Explanation of Apparent Superluminal Neutrino Velocity in the CERN-OPERA Experiment

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Abstract

The CERN-OPERA neutrino experiment at the Gran Sasso Laboratory obtained a measurement, v_n, of the muon neutrino velocity with respect to the speed of light, c, of $(v_n-c)/c = (2.48 \pm 0.28 \text{ (stat.)} \pm 0.30 \text{ (sys.)}) \times 10^{-5}$. The neutrino flight path from CERN to OPERA was established using distances and timings based on "round-trip" light speed signals. These are incommensurate with the reference frame dependent "one-way" flight times of neutrinos over the same path. We perform a Lorentz transformation to demonstrate the frame-dependence of the result. We conclude that an Earth system (ES) reference frame defined by a timing system which assumes isotropic light speed, such as the UTC, is not able to support experiments requiring accurate one-way light speed measurement. We hypothesise that $v_n = c$ and consider the 2.7K CMB as a possible candidate for the isotropic frame of reference where round-trip and one-way light speeds are equal. On this basis we find that the CERN-OPERA experiment would be expected to measure deviations in neutrino arrival times compared to the expected light speed transmission of up to ±~2ns/km of neutrino flight path, but usually of less magnitude and with a bias towards early arrival. Only the N-S component (relative to the Earth's axis) of the motion of the neutrino flight path relative to the isotropic frame would be statistically significant in the CERN-OPERA experiment. Assuming no bias in the mean of the other components of the experiment's motion against the isotropic frame in the neutrino timing, because of the Earth's rotation and orbit, we find a mean early neutrino arrival time of ~113ns would be expected were the CMB the isotropic frame. That is, the potential error is of the same order as the early arrival time of the neutrinos of (60.7 ± 6.9 (stat.) ± 7.4 (sys.)) ns, suggesting further analysis of possible sources of deviation from our theoretical estimate may be worthwhile. We propose further statistical methods to test the hypotheses that $v_n = c$ and that the CMB represents the isotropic frame, using the existing OPERA neutrino velocity measurement data.

Introduction

This is a comment on the apparent superluminal neutrino speed measurement in the CERN-OPERA experiment, [1].

We suggest that the early arrival time of neutrinos, sent from CERN, at the OPERA detector at Gran Sasso arises from the assumptions that (a) the Earth system (ES) as defined by Coordinated Universal Time (UTC) is an isotropic reference frame where all "one-way" light speeds are equal to c; and (b) the experimental methodology, which mixes reference frame-dependent empirical one-way flight timings of neutrinos and with frame-independent "round-trip" time delays established by the GPS "common view" method and by timing transmission through fibre optic cables and experimental equipment. These are different problems to those suggested in [2] and [3].

The CERN-OPERA neutrino experiment timed the flight of neutrinos between two specific points. The simultaneous arrival of the neutrinos and a timing signal based on light travelling over the same path (i.e. not affected by the "relativity of simultaneity") would be expected to be seen by any observer in motion relative to the ES, though the flight time taken would be different. We thus perform a Lorentz transformation of the timings to test the frame-independence of the methodology.

Analysis

The experiment (see [1]), as shown schematically in Fig 1, compared the neutrino flight time from A to C (t_3) with anticipated light speed flight times between GPS synchronised clocks at A and B ($t_1 = x_1/c$), and from the clock at B to the neutrino detector at C ($t_2 = x_2/c$), with the null hypothesis that:

$$t_3 = t_1 + t_2 \tag{1}$$

This hypothesis would be expected to be true for any observer.

We therefore expect (1) also to be true under any Lorentz transformation.

Consider first t_1. As described in [4], GPS "common view" clock synchronisation is established and maintained between A and B by the periodic receipt by both clocks of GPS signals with the same timestamp. Viewed by an observer moving at velocity v_E with respect to the ES, signals seen as simultaneous within the ES would no longer be seen as simultaneous. However, a timing signal is being transmitted, and the time at the GPS satellite, A and B, defined as simultaneous within the ES, would not be seen as simultaneous by an observer in motion with respect to the ES. We demonstrate this by means of a Lorentz transformation of the GPS common-view synchronisation process.

The times at A and B, t_A and t_B, are determined during the "common-view" synchronisation process with reference to the time at the GPS satellite, t_{SAT}. We define x_A and x_B as the distance between the GPS satellite and the clocks at A and B, respectively, as measured in the ES reference frame.

In practice, x_A/c and x_B/c are presumed from the satellite's position data and subtracted, such that the clocks at A and B are assumed to show the same time. This process includes an implicit assumption that time at A is the same as at the satellite, as defined by UTC. That is:

$$t_A = t_{ASIG} - t_{A-SAT} - x_A/c \tag{2}$$

where:

- t_{ASIG} is the satellite time, t_{SAT}, received at A in a GPS common-view signal, i.e. t_{SAT} plus the signal transmission time – the signal transmission time will not be the same as seen in the ES in the reference frame of an observer in motion relative to the ES. Thus:

$$t_{ASIG} = (t_{SAT} + x_A/c) \tag{3}$$

- t_{A-SAT} is the difference between time at A and in the GPS satellite, defined as the same under UTC (so $t_{A-SAT}= 0$) in the ES reference frame, but *not* the same when viewed by an observer moving relative to the ES, so necessary in the calculation.

Under Special Relativity (SR), events that we see simultaneously at A and B (i.e. spatially separated – this *is* the "relativity of simultaneity") are not generally simultaneous when viewed by an observer in motion relative to the ES. We therefore test whether the process of transmission of the common view signals would be seen to result in time synchronisation between A and the GPS satellite when viewed by an observer in motion at v_E relative to the ES, by applying the Lorentz boost function to (2), (3) and to t_{A-SAT}, with $\gamma = 1/\sqrt{(1 - v_E^2/c^2)}$, such that:

$$t'_A = t'_{ASIG} - t'_{A-SAT} - x'_A/c \tag{4}$$

$$t'_{ASIG} = \gamma(t_{SAT} + x_A/c + x_A v_E/c^2) \tag{5}$$

because t_{ASIG} represents a communication over distance $x_{A.}$

$$t'_{A-SAT} = \gamma(t_{A-SAT} + x_A v_E/c^2) \tag{6}$$

because the clocks at A and in the GPS satellite are distance x_A apart.

Combining (4), (5) and (6):

$$\begin{aligned} t'_A &= \gamma(t_{SAT} + x_A/c + x_A v_E/c^2) - \gamma(t_{A-SAT} + x_A v_E/c^2) - \gamma x_A/c \\ &= \gamma t_{ASIG} - \gamma(t_{A-SAT}) - \gamma x_A/c = \gamma t_A \end{aligned} \tag{7}$$

Similarly:

$$t'_B = \gamma t_B \tag{8}$$

Thus:

$$t'_1 = \gamma t_1 \tag{9}$$

Only the time dilation element γ, and not the full Lorentz transformation applies in the case of $t_{1.}$

Fig 1 includes a simplified representation of the optical fibre timing which was used several times to determine delays in the transmission of the clock timing signals (and thus of the GPS "common view" timing signal) at both CERN and OPERA, including over cables of lengths 2.1km at CERN [5] and 8.3km at OPERA [1]. The total neutrino flight distance (BC or x_2 in Fig 1) not covered by the GPS timing signal is not specified in [1], but is implied to be of the order of 10km.

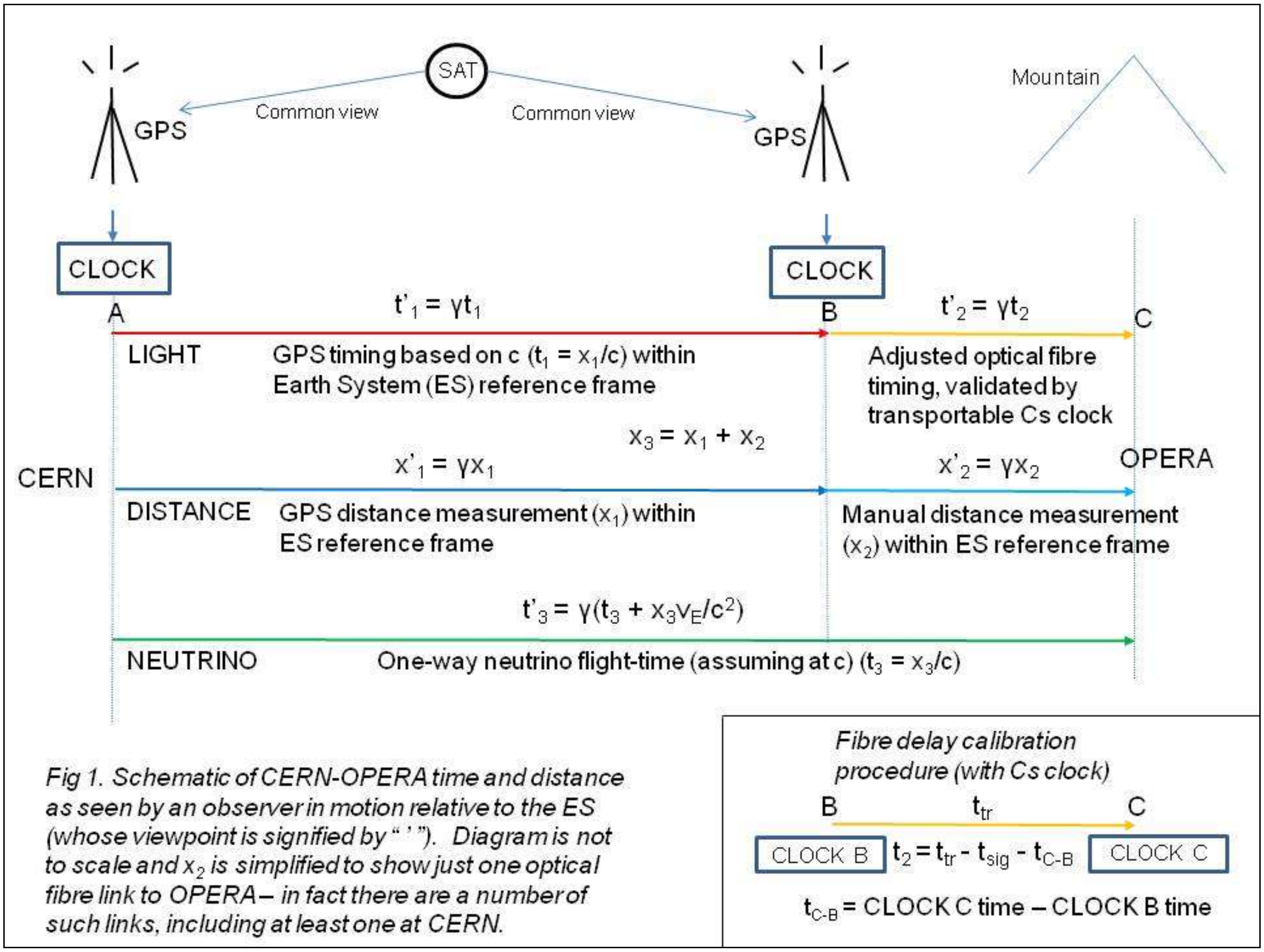

Fig 1. Schematic of CERN-OPERA time and distance as seen by an observer in motion relative to the ES (whose viewpoint is signified by " ' "). Diagram is not to scale and x_2 is simplified to show just one optical fibre link to OPERA– in fact there are a number of such links, including at least one at CERN.

The expected flight time, t_2, at c, over the distance x_2, representing the neutrino flight path other than between the GPS synchronised clocks at CERN and Gran Sasso (i.e. $x_2 = x_3 - x_1$), could not be determined empirically with reference to light speed transmission such as GPS signals, since the OPERA detector is inside the Gran Sasso mountain. Instead, it was determined by two separate procedures yielding the same result. These procedures were a two-way fibre delay calibration procedure [5], and the use of a transportable clock ([1], p.13), to synchronise the time between the two ends of each of a number of optical fibre cables (simplified to one cable here) prior to timing signal transmission. We consider the simpler case, the use of a transportable clock (inset, Fig 1).

The aim of the procedure is to establish the one-way time of transmission of light through each optical fibre link, corrected for signal delays, but this is very difficult to achieve, as discussed in [6] and [7], and has, in fact, not been done here. What has actually been established is, in fact, half the round-trip time. The time delay required to compare against neutrino flight time, t_2, is described by:

$$t_2 = t_{tr} - t_{C\text{-}B} - t_{sig} \qquad (10)$$

where:

- t_2 is the expected neutrino flight time, assuming neutrinos travel at c, over the distance from B to C (that is, $t_2 = x_2/c$);

- t_{tr} is the transmission time of the timing signal from the GPS clock at B to the OPERA neutrino detector at C, as measured on a Cs clock transported from B to C;
- $t_{C\text{-}B}$ is the difference between the time bases of the clocks at each end of the cable. As the clocks were synchronised, $t_{C\text{-}B}$ is 0 in the ES. As will be seen, due to the effects of SR, $t_{C\text{-}B}$ is not generally 0 when viewed by an observer in motion relative to the ES, i.e. following a Lorentz transformation, so must be included in the analysis;
- t_{sig} is a signalling delay included in the measurement, t_{tr}, compared to direct transmission at c from B to C. It includes all delays in the equipment to convert the timing signal to and from an optical fibre signal; to carry out any other processing of the signal; any delay in the fibre optic cable itself compared to signal transmission at c; and any delays due to the cable not being a direct straight path from B to C. t_{sig} is a pure time delay.

In the experiment, neutrino flight time to the OPERA detector is calculated with reference to timing signals originating at the GPS clock at B, which is itself synchronised with that at A. The optical fibre transmission time, t_{tr}, is determined in order to perform a subtraction from the signal arrival time at C, in order to derive the total expected flight time over A to C at c (that is, x_3/c). The steps that are taken to establish the neutrino flight velocity from CERN (A) to the OPERA detector at Gran Sasso (C) can be summarised as:

1. Accurate measurement of the distance from A to C, $x_3 = x_1 + x_2$.
2. Production of a timing signal at B, which is synchronised with A by GPS common-view and therefore assumed synchronous with the neutrino origin time at A. The expected neutrino flight time from A to B at c would be delayed by $t_1 = x_1/c$ relative to this timing signal. t_1 is reintroduced in step 5.
3. Transmission of the timing signal from B to the OPERA detector at C via optical fibre cables.
4. Subtraction of t_{tr} from the timing signal at the OPERA detector, giving a signal at C assumed to be synchronised with B and therefore A. The expected neutrino flight time from B to C at c would be delayed by $t_2 = x_2/c$ relative to this timing signal. t_2 is reintroduced in step 5.
5. Calculation of the neutrino velocity based on the delay in its arrival time compared to the signal. On average, this arrival time was ~60ns earlier than would have been expected for neutrinos travelling at c, that is if $t_3 = t_1 + t_2 = x_3/c$.

Accurate measurement of the neutrino velocity is therefore critically dependent on correct calibration of the timing signal.

It is arbitrary that $t_{C\text{-}B} = 0$ since we could have adopted a different timing convention such as one based on signals transmitted at c, for example, such that $t_{C\text{-}B} = x_2/c$, where x_2 is the distance from B to C. Simply adopting a convention that $t_{C\text{-}B} = 0$ does not enable a one-way light speed measurement [6]. Here, we provide a proof of the assertion in [6].

We consider a Lorentz transformation of the calibration process itself, to the reference frame of an observer moving at v_E relative to the ES:

$$t'_{tr} = \gamma(t_{tr} + x_2 v_E/c^2) \quad (11)$$

because the ends of the cable from B to C are distance x_2 apart (see Fig 1);

$$t'_{C\text{-}B} = \gamma(t_{C\text{-}B} + x_2 v_E/c^2) \quad (12)$$

because the clocks at B and C are distance x_2 apart (see Fig 1);

$$t'_{sig} = \gamma t_{sig} \quad (13)$$

because t_{sig} is, by definition, a pure time delay relative to light speed transmission - the distance element of the timing signal transmission from B to C is included in (11).

Combining (10), (11), (12) and (13):

$$t_2' = t_{tr}' - t'_{C\text{-}B} - t'_{sig} = \gamma(t_{tr} - t_{C\text{-}B} - t_{sig}) = \gamma t_2 \quad (14)$$

Exactly the same logic as in (11) – (14), and as for the GPS common view signal, (2) – (9), applies when the timing signal is transmitted during the actual experiment, rather than as part of calibration, to determine the arrival time of a neutrino detected at C. In this case, the information about the time at B (such that $t_{C\text{-}B} = 0$) is included in the timing signal itself, rather than established by a transportable clock.

Only the time dilation, γ, and not the full Lorentz transformation applies in the case of t_2. The round-trip light-speed timing of t_2 is frame independent and not compatible with the one-way empirical timings of the neutrino from A to C.

Finally, we also apply the Lorentz boost function to obtain the observed neutrino flight time, t'_3, assuming the null hypothesis that the neutrino travels at c:

$$t'_3 = \gamma(t_3 + x_3 v_E/c^2) \quad (15)$$

where x_3 is the entire neutrino flight distance from CERN to the OPERA detector.

The full Lorentz boost transformation applies to the neutrino flight time, t_3.

We now carry out a full Lorentz transformation of the experiment to determine whether a successful test of the null hypothesis (1) would be seen by an observer moving relative to the ES. Combining (1), (9), (14) and (15), the null hypothesis would be true if:

$$\gamma(t_3 + x_3 v_E/c^2) = \gamma t_1 + \gamma t_2 \quad (16)$$

that is, since $x_3 = x_1 + x_2$, when:

$$t_3 = t_1 + t_2 - x_3 v_E/c^2 \quad (17)$$

That is, the null hypothesis that the neutrino flight time will be measured at c in the OPERA experiment [1] is not, in general, true, that is, true for an observer in motion relative to the experiment. *Thus, the experiment is not a valid test that neutrinos travel at c.*

The null hypothesis (1) in fact assumes that the experiment was conducted in what has been termed the "isotropic system" [7] (or isotropic frame of reference). The failure to confirm (1) empirically should be no surprise. As has been pointed out:

> "...we do not know whether the earth is the isotropic system (most probably it is not)...". ([7], p.6).

An alternative interpretation of the CERN-OPERA neutrino velocity measurement experiment is that $v_n = c$ and the ES is not isotropic in the sense of [7]. Result (17) therefore allows us to assume that v_n = c and instead calculate v_E, the velocity of an observer in the isotropic frame of reference who *would* observe the early neutrino flight time determined by the CERN-OPERA experiment.

The OPERA experiment found an early neutrino arrival time of ~60ns. Thus, from (17):

$$\sim 60\text{ns} = x_3 v_E / c^2 \qquad (18)$$

which, with x_3 being 730km, yields $v_E \approx$ 7.4km/s, that is, suggests that the OPERA experiment neutrinos would have been observed to be travelling at the speed of light, c, by an observer moving at ~7.4km/s towards the ES in the reverse (given the sign convention used here) of the flight path of the neutrinos (the other axes of motion are not known from this calculation).

Discussion

The analysis shows that the hypothesis that neutrinos travel at c has not been falsified by the CERN-OPERA experiment [1].

The analysis also shows that the use of the UTC or any system of "universal time" invariant over a volume of space (in this case the ES) will not, in the general case, create a consistent ES reference frame where the second principle of SR applies, that is, that of the Constancy of the Speed of Light. The problem is that not only may we observe *from* the ES, we may also *be* observed - and the Lorentz transforms create a second-order term, xv/c^2, in the description of time and velocity in the *observed* system. Implementation requires a system of time that adjusts for distance depending on the motion of the Earth, not just the Sagnac effect (see [8] and [9] for discussion of the Sagnac effect).

Clearly, though, there must be a special case where v = 0, that is, an "isotropic", "stationary", "rest" or "preferred" frame of reference.

The question arises as to how to identify the isotropic frame of reference against which all clocks can be adjusted.

Once such an isotropic frame is identified it would be perfectly feasible to implement a modified UTC consistent with the second principle of SR, with a time adjustment (or adjustments) additional to the Sagnac effect to correct for aspects of the Earth's motion other than its rotation, for experimental and other applications. The ability to determine one-way light speeds more accurately may allow a better determination of distance from observed time difference and vice versa. Note that applications such as the GPS are not affected by a failure to take account of the motion of the ES in their clocking because the errors generally cancel out, for example, in the determination of position and in the case of common view time synchronisation.

We hypothesise that the cosmic microwave background (CMB) provides the isotropic frame of reference.

Measurements of the red-shift/blue-shift dipole of the CMB suggest a motion relative to the CMB of the Earth (and the rest of the Solar System) of ~300km/s (~0.001c). Other components of the Earth's motion, such as its rotation (<0.5km/s even at the Equator) and the Earth's orbit about the Sun (~30km/s), are less significant in terms of velocity but affect the orientation of the planet in relation to the CMB over relatively short timescales.

We can carry out a Lorentz transformation similar to (15) to determine the time delay expected given the Earth's velocity, v_C, relative to the CMB. Given v_C of ~0.001c/, the difference from unity of $\gamma = 1/\sqrt{(1 - v_c^2/c^2)}$ is insignificant. To a reasonable approximation, therefore, the relationship between time measurements in an experiment in the ES, T_{EXP}, to those in the isotropic frame of reference of the CMB ("absolute time", T_{ABS}) is given by:

$$T_{EXP} = T_{ABS} \pm xv_C/c^2 \qquad (19)$$

where x is distance as measured in the ES.

(19) yields:

$$3.3\text{ns/km} \geq (T_{EXP} - T_{ABS}) \geq -3.3\text{ns/km} \qquad (20)$$

that is, for $v_C \approx 0.001c$, we can say that, in general, it will be necessary to adjust Earthbound clocks by up to ~3.3ns/km, in order to convert individual measurements of the one-way speed of light to c or vice versa.

The value of 3.3ns/km is not possible in the CERN-OPERA experiment because the neutrino transmission path can never be directly aligned with the Earth's motion relative to the CMB. The maximum deviation in arrival time from c would be expected to be ~±2ns/km, but usually much less. This implies a small proportion of neutrinos could arrive as much as ~1.5μs earlier or later than expected. It is not clear to us from [1], Fig 12, whether such a possibility is excluded by the statistical analysis of the data. Fig 12 shows a range of ~11μs in neutrino arrival times from pulses of duration 10.5μs.

The sign and magnitude (up to the maximum value) of the actual relationship at any moment will depend on the orientation of the specific direction of the motion being measured (not of the whole Earth) in relation to the CMB. It is feasible to establish $(T_{EXP} - T_{ABS})$ for a given experimental configuration to accurately measure particle velocities relative to c, although the adjustment $(T_{EXP} - T_{ABS})$ will continually vary with time because of the Earth's rotation and orbit about the Sun.

The CMB is seen as blue-shifted towards the south, however, so we further hypothesise that the ~60ns early neutrino arrival time in the CERN-OPERA experiment represents the N-S component of the earth's motion in relation to the CMB rest frame.

The expected mean neutrino early arrival time is given by:

$$t_{NS} = x_{NS}v_{C(NS)}/c^2 \qquad (21)$$

where

- t_{NS} is the north to south component of the neutrino early arrival time, which is also the mean time of early arrival assuming no other systematic bias in the neutrino timing data;
- x_{NS} is the north to south component of the neutrino trajectory;
- $v_{C(NS)}$ is the invariant north to south component of the Earth's motion relative to the CMB.

The longitudinal distance from CERN (46° 14'N) to Gran Sasso (42° 28'N) is ~400km.

The latitude of the experiment is around 45°N, so the distance parallel to the Earth's axis, $x_{(NS)} \approx 400/\sqrt{2} \approx 285$km.

The CMB dipole is at 6.93deg to the south [11], so the trigonometry suggests $v_{c(NS)} \approx 0.12 v_c$.

Thus, the expectation would be that:

$$t_{NS} \approx 285 \times 0.12 \times 3.3\text{ns} \approx 113\text{ns} \quad (22)$$

Result (22) is of the same order as result (18), giving weight to the hypothesis that the motion of the experiment relative to the CMB is at least part of the explanation for the observed early neutrino arrival time in the CERN-OPERA experiment.

Result (22) is only an approximate calculation and may require correction for factors that have not been fully or accurately taken into account.

And, of course, the possibility should not be excluded that there are additional unidentified contributory factors to the early neutrino arrival time measurement or that known uncertainties will be re-quantified, such as those highlighted in [2].

The hypothesis, that neutrinos travel at c and that this can be verified by adjusting the frame of reference to that of the CMB, can also be tested by carrying out statistical tests of systematic dependences additional to those discussed in [1], such as comparisons of subsamples of events taken during:

- the day and night in each of the three seasons represented (simplest);
- the day in spring vs. during the night in autumn and vice versa;
- days and times when the Earth is calculated to be similarly oriented vs. when its orientation is approximately opposite.

A significant difference between any pair of such bins would lend support to the hypothesis proposed in this paper.

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