Special relativity is theorem based

Hamid Bassera[1], Freelance.
Paris, France

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Abstract

In this work I show that special relativity is mathematical theorem based on just Chasles relation in Euclidian space. So special relativity is just a direct consequence of Euclidean geometry no more, no less.
I show then definitely, there is no mean to doubt about special relativity and it must be engraved in the marble.

Lorentz transformations

Within the framework of Euclidean geometry, let $R = (O, \overrightarrow{e_x}, \overrightarrow{e_y}, \overrightarrow{e_z})$ be an ortho-normal referential and $R' = (O, \overrightarrow{e_{x'}}, \overrightarrow{e_{y'}}, \overrightarrow{e_{z'}})$ be another ortho-normal frame in uniform translation along Ox axe with velocity v with respect to R. All R' axes remain parallel to those of R.

Let M be a point of space with (x, y, z) coordinates relatively to R and (x', y', z') coordinates relatively to R'.

a) Let us just write Chasles relation in R: $\overrightarrow{OM} - \overrightarrow{OO'} = \overrightarrow{O'M}$

then let us project on $\overrightarrow{e_x}$ $\qquad \overrightarrow{OM}.\overrightarrow{e_x} - \overrightarrow{OO'}.\overrightarrow{e_x} = \overrightarrow{O'M}.\overrightarrow{e_x}\big|_R$

or $\quad x - vt = x'(\overrightarrow{e_{x'}}.\overrightarrow{e_x})|_R$ $\qquad (\overrightarrow{e_{x'}}.\overrightarrow{e_x})|_R$ stand for scalar product of $\overrightarrow{e_{x'}}$ and $\overrightarrow{e_x}$ in R.

as there is no mathematical raison at all to impose $\overrightarrow{e_{x'}}.\overrightarrow{e_x}|_R = 1$, then I put $\overrightarrow{e_{x'}}.\overrightarrow{e_x}|_R = \frac{1}{\gamma}$.

then $\quad x' = \gamma(x - vt) \qquad (1).$

b) Now let us do the same in R'

Chasles relation in R' gives : $\qquad \overrightarrow{O'M} - \overrightarrow{O'O} = \overrightarrow{OM}$

let us project this time on $\overrightarrow{e_{x'}}$ $\qquad \overrightarrow{O'M}.\overrightarrow{e_{x'}} - \overrightarrow{O'O}.\overrightarrow{e_{x'}} = \overrightarrow{OM}.\overrightarrow{e_{x'}}\big|_{R'}$

or $\quad x' - v't' = x(\overrightarrow{e_x}.\overrightarrow{e_{x'}})|_{R'}$ $\qquad v'$ stands for R velocity with respect to R' and t' time in R' [3].

same, there is no mathematical raison at all to impose $\overrightarrow{e_x}.\overrightarrow{e_{x'}}|_{R'} = 1$, then I put $\overrightarrow{e_x}.\overrightarrow{e_{x'}}|_{R'} = \frac{1}{\gamma'}$

so $\quad x = \gamma'(x' - v't') \qquad (2)$

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[1] Bh4ha@yahoo.fr

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[3] Indeed if we suppose that time is the same in all frames $t' = t$ then

$\begin{Bmatrix} x' = \gamma(x - vt) \\ x = \gamma'(x' - v't) \end{Bmatrix}$ so $\begin{Bmatrix} \gamma\gamma' = 1 \\ v' = -\gamma v \end{Bmatrix}$. But as $v' = \frac{dO'O}{dt} = -\frac{dOO'}{dt} = -v$ then $\gamma = \gamma' = 1$

which leads to $x' = x - vt$ or $v'_x = v_x - v$ then no invariant speed is allowed and this contradicts observation facts.

c) (1) and (2) lead to

$$\left\{ \begin{array}{l} x^{'} = \gamma(x - vt) \\ t^{'} = -\gamma \frac{v}{v'}\left[t + \left(\frac{1}{\gamma\gamma'} - 1\right)\frac{x}{v}\right] \end{array} \right\}$$

for sake of simplicity, let $\alpha = \frac{1}{\gamma\gamma'} - 1$ and $\varepsilon = \frac{v'}{v}$

so $$\left\{ \begin{array}{l} x^{'} = \gamma(x - vt) \\ t^{'} = -\frac{\gamma}{\varepsilon}\left[t + \alpha\frac{x}{v}\right] \end{array} \right\}$$

d) **What about y and z?**

- For y, in R; let us project the expression $\overrightarrow{OM} - \overrightarrow{OO'} = \overrightarrow{O'M}$ on $\overrightarrow{e_y}$

I get, (a) $y = y^{'}(\overrightarrow{e_{y'}}.\overrightarrow{e_y})\big|_R = \beta' y'$ where $\beta^{'} = (\overrightarrow{e_{y'}}.\overrightarrow{e_y})\big|_R$

Now in R': let us project the expression $\overrightarrow{O'M} - \overrightarrow{O'O} = \overrightarrow{OM}$ on $\overrightarrow{e_{y'}}$

this leads to, (b) $y^{'} = y\left(\overrightarrow{e_y}.\overrightarrow{e_{y'}}\right)\big|_{R'} = \beta y$ where $\beta = \left(\overrightarrow{e_y}.\overrightarrow{e_{y'}}\right)\big|_{R'}$

(a)x(b) gives $yy' = y'y\beta\beta'$ then $\beta\beta' = 1$

- For z, as y and z axes play same roles, then by symmetry we have also : $z' = \beta z$

so we can write

$$\left\{ \begin{array}{l} x' = \gamma(x - vt) \\ t' = -\frac{\gamma}{\varepsilon}\left[t + \alpha\frac{x}{v}\right] \\ y' = \beta y \\ z' = \beta z \end{array} \right\}$$

There exists an invariant and isotropic speed

Space isotropy

The movement seen by R' is exactly the same as the one seen by R observer, means the transformation form doesn't change if we exchange x' by $-x'$ and x by $-x$, so

$$\left\{ \begin{array}{l} -x' = \gamma' \, (-x - v't) \\ t' = -\frac{\gamma'}{\varepsilon}\left[t - \alpha\frac{x}{v}\right] \end{array} \right. \quad \text{i.e.} \quad \{x' = \gamma' \, (x + v't) = \gamma(x - vt) \quad \text{then} \ \left\{ \begin{array}{l} \gamma' = \gamma \\ v' = -v \end{array} \right. \quad \text{or} \ \left\{ \begin{array}{l} \gamma' = \gamma \\ \varepsilon = -1 \end{array} \right.$$

So

$$\left\{ \begin{array}{l} x' = \gamma(x - vt) \\ t' = \gamma\left[t + \alpha\frac{x}{v}\right] \\ y' = \beta y \\ z' = \beta z \end{array} \right\}$$

Composition of movements

Let $R"$ be an ortho-normal frame in uniform translation along Ox axe with velocity w with respect to R', so

$$\left\{\begin{array}{c} x" = \gamma(w)(x' - wt') = \gamma(w)\gamma(v)(1 - \frac{\alpha(v)w}{v})\ \left[x - \frac{v+w}{1-\frac{\alpha(v)w}{v}}\right] \\ t" = \gamma(w)\left[t' + \alpha(w)\frac{x'}{w}\right] = \gamma(w)\gamma(v)(1 - \frac{\alpha(w)v}{w})\ \left[t - \frac{\frac{\alpha(v)}{v}+\frac{\alpha(w)}{w}}{1-\frac{\alpha(w)v}{w}}\right] \\ y" = \beta y' = \beta^2 y = \beta y \\ z" = \beta z' = \beta^2 z = \beta z \end{array}\right\}$$ then

$$\begin{cases} \gamma(w)\gamma(v)\left(1 - \frac{\alpha(v)w}{v}\right) = \gamma(w)\gamma(v)(1 - \frac{\alpha(w)v}{w}) \\ \beta^2 = \beta \end{cases}$$ leads to

$$\begin{cases} \frac{\alpha(v)}{v^2} = \frac{\alpha(w)}{w^2} \\ \beta = 1 \end{cases}$$ so $\frac{\alpha(v)}{v^2} = K$ whichever is the referential .

If $K = 0$ then $\alpha(v) = 0$ and $\gamma = \gamma' = 1$ leads to Galilean's and this contradicts observation facts.

Now, suppose $K > 0$

$t' = \gamma[t + Kvx] \Rightarrow \Delta t' = \gamma[\Delta t + Kv\Delta x]$ where v can take any real value.

So, for every $\Delta t > 0$ and for any Δx, we can find a value of v which allows $\Delta t' < 0$! A situation that breaks up the causality!

then K with all the more reason is negative. Let us write it as $K = -\frac{1}{u^2}$ where u is a speed, so the equations shape as

$$\left\{\begin{array}{l} x' = \gamma(x - vt) \\ t' = \gamma\left[t - \frac{vx}{u^2}\right] \\ y' = y \\ z' = z \end{array}\right\}$$ where $\frac{1}{\gamma^2} = 1 - \frac{v^2}{u^2}$ for $|v| < u$

For speed transformation we have

$v_{x'} = \frac{v_x - v}{1-\frac{vv_x}{v}}$, $v_{y'} = \frac{v_y}{\gamma(1-\frac{vv_x}{v})}$ and $v_{z'} = \frac{v_z}{\gamma(1-\frac{vv_x}{v})}$

Let $\vec{V}$ be a speed such as $V^2 = u^2$, it transform $\overrightarrow{V'}$ is :

$D^2V'^2 = D^2\left(v_x'^2 + v_y'^2 + v_z'^2\right) = (v_x^2 - v)^2 + \frac{v_y^2+v_z^2}{\gamma^2} = (v_x^2 - v)^2 + \frac{u^2-v_x^2}{\gamma^2}$ where $D = 1 - \frac{vv_x}{v}$

$D^2V'^2 = (v_x^2 - v)^2 + \left(1 - \frac{v^2}{u^2}\right)(u^2 - v_x^2) = D^2u^2$, then $V'^2 = u^2$ also.

So $\vec{V}$ is invariant and isotropic. This shows that there exists such a speed indeed.

Conversely

Let us again take up the transformations in their general form.

$$\left\{ \begin{array}{l} x' = \gamma(x - vt) \\ t' = -\frac{\gamma}{\varepsilon}\left[t + \alpha \frac{x}{v}\right] \\ y' = \beta y \\ z' = \beta z \end{array} \right\}$$

and suppose there is a velocity $\vec{u}$ which is isotropic and invariant.

before proceeding, let us express velocities in R' in terms of those in R

$\frac{dx\prime}{dt\prime} = -\varepsilon \frac{dx - vdt}{dt + \alpha\frac{dx}{v}}$ or $v_{x\prime} = -\varepsilon \frac{v_x - v}{1 + \alpha\frac{v_x}{v}}$

$\frac{dy\prime}{dt\prime} = -\varepsilon\beta \frac{dy}{\gamma\left(dt + \alpha\frac{v_x}{v}\right)}$ or $v_{y\prime} = -\varepsilon\beta \frac{v_y}{\gamma(1 + \alpha\frac{v_x}{v})}$

same for z $v_{z\prime} = -\varepsilon\beta \frac{v_z}{\gamma(1 + \alpha\frac{v_x}{v})}$

we can write :

$$u^2 = (u_x)^2 + (u_y)^2 + (u_z)^2 = (u_{x'})^2 + (u_{y'})^2 + (u_{z'})^2$$

$$u^2 = (u_{x'})^2 + (u_{y'})^2 + (u_{z'})^2 = \frac{\varepsilon^2}{(1 + \alpha\frac{u_x}{v})^2}\left[(u_x - v)^2 + ((u_y)^2 + (u_z)^2)\frac{\beta^2}{\gamma^2}\right]$$

$$u^2 = \frac{\varepsilon^2}{(1 + \alpha\frac{u_x}{v})^2}\left[(u_x - v)^2 + (u^2 - (u_x)^2)\frac{\beta^2}{\gamma^2}\right]$$

or $\left(\varepsilon^2\left(1 - \frac{\beta^2}{\gamma^2}\right) - \frac{\alpha^2 u^2}{v^2}\right){u_x}^2 - 2v\left(\frac{\alpha u^2}{v^2} + \varepsilon^2\right)u_x + \varepsilon^2\left(v^2 + \frac{u^2\beta^2}{\gamma^2}\right) - u^2 = 0$

The velocity $\vec{u}$ is isotropic by hypothesis, it may have any direction, then u_x varies continuously, so all coefficients of the preceding trinomial must be null:

$\varepsilon^2\left(1 - \frac{\beta^2}{\gamma^2}\right) - \frac{\alpha^2 u^2}{v^2} = 0$ (E1)

$\frac{\alpha u^2}{v^2} + \varepsilon^2 = 0$ (E2)

$\varepsilon^2\left(v^2 + \frac{u^2\beta^2}{\gamma^2}\right) - u^2 = 0$ (E3)

(E2) in (E1) leads to $\frac{\alpha^2 u^2}{v^2} = -\frac{\alpha u^2}{v^2}\left(1 - \frac{\beta^2}{\gamma^2}\right)$ otherwise $\alpha = \frac{\beta^2}{\gamma^2} - 1$

but $\alpha = \frac{1}{\gamma\gamma\prime} - 1$ then $\beta^2 = \frac{\gamma}{\gamma\prime}$

(E2) in (E3) leads to $u^2 = -\frac{\alpha u^2}{v^2}\left(v^2 + \frac{u^2}{\gamma\gamma'}\right) = -\frac{\alpha u^2}{v^2}(v^2 + (1+\alpha)u^2) = -\alpha u^2 - \alpha(1+\alpha)\frac{u^4}{v^2}$

or $1+\alpha = -\alpha\frac{u^2}{v^2}(1+\alpha)$

which gives $\alpha = -\frac{v^2}{u^2}$ and from (E2) $\varepsilon^2 = 1$

NB. The solution $\alpha = -1$ leads to $\gamma\gamma' = \infty$ which is impossible for γ and γ' are finite.

From (1) et (2) and for $x' = 0$ we have

$t = -\gamma'\frac{v'}{v}t'$ It implies $vv' < 0$

Because t' and t must have same signs (time arrow), and $\gamma' > 0$ for ox and o'x' are parallel and in the same sense by hypothesis

But $\varepsilon^2 = 1$ implies $|v| = |v'|$

Then $\left\{\begin{matrix} vv' < 0 \\ |v| = |v'| \end{matrix}\right\}$ implies $v' = -v$ or $\varepsilon = -1$

Now from $\varepsilon^2 = 1$, $\beta^2 = \frac{\gamma}{\gamma'}$, $\alpha = -\frac{v^2}{u^2}$

Equation (E1) leads to $\left(1 - \frac{1}{\gamma'^2}\right) = \frac{v^2}{u^2}$ otherwise $\frac{1}{\gamma'^2} = 1 - \frac{v^2}{u^2}$

also from $\alpha = \frac{1}{\gamma\gamma'} - 1 = -\frac{v^2}{u^2}$ $\frac{1}{\gamma\gamma'} = 1 - \frac{v^2}{u^2} = \frac{1}{\gamma'^2}$ then $\gamma = \gamma'$

so for $v \neq u$ $\gamma^2 = \gamma'^2 = \frac{1}{1-\frac{v^2}{u^2}}$

and finally for $-u < v < u$ $\gamma = \gamma' = \frac{\pm 1}{\sqrt{1-\frac{v^2}{u^2}}}$

but when $v = 0$, $R \equiv R'$, $\gamma = \gamma' = 1$ then for $-u < v < u$ $\gamma = \gamma' = \frac{1}{\sqrt{1-\frac{v^2}{u^2}}}$

Elsewhere we found $\beta^2 = \frac{\gamma}{\gamma'}$ so $\beta^2 = 1$ or $\beta = \pm 1$

but also when $v = 0$, $\overrightarrow{e_{y'}} = \overrightarrow{e_y}$, $\beta = \left(\overrightarrow{e_y}.\overrightarrow{e_{y'}}\right)\Big|_{R'} = 1$ and $\beta' = 1$ for $\beta\beta' = 1$

We obtain once and for all

$$\left\{\begin{aligned} x' &= \gamma(x - vt) \\ t' &= \gamma\left[t - \frac{vx}{u^2}\right] \\ y' &= y \\ z' &= z \end{aligned}\right\} \quad \text{where } \frac{1}{\gamma^2} = 1 - \frac{v^2}{u^2} \quad \text{for} \quad |v| < u$$

Theorem:

Within the framework of Euclidean geometry, let R' be a frame in uniform translation along (ox) axe with velocity v with respect to another ortho-normal referential R, all $R^{'}$ axes remain parallel to those of R.
Let M be a point of space with (x, y, z) coordinates relatively to R and $(x^{'}, y^{'}, z^{'})$ coordinates relatively to R', the transformation connecting coordinates is :

$$\left\{ \begin{array}{l} t' = \gamma\left(t - \frac{vx}{u^2}\right) \\ x' = \gamma(x - vt) \\ y' = y \\ z' = z \end{array} \right\}$$

with $\gamma = \frac{1}{\sqrt{1-\frac{v^2}{u^2}}}$ for $-u < v < u$ and where $\vec{u}$ is an invariant and isotropic velocity.

Finally, I obtain theoretically Lorentz transformations with a limit velocity u.
Nothing in the above theorem points towards the velocity of light and this fact has an important implication in the sequel.

Invariance of Lorentz transformations with scale changing

Consider for a while, we plunge in a new environment where time scale is no longer t but $\bar{t} = kt$ and where space scale is $\bar{x}^i = k'x^i$ where $k,\ k'$ are a positive numbers

So for velocities $\bar{v}^i = \frac{d\bar{x}^i}{d\bar{t}} = \frac{k'}{k}\frac{dx^i}{dt} = \frac{k'}{k}v^i$ or $\bar{v} = \frac{k'}{k}v$

and for x' and t'

$$x' = \gamma_u(x - vt) = \gamma_{\bar{u}}\left(\frac{\bar{x}}{k'} - \frac{\bar{v}\bar{t}}{k'}\right) \quad \Leftrightarrow \quad \bar{x}' = \gamma_{\bar{u}}(\bar{x} - \bar{v}\bar{t})$$

$$\gamma_u = \frac{1}{\sqrt{1-\frac{v^2}{u^2}}} = \frac{1}{\sqrt{1-\frac{\bar{v}^2}{\bar{u}^2}}} = \gamma_{\bar{u}} \quad \text{where} \quad \bar{v} = \frac{k'}{k}v \quad \text{and} \quad \bar{u} = \frac{k'}{k}u$$

$$t' = \gamma_u\left(t - \frac{vx}{u^2}\right) = \gamma_{\bar{u}}\left(\frac{\bar{t}}{k} - \frac{\bar{v}\bar{x}}{k\bar{u}^2}\right) \quad \Leftrightarrow \quad \bar{t}' = \gamma_{\bar{u}}\left(\bar{t} - \frac{\bar{v}\bar{x}}{\bar{u}^2}\right)$$

Also $y' = y \Leftrightarrow \bar{y}' = \bar{y}$ and $z' = z \Leftrightarrow \bar{z}' = \bar{z}$

So $$\left\{ \begin{array}{l} t' = \gamma_u\left(t - \frac{vx}{u^2}\right) \\ x' = \gamma_u(x - vt) \\ y' = y \\ z' = z \end{array} \right\} \Leftrightarrow \left\{ \begin{array}{l} \bar{t}' = \gamma_{\bar{u}}\left(\bar{t} - \frac{\bar{v}\bar{x}}{\bar{u}^2}\right) \\ \bar{x}' = \gamma_{\bar{u}}(\bar{x} - \bar{v}\bar{t}) \\ \bar{y}' = \bar{y} \\ \bar{z}' = \bar{z} \end{array} \right\}$$

From this equivalence, the limit velocity $\bar{u}$ in bared environment is seen as limit velocity u from unbarred one. It depends then in which environment the observer resides.

Conclusion : “**Lorentz transformations are true at all scales**”

Now that for certain, Lorentz transformations are shape invariant with scale changing, we have to extend the relativity principle and state:

"**All lows of nature are the same in all inertial reference frames and in all environments**"

So, as in the empty space as particular environment special relativity is firmly proved with light speed c, then inside any environment $\bar{u} = c$ in accordance with the extended principle of relativity.

$$\left\{ \begin{array}{l} \bar{t}' = \gamma_c \left(\bar{t} - \frac{\bar{v}\bar{x}}{c^2} \right) \\ \bar{x}' = \gamma_c (\bar{x} - \bar{v}\bar{t}) \\ \bar{y}' = \bar{y} \\ \bar{z}' = \bar{z} \end{array} \right\}$$

for any observer totally inside an environment.

$$\left\{ \begin{array}{l} t' = \gamma_u \left(t - \frac{vx}{u^2} \right) \\ x' = \gamma_u (x - vt) \\ y' = y \\ z' = z \end{array} \right\}$$

as seen by an observer from outside this environment.

So the velocity of light as limit speed inside an environment emerges not from the preceding theorem but from the extended principle of relativity and experimental observations, i.e. from the fact that relativity in empty space with light velocity as limit speed is firmly proved by experiment.

Restatement of the second principle of relativity

" Inside any environment there is a limit speed which is the light velocity c "