0$ then we have asymptotic formula [1,2]

$$\pi_k(N, 2) = \left| \left\{ P \leq N : jp^{2606} + k - j = \text{prime} \right\} \right| \sim \frac{J_2(\omega)\omega^{k-1}}{(2606)^{k-1}\phi^k(\omega)} \frac{N}{\log^k N}, \tag{6}$$

where $\phi(\omega) = \prod_P (P-1)$.

From (6) we are able to find the smallest solution $\pi_k(N_0, 2) \geq 1$.

Example 1. Let $k = 3$

From (2) and (3) we have

$$J_2(\omega) = 0. \tag{7}$$

We prove that for $k = 3$

（1）contain no prime solutions. 1 is not a prime.

Example 2. Let $k \neq 3$

From (2) and (3) we have

$$J_2(\omega) \neq 0. \tag{8}$$

We prove that for $k \neq 3$

(1) contain infinitely many prime solutions.

The New Prime Theorem (1334)

$$p, jp^{2608} + k - j(j = 1, 2, \cdots, k-1)$$

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Abstract

Using Jiang function we prove that $jp^{2608} + k - j$ contain infinitely many prime solutions and no prime solutions.

Theorem. Let k be a given odd prime.

$$p, jp^{2608} + k - j(j = 1, \cdots, k-1) \tag{1}$$

Contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega) = \prod_{P>2} [P - 1 \chi \quad P]. \tag{2}$$

where $\omega = \prod_P P$, $\chi(P)$ is the number of solutions of congruence

$$\prod_{j=1}^{k-1}(jq^{2608} + k - j) \equiv 0 \pmod{p}, q = 1, \cdots, p-1. \tag{3}$$

If $\chi(P) \le P-2$ then from (2) and (3) we have

$$J_2(\omega) \ne (\tag{4}$$

We prove that (1) contain infinitely many prime solutions that is for any k there are infinitely many primes P such that each of $jp^{2506} + k - j$ is a prime.

Using Fermat's little theorem from (3) we have $\chi(P) = P-1$. Substituting it into (2) we have

$$J_2(\omega) = (\tag{5}$$

We prove that (1) contain no prime solutions [1,2]

If $J_2(\omega) \ne 0$ then we have asymptotic formula [1,2]

$$\pi_k(N,2) = \left| \{P \le N : jp^{2608} + k - j = \text{prime}\} \right| \sim \frac{J_2(\omega)\omega^{k-1}}{(2608)^{k-1}\phi^k(\omega)} \frac{N}{\log^k N}, \tag{6}$$

where $\phi(\omega) = \prod_P (P-1)$.

From (6) we are able to find the smallest solution $\pi_k(N_0, 2) \ge 1$.

Example 1. Let $k = 3, 5, 17, 653, 2609$

From (2) and (3) we have

$$J_2(\omega) = (. \tag{7}$$

We prove that for $k = 3, 5, 17, 653, 2609$

（1）contain no prime solutions. 1 is not a prime.

Example 2. Let $k \ne 3, 5, 17, 653, 2609$

From (2) and (3) we have

$$J_2(\omega) \ne (. \tag{8}$$

We prove that for $k \ne 3, 5, 17, 653, 2609$

(1) contain infinitely many prime solutions.

The New Prime Theorem (1335)

$$p, jp^{2610} + k - j(j = 1, 2, \cdots, k-1)$$

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Abstract

Using Jiang function we prove that $jp^{2610} + k - j$ contain infinitely many prime solutions and no prime solutions.

Theorem. Let k be a given odd prime.

$$p, jp^{2610} + k - j(j = 1, \cdots, k-1) \tag{1}$$

Contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega) = \prod_{P>2} [P - 1 - \chi(P)]. \tag{2}$$

where $\omega = \prod_P P$, $\chi(P)$ is the number of solutions of congruence

$$\prod_{j=1}^{k-1}(jq^{2610} + k - j) \equiv 0 \pmod{p}, q = 1, \cdots, p-1. \tag{3}$$

If $\chi(P) \leq P - 2$ then from (2) and (3) we have

$$J_2(\omega) \neq 0 \tag{4}$$

We prove that (1) contain infinitely many prime solutions that is for any k there are infinitely many primes P such that each of $jp^{2610} + k - j$ is a prime.

Using Fermat's little theorem from (3) we have $\chi(P) = P - 1$. Substituting it into (2) we have

$$J_2(\omega) = 0 \tag{5}$$

We prove that (1) contain no prime solutions [1,2]

If $J_2(\omega) \neq 0$ then we have asymptotic formula [1,2]

$$\pi_k(N,2) = \left|\left\{P \leq N : jp^{2610} + k - j = \text{prime}\right\}\right| \sim \frac{J_2(\omega)\omega^{k-1}}{(2610)^{k-1}\phi^k(\omega)} \frac{N}{\log^k N}, \tag{6}$$

where $\phi(\omega) = \prod_P (P-1)$.

From (6) we are able to find the smallest solution $\pi_k(N_0, 2) \geq 1$.

Example 1. Let $k = 3, 7, 11, 19, 31, 59, 523$

From (2) and (3) we have

$$J_2(\omega) = 0 \tag{7}$$

We prove that for $k = 3, 7, 11, 19, 31, 59, 523$

（1）contain no prime solutions. 1 is not a prime.

Example 2. Let $k \neq 3, 7, 11, 19, 31, 59, 523$

From (2) and (3) we have

$$J_2(\omega) \neq 0 \tag{8}$$

We prove that for $k \neq 3, 7, 11, 19, 31, 59, 523$

(1) contain infinitely many prime solutions.

The New Prime Theorem (1336)

$$p, jp^{2612} + k - j (j = 1, 2, \cdots, k-1)$$

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Abstract

Using Jiang function we prove that $jp^{2612} + k - j$ contain infinitely many prime solutions and no prime solutions.

Theorem. Let k be a given odd prime.

$$p, jp^{2612} + k - j (j = 1, \cdots, k-1) \tag{1}$$

Contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega) = \prod_{P>2} [P - 1 - \chi(P)]. \tag{2}$$

where $\omega = \prod_P P$, $\chi(P)$ is the number of solutions of congruence

$$\prod_{j=1}^{k-1}(jq^{2612} + k - j) \equiv 0 \pmod{p}, q = 1, \cdots, p-1. \tag{3}$$

If $\chi(P) \le P - 2$ then from (2) and (3) we have

$$J_2(\omega) \ne 0 \tag{4}$$

We prove that (1) contain infinitely many prime solutions that is for any k there are infinitely many primes P such that each of $jp^{2612} + k - j$ is a prime.

Using Fermat's little theorem from (3) we have $\chi(P) = P - 1$. Substituting it into (2) we have

$$J_2(\omega) = 0 \tag{5}$$

We prove that (1) contain no prime solutions [1,2]

If $J_2(\omega) \ne 0$ then we have asymptotic formula [1,2]

$$\pi_k(N,2) = \left| \{ P \le N : jp^{2612} + k - j = \text{prime} \} \right| \sim \frac{J_2(\omega)\omega^{k-1}}{(2612)^{k-1}\phi^k(\omega)} \frac{N}{\log^k N}, \tag{6}$$

where $\phi(\omega) = \prod_P (P-1)$.

From (6) we are able to find the smallest solution $\pi_k(N_0, 2) \ge 1$.

Example 1. Let $k = 3, 5, 1307$

From (2) and (3) we have

$$J_2(\omega) = 0 \tag{7}$$

We prove that for $k = 3, 5, 1307$

（1）contain no prime solutions. 1 is not a prime.

Example 2. Let $k \ne 3, 5, 1307$

From (2) and (3) we have

$$J_2(\omega) \ne 0 \tag{8}$$

We prove that for $k \ne 3, 5, 1307$

(1) contain infinitely many prime solutions.

The New Prime Theorem (1337)

$$p, jp^{2614} + k - j(j = 1, 2, \cdots, k-1)$$

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Abstract

Using Jiang function we prove that $jp^{2614} + k - j$ contain infinitely many prime solutions and no prime solutions.

Theorem. Let k be a given odd prime.

$$p, jp^{2614} + k - j(j = 1, \cdots, k-1) \qquad (1)$$

Contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega) = \prod_{P>2} [P - 1 - \chi(P)]. \qquad (2)$$

where $\omega = \prod_P P$, $\chi(P)$ is the number of solutions of congruence

$$\prod_{j=1}^{k-1} (jq^{2614} + k - j) \equiv 0 \pmod{p}, q = 1, \cdots, p-1. \qquad (3)$$

If $\chi(P) \le P - 2$ then from (2) and (3) we have

$$J_2(\omega) \ne 0 \qquad (4)$$

We prove that (1) contain infinitely many prime solutions that is for any k there are infinitely many primes P such that each of $jp^{2614} + k - j$ is a prime.

Using Fermat's little theorem from (3) we have $\chi(P) = P - 1$. Substituting it into (2) we have

$$J_2(\omega) = 0 \qquad (5)$$

We prove that (1) contain no prime solutions [1,2]

If $J_2(\omega) \ne 0$ then we have asymptotic formula [1,2]

$$\pi_k(N, 2) = \left| \left\{ P \le N : jp^{2614} + k - j = \text{prime} \right\} \right| \sim \frac{J_2(\omega)\omega^{k-1}}{(2614)^{k-1}\phi^k(\omega)} \frac{N}{\log^k N}, \qquad (6)$$

where $\phi(\omega) = \prod_P (P-1)$.

From (6) we are able to find the smallest solution $\pi_k(N_0, 2) \ge 1$.

Example 1. Let $k = 3$

From (2) and (3) we have

$$J_2(\omega) = 0. \qquad (7)$$

We prove that for $k = 3$

（1）contain no prime solutions. 1 is not a prime.

Example 2. Let $k \ne 3$

From (2) and (3) we have

$$J_2(\omega) \ne 0. \qquad (8)$$

We prove that for $k \ne 3$

(1) contain infinitely many prime solutions.

The New Prime Theorem (1338)

$$p, jp^{2616} + k - j(j = 1, 2, \cdots, k-1)$$

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Abstract

Using Jiang function we prove that $jp^{2616} + k - j$ contain infinitely many prime solutions and no prime solutions.

Theorem. Let k be a given odd prime.

$$p, jp^{2616} + k - j(j = 1, \cdots, k-1) \qquad (1)$$

Contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega) = \prod_{P>2} [P - 1 - \chi(P)]. \qquad (2)$$

where $\omega = \prod_P P$, $\chi(P)$ is the number of solutions of congruence

$$\prod_{j=1}^{k-1}(jq^{2616} + k - j) \equiv 0 \pmod{p}, q = 1, \cdots, p-1. \qquad (3)$$

If $\chi(P) \leq P - 2$ then from (2) and (3) we have

$$J_2(\omega) \neq 0 \qquad (4)$$

We prove that (1) contain infinitely many prime solutions that is for any k there are infinitely many primes P such that each of $jp^{2616} + k - j$ is a prime.

Using Fermat's little theorem from (3) we have $\chi(P) = P - 1$. Substituting it into (2) we have

$$J_2(\omega) = 0 \qquad (5)$$

We prove that (1) contain no prime solutions [1,2]

If $J_2(\omega) \neq 0$ then we have asymptotic formula [1,2]

$$\pi_k(N,2) = \left| \left\{ P \leq N : jp^{2616} + k - j = \text{prime} \right\} \right| \sim \frac{J_2(\omega)\omega^{k-1}}{(2616)^{k-1}\phi^k(\omega)} \frac{N}{\log^k N}, \qquad (6)$$

where $\phi(\omega) = \prod_P (P-1)$.

From (6) we are able to find the smallest solution $\pi_k(N_0, 2) \geq 1$.

Example 1. Let $k = 3, 5, 7, 13, 2617$

From (2) and (3) we have

$$J_2(\omega) = 0. \qquad (7)$$

We prove that for $k = 3, 5, 7, 13, 2617$

（1）contain no prime solutions. 1 is not a prime.

Example 2. Let $k \neq 3, 5, 7, 13, 2617$

From (2) and (3) we have

$$J_2(\omega) \neq 0. \qquad (8)$$

We prove that for $k \neq 3, 5, 7, 13, 2617$

(1) contain infinitely many prime solutions.

The New Prime Theorem (1339)

$$p, jp^{2618} + k - j(j = 1, 2, \cdots, k-1)$$

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Abstract

Using Jiang function we prove that $jp^{2618} + k - j$ contain infinitely many prime solutions and no prime solutions.

Theorem. Let k be a given odd prime.

$$p, jp^{2618} + k - j(j = 1, \cdots, k-1) \tag{1}$$

Contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega) = \prod_{P>2} [P - 1 - \chi(P)]. \tag{2}$$

where $\omega = \prod_P P$, $\chi(P)$ is the number of solutions of congruence

$$\prod_{j=1}^{k-1}(jq^{2618} + k - j) \equiv 0 \pmod{p}, q = 1, \cdots, p-1. \tag{3}$$

If $\chi(P) \le P - 2$ then from (2) and (3) we have

$$J_2(\omega) \ne 0 \tag{4}$$

We prove that (1) contain infinitely many prime solutions that is for any k there are infinitely many primes P such that each of $jp^{2618} + k - j$ is a prime.

Using Fermat's little theorem from (3) we have $\chi(P) = P - 1$. Substituting it into (2) we have

$$J_2(\omega) = 0 \tag{5}$$

We prove that (1) contain no prime solutions [1,2]

If $J_2(\omega) \ne 0$ then we have asymptotic formula [1,2]

$$\pi_k(N, 2) = \left| \left\{ P \le N : jp^{2618} + k - j = \text{prime} \right\} \right| \sim \frac{J_2(\omega)\omega^{k-1}}{(2618)^{k-1}\phi^k(\omega)} \frac{N}{\log^k N}, \tag{6}$$

where $\phi(\omega) = \prod_P (P-1)$.

From (6) we are able to find the smallest solution $\pi_k(N_0, 2) \ge 1$.

Example 1. Let $k = 3, 23, 239$

From (2) and (3) we have

$$J_2(\omega) = 0 \tag{7}$$

We prove that for $k = 3, 23, 239$ $k = 3$

（1）contain no prime solutions. 1 is not a prime.

Example 2. Let $k \ne 3, 23, 239$

From (2) and (3) we have

$$J_2(\omega) \ne 0 \tag{8}$$

We prove that for $k \ne 3, 23, 239$

(1) contain infinitely many prime solutions.

The New Prime Theorem (1340)

$$p, jp^{2620} + k - j(j = 1, 2, \cdots, k-1)$$

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Abstract

Using Jiang function we prove that $jp^{2620} + k - j$ contain infinitely many prime solutions and no prime solutions.

Theorem. Let k be a given odd prime.

$$p, jp^{2620} + k - j(j = 1, \cdots, k-1) \tag{1}$$

Contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega) = \prod_{P>2} [P - 1 - \chi(P)]. \tag{2}$$

where $\omega = \prod_P P$, $\chi(P)$ is the number of solutions of congruence

$$\prod_{j=1}^{k-1}(jq^{2620} + k - j) \equiv 0 \pmod{p}, q = 1, \cdots, p-1. \tag{3}$$

If $\chi(P) \le P - 2$ then from (2) and (3) we have

$$J_2(\omega) \ne 0 \tag{4}$$

We prove that (1) contain infinitely many prime solutions that is for any k there are infinitely many primes P such that each of $jp^{2620} + k - j$ is a prime.

Using Fermat's little theorem from (3) we have $\chi(P) = P - 1$. Substituting it into (2) we have

$$J_2(\omega) = 0 \tag{5}$$

We prove that (1) contain no prime solutions [1,2]

If $J_2(\omega) \ne 0$ then we have asymptotic formula [1,2]

$$\pi_k(N, 2) = \left| \left\{ P \le N : jp^{2620} + k - j = \text{prime} \right\} \right| \sim \frac{J_2(\omega)\omega^{k-1}}{(2620)^{k-1}\phi^k(\omega)} \frac{N}{\log^k N}, \tag{6}$$

where $\phi(\omega) = \prod_P (P-1)$.

From (6) we are able to find the smallest solution $\pi_k(N_0, 2) \ge 1$.

Example 1. Let $k = 3, 5, 11, 263, 2621$

From (2) and (3) we have

$$J_2(\omega) = 0 \tag{7}$$

We prove that for $k = 3, 5, 11, 263, 2621$

（1）contain no prime solutions. 1 is not a prime.

Example 2. Let $k \ne 3, 5, 11, 263, 2621$

From (2) and (3) we have

$$J_2(\omega) \ne 0 \tag{8}$$

We prove that for $k \ne 3, 5, 11, 263, 2621$

(1) contain infinitely many prime solutions.

Remark. The prime number theory is basically to count the Jiang function $J_{n+1}(\omega)$

and Jiang prime k-tuple singular series $\sigma(J) = \dfrac{J_2(\omega)\omega^{k-1}}{\phi^k(\omega)} = \prod_P \left(1 - \dfrac{1+\chi(P)}{P}\right)(1 - \dfrac{1}{P})^{-k}$

[1,2], which can count the number of prime numbers. The prime distribution is not random. But

Hardy-Littlewood prime k-tuple singular series $\sigma(H) = \prod_P \left(1 - \dfrac{\nu(P)}{P}\right)(1 - \dfrac{1}{P})^{-k}$ is false

[3-17], which cannot count the number of prime numbers[3].

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Szemerédi's theorem does not directly to the primes, because it cannot count the number of primes. Cramér's random model cannot prove any prime problems. The probability of $1/\log N$ of being prime is false. Assuming that the events " P is prime", " $P+2$ is prime" and " $P+4$ is prime" are independent, we conclude that P , $P+2$, $P+4$ are simultaneously prime with probability about $1/\log^3 N$. There are about $N/\log^3 N$ primes less than N . Letting $N \to \infty$ we obtain the prime conjecture, which is false. The tool of additive prime number theory is basically the Hardy-Littlewood prime tuples conjecture, but cannot prove and count any prime problems[6].

Mathematicians have tried in vain to discover some order in the sequence of prime numbers but we have every reason to believe that there are some mysteries which the human mind will never penetrate.

Leonhard Euler(1707-1783)

It will be another million years, at least, before we understand the primes.

Paul Erdos(1913-1996)

Jiang's function $J_{n+1}(\omega)$ in prime distribution

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Dedicated to the 30-th anniversary of hadronic mechanics

Abstract

We define that prime equations

$$f_1(P_1,\cdots ,P_n); \quad f_k(P_1 \cdots , P_n) \tag{5}$$

are polynomials (with integer coefficients) irreducible over integers, where $P_1,\cdots,P_n$ are all prime. If Jiang's function $J_{n+1}(\omega)=0$ then （5）has finite prime solutions. If $J_{n+1}(\omega) \neq 0$ then there are infinitely many primes $P_1,\cdots,P_n$ such that $f_1,\cdots f_k$ are primes. We obtain a unite prime formula in prime distribution

$$\pi_{k+1}(N,n+1) = |\{P_1,\cdots,P_n \leq N : f_1,\cdots,f_k \text{ are prime}\}|$$

$$= \prod_{i=1}^{k} \ (\deg f_i)^{-1} \times \frac{J_{n+1}(\omega)\omega^k}{n!\phi^{k+n}(\omega)} \frac{N^n}{\log^{k+n} N}(1+o(1)). \qquad (8)$$

Jiang's function is accurate sieve function. Using Jiang's function we prove about 600 prime theorems [6]. Jiang's function provides proofs of the prime theorems which are simple enough to understand and accurate enough to be useful.

Mathematicians have tried in vain to discover some order in the sequence of prime numbers but we have every reason to believe that there are some mysteries which the human mind will never penetrate.

Leonhard Euler

It will be another million years, at least, before we understand the primes.

Paul Erdös

Suppose that Euler totient function

$$\phi(\omega)= \prod_{2 \le P} \ P-\ 1)\infty \quad \text{as} \quad \omega \to \infty, \qquad (1)$$

where $\omega = \prod_{2 \le P} P$ is called primorial.

Suppose that $(\omega, h_i)=1$, where $i = 1, \cdots, \phi(\omega)$. We have prime equations

$$P_1 = \omega n+1; \cdots, P_{\phi(\omega)} = \omega \ n \ h_{(} \qquad (2)$$

where $n = 0,1,2,\cdots$.

(2) is called infinitely many prime equations (IMPE). Every equation has infinitely many prime solutions. We have

$$\pi_{h_i} = \sum_{\substack{P_i \le N \\ P_i \equiv h_i (\ \text{m} \ \text{od} \ )}} 1 \ = \frac{\pi(N)}{\phi(\omega)}(1+o(1)). , \qquad (3)$$

where π_{h_i} denotes the number of primes $P_i \le N$ in $P_i = \omega n + h_i$ $n = 0,1,2,\cdots$, $\pi(N)$ the number of primes less than or equal to N.

We replace sets of prime numbers by IMPE. (2) is the fundamental tool for proving the prime theorems in prime distribution.

Let $\omega = 30$ and $\phi(30)=8$. From (2) we have eight prime equations

$P_1 = 30n+1$, $P_2 = 30n+7$, $P_3 = 30n+11$, $P_4 = 30n+13$, $P_5 = 30n+17$,

$P_6 = 30n+19$, $P_7 = 30n+23$, $P_8 = 30n+29$, $n = 0,1,2,\cdots$ $\qquad (4)$

Every equation has infinitely many prime solutions.

THEOREM. We define that prime equations

$$f_1(P_1; \cdots ,P_n); \quad f \quad P \cdots , \ _{n^{\downarrow}} \qquad (5)$$

are polynomials (with integer coefficients) irreducible over integers, where $P_1, \cdots, P_n$ are primes. If Jiang's function $J_{n+1}(\omega)=0$ then (5) has finite prime solutions. If

$J_{n+1}(\omega) \neq 0$ then there exist infinitely many primes $P_1, \cdots, P_n$ such that each f_k is a prime.

PROOF. Firstly, we have Jiang's function [1-11]

$$J_{n+1}(\omega) = \prod_{3 \leq P} [(P-1)^n - \chi(P)], \qquad (6)$$

where $\chi(P)$ is called sieve constant and denotes the number of solutions for the following congruence

$$\prod_{i=1}^{k} f_i(q_1, \cdots, q_n) \equiv 0 \pmod{P}, \qquad (7)$$

where $q_1 = 1, \cdots, P-1, \cdots, q_n = 1, \cdots, P-1$.

$J_{n+1}(\omega)$ denotes the number of sets of $P_1, \cdots, P_n$ prime equations such that $f_1(P_1, \cdots, P_n), \cdots, f_k(P_1, \cdots, P_n)$ are prime equations. If $J_{n+1}(\omega) = 0$ then (5) has finite prime solutions. If $J_{n+1}(\omega) \neq 0$ using $\chi(P)$ we sift out from (2) prime equations which can not be represented $P_1, \cdots, P_n$, then residual prime equations of (2) are $P_1, \cdots, P_n$ prime equations such that $f_1(P_1, \cdots, P_n), \cdots, f_k(P_1, \cdots, P_n)$ are prime equations. Therefore we prove that there exist infinitely many primes $P_1, \cdots, P_n$ such that $f_1(P_1, \cdots, P_n), \cdots, f_k(P_1, \cdots, P_n)$ are primes.

Secondly, we have the best asymptotic formula [2,3,4,6]

$$\pi_{k+1}(N, n+1) = |\{P_1, \cdots, P_n \leq N : f_1, \cdots, f_k \text{ are prime}\}|$$

$$= \prod_{i=1}^{k} (\deg f_i)^{-1} \times \frac{J_{n+1}(\omega)\omega^k}{n!\phi^{k+n}(\omega)} \frac{N^n}{\log^{k+n} N}(1+o(1)). \qquad (8)$$

(8) is called a unite prime formula in prime distribution. Let $n = 1, k = 0$, $J_2(\omega) = \phi(\omega)$. From (8) we have prime number theorem

$$\pi_1(N, 2) = |\{P_1 \leq N : P_1 \text{ is prime}\}| = \frac{N}{\log N} + o(1). \qquad (9)$$

Number theorists believe that there are infinitely many twin primes, but they do not have rigorous proof of this old conjecture by any method. All the prime theorems are conjectures except the prime number theorem, because they do not prove that prime equations have infinitely many prime solutions. We prove the following conjectures by this theorem.

Example 1. Twin primes $P, P+2$ (300BC)**.**

From (6) and (7) we have Jiang's function

$$J_2(\omega) = \prod_{3 \leq P} (P-2) \neq 0 .$$

Since $J_2(\omega) \neq 0$ in (2) exist infinitely many P prime equations such that $P+2$ is a prime equation. Therefore we prove that there are infinitely many primes P such that $P+2$ is a prime.

Let $\omega = 30$ and $J_2(30) = 3$. From (4) we have three P prime equations

$$P_3 = 3\,0n + 1\,1, \quad P_5 = 3\,n0 + 1\,7, \quad P_8 = 3\,n + .$$

From (8) we have the best asymptotic formula

$$\pi_2(N,2) = \left|\{ P \le N : P + 2 \text{ prime} \}\right| = \frac{J_2(\omega)\omega}{\phi^2(\omega)} \frac{N}{\log^2 N}(1 + o(1))$$

$$= 2 \prod_{3 \le P} \left(1 - \frac{1}{(P-1)^2} \right) \frac{N}{\log^2 N}(1 + o(1)).$$

In 1996 we proved twin primes conjecture [1]

Remark. $J_2(\omega)$ denotes the number of P prime equations, $\dfrac{\omega}{\phi^2(\omega)} \dfrac{N}{\log^2 N}(1 + o(1))$

the number of solutions of primes for every P prime equation.

Example 2. Even Goldbach's conjecture $N = P_1 + P_2$. Every even number $N \ge 6$ is the sum of two primes.

From (6) and (7) we have Jiang's function

$$J_2(\omega) = \prod_{3 \le P} P - 2 \prod_{P \mid N} \frac{P-1}{P-2} \ne .$$

Since $J_2(\omega) \ne 0$ as $N \to \infty$ in (2) exist infinitely many P_1 prime equations such that $N - P_1$ is a prime equation. Therefore we prove that every even number $N \ge 6$ is the sum of two primes.

From (8) we have the best asymptotic formula

$$\pi_2(N,2) = \left|\{ P_1 \le N, N - P_1 \text{ prime} \}\right| = \frac{J_2(\omega)\omega}{\phi^2(\omega)} \frac{N}{\log^2 N}(1 + o(1)).$$

$$= 2 \prod_{3 \le P} \left(1 - \frac{1}{(P-1)^2} \right) \prod_{P \mid N} \frac{P-1}{P-2} \frac{N}{\log^2 N}(1 + o(1)).$$

In 1996 we proved even Goldbach's conjecture [1]

Example 3. Prime equations $P, P+2, P+6$.

From (6) and (7) we have Jiang's function

$$J_2(\omega) = \prod_{5 \le P} P - 3 \ne) ,$$

$J_2(\omega)$ is denotes the number of P prime equations such that $P+2$ and $P+6$ are prime equations. Since $J_2(\omega) \ne 0$ in (2) exist infinitely many P prime equations such that $P+2$ and $P+6$ are prime equations. Therefore we prove that there are infinitely many primes P such that $P+2$ and $P+6$ are primes.

Let $\omega = 30$, $J_2(30) = 2$. From (4) we have two P prime equations

$$P_3 = 30n + 11, \quad P_5 = 30n + 17.$$

From (8) we have the best asymptotic formula

$$\pi_3(N,2) = \left| \{ P \leq N : P+2, P+6 \text{ are primes} \} \right| = \frac{J_2(\omega)\omega^2}{\phi^3(\omega)} \frac{N}{\log^3 N}(1+o(1)).$$

Example 4. Odd Goldbach's conjecture $N = P_1 + P_2 + P_3$. Every odd number $N \geq 9$ is the sum of three primes.

From (6) and (7) we have Jiang's function

$$J_3(\omega) = \prod_{3 \leq P}\left(P^2 - 3P + 3 \right)\prod_{P|N}\left(1 - \frac{1}{P^2 - 3P + 3} \right) \neq .$$

Since $J_3(\omega) \neq 0$ as $N \to \infty$ in (2) exist infinitely many pairs of P_1 and P_2 prime equations such that $N - P_1 - P_2$ is a prime equation. Therefore we prove that every odd number $N \geq 9$ is the sum of three primes.

From (8) we have the best asymptotic formula

$$\pi_2(N,3) = \left| \{ P_1, P_2 \leq N : N - P_1 - P_2 \text{ prime} \} \right| = \frac{J_3(\omega)\omega}{2\phi^3(\omega)} \frac{N^2}{\log^3 N}(1+o(1)) .$$

$$= \prod_{3 \leq P}\left(1 + \frac{1}{(P-1)^3} \right)\prod_{P|N}\left(1 - \frac{1}{P^3 - 3P + 3} \right)\frac{N^2}{\log^3 N}(1+o(1)) .$$

Example 5. Prime equation $P_3 = P_1 P_2 + 2$.

From (6) and (7) we have Jiang's function

$$J_3(\omega) = \prod_{3 \leq P}\left(P^3 - 3P + 2 \right) \neq$$

$J_3(\omega)$ denotes the number of pairs of P_1 and P_2 prime equations such that P_3 is a prime equation. Since $J_3(\omega) \neq 0$ in (2) exist infinitely many pairs of P_1 and P_2 prime equations such that P_3 is a prime equation. Therefore we prove that there are infinitely many pairs of primes P_1 and P_2 such that P_3 is a prime.

From (8) we have the best asymptotic formula

$$\pi_2(N,3) = \left| \{ P_1, P_2 \leq N : P_1 P_2 + 2 \text{ prime} \} \right| = \frac{J_3(\omega)\omega}{4\phi^3(\omega)} \frac{N^2}{\log^3 N}(1+o(1)).$$

Note. $\deg(P_1 P_2) = 2$.

Example 6 [12].   Prime equation $P_3 = P_1^3 + 2P_2^3$.

From (6) and (7) we have Jiang's function

$$J_3(\omega) = \prod_{3 \leq P}\left[(P-1)^2 + \chi(P) \right] \neq ,$$

where $\chi(P) = 3(P-1)$ if $2^{\frac{P-1}{3}} \equiv 1 \pmod P$; $\chi(P) = 0$ if $2^{\frac{P-1}{3}} \not\equiv 1 \pmod P$;

$\chi(P) = P - 1$ otherwise.

Since $J_3(\omega) \neq 0$ in (2) there are infinitely many pairs of P_1 and P_2 prime equations such that P_3 is a prime equation. Therefore we prove that there are

infinitely many pairs of primes P_1 and P_2 such that P_3 is a prime.

From (8) we have the best asymptotic formula

$$\pi_2(N,3) = \left| \{P_1, P_2 \le N : P_1^3 + 2P_2^3 \text{ prime}\} \right| = \frac{J_3(\omega)\omega}{6\phi^3(\omega)} \frac{N^2}{\log^3 N}(1+o(1)).$$

Example 7 [13].   Prime equation $P_3 = P_1^4 + (P_2+1)^2$.

From (6) and (7) we have Jiang's function

$$J_3(\omega) = \prod_{3 \le P} \left[(P-1) + \chi(R) \right] (\ne)$$

where $\chi(P) = 2(P-1)$ if $P \equiv 1 \pmod 4$; $\chi(P) = 2(P-3)$ if $P \equiv 1 \pmod 8$; $\chi(P) = 0$ otherwise.

Since $J_3(\omega) \ne 0$ in (2) there are infinitely many pairs of P_1 and P_2 prime equations such that P_3 is a prime equation. Therefore we prove that there are infinitely many pairs of primes P_1 and P_2 such that P_3 is a prime.

From (8) we have the best asymptotic formula

$$\pi_2(N,3) = \left| \{P_1, P_2 \le N : P_3 \text{ prime}\} \right| = \frac{J_3(\omega)\omega}{8\phi^3(\omega)} \frac{N^2}{\log^3 N}(1+o(1)).$$

Example 8 [14-20]. Arithmetic progressions consisting only of primes. We define the arithmetic progressions of length k.

$$P_1, \quad P_2 = P_1 + d, \quad P_3 = P_1 + 2 \cdot d, \quad _k, \quad P = _1 P (k-1) d (P, . \qquad (10)$$

From (8) we have the best asymptotic formula

$$\pi_2(N,2) = \left| \{P_1 \le N : P_1, P_1 + d, \cdots, P_1 + (k-1)d \text{ are prime} \} \right.$$

$$= \frac{J_2(\omega)\omega^{k-1}}{\phi^k(\omega)} \frac{N}{\log^k N}(1+o(1))..$$

If $J_2(\omega) = 0$ then (10) has finite prime solutions. If $J_2(\omega) \ne 0$ then there are infinitely many primes P_1 such that $P_2, \cdots, P_k$ are primes.

To eliminate d from (10) we have

$$P_3 = 2P_2 - P_1, \quad _j P = (j-1)P_2 - (j-2)P_1, \quad 3 \le .$$

From (6) and (7) we have Jiang's function

$$J_3(\omega) = \prod_{3 \le P < k} (P-1) \prod_{k \le P} (R - 1)(-k + 1)$$

Since $J_3(\omega) \ne 0$ in (2) there are infinitely many pairs of P_1 and P_2 prime equations such that $P_3, \cdots, P_k$ are prime equations. Therefore we prove that there are infinitely many pairs of primes P_1 and P_2 such that $P_3, \cdots, P_k$ are primes.

From (8) we have the best asymptotic formula

$$\pi_{k-1}(N,3) = \left| \{P_1, P_2 \le N : (j-1)P_2 - (j-2)P_1 \text{ prime}, 3 \le j \le k\} \right|$$

$$= \frac{J_3(\omega)\omega^{k-2}}{2\phi^k(\omega)} \frac{N^2}{\log^k N}(1+o(1))$$

$$= \frac{1}{2} \prod_{2 \leq P < k} \frac{P^{k-2}}{(P-1)^{k-1}} \prod_{k \leq P} \frac{P^{k-2}(P-k+1)}{(P-1)^{k-1}} \frac{N^2}{\log^k N} (1+o(1)).$$

Example 9. It is a well-known conjecture that one of $P, P+2, P+2^2$ is always divisible by 3. To generalize above to the $k-$ primes, we prove the following conjectures. Let n be a square-free even number.

1. $P, P+n, P+n^2$,

where $3|(n+1)$.

From (6) and (7) we have $J_2(3) = 0$, hence one of $P, P+n, P+n^2$ is always divisible by 3.

2. $P, P+n, P+n^2, \cdots, P+n^4$,

where $5|(n+b), b = 2, 3$.

From (6) and (7) we have $J_2(5) = 0$, hence one of $P, P+n, P+n^2, \cdots, P+n^4$ is always divisible by 5.

3. $P, P+n, P+n^2, \cdots, P+n^6$,

where $7|(n+b), b = 2, 4$.

From (6) and (7) we have $J_2(7) = 0$, hence one of $P, P+n, P+n^2, \cdots, P+n^6$ is always divisible by 7.

4. $P, P+n, P+n^2, \cdots, P+n^{10}$,

where $11|(n+b), b = 3, 4, 5, 9$.

From (6) and (7) we have $J_2(11) = 0$, hence one of $P, P+n, P+n^2, \cdots, P+n^{10}$ is always divisible by 11.

5. $P, P+n, P+n^2, \cdots, P+n^{12}$,

where $13|(n+b), b = 2, 6, 7, 11$.

From (6) and (7) we have $J_2(13) = 0$, hence one of $P, P+n, P+n^2, \cdots, P+n^{12}$ is always divisible by 13.

6. $P, P+n, P+n^2, \cdots, P+n^{16}$,

where $17|(n+b), b = 3, 5, 6, 7, 10, 11, 12, 14, 15$.

From (6) and (7) we have $J_2(17) = 0$, hence one of $P, P+n, P+n^2, \cdots, P+n^{16}$ is always divisible by 17.

7. $P, P+n, P+n^2, \cdots, P+n^{18}$,

where $19|(n+b), b = 4, 5, 6, 9, 16.17$.

From (6) and (7) we have $J_2(19) = 0$, hence one of $P, P+n, P+n^2, \cdots, P+n^{18}$ is always divisible by 19.

Example 10. Let n be an even number.

1. $P, P+n^i, i = 1, 3, 5, \cdots, 2k+1$,

From (6) and (7) we have $J_2(\omega) \neq 0$. Therefore we prove that there exist infinitely many primes P such that $P, P+n^i$ are primes for any k.

2. $P, P+n^i, i = 2, 4, 6, \cdots, 2k$.

From (6) and (7) we have $J_2(\omega) \neq 0$. Therefore we prove that there exist infinitely

many primes P such that $P, P + n^i$ are primes for any k.

Example 11. Prime equation $2P_2 = P_1 + P_3$

From (6) and (7) we have Jiang's function

$$J_3(\omega) = \prod_{3 \leq P} (P^3 - 3P + 2) \neq.$$

Since $J_3(\omega) \neq 0$ in (2) there are infinitely many pairs of P_1 and P_2 prime equations such that P_3 is prime equations. Therefore we prove that there are infinitely many pairs of primes P_1 and P_2 such that P_3 is a prime.
From (8) we have the best asymptotic formula

$$\pi_2(N,3) = \left| \{P_1, P_2 \leq N : P_3 \text{ prime}\} \right| = \frac{J_3(\omega)\omega}{2\phi^3(\omega)} \frac{N^2}{\log^3 N} (1 + o(1)).$$

In the same way we can prove $2P_2^2 = P_3 + P_1$ which has the same Jiang's function. Jiang's function is accurate sieve function. Using it we can prove any irreducible prime equations in prime distribution. There are infinitely many twin primes but we do not have rigorous proof of this old conjecture by any method [20]. As strong as the numerical evidence may be, we still do not even know whether there are infinitely many pairs of twin primes [21]. All the prime theorems are conjectures except the prime number theorem, because they do not prove the simplest twin primes. They conjecture that the prime distribution is randomness [12-26], because they do not understand theory of prime numbers.

Acknowledgements

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The Hardy-Littlewood prime k-tuple conjecture is false

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Abstract

Using Jiang function we prove Jiang prime k-tuple theorem. We prove that the Hardy-Littlewood prime k-tuple conjecture is false. Jiang prime k-tuple theorem can replace the Hardy-Littlewood prime k-tuple conjecture.

(A) Jiang prime k-tuple theorem [1, 2].

We define the prime k-tuple equation

$$p, \quad p+ n_i, \tag{1}$$

where $2|n_i, i = 1, \cdots k-1$.

we have Jiang function [1, 2]

$$J_2(\omega) = \prod_P (P-1-\chi(P)), \tag{2}$$

where $\omega = \prod_P P$, $\chi(P)$ is the number of solutions of congruence

$$\prod_{i=1}^{k-1}(q+n_i) \equiv 0 \pmod{P}, \quad q = 1, \cdots, p-1. \tag{3}$$

If $\chi(P) < P-1$ then $J_2(\omega) \neq 0$. There exist infinitely many primes P such that each of $P+n_i$ is prime. If $\chi(P) = P-1$ then $J_2(\omega) = 0$. There exist finitely many primes P such that each of $P+n_i$ is prime. $J_2(\omega)$ is a subset of Euler function $\phi(\omega)$ [2].

If $J_2(\omega) \neq 0$, then we hae the best asymptotic formula of the number of prime P [1, 2]

$$\pi_k(N,2) = \left|\{P \leq N : P+n_i = prime\}\right| \sim \frac{J_2(\omega)\omega^{k-1}}{\phi^k(\omega)} \frac{N}{\log^k N} = C(k)\frac{N}{\log^k N} \tag{4}$$

$$\phi(\omega) = \prod_P(P-1),$$

$$C(k) = \prod_P \left(1 - \frac{1+\chi(P)}{P}\right)\left(1 - \frac{1}{P}\right)^{-k} \tag{5}$$

Example 1. Let $k = 2, P, P+2$, twin primes theorem.

From (3) we have

$$\chi(2) = 0, \chi(P) = 1 \text{ if } P > 2, \tag{6}$$

Substituting (6) into (2) we have

$$J_2(\omega) = \prod_{P \geq 3} (P - 2) \neq 0 \tag{7}$$

There exist infinitely many primes P such that $P+2$ is prime. Substituting (7) into (4) we have the best asymptotic pormula

$$\pi_k(N,2) = \left|\{P \leq N : P+2 = prime\}\right| \sim 2 \prod_{P \geq 3}(1 - \frac{1}{(P-1)^2}) \frac{N}{\log^2 N}. \tag{8}$$

Example 2. Let $k = 3, P, P+2, P+4$.

From (3) we have

$$\chi(2) \neq 0, \chi(3) = 3 \tag{9}$$

From (2) we have

$$J_2(\omega) = 0. \tag{10}$$

It has only a solution $P = 3$, $P+2 = 5$, $P+4 = 7$. One of $P, P+2, P+4$ is always divisible by 3.

Example 3. Let $k = 4, P, P+n$, where $n = 2, 6, 8$.

From (3) we have

$$\chi(2) = 0, \chi(3) = 1, \chi(P) = 3 \text{ if } P > 3. \tag{11}$$

Substituting (11) into (2) we have

$$J_2(\omega) = \prod_{P \geq 5} (P - 4) \neq 0, \tag{12}$$

There exist infinitely many primes P such that each of $P+n$ is prime.

Substituting (12) into (4) we have the best asymptotic formula

$$\pi_4(N,2) = \left|\{P \leq N : P+n = prime\}\right| \sim \frac{27}{3} \prod_{P \geq 5} \frac{P^3(P-4)}{(P-1)^4} \frac{N}{\log^4 N} \tag{13}$$

Example 4. Let $k = 5$, P, $P+n$, where $n = 2, 6, 8, 12$.

From (3) we have

$$\chi(2) \neq 0, \chi(3) = 1, \chi(5) = 3, \text{ if } P > 5 \tag{14}$$

Substituting (14) into (2) we have

$$J_2(\omega) = \prod_{P \geq 7} (P - 5) \neq 0 \tag{15}$$

There exist infinitely many primes P such that each of $P+n$ is prime. Substituting (15) into (4) we have the best asymptotic formula

$$\pi_5(N,2) = \left|\{P \leq N : P+n = prime\}\right| \sim \frac{15^4}{2^{11}} \prod_{P \geq 7} \frac{(P-5)P^4}{(P-1)^5} \frac{N}{\log^5 N} \tag{16}$$

Example 5. Let $k = 6$, P, $P+n$, where $n = 2, 6, 8, 12, 14$.

From (3) and (2) we have

$$\chi(2) \neq 0, \chi(3) = 1, \chi(5) = 4, \quad (17)$$

It has only a solution $P = 5$, $P+2 = 7$, $P+6 = 11$, $P+8 = 13$, $P+12 = 17$, $P+14 = 19$. One of $P+n$ is always divisible by 5.

（**B**）**The Hardy-Littlewood prime k -tuple conjecture[3-14].**

This conjecture is generally believed to be true, but has not been proved(Odlyzko et al.1999).

We define the prime k -tuple equation

$$P, \quad P + n_i \quad\quad (18)$$

where $2 \mid n_i, i = 1, \cdots, k-1$.

In 1923 Hardy and Littlewood conjectured the asymptotic formula

$$\pi_k(N, 2) = |\{P \leq N : P+n_i = prime\}| = H(k) \frac{N}{\log^k N}, \quad (19)$$

where

$$H(k) = \prod_P \left(1 + \frac{\nu(P)}{P}\right)\left(1 - \frac{1}{P}\right)^{-k} \quad (20)$$

$\nu(P)$ is the number of solutions of congruence

$$\prod_{i=1}^{k-1}(q + n_i) \equiv 0 \pmod{P}, \quad q = 1, \cdots, P. \quad (21)$$

From (21) we have $\nu(P) < P$ and $H(k) \neq 0$. For any prime k -tuple equation there exist infinitely many primes P such that each of $P+n_i$ is prime, which is false.

Conjectore 1. Let $k = 2, P, P+2$, twin primes theorem

Frome (21) we have

$$\nu(P) = \quad (22)$$

Substituting (22) into (20) we have

$$H(2) = \prod_P \frac{P}{P-1} \quad (23)$$

Substituting (23) into (19) we have the asymptotic formula

$$\pi_2(N, 2) = |\{P \leq N : P+2 = prime\}| = \prod_P \frac{P}{P-1} \frac{N}{\log^2 N} \quad (24)$$

which is false see example 1.

Conjecture 2. Let $k = 3, P, P+2, P+4$.

From (21) we have

$$\nu(2) = 1, \ \nu(P) = 0 \ \text{ if } \ P > 2 \qquad (25)$$

Substituting (25) into (20) we have

$$H(3) = \prod_{P \geq 3} \frac{P^2(P-2)}{(P-1)^3} \qquad (26)$$

Substituting (26) into (19) we have asymptotic formula

$$\pi_3(N,2) = \left|\{P \leq N : P+2 = prime, P+4 = prim\}\right| \sim 4 \prod_{P \geq 3} \frac{P^2(P-2)}{(P-1)^3} \frac{N}{\log^3 N} \qquad (27)$$

which is false see example 2.

Conjecutre 3. Let $k=4$, $P, P+n$, where $n=2,6,8$.

From (21) we have

$$\nu(2) = 1, \ \nu(3) = 2, \ \nu(P) = 3 \ \text{ if } \ P > 3 \qquad (28)$$

Substituting (28) into (20) we have

$$H(4) = \frac{27}{2} \prod_{P > 3} \frac{P^3(P-3)}{(P-1)^4} \qquad (29)$$

Substituting (29) into (19) we have asymptotic formula

$$\pi_4(N,2) = \left|\{P \leq N : P+n = prime\}\right| \sim \frac{27}{2} \prod_{P > 3} \frac{P^3(P-3)}{(P-1)^4} \frac{N}{\log^4 N} \qquad (30)$$

Which is false see example 3.

Conjecutre 4. Let $k=5$, $P, P+n$, where $n=2,6,8,12$

From (21) we have

$$\nu(2) = 1, \ \nu(3) = 2, \ \nu(5) = P-3 \ \text{ if } \ P > 5 \qquad (31)$$

Substituting (31) into (20) we have

$$H(5) = \frac{15^4}{4^5} \prod_{P > 5} \frac{P^4(P-4)}{(P-1)^5} \qquad (32)$$

Substituting (32) into (19) we have asymptotic formula

$$\pi_5(N,2) = \left|\{P \leq N : P+n = prime\}\right| \sim \frac{15^4}{4^5} \prod_{P > 5} \frac{P^4(P-4)}{(P-1)^5} \frac{N}{\log^5 N} \qquad (33)$$

Which is false see example 4.

Conjecutre 5. Let $k=6$, $P, \ P+n$, where $n=2,6,8,12,14$.

From (21) we have

$$\nu(2) = 1, \ \nu(3) = 2, \ \nu(5) = P-4 \ \text{ if } \ P > 5 \qquad (34)$$

Substituting (34) into (20) we have

$$H(6) = \frac{15^5}{2^{13}} \prod_{P>5} \frac{(P-5)P^5}{(P-1)^6}$$ （35）

Substituting (35) into (19) we have asymptotic formula

$$\pi_6(N,2) = \left|\{P \leq N : P+n = prime\}\right| \sim \frac{15^5}{2^{13}} \prod_{P>5} \frac{(P-5)P^5}{(P-1)^6} \frac{N}{\log^6 N}$$ （36）

which is false see example 5.

Conclusion. The Hardy-Littlewood prime k-tuple conjecture is false. The tool of addive prime number theory is basically the Hardy-Littlewood prime tuples conjecture. Jiang prime k-tuple theorem can replace Hardy-Littlewood prime k-tuple Conjecture. There cannot be really modern prime theory without Jiang function.

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I

Riemann Paper (1859) Is False

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Abstract

In 1859 Riemann defined the zeta function $\zeta(s)$. From Gamma function he derived the zeta function with Gamma function $\overline{\zeta}(s)$. $\overline{\zeta}(s)$ and $\zeta(s)$ are the two different functions. It is false that $\overline{\zeta}(s)$ replaces $\zeta(s)$. After him later mathematicians put forward Riemann hypothesis(RH) which is false. The Jiang function $J_n(\omega)$ can replace RH.

AMS   mathematics subject classification: Primary 11M26.

In 1859 Riemann defined the Riemann zeta function (RZF)[1]

$$\zeta(s) = \prod_{P}(1-P^{-s})^{-1} = \sum_{n=1}^{\infty}\frac{1}{n^s}\ ,\qquad(1)$$

where $s = \sigma + ti, i = \sqrt{-1}$, σ and t are real, P ranges over all primes. RZF is the function of the complex variable s in $\sigma \geq 0, t \neq 0$, which is absolutely convergent.

In 1896 J. Hadamard and de la Vallee Poussin proved independently [2]

$$\zeta(1+ti) \neq 0.\qquad(2)$$

In 1998 Jiang proved [3]

$$\zeta(s) \neq 0,\qquad(3)$$

where $0 \leq \sigma \leq 1$.

Riemann paper (1859) is false [1]   We define Gamma function [1, 2]

$$\Gamma\left(\frac{s}{2}\right) = \int_0^{\infty} e^{-t}t^{\frac{s}{2}-1}dt\ .\qquad(4)$$

For $\sigma > 0$. On setting $t = n^2\pi x$, we observe that

$$\pi^{-\frac{s}{2}}\Gamma\left(\frac{s}{2}\right)n^{-s} = \int_0^{\infty} x^{\frac{s}{2}-1}e^{-n^2\pi x}dx\ .\qquad(5)$$

Hence, with some care on exchanging summation and integration, for $\sigma > 1$,

$$\pi^{-\frac{s}{2}}\Gamma\left(\frac{s}{2}\right)\overline{\zeta}(s) = \int_0^{\infty} x^{\frac{s}{2}-1}\left(\sum_{n=1}^{\infty} e^{-n^2\pi x}\right)dx$$

$$= \int_0^{\infty} x^{\frac{s}{2}-1}\left(\frac{\vartheta(x)-1}{2}\right)dx,\qquad(6)$$

where $\overline{\zeta}(s)$ is called Riemann zeta function with gamma function rather than $\zeta(s)$,

$$\vartheta(x) := \sum_{n=-\infty}^{\infty} e^{-n^2\pi x}\ ,\qquad(7)$$

is the Jacobi theta function. The functional equation for $\vartheta(x)$ is

$$x^{\frac{1}{2}}\vartheta(x) = \vartheta(x^{-1}),\qquad(8)$$

and is valid for $x > 0$.

Finally, using the functional equation of $\vartheta(x)$, we obtain

$$\overline{\zeta}(s) = \frac{\pi^{\frac{s}{2}}}{\Gamma\left(\frac{s}{2}\right)}\left\{\frac{1}{s(s-1)} + \int_1^{\infty}\left(x^{\frac{s}{2}-1} + x^{-\frac{s}{2}-\frac{1}{2}}\right)\cdot\left(\frac{\vartheta(x)-1}{2}\right)dx\right\}\qquad(9)$$

From (9) we obtain the functional equation

$$\pi^{-\frac{s}{2}}\Gamma\left(\frac{s}{2}\right)\overline{\zeta}(s) = \pi^{-\frac{1-s}{2}}\Gamma\left(\frac{1-s}{2}\right)\overline{\zeta}(1-s). \qquad (10)$$

The function $\overline{\zeta}(s)$ satisfies the following

1. $\overline{\zeta}(s)$ has no zero for $\sigma > 1$;

2. The only pole of $\overline{\zeta}(s)$ is at $s = 1$; it has residue 1 and is simple;

3. $\overline{\zeta}(s)$ has trivial zeros at $s = -2, -4, \dots$ but $\zeta(s)$ has no zeros;

4. The nontrivial zeros lie inside the region $0 \le \sigma \le 1$ and are symmetric about both the vertical line $\sigma = 1/2$.

The strip $0 \le \sigma \le 1$ is called the critical strip and the vertical line $\sigma = 1/2$ is called the critical line.

Conjecture (The Riemann Hypothesis). All nontrivial zeros of $\overline{\zeta}(s)$ lie on the critical line $\sigma = 1/2$, which is false. [3]

$\overline{\zeta}(s)$ and $\zeta(s)$ are the two different functions. It is false that $\overline{\zeta}(s)$ replaces $\zeta(s)$, Pati proved that is not all complex zeros of $\overline{\zeta}(s)$ lie on the critical line: $\sigma = 1/2$ [4].

Schadeck pointed out that the falsity of RH implies the falsity of RH for finite fields [5, 6]. RH is not directly related to prime theory. Using RH mathematicians prove many prime theorems which is false. In 1994 Jiang discovered Jiang function $J_n(\omega)$ which can replace RH, Riemann zeta function and L-function in view of its proved feature: if $J_n(\omega) \ne 0$ then the prime equation has infinitely many prime solutions; and if $J_n(\omega) = 0$, then the prime equation has finitely many prime solutions. By using $J_n(\omega)$ Jiang proves about 600 prime theorems including the Goldbach's theorem, twin prime theorem and theorem on arithmetic progressions in primes[7,8].

In the same way we have a general formula involving $\overline{\zeta}(s)$

$$\int_0^\infty x^{s-1} \sum_{n=1}^\infty F(nx)dx = \sum_{n=1}^\infty \int_0^\infty x^{s-1} F(nx)dx$$

$$= \sum_{n=1}^\infty \frac{1}{n^s} \int_0^\infty y^{s-1} F(y)dy = \overline{\zeta}(s) \int_0^\infty y^{s-1} F(y)dy, \qquad (11)$$

where $F(y)$ is arbitrary.

From (11) we obtain many zeta functions $\overline{\zeta}(s)$ which are not directly related to the number theory.

The prime distributions are order rather than random. The arithmetic progressions in primes are not directly related to ergodic theory, harmonic analysis, discrete geometry, and combinatories. Using the ergodic theory Green and Tao prove that there exist infinitely many arithmetic progressions of length k consisting only of primes which is false [9, 10, 11]. Fermat's last theorem (FLT) is not directly related to elliptic curves. In 1994 using elliptic curves Wiles proved FLT which is false [12]. There are Pythagorean theorem and FLT in the complex hyperbolic functions and complex trigonometric functions. In 1991 without using any number theory Jiang proved FLT which is Fermat's marvelous proof[7, 13].

Primes Represented by $P_1^n + mP_2^n$ [14]

（1）Let $n = 3$ and $m = 2$. We have

$$P_3 = P_1^3 + 2P_2^3 .$$

We have Jiang function

$$J_3(\omega) = \prod_{\substack{3 \leq P \\ P-1}} (P^2 - 3P + 3 - \chi(P)) \neq 0 ,$$

Where $\chi(P) = 2P - 1$ if $2^{\frac{P-1}{3}} \equiv 1 \pmod{P}$; $\chi(P) = -P + 2$ if $2^{\frac{P-1}{3}} \not\equiv 1 \pmod{P}$; $\chi(P) = 1$ otherwise.

Since $J_n(\omega) \neq 0$, there exist infinitely many primes P_1 and P_2 such that P_3 is a prime.
We have the best asymptotic formula

$$\pi_2(N,3) = \left| \{P_1, P_2 : P_1, P_2 \leq N, P_1^3 + 2P_2^3 = P_3 \text{ prime}\} \right|$$

$$\sim \frac{J_3(\omega)\omega}{6\Phi^3(\omega)} \frac{N^2}{\log^3 N} = \frac{1}{3} \prod_{3 \leq P} \frac{P(P^2 - 3P + 3 - \chi(P))}{(P-1)^3} \frac{N^2}{\log^3 N} .$$

where $\omega = \prod_{2 \leq P} P$ is called primorial, $\Phi(\omega) = \prod_{2 \leq P} (P-1)$.

It is the simplest theorem which is called the Heath-Brown problem [15].

（2）Let $n = P_0$ be an odd prime, $2 | m$ and $m \neq \pm b^{P_0}$.
we have

$$P_3 = P_1^{P_0} + mP_2^{P_0}$$

We have

$$J_3(\omega) = \prod_{3 \leq P} (P^2 - 3P + 3 - \chi(P)) \neq 0 ,$$

where $\chi(P) = -P + 2$ if $P | m$; $\chi(P) = (P_0 - 1)P - P_0 + 2$ if $m^{\frac{P-1}{P_0}} \equiv 1 \pmod{P}$;

$\chi(P) = -P + 2$ if $m^{\frac{P-1}{P_0}} \not\equiv 1 \pmod{P}$; $\chi(P) = 1$ otherwise.

Since $J_n(\omega) \neq 0$, there exist infinitely many primes P_1 and P_2 such that P_3 is a prime.
We have

$$\pi_2(N,3) \sim \frac{J_3(\omega)\omega}{2P_0\Phi^3(\omega)} \frac{N^2}{\log^3 N} .$$

The Polynomial $P_1^n + (P_2 + 1)^2$ Captures Its Primes [14]

（1）Let $n = 4$, We have

$$P_3 = P_1^4 + (P_2 + 1)^2 ,$$

We have Jiang function

$$J_3(\omega) = \prod_{3 \leq P} (P^2 - 3P + 3 - \chi(P)) \neq 0 ,$$

Where $\chi(P) = P$ if $P \equiv 1 \pmod{4}$; $\chi(P) = P - 4$ if $P \equiv 1 \pmod{8}$; $\chi(P) = -P + 2$ otherwise.

Since $J_n(\omega) \neq 0$, there exist infinitely many primes P_1 and P_2 such that P_3 is a prime.
We have the best asymptotic formula

$$\pi_2(N,3) = \left| \{P_1, P_2 : P_1, P_2 \le N, P_1^4 + (P_2+1)^2 = P_3 \text{ prime}\} \right|$$

$$\sim \frac{J_3(\omega)\omega}{8\Phi^3(\omega)} \frac{N^2}{\log^3 N} .$$

It is the simplest theorem which is called Friedlander-Iwaniec problem [16].

（2）Let $n = 4m$, We have

$$P_3 = P_1^{4m} + (P_2+1)^2 ,$$

where $m = 1, 2, 3, \cdots$.

We have Jiang function

$$J_3(\omega) = \prod_{3 \le P \le P_i} (P^2 - 3P + 3 - \chi(P)) \ne 0,$$

where $\chi(P) = P - 4m$ if $8m \mid (P-1)$; $\chi(P) = P - 4$ if $8 \mid (P-1)$; $\chi(P) = P$ if $4 \mid (P-1)$; $\chi(P) = -P + 2$ otherwise.

Since $J_3(\omega) \ne 0$, there exist infinitely many primes P_1 and P_2 such that P_3 is a prime. It is a generalization of Euler proof for the existence of infinitely many primes.

We have the best asymptotic formula

$$\pi_2(N,3) \sim \frac{J_3(\omega)\omega}{8m\Phi^3(\omega)} \frac{N^2}{\log^3 N} .$$

（3）Let $n = 2b$. We have

$$P_3 = P_1^{2b} + (P_2+1)^2 ,$$

where b is an odd.

We have Jiang function

$$J_3(\omega) = \prod_{3 \le P} (P^2 - 3P + 3 - \chi(P)) \ne 0,$$

Where $\chi(P) = P - 2b$ if $4b \mid (P-1)$; $\chi(P) = P - 2$ if $4 \mid (P-1)$; $\chi(P) = -P + 2$ otherwise.

We have the best asymptotic formula

$$\pi_2(N,3) \sim \frac{J_3(\omega)\omega}{4b\Phi^3(\omega)} \frac{N^2}{\log^3 N} .$$

（4）Let $n = P_0$, We have

$$P_3 = P_1^{P_0} + (P_2+1)^2 .$$

where P_0 is an odd. Prime.

we have Jiang function

$$J_3(\omega) = \prod_{3 \le P} (P^2 - 3P + 3 - \chi(P)) \ne 0,$$

where $\chi(P) = P_0 + 1$ if $P_0 \mid (P-1)$; $\chi(P) = 0$ otherwise.

Since $J_3(\omega) \ne 0$, there exist infinitely many primes P_1 and P_2 such that P_3 is also a prime.

We have the best asymptotic formula

$$\pi_2(N,3) \sim \frac{J_3(\omega)\omega}{2P_0\Phi^3(\omega)} \frac{N^2}{\log^3 N}.$$

The Jiang function $J_n(\omega)$ is closely related to the prime distribution. Using $J_n(\omega)$ we are able to tackle almost all prime problems in the prime distributions.

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Automorphic Functions
And
Fermat's Last Theorem(1)

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Abstract

In 1637 Fermat wrote: "*It is impossible to separate a cube into two cubes, or a biquadrate into two biquadrates, or in general any power higher than the second into powers of like degree: I have discovered a truly marvelous proof, which this margin is too small to contain.*"

This means: $x^n + y^n = z^n (n > 2)$ has no integer solutions, all different from 0(i.e., it has only the trivial solution, where one of the integers is equal to 0). It has been called Fermat's last theorem (FLT). It suffices to prove FLT for exponent 4. and every prime exponent P. Fermat proved FLT for exponent 4. Euler proved FLT for exponent 3.

In this paper using automorphic functions we prove FLT for exponents $3P$ and P, where P is an odd prime. The proof of FLT must be direct. But indirect proof of FLT is disbelieving.

In 1974 Jiang found out Euler formula of the cyclotomic real numbers in the cyclotomic fields

$$\exp\left(\sum_{i=1}^{n-1} t_i J^i\right) = \sum_{i=1}^{n} S_i J^{i-1} \qquad (1)$$

where J denotes a n th root of unity, $J^n = 1$, n is an odd number, t_i are the real numbers.

S_i is called the automorphic functions(complex hyperbolic functions) of order n with $n-1$ variables [1-7].

$$S_i = \frac{1}{n}\left[e^A + 2\sum_{j=1}^{\frac{n-1}{2}} (-1)^{(i-1)j} e^{B_j} \cos\left(\theta_j + \frac{(i-1)j\pi}{n}\right) \right] \qquad (2)$$

where $i=1,2,\ldots,n$;

$$A = \sum_{\alpha-1}^{n-1} t_\alpha, \qquad B_j = \sum_{\alpha=1}^{n-1} t_\alpha (-1)^{\alpha j} \cos\frac{\alpha j\pi}{n},$$

$$\theta_j = (-1)^{j+1}\sum_{\alpha=1}^{n-1} t_\alpha (-1)^{\alpha j} \sin\frac{\alpha j\pi}{n}, \qquad A + 2\sum_{j=1}^{\frac{n-1}{2}} B_j = 0 \qquad (3)$$

(2) may be written in the matrix form

$$\begin{bmatrix} S_1 \\ S_2 \\ S_3 \\ \cdots \\ S_n \end{bmatrix} = \frac{1}{n} \begin{bmatrix} 1 & 1 & 0 & \cdots & 0 \\ 1 & -\cos\dfrac{\pi}{n} & -\sin\dfrac{\pi}{n} & \cdots & -\sin\dfrac{(n-1)\pi}{2n} \\ 1 & \cos\dfrac{2\pi}{n} & \sin\dfrac{2\pi}{n} & \cdots & -\sin\dfrac{(n-1)\pi}{n} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 1 & \cos\dfrac{(n-1)\pi}{n} & \sin\dfrac{(n-1)\pi}{n} & \cdots & -\sin\dfrac{(n-1)^2\pi}{2n} \end{bmatrix} \begin{bmatrix} e^A \\ 2e^{B_1}\cos\theta_1 \\ 2e^{B_1}\sin\theta_1 \\ \cdots \\ 2\exp B_{\frac{n-1}{2}}\sin\theta_{\frac{n-1}{2}} \end{bmatrix} \quad (4)$$

where $(n-1)/2$ is an even number.

From (4) we have its inverse transformation

$$\begin{bmatrix} e^A \\ e^{B_1}\cos\theta_1 \\ e^{B_1}\sin\theta_1 \\ \cdots \\ \exp(B_{\frac{n-1}{2}})\sin(\theta_{\frac{n-1}{2}}) \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & \cdots & 1 \\ 1 & -\cos\dfrac{\pi}{n} & \cos\dfrac{2\pi}{n} & \cdots & \cos\dfrac{(n-1)\pi}{n} \\ 0 & -\sin\dfrac{\pi}{n} & \sin\dfrac{2\pi}{n} & \cdots & \sin\dfrac{(n-1)\pi}{n} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & -\sin\dfrac{(n-1)\pi}{2n} & -\sin\dfrac{(n-1)\pi}{n} & \cdots & -\sin\dfrac{(n-1)^2\pi}{2n} \end{bmatrix} \begin{bmatrix} S_1 \\ S_2 \\ S_3 \\ \cdots \\ S_n \end{bmatrix}$$

$$(5)$$

From (5) we have

$$e^A = \sum_{i=1}^{n} S_i, \qquad e^{B_j}\cos\theta_j = S_1 + \sum_{i=1}^{n-1} S_{1+i}(-1)^{ij}\cos\frac{ij\pi}{n}$$

$$e^{B_j}\sin\theta_j = (-1)^{j+1}\sum_{i=1}^{n-1} S_{1+i} - (-1)^{ij}\sin\frac{ij\pi}{n}, \qquad (6)$$

In (3) and (6) t_i and S_i have the same formulas. (4) and (5) are the most critical formulas of proofs for FLT. Using (4) and (5) in 1991 Jiang invented that every factor of exponent n has the Fermat equation and proved FLT [1-7] Substituting (4) into (5) we prove (5).

$$\begin{bmatrix} e^A \\ e^{B_1}\cos\theta_1 \\ e^{B_1}\sin\theta_1 \\ \cdots \\ \exp(B_{\frac{n-1}{2}})\sin(\theta_{\frac{n-1}{2}}) \end{bmatrix} = \frac{1}{n} \begin{bmatrix} 1 & 1 & 1 & \cdots & 1 \\ 1 & -\cos\dfrac{\pi}{n} & \cos\dfrac{2\pi}{n} & \cdots & \cos\dfrac{(n-1)\pi}{n} \\ 0 & -\sin\dfrac{\pi}{n} & \sin\dfrac{2\pi}{n} & \cdots & \sin\dfrac{(n-1)\pi}{n} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & -\sin\dfrac{(n-1)\pi}{2n} & -\sin\dfrac{(n-1)\pi}{n} & \cdots & -\sin\dfrac{(n-1)^2\pi}{2n} \end{bmatrix} \times$$

$$
\begin{bmatrix}
1 & 1 & 0 & \cdots & 0 \\
1 & -\cos\dfrac{\pi}{n} & -\sin\dfrac{\pi}{n} & \cdots & -\sin\dfrac{(n-1)\pi}{2n} \\
1 & \cos\dfrac{2\pi}{n} & \sin\dfrac{2\pi}{n} & \cdots & -\sin\dfrac{(n-1)\pi}{n} \\
\cdots & \cdots & \cdots & \cdots & \cdots \\
1 & \cos\dfrac{(n-1)\pi}{n} & \sin\dfrac{(n-1)\pi}{n} & \cdots & -\sin\dfrac{(n-1)^2\pi}{2n}
\end{bmatrix}
\begin{bmatrix}
e^A \\
2e^{B_1}\cos\theta_1 \\
2e^{B_1}\sin\theta_1 \\
\cdots \\
2\exp(B_{\frac{n-1}{2}})\sin(\theta_{\frac{n-1}{2}})
\end{bmatrix}
$$

$$
= \frac{1}{n}
\begin{bmatrix}
n & 0 & 0 & \cdots & 0 \\
0 & \dfrac{n}{2} & 0 & \cdots & 0 \\
0 & 0 & \dfrac{n}{2} & \cdots & 0 \\
\cdots & \cdots & \cdots & \cdots & \cdots \\
0 & 0 & 0 & \cdots & \dfrac{n}{2}
\end{bmatrix}
\begin{bmatrix}
e^A \\
2e^{B_1}\cos\theta_1 \\
2e^{B_1}\sin\theta_1 \\
\cdots \\
2\exp(B_{\frac{n-1}{2}})\sin(\theta_{\frac{n-1}{2}})
\end{bmatrix}
$$

$$
=
\begin{bmatrix}
e^A \\
e^{B_1}\cos\theta_1 \\
e^{B_1}\sin\theta_1 \\
\cdots \\
\exp(B_{\frac{n-1}{2}})\sin(\theta_{\frac{n-1}{2}})
\end{bmatrix},
\tag{7}
$$

where $1+\displaystyle\sum_{j=1}^{n-1}(\cos\frac{j\pi}{n})^2=\frac{n}{2},\quad \sum_{j=1}^{n-1}(\sin\frac{j\pi}{n})^2=\frac{n}{2}.$

From (3) we have

$$
\exp\left(A+2\sum_{j=1}^{\frac{n-1}{2}}B_j\right)=1.
\tag{8}
$$

From (6) we have

$$
\exp\left(A+2\sum_{j=1}^{\frac{n-1}{2}}B_j\right)=
\begin{vmatrix}
S_1 & S_n & \cdots & S_2 \\
S_2 & S_1 & \cdots & S_3 \\
\cdots & \cdots & \cdots & \cdots \\
S_n & S_{n-1} & \cdots & S_1
\end{vmatrix}
=
\begin{vmatrix}
S_1 & (S_1)_1 & \cdots & (S_1)_{n-1} \\
S_2 & (S_2)_1 & \cdots & (S_2)_{n-1} \\
\cdots & \cdots & \cdots & \cdots \\
S_n & (S_n)_1 & \cdots & (S_n)_{n-1}
\end{vmatrix},
\tag{9}
$$

where $(S_i)_j=\dfrac{\partial S_i}{\partial t_j}$ [7].

From (8) and (9) we have the circulant determinant

$$\exp(A+2\sum_{j=1}^{\frac{n-1}{2}} B_j) = \begin{vmatrix} S_1 & S_n & \cdots & S_2 \\ S_2 & S_1 & \cdots & S_3 \\ \cdots & \cdots & \cdots & \vdots \\ S_n & S_{n-1} & \cdots & S_1 \end{vmatrix} = 1 \qquad (10)$$

If $S_i \neq 0$, where $i = 1, 2, \cdots, n$, then (10) has infinitely many rational solutions.

Assume $S_1 \neq 0$, $S_2 \neq 0$, $S_i = 0$ where $i = 3, 4, \cdots, n$. $S_i = 0$ are $n-2$ indeterminate equations with $n-1$ variables. From (6) we have

$$e^A = S_1 + S_2, \quad e^{2B_j} = S_1^2 + S_2^2 + 2S_1 S_2 (-1)^j \cos\frac{j\pi}{n}. \qquad (11)$$

From (10) and (11) we have the Fermat equation

$$\exp(A + 2\sum_{j=1}^{\frac{n-1}{2}} B_j) = (S_1 + S_2)\prod_{j=1}^{\frac{n-1}{2}}(S_1^2 + S_2^2 + 2S_1 S_2(-1)^j \cos\frac{j\pi}{n}) = S_1^n + S_2^n = 1 \qquad (12)$$

Example[1]. Let $n = 15$. From (3) we have

$$A = (t_1 + t_{14}) + (t_2 + t_{13}) + (t_3 + t_{12}) + (t_4 + t_{11}) + (t_5 + t_{10}) + (t_6 + t_9) + (t_7 + t_8)$$

$$B_1 = -(t_1 + t_{14})\cos\frac{\pi}{15} + (t_2 + t_{13})\cos\frac{2\pi}{15} - (t_3 + t_{12})\cos\frac{3\pi}{15} + (t_4 + t_{11})\cos\frac{4\pi}{15}$$
$$\quad -(t_5 + t_{10})\cos\frac{5\pi}{15} + (t_6 + t_9)\cos\frac{6\pi}{15} - (t_7 + t_8)\cos\frac{7\pi}{15},$$

$$B_2 = (t_1 + t_{14})\cos\frac{2\pi}{15} + (t_2 + t_{13})\cos\frac{4\pi}{15} + (t_3 + t_{12})\cos\frac{6\pi}{15} + (t_4 + t_{11})\cos\frac{8\pi}{15}$$
$$\quad +(t_5 + t_{10})\cos\frac{10\pi}{15} + (t_6 + t_9)\cos\frac{12\pi}{15} + (t_7 + t_8)\cos\frac{14\pi}{15},$$

$$B_3 = -(t_1 + t_{14})\cos\frac{3\pi}{15} + (t_2 + t_{13})\cos\frac{6\pi}{15} - (t_3 + t_{12})\cos\frac{9\pi}{15} + (t_4 + t_{11})\cos\frac{12\pi}{15}$$
$$\quad -(t_5 + t_{10})\cos\frac{15\pi}{15} + (t_6 + t_9)\cos\frac{18\pi}{15} - (t_7 + t_8)\cos\frac{21\pi}{15},$$

$$B_4 = (t_1 + t_{14})\cos\frac{4\pi}{15} + (t_2 + t_{13})\cos\frac{8\pi}{15} + (t_3 + t_{12})\cos\frac{12\pi}{15} + (t_4 + t_{11})\cos\frac{16\pi}{15}$$
$$\quad +(t_5 + t_{10})\cos\frac{20\pi}{15} + (t_6 + t_9)\cos\frac{24\pi}{15} + (t_7 + t_8)\cos\frac{28\pi}{15},$$

$$B_5 = -(t_1 + t_{14})\cos\frac{5\pi}{15} + (t_2 + t_{13})\cos\frac{10\pi}{15} - (t_3 + t_{12})\cos\frac{15\pi}{15} + (t_4 + t_{11})\cos\frac{20\pi}{15}$$
$$\quad -(t_5 + t_{10})\cos\frac{25\pi}{15} + (t_6 + t_9)\cos\frac{30\pi}{15} - (t_7 + t_8)\cos\frac{35\pi}{15},$$

$$B_6 = (t_1 + t_{14})\cos\frac{6\pi}{15} + (t_2 + t_{13})\cos\frac{12\pi}{15} + (t_3 + t_{12})\cos\frac{18\pi}{15} + (t_4 + t_{11})\cos\frac{24\pi}{15}$$
$$\quad +(t_5 + t_{10})\cos\frac{30\pi}{15} + (t_6 + t_9)\cos\frac{36\pi}{15} + (t_7 + t_8)\cos\frac{42\pi}{15},$$

$$B_7 = -(t_1 + t_{14})\cos\frac{7\pi}{15} + (t_2 + t_{13})\cos\frac{14\pi}{15} - (t_3 + t_{12})\cos\frac{21\pi}{15} + (t_4 + t_{11})\cos\frac{28\pi}{15}$$

$$-(t_5+t_{10})\cos\frac{35\pi}{15}+(t_6+t_9)\cos\frac{42\pi}{15}-(t_7+t_8)\cos\frac{49\pi}{15},$$

$$A+2\sum_{j=1}^{7}B_j=0,\qquad A+2B_3+2B_6=5(t_5+t_{10}).\tag{13}$$

Form (12) we have the Fermat equation

$$\exp(A+2\sum_{j=1}^{7}B_j)=S_1^{15}+S_2^{15}=(S_1^5)^3+(S_2^5)^3=1.\tag{14}$$

From (13) we have

$$\exp(A+2B_3+2B_6)=[\exp(t_5+t_{10})]^5.\tag{15}$$

From (11) we have

$$\exp(A+2B_3+2B_6)=S_1^5+S_2^5.\tag{16}$$

From (15) and (16) we have the Fermat equation

$$\exp(A+2B_3+2B_6)=S_1^5+S_2^5=[\exp(t_5+t_{10})]^5.\tag{17}$$

Euler proved that (14) has no rational solutions for exponent 3[8]. Therefore we prove that (17) has no rational solutions for exponent 5[1].

Theorem 1. [1-7]. Let $n=3P$, where $P>3$ is odd prime. From (12) we have the Fermat's equation

$$\exp(A+2\sum_{j=1}^{3P-1}B_j)=S_1^{3P}+S_2^{3P}=(S_1^P)^3+(S_2^P)^3=1.\tag{18}$$

From (3) we have

$$\exp(A+2\sum_{j=1}^{\frac{P-1}{2}}B_{3j})=[\exp(t_P+t_{2P})]^P.\tag{19}$$

From (11) we have

$$\exp(A+2\sum_{j=1}^{\frac{P-1}{2}}B_{3j})=S_1^P+S_2^P.\tag{20}$$

From (19) and (20) we have the Fermat equation

$$\exp(A+2\sum_{j=1}^{\frac{P-1}{2}}B_{3j})=S_1^P+S_2^P=[\exp(t_P+t_{2P})]^P.\tag{21}$$

Euler proved that (18) has no rational solutions for exponent 3[8]. Therefore we prove that (21) has no rational solutions for $P>3$ [1, 3-7].

Theorem 2. In 1847 Kummer write the Fermat's equation

$$x^P+y^P=z^P\tag{22}$$

in the form

$$(x+y)(x+ry)(x+r^2y)\cdots(x+r^{P-1}y)=z^P\tag{23}$$

where P is odd prime, $r = \cos\dfrac{2\pi}{P} + i\sin\dfrac{2\pi}{P}$.

Kummer assume the divisor of each factor is a P th power. Kummer proved FLT for prime exponent p<100 [8]..

We consider the Fermat's equation

$$x^{3P} + y^{3P} = z^{3P} \qquad (24)$$

we rewrite (24)

$$(x^P)^3 + (y^P)^3 = (z^P)^3 \qquad (25)$$

From (24) we have

$$(x^P + y^P)(x^P + ry^P)(x^P + r^2 y^P) = z^{3P} \qquad (26)$$

where $r = \cos\dfrac{2\pi}{3} + i\sin\dfrac{2\pi}{3}$

We assume the divisor of each factor is a P th power.

Let $S_1 = \dfrac{x}{z}$, $S_2 = \dfrac{y}{z}$. From (20) and (26) we have the Fermat's equation

$$x^P + y^P = [z \times \exp(t_P + t_{2P})]^P \qquad (27)$$

Euler proved that (25) has no integer solutions for exponent 3[8]. Therefore we prove that (27) has no integer solutions for prime exponent P.

Fermat Theorem. It suffices to prove FLT for exponent 4. We rewrite (24)

$$(x^3)^P + (y^3)^P = (z^3)^P \qquad (28)$$

Euler proved that（25）has no integer solutions for exponent 3 [8]. Therefore we prove that (28) has no integer solutions for all prime exponent P [1-7].

We consider Fermat equation

$$x^{4P} + y^{4P} = z^{4P} \qquad (29)$$

We rewrite (29)

$$(x^P)^4 + ((y^P)^4 = (z^P)^4 \qquad (30)$$

$$(x^4)^P + (y^4)^P = (z^4)^P \qquad (31)$$

Fermat proved that (30) has no integer solutions for exponent 4 [8]. Therefore we prove that (31) has no integer solutions for all prime exponent P [2,5,7].This is the proof that Fermat thought to have had.

Remark. It suffices to prove FLT for exponent 4. Let $n = 4P$, where P is an odd prime. We have the Fermat's equation for exponent $4P$ and the Fermat's equation for exponent P [2,5,7]. This is the proof that Fermat thought to have had. In complex hyperbolic functions let exponent n be $n = \Pi P$, $n = 2\Pi P$ and $n = 4\Pi P$. Every factor of exponent n has the Fermat's equation [1-7]. In complex trigonometric functions let exponent n be $n = \Pi P$, $n = 2\Pi P$ and $n = 4\Pi P$. Every factor of exponent n has Fermat's equation [1-7].Using modular elliptic curves Wiles and Taylor prove FLT[9,10].This is not the proof that Fermat thought to have had. The classical theory of automorphic functions, created by Klein and Poincare, was concerned with

the study of analytic functions in the unit circle that are invariant under a discrete group of transformations. Automorphic functions are generalization of the trigonometric ,hyperbolic,elliptic, and certain other functions of elementary analysis. The complex trigonometric functions and complex hyperbolic functions have a wide application in mathematics and physics.

Acknowledgments.We thank Chenny and Moshe Klein for their help and suggestion.

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An equation that changed the universe: $\overline{F} = -mc^2/R$

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Abstract

This paper explains the behavior of the entire universe from the smallest to the largest scales, found an equation that changed the universe: $\overline{F} = -mc^2/R$, established the expansion theory of the universe without dark matter and dark energy,and obtained the expansion acceleration: $g_e = u^4/C^2R$. It shows that gravity is action-at-a-distance and that a gravitational wave is unobservable. Thus,a new universe model is suggested that the universe has a centre consisting of the tachyonic matter.

Keywords: The universe equation; the universe expansion theory

Introduction

According to Jiang idea[1],in the Universe there are two kinds of matter: (1) observable subluminal matter called tardyons(locality) and (2) unobservable superluminal matter called tachyons(nonlocality). They coexist in motion. What are tachyons? Historically tachyons are described as particles which travel faster than light. Describing tachyon as a particle with an imaginary mass is wrong[2]. In our theory[1] tachyon has no rest time and no rest mass. It is unobservable. Tachyons can be converted into tardyons and vice versa. Tardyonic rotating motion produces the centrifugal force but tachyonic rotating motion produces the centripetal force which is force of gravity. Using the coexistence principle of tardyons and tachyons it follows that an equation that changed the universe: $\overline{F} = -mc^2/R$. We establish the expansion theory of a universe without dark matter and dark energy. We obtain the expansion acceleration: $g_e = u^4/C^2R$. We unify the gravitational theory and particle theory and explain the behavior of the entire universe from the smallest to the largest scales. In this universe there are no quarks , no Higgs particles ,and no black holes. The geometrization of all physical fields is a mathematical guess which has no basis in physical reality, because it does not consider and understand the tachyonic theory. It shows that gravity is action-at-a-distance and that a gravitational wave is unobservable.We suggest a new universe model that the universe has a centre consisting of the tachyonic matter.

An equation that Changed the Universe: $\overline{F} = -mc^2/R$

We first define two-dimensional space and time ring[1]

$$z = \begin{pmatrix} ct & x \\ x & ct \end{pmatrix} = ct + jx, \qquad (1)$$

where x and t are the tardyonic space and time coordinates, c is light velocity in vacuum,

$$j = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}.$$

(1) can be written in Euler form

$$z = ct_0 e^{j\theta} = ct_0 (\operatorname{ch}\theta + j\operatorname{sh}\theta), \qquad (2)$$

where ct_0 is the tardyonic invariance, and θ is the tardyonic hyperbolical angle.

From (1) and (2) it follows

$$ct = ct_0 \operatorname{ch}\theta, \quad x = ct_0 \operatorname{sh}\theta \qquad (3)$$

$$ct_0 = \sqrt{(ct)^2 - x^2}. \qquad (4)$$

From (3) it follows

$$\theta = \operatorname{th}^{-1} \frac{x}{ct} = \operatorname{th}^{-1} \frac{u}{c}. \qquad (5)$$

where $c \geq u$ is the tardyonic velocity, $\operatorname{ch}\theta = \dfrac{1}{\sqrt{1-(u/c)^2}}$ and $\operatorname{sh}\theta = \dfrac{u/c}{\sqrt{1-(u/c)^2}}$.

The z denotes space-time of the tardyonic theory.

Using the morphism $j : z \rightarrow jz$, it follows

$$jz = \bar{x} + jc\bar{t} = \bar{x}_0 e^{j\bar{\theta}} = \bar{x}_0 (\operatorname{ch}\bar{\theta} + j\operatorname{sh}\bar{\theta}), \qquad (6)$$

where $\bar{x}$ and $\bar{t}$ are the tachyonic space and time coordinates, $\bar{x}_0$ is tachyonic invariance, $\bar{\theta}$ tachyonic hyperbolical angle.

From (6) it follows

$$\bar{x} = \bar{x}_0 \operatorname{ch}\bar{\theta}, \quad c\bar{t} = \bar{x}_0 \operatorname{sh}\bar{\theta}. \qquad (7)$$

$$\bar{x}_0 = \sqrt{(\bar{x})^2 - (c\bar{t})^2}. \qquad (8)$$

From (7) it follows

$$\bar{\theta} = \operatorname{th}^{-1} \frac{c\bar{t}}{\bar{x}} = \operatorname{th}^{-1} \frac{c}{\bar{u}}. \qquad (9)$$

where $\bar{u} \geq c$ is the tachyonic velocity, $\operatorname{ch}\bar{\theta} = \dfrac{1}{\sqrt{1-(c/\bar{u})^2}}$ and

$$\operatorname{sh}\bar{\theta} = \frac{c/\bar{u}}{\sqrt{1-(c/\bar{u})^2}}.$$

The jz denotes space-time of the tachyonic theory. Both the z and the jz form the entire world but the jz world is unexploited and unstudied.
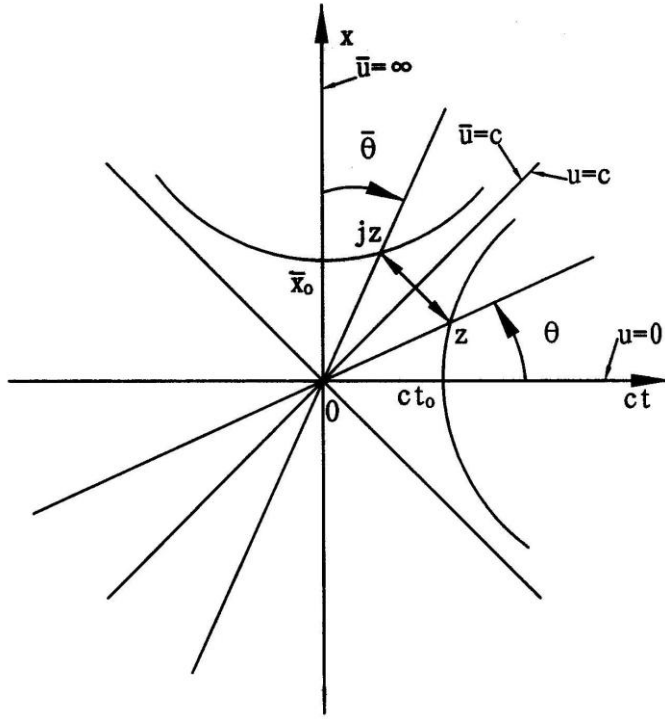
Fig. 1. Minkowskian spacetime diagram

Figure 1 shows the formulas (1)-(9). $j : z \to jz$ shows that a tardyon can be converted into a tachyon, but $j : jz \to z$ shows that a tachyon can be converted into a tardyon. $u = 0 \to u = c$ is a tardyonic velocity, but $\bar{u} = \infty \to \bar{u} = c$ is a tachyonic velocity, which coexist. At the $x-$axis we define the tachyonic string length

$$\bar{x}_0 = \lim_{\substack{\bar{u} \to \infty \\ t \to 0}} \bar{u}t = \text{constant}. \tag{10}$$

where t is the rest time.

Since at rest the tachyonic string time $t = 0$ and $\bar{u} = \infty$, it shows that the tachyon is a string which is unobservable. In the rest system the tachyonic string motion is an action-at-a distance motion. This simple thought made a deep impression on me. It impelled me toward the only string theory[1]. Other string theories all are guesses.

Assume $\theta = \bar{\theta}$, from (5) and (9) it follows that the tardyonic and tachyonic coexistence principle[1,3,4]

$$u\bar{u} = c^2 \qquad . \tag{11}$$

Differentiating (11) by the time, it follows

$$\frac{d\bar{u}}{dt} = -\left(\frac{c}{u}\right)^2 \frac{du}{dt}. \tag{12}$$

$\dfrac{du}{dt}$ and $\dfrac{d\bar{u}}{dt}$ can coexist in motion, but their directions are opposite.

We study the tardyonic and tachyonic rotating motions. The tardyonic rotation produces centripetal acceleration

$$\frac{du}{dt} = \frac{u^2}{R} , \tag{13}$$

where R is rotating radius.

Substituting (13) into (12) it follows that the tachyonic rotating produces centrifugal acceleration

$$\frac{d\bar{u}}{dt} = -\frac{c^2}{R} . \tag{14}$$

It is independent of tachyonic velocity $\bar{u}$, only inversely proportional to radius R.

(13) and (14) are dual formulas, which have the same form. It is unique and perfect. From (13) it follows the tardyonic centrifugal force

$$F = \frac{Mu^2}{R} , \tag{15}$$

where M is the inertial mass.

From (14) it follows the tachyonic centripetal force, that is gravity

$$\bar{F} = -\frac{mc^2}{R} , \tag{16}$$

where m is the gravitational mass converted into by tachyonic mass $\bar{m}$ which is unobservable but m is observable.

Whether $u = 0$ or $u \neq 0$, all matter produces gravity. (15) and (16) are dual formulas, which have the same form. (16) is a new gravitational formula called an equation that changed the universe. This simple thought made a deep impression on me. It impelled me toward a theory of gravitation. It has simplicity, elegance and mathematical beauty. It is the foundations of gravitational theory and cosmology. In the universe there are two main forces: the tardyonic centrifugal force (15) and tachyonic centripetal force (16) which make structure formation of the universe.

Now we study the freely falling body. Tachyonic mass $\bar{m}$ can be converted into tardyonic mass m, which acts on the freely falling body and produces the gravitational force

$$\bar{F} = -\frac{mc^2}{R} , \tag{17}$$

where R is the Earth radius.

We have the equation of motion

$$\frac{mc^2}{R} = Mg , \tag{18}$$

where g is gravitational acceleration, M is mass of freely falling body.

From (18) it follows the gravitational coefficient

$$\eta = \frac{m}{M} = \frac{Rg}{c^2} = 6.9 \times 10^{-10} . \tag{19}$$

Eötvös(1922) experiment $\eta \sim 5 \cdot 10^{-9}$ and Dicke experiment $\eta \sim 10^{-11}$ [5]. Since the gravitational mass m can be transformed into the rest mass in freely falling body, we define Einstein's gravitational mass $M_g = M_i + m$ and inertial mass $M_i = M$ [6]. It follows

$$M_g > M_i. \tag{20}$$

Therefore it shows that the principle of equivalence is nonexistent.

The expansion theory of the universe without dark matter and dark energy

The Big Bang threw all the matter in the universe outwards. Both Newton's and Einstein's theories of gravity predict that the expansion must be slowing down to some degree: the mutual gravitational attraction of all the matter in all the galaxies should be pulling them inwards. But measurements of distant supernovae show just the opposite[7] . All the matter in the universe appears to be accelerating outwards. Its speed is picking up. There is no agreement yet about how to explain these mysterious observations. Now we explain our accelerating universe.

Using (16) we study the expansion theory of the Universe. Figure 2 shows a expansion model of the Universe. The rotation ω_1 of body A emits tachyonic flow, which forms the tachyonic field. Tachyonic mass $\overline{m}$ acts on body B, which produces its rotation ω_2, revolution u and gravitational force

$$\overline{F}_1 = -\frac{mc^2}{R} , \tag{21}$$

where R denotes the distance between body A and body B, m is gravitational mass converted into by tachyonic mass $\overline{m}$ which is unobservable but m is observable.

The revolution of the body B around body A produces the centrifugal force
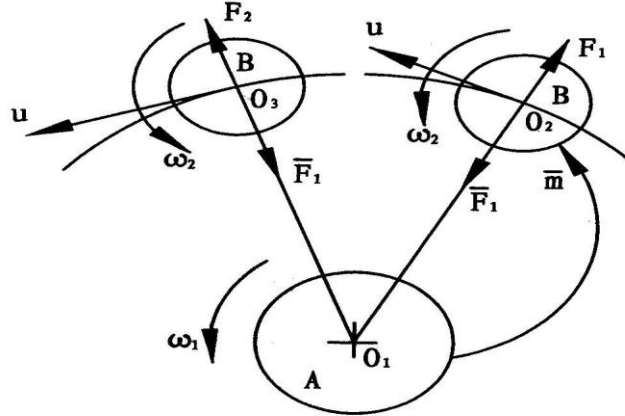
$$F_1 = \frac{M_B u^2}{R} , \tag{22}$$

Fig. 2. A expansion model of the Universe

where M_B is the inertial mass of body B, u is the orbital velocity of body B.

At the O_2 point we assume

$$F_1 + \overline{F}_1 = 0 . \tag{23}$$

From (23) it follows that the coexistence of the gravitational force and centrifugal force.

From (21)-(23) it follows the gravitational coefficient

$$\eta = \frac{m}{M_B} = \left(\frac{u}{c}\right)^2 . \qquad (24)$$

At the O_3 point the tachyonic mass $\overline{m}$ can be converted into the rest mass m in body B, it follows

$$F_2 = \frac{M_B u^2}{R} + \frac{mu^2}{R} . \qquad (25)$$

Since $F_2 + \overline{F}_1 > 0$, centrifugal force F_2 is greater than gravitational force $\overline{F}_1$, then the body B expands outwards and its mass increases. This is a expansion mechanism of the Universe. From (21)-(23) we have

$$F_2 + \overline{F}_1 = \frac{mu^2}{R} = M_B g_e \qquad . \qquad (26)$$

From (26) we obtain the expansion acceleration

$$g_e = mu^2 \Big/ M_B R \qquad . \qquad (27)$$

Substuting (24) in (27) we obtain

$$g_e = \frac{u^4}{C^2 R} \qquad . \qquad (28)$$

If body A is the Earth, then body B is the Moon; if body A is the Sun, then body B is the Earth; $\cdots$. It can explain our accelerating universe. In this model universe there are no dark matter and no dark energy. This simple thought made a deep impression on me. It impelled me toward a expansion theory of the universe without dark matter and dark energy.

If the body A is the Sun and body B is the planet. We calculate the gravitational coefficients η as shown in table 1.

Table 1:Values of the gravitational coefficients η

Planet	u (km/sec)	$\eta(10^{-10})$
Mercury	47.89	255.2
Venus	35.03	136.5
Earth	29.79	98.7
Mars	24.13	64.8
Jupiter	13.06	19.0
Saturn	9.64	10.3
Uranus	6.81	5.2
Neptune	5.43	3.3
Pluto	4.74	2.5

Since gravitational mass m can be transformed into the rest mass in body B, we define Einstein's gravitational mass $M_g = M_i + m$ and inertial mass $M_i = M_B$ [6]..

It follows

$$M_g > M_i. \qquad\qquad (29)$$

Therefore it shows that the principle of equivalence in the Solar system is nonexistent. Of all the

principles at work in gravitation, none is more central than the principles of equivalence[5], which could be wrong.

The tachyonic mass $\overline{m}$ can be converted into electrons and positrons which are the basic building-blocks of elementary particles [8,9]. In this universe there are no Higgs particles. They have not been produced at the Large Hadron Collider and other particle accelerators.

From (21) it follows Newtonian gravitational formula. The m is proportional to M_A, which denotes inertial mass of body A, in (24) m is proportional to M_B, is inversely proportional to the distance R between body A and body B. It follows

$$m = k \frac{M_A M_B}{R},$$ (30)

where k is a constant.

Substituting (30) into (21) it follows Newtonian gravitational formula[3,4]

$$\overline{F}_1 = -G \frac{M_A M_B}{R^2},$$ (31)

where $G = kc^2$ is a gravitational constant.

We have Einstein's gravitational mass

$$M_g = M_i + m = M_i(1 + \eta).$$ (32)

Substituting (32) into (31) it follows Newtonian generalized gravitational formula

$$\overline{F}_1 = -G \frac{M_A(1 + \eta_A)M_B(1 + \eta_B)}{R^2},$$ (33)

where η_A and η_B denote gravitational coefficients of body A and body B separately. Assume ρ_A and ρ_B denote the densities of body A and body B separately. In the same way from (21) it follows unified formula of the gravitational and strong forces [4]

$$\overline{F}_1 = -G_0 \frac{\rho_A M_A(1 + \eta_A)\rho_B M_B(1 + \eta_B)}{R^2}$$ (34)

where $G_0 = 5.2 \times 10^{-10} \, \mathrm{cm}^9/\mathrm{g}^3 \cdot \sec^2$ is a new gravitational constant.

In the nucleus exists the strong interactions. It follows[4]

$$\frac{\text{Strong interaction}}{\text{Gravitational interaction}} = \frac{G_s}{G_g} = 10^{38}$$ (35)

where $G_g = 6.7 \times 10^{-8} \, \mathrm{cm}^3/\mathrm{g} \cdot \sec^2$ and $G_s = 6.7 \times 10^{30} \, \mathrm{cm}^3/\mathrm{g} \cdot \sec^2$

In the nucleus we assume $\rho_A = \rho_B = \rho$. From (34) it follows

$$G_s = G_0 \rho^2$$ (36)

From (36) it follows the formula of the particle radii

$$r = 1.55[m(\mathrm{Gev})]^{1/3} \, \mathrm{jn},$$ (37)

where 1 jn=10^{-15} cm and m (Gev) is the mass of the particles.

From (37) it follows that the proton and neutron radii are 1.5 jn[4,10].Pohl et al measure the proton diameter 3 jn[11].

We have the formula of the nuclear radii[12]

$$r = 1.2(A)^{1/3} \text{ fm,} \tag{38}$$

where 1 fm=10^{-13} cm and A is its mass number.

It shows that (37) and (38) have the same form. The particle radii $r < 5$ jn and the nuclear radii $r < 7$ fm.

Similar to equation (10) we define the tachyonic momentum of a string length $\bar{x}_0$ [1,4].

$$\overline{P}_0 = \lim_{\substack{m_0 \to 0 \\ \bar{u} \to \infty}} m_0 \bar{u} = const, \tag{39}$$

where m_0 is tachyonic string rest mass.

Since $\bar{u} \to \infty$ and $t = 0$, tachyonic string has no rest mass and no rest time, it shows that tachyon is unobservable, that gravity is action-at-a-distance and gravitational wave is unobservable. If quantum teleportation, quantum computation and quantum information are the tachyonic motion[13], then they are unobservable.

A new universe model

From above we suggest a new universe model. The universe has no beginning and no end. The universe is infinite, but it has a centre consisting of the tachyonic matter, which dominates motion of the entire universe. Therefore the universe is stable…..In the sun there is a centre consisting of the tachyonic matter, which dominates motion of the sun system. In the earth there is a centre consisting of the tachyonic matter, which dominates motion of the earth and the moon. In the moon there is a centre consisting of the tachyonic matter, which dominates motion of the moon. In atomic nucleus there is a centre consisting of the tachyonic matter, which dominates motion of the nucleus. Therefore atomic nuclei are stable.

Conclusion

Special relativity is the tardyonic theory. Einstein pointed out that velocities greater than that of light have –as in our previous results-no possibility of existence [14], which could be wrong. But gravitation is the tachyonic theory and an action-at-a-distance.

What is gravity? Newton wrote, "I have not been able to discover the cause of those properties of gravity from phenomena, and I frame no hypotheses …". Einstein's theory of general relativity answered Newton's question: mass causes space-time curvature which is wrong. Gravity is the tachyonic centripetal force.

Where did we come from? Where are we going ? What makes up the universe? These questions have occupied mankind for thousands of years. Over the course of history, our view of the world has changed. Theologians and philosophers, physicists and astronomers have given us very different answers. Where did we come from ? We answer this questions this way $\bar{m} \to m$, tachyons $\to$ tardyons, that is gravitons can be converted into the electrons and positrons which are the basic building-blocks of particles. In this model Universe there are no

quarks and no Higgs particles. Where are we going? We answer this question this way $m \rightarrow \bar{m}$, that is the tardyons produce tachyons. The tardyons and tachyons make up the Universe.

Jiang found a gravitational formula[3] : $\bar{F} = -\bar{m}c^2 \big/ R$, where $\bar{m}$ is the tachyonic mass. In 2004 Jiang studied the Universe expansion and found $\bar{F} = -mc^2 \big/ R$, where m is gravitational mass converted into by tachyonic mass $\bar{m}$.

The author thanks Yong-Shi Wu to put forward this research.

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