POSITIVE ENERGY SOLUTION TO EXOTIC ENERGY REQUIREMENT OF ANY GENERIC WARP DRIVE METRIC

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Abstract: In this article I look at some of the math behind replacing the exotic energy of any warp metric with an inflation field with a focus on a simple generic solution to the frame switch in the recent CERN superluminal neutrino detection to that of a Newtonian metric.

A time-like vector field coupled to Curvature does break the Lorentz invariance as well as the spatial fields, since picking up the time coordinate introduces a preferred frame and the expansion takes our nearly flat vacuum state and essentially forms a Newtonian frame out of it. Here we allow also a potential for the vector field, and do not couple the kinetic term of the field but add an interaction with the Ricci scalar R and the field Aμ,

$$S = \int d^4x\sqrt{-g}\left[\frac{1}{2}\left(\frac{1}{8\pi G} + \omega(A^2)\right)R - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - V(A^2) + L_m\right],$$

There are three things this satisfies:
1) The Lagrangian density is a four-scalar.
2) The resulting theory is metric .
3) There are no higher than second derivatives in the resulting field equations.

In general we would allow also a coupling of the form

$$A^\mu A^\nu R_{\mu\nu}\|.$$

The contribution of the coupling term ω to the field equations can be presented as an effective energy-momentum tensor,

$$G_{\mu\nu} = 8\pi G\left(T^m_{\mu\nu} + T^A_{\mu\nu} + T^\omega_{\mu\nu}\right),$$

where

$$T^A_{\mu\nu}$$

is given by

$$T_{00}^{A} = \frac{1}{2}\sum_{i=1}^{3}\frac{1}{a_i^2}\dot{A}_i^2 + V(A^2) + 2V'(A^2)\phi^2,$$

And

$$T_{\mu\nu}^{\omega}$$

Reads

$$T_{\mu\nu}^{\omega} = -\omega G_{\mu\nu} + \left(\nabla_\mu \nabla_\nu - g_{\mu\nu}\Box\right)\omega - \omega' A_\mu A_\nu R.$$

If we introduce

$$A_\mu = (\phi, a_i\Lambda_i)$$

For the field we find

$$\begin{aligned} T_{00}^{\omega} =& -6H\left(\mathbf{\Lambda}\cdot\dot{\mathbf{\Lambda}} - \dot{\phi}\phi\right)\omega' - \frac{1}{2}\left(9H^2 - \mathbf{H}\cdot\mathbf{H}\right)\omega - \phi^2\omega' R, \\ T_{0i}^{\omega} =& -\phi A_i \omega' R, \end{aligned}$$

If we stipulate that the free-field energies are positive for both the metric and the vector we impose further constraints on the form of ω, which equals

$$A^\mu A^\nu R_{\mu\nu} \rightarrow (\nabla_\alpha A^\alpha)^2 - \nabla_\alpha A^\beta \nabla_\beta A^\alpha,$$

after a partial integration of the action:

$$\begin{aligned} T_{ij}^{\omega} =& 2a_i a_j\left[\left(\mathbf{\Lambda}\cdot\dot{\mathbf{\Lambda}} - \dot{\phi}\phi\right)(3H - H_i) + \dot{\mathbf{\Lambda}}\cdot\dot{\mathbf{\Lambda}} + \ddot{\mathbf{\Lambda}}\cdot\mathbf{\Lambda} - \dot{\phi}^2 - \ddot{\phi}\phi\right]\omega'\delta_{ij} \\ & - a_i a_j\left(3\dot{H} + \frac{9}{2}H^2 + \frac{1}{2}\mathbf{H}\cdot\mathbf{H} - \dot{H}_i - 3HH_i\right)\delta_{ij}\omega \\ & + 4a^2\left(\mathbf{\Lambda}\cdot\dot{\mathbf{\Lambda}}\right)\delta_{ij}\omega'' - A_i A_j \omega' R, \end{aligned}$$

Where

$$R = 9H^2 + 6\dot{H} + \mathbf{H}\cdot\mathbf{H},$$

The equation of motion for the time component of the field is

$$\phi\left(2V'(A^2) - \omega(A^2)R\right) = 0.$$

The additional condition

$G_{ij} = 0$

Then yields

$$-\dot{A}_i\dot{A}_j + \left(2V'(A^2) - \omega(A^2)R\right) A_iA_j = 0.$$

Going back to

$$T^A_{00} = \frac{1}{2}\sum_{i=1}^{3}\frac{1}{a_i^2}\dot{A}_i^2 + V(A^2) + 2V'(A^2)\phi^2,$$
$$T^A_{0i} = 2V'(A^2)\phi A_i,$$
$$T^A_{ij} = -\dot{A}_i\dot{A}_j + 2V'(A^2)A_iA_j + a_ia_j\left(\frac{1}{2}\sum_{k=1}^{3}\frac{1}{a_k^2}\dot{A}_k^2 - V(A^2)\right)\delta_{ij}.$$

The energy density of our tensor driven inflation field is

$$\rho_\phi = 6H\phi\dot{\phi} - 3H^2\omega + V,$$

Only if our field is not massive vector field and the pressure of our field is

$$p_\phi = -V - 2\left(2H\dot{\phi}\phi - \dot{\phi}^2 - \ddot{\phi}\dot{\phi}\right)\omega' + 4\left(\dot{\phi}\phi\right)^2\omega'' + \left(2\frac{\ddot{a}}{a} + H^2\right)\omega.$$

One notes that

$$\rho_\phi + p_\phi$$

Satisfies the relation

$$\rho_\phi + p_\phi = -\dot{\rho}_\phi/(3H),$$

Which fits observationally with our current universe and the vector field only changes the geometry. We then have the conservation law of

$$\rho_\phi + p_\phi = \ddot{\omega} + H^3\left(\frac{\omega}{H^2}\right)^{\bullet}.$$

Again we note that ϖ is constrained to be only positive energy.

My point in all this there are ways to replace the exotic energy of any warp metric field with a positive energy solution involving inflation. In the example I choose one that has a time-like coupling because it also fits with the frame switch to a Newtonian metric I think could have been involved in the recent CERN experiments. But this also generically either with a time-like coupling or a space-like coupled vector field for all versions of warp drive as a replacement for exotic energy.