Special relativity is theorem based[1]

and some suggestions about OPERA experiment

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Abstract

In this work I show that special relativity is mathematical theorem based on just Chasles relation in Euclidian space. So special relativity will appears just as a direct consequence of Euclidean geometry no more, no less.
In the end of this paper some light is brought about OPERA experiment.

Lorentz transformations

Within the framework of Euclidean geometry, let $R = (O, \overrightarrow{e_x}, \overrightarrow{e_y}, \overrightarrow{e_z})$ be an ortho-normal referential and $R' = (O, \overrightarrow{e_{x'}}, \overrightarrow{e_{y'}}, \overrightarrow{e_{z'}})$ be another ortho-normal frame in uniform translation along Ox axe with velocity v with respect to R. All R' axes remain parallel to those of R.

Let M be a point of space with (x, y, z) coordinates relatively to R and (x', y', z') coordinates relatively to R'.

a) Let us just write Chasles relation in R: $\overrightarrow{OM} - \overrightarrow{OO'} = \overrightarrow{O'M}$

then let us project on $\overrightarrow{e_x}$ $\quad \overrightarrow{OM}.\overrightarrow{e_x} - \overrightarrow{OO'}.\overrightarrow{e_x} = \overrightarrow{O'M}.\overrightarrow{e_x}\big|_R$

or $\quad x - vt = x'(\overrightarrow{e_{x'}}.\overrightarrow{e_x})|_R$ $\quad (\overrightarrow{e_{x'}}.\overrightarrow{e_x})|_R$ stand for scalar product of $\overrightarrow{e_{x'}}$ and $\overrightarrow{e_x}$ in R.

as there is no mathematical raison at all to impose $\overrightarrow{e_{x'}}.\overrightarrow{e_x}|_R = 1$, then I put $\overrightarrow{e_{x'}}.\overrightarrow{e_x}|_R = \frac{1}{\gamma}$.

then $\quad x' = \gamma(x - vt) \quad$ (1).

b) Now let us do the same in R'

Chasles relation in R' gives : $\quad \overrightarrow{O'M} - \overrightarrow{O'O} = \overrightarrow{OM}$

let us project this time on $\overrightarrow{e_{x'}}$ $\quad \overrightarrow{O'M}.\overrightarrow{e_{x'}} - \overrightarrow{O'O}.\overrightarrow{e_{x'}} = \overrightarrow{OM}.\overrightarrow{e_{x'}}\big|_{R'}$

[1] This theorem I established during summer, 16 August 2009

or $\quad x' - v't' = x(\overrightarrow{e_x}.\overrightarrow{e_{x'}})|_{R'}$ $\quad v'$ stands for R velocity with respect to R' and t' time in R' [2].

same, there is no mathematical raison at all to impose $\overrightarrow{e_x}.\overrightarrow{e_{x'}}|_{R'} = 1$, then I put $\overrightarrow{e_x}.\overrightarrow{e_{x'}}|_{R'} = \frac{1}{\gamma'}$

so $\quad x = \gamma'(x' - v't') \quad$ (2)

c) (1) and (2) lead to

$$\begin{Bmatrix} x' = \gamma(x - vt) \\ t' = -\gamma\frac{v}{v'}\left[t + \left(\frac{1}{\gamma\gamma'} - 1\right)\frac{x}{v}\right] \end{Bmatrix}$$

for sake of simplicity, let $\alpha = \frac{1}{\gamma\gamma'} - 1$ and $\varepsilon = \frac{v'}{v}$

so $\begin{Bmatrix} x' = \gamma(x - vt) \\ t' = -\frac{\gamma}{\varepsilon}\left[t + \alpha\frac{x}{v}\right] \end{Bmatrix}$

d) What about y and z?

- For y, in R; let us project the expression $\overrightarrow{OM} - \overrightarrow{OO'} = \overrightarrow{O'M}$ on $\overrightarrow{e_y}$

I get, (a) $\quad y = y'(\overrightarrow{e_{y'}}.\overrightarrow{e_y})\big|_R = \beta'y'$ where $\beta' = (\overrightarrow{e_{y'}}.\overrightarrow{e_y})\big|_R$

Now in R': let us project the expression $\overrightarrow{O'M} - \overrightarrow{O'O} = \overrightarrow{OM}$ on $\overrightarrow{e_{y'}}$

this leads to, (b) $\quad y' = y(\overrightarrow{e_y}.\overrightarrow{e_{y'}})\big|_{R'} = \beta y$ where $\beta = (\overrightarrow{e_y}.\overrightarrow{e_{y'}})\big|_{R'}$

(a)x(b) gives $\quad yy' = y'y\beta\beta'$ then $\beta\beta' = 1$

- For z, as y and z axes play same roles, then by symmetry we have also : $z' = \beta z$

so we can write

$$\begin{Bmatrix} x' = \gamma(x - vt) \\ t' = -\frac{\gamma}{\varepsilon}\left[t + \alpha\frac{x}{v}\right] \\ y' = \beta y \\ z' = \beta z \end{Bmatrix}$$

Before proceeding, let us express velocities in R' in terms of those in R

$\frac{dx'}{dt'} = -\varepsilon\frac{dx - vdt}{dt + \alpha\frac{dx}{v}}$ $\quad$ or $\quad v_{x'} = -\varepsilon\frac{v_x - v}{1 + \alpha\frac{v_x}{v}}$

[2] Indeed if we suppose that time is the same in all frames $t' = t$ then
$\begin{Bmatrix} x' = \gamma(x - vt) \\ x = \gamma'(x' - v't) \end{Bmatrix}$ so $\begin{Bmatrix} \gamma\gamma' = 1 \\ v' = -\gamma v \end{Bmatrix}$. But as $v' = \frac{dO'O}{dt} = -\frac{dOO'}{dt} = -v$ then $\gamma = \gamma' = 1$

which leads to $x' = x - vt$ or $v'_x = v_x - v$ then no invariant speed is allowed and this contradicts observation facts.

$\frac{dy\prime}{dt\prime} = -\varepsilon\beta \frac{dy}{\gamma\left(dt+\alpha\frac{v_x}{v}\right)}$ or $v_{y\prime} = -\varepsilon\beta \frac{v_y}{\gamma(1+\alpha\frac{v_x}{v})}$

same for z $v_{z\prime} = -\varepsilon\beta \frac{v_z}{\gamma(1+\alpha\frac{v_x}{v})}$

Now, what is the relation between v and v' and what is the α value?

There is a velocity $\vec{u}$ which is isotropic and invariant[3], then we can write :

$$u^2 = (u_x)^2 + (u_y)^2 + (u_z)^2 = (u_{x'})^2 + (u_{y'})^2 + (u_{z'})^2$$

$$u^2 = (u_{x'})^2 + (u_{y'})^2 + (u_{z'})^2 = \frac{\varepsilon^2}{(1+\alpha\frac{u_x}{v})^2}\left[(u_x - v)^2 + ((u_y)^2 + (u_z)^2)\frac{\beta^2}{\gamma^2}\right]$$

$$u^2 = \frac{\varepsilon^2}{(1+\alpha\frac{u_x}{v})^2}\left[(u_x - v)^2 + (u^2 - (u_x)^2)\frac{\beta^2}{\gamma^2}\right]$$

or $\left(\varepsilon^2\left(1-\frac{\beta^2}{\gamma^2}\right) - \frac{\alpha^2 u^2}{v^2}\right){u_x}^2 - 2v\left(\frac{\alpha u^2}{v^2} + \varepsilon^2\right)u_x + \varepsilon^2\left(v^2 + \frac{u^2\beta^2}{\gamma^2}\right) - u^2 = 0$

The velocity $\vec{u}$ is isotropic by hypothesis, it may have any direction, then u_x varies continuously, so all coefficients of the preceding trinomial must be null:

$\varepsilon^2\left(1-\frac{\beta^2}{\gamma^2}\right) - \frac{\alpha^2 u^2}{v^2} = 0$ (E1)

$\frac{\alpha u^2}{v^2} + \varepsilon^2 = 0$ (E2)

$\varepsilon^2\left(v^2 + \frac{u^2\beta^2}{\gamma^2}\right) - u^2 = 0$ (E3)

(E2) in (E1) leads to $\frac{\alpha^2 u^2}{v^2} = -\frac{\alpha u^2}{v^2}\left(1-\frac{\beta^2}{\gamma^2}\right)$ otherwise $\alpha = \frac{\beta^2}{\gamma^2} - 1$

but $\alpha = \frac{1}{\gamma\gamma\prime} - 1$ then $\beta^2 = \frac{\gamma}{\gamma\prime}$

(E2) in (E3) leads to $u^2 = -\frac{\alpha u^2}{v^2}\left(v^2 + \frac{u^2}{\gamma\gamma'}\right) = -\frac{\alpha u^2}{v^2}(v^2 + (1+\alpha)u^2) = -\alpha u^2 - \alpha(1+\alpha)\frac{u^4}{v^2}$

or $1 + \alpha = -\alpha\frac{u^2}{v^2}(1+\alpha)$

which gives $\alpha = -\frac{v^2}{u^2}$ and from (E2) $\varepsilon^2 = 1$

NB. The solution $\alpha = -1$ leads to $\gamma\gamma' = \infty$ which is impossible for γ and γ' are finite.

From (1) et (2) and for $x' = 0$ we have

[3] In the concern not to load down this paper, I decided to publish a mathematical demonstration about the existence and the invariance of $\vec{u}$ in a next paper.

$t = -\gamma'\frac{v'}{v}t'$ It implies $vv' < 0$

Because t' and t must have same signs (time arrow), and $\gamma' > 0$ for ox and o'x' are parallel and in the same sense by hypothesis

But $\varepsilon^2 = 1$ implies $|v| = |v'|$

Then $\left\{\begin{array}{l} vv' < 0 \\ |v| = |v'| \end{array}\right\}$ implies $v' = -v$ or $\varepsilon = -1$

Now from $\varepsilon^2 = 1$, $\beta^2 = \frac{\gamma}{\gamma'}$, $\alpha = -\frac{v^2}{u^2}$

Equation (E1) leads to $\left(1 - \frac{1}{\gamma'^2}\right) = \frac{v^2}{u^2}$ otherwise $\frac{1}{\gamma'^2} = 1 - \frac{v^2}{u^2}$

also from $\alpha = \frac{1}{\gamma\gamma'} - 1 = -\frac{v^2}{u^2}$ $\frac{1}{\gamma\gamma'} = 1 - \frac{v^2}{u^2} = \frac{1}{\gamma'^2}$ then $\gamma = \gamma'$

so for $v \neq u$ $\gamma^2 = \gamma'^2 = \frac{1}{1-\frac{v^2}{u^2}}$

and finally for $-u < v < u$ $\gamma = \gamma' = \frac{\pm 1}{\sqrt{1-\frac{v^2}{u^2}}}$

but when $v = 0$, $R \equiv R'$, $\gamma = \gamma' = 1$ then for $-u < v < u$ $\gamma = \gamma' = \frac{1}{\sqrt{1-\frac{v^2}{u^2}}}$

Elsewhere we found $\beta^2 = \frac{\gamma}{\gamma'}$ so $\beta^2 = 1$ or $\beta = \pm 1$

but also when $v = 0$, $\overrightarrow{e_{y'}} = \overrightarrow{e_y}$, $\beta = \left(\overrightarrow{e_y}.\overrightarrow{e_{y'}}\right)\Big|_{R'} = 1$ and $\beta' = 1$ for $\beta\beta' = 1$

Theorem:
Within the framework of Euclidean geometry, let R' be a frame in uniform translation along (ox) axe with velocity v with respect to another ortho-normal referential R, all R' axes remain parallel to those of R.
Let M be a point of space with (x, y, z) coordinates relatively to R and (x', y', z') coordinates relatively to R', the transformation connecting coordinates is :

$$\left\{\begin{array}{l} t' = \gamma\left(t - \frac{vx}{u^2}\right) \\ x' = \gamma(x - vt) \\ y' = y \\ z' = z \end{array}\right\}$$

with $\gamma = \frac{1}{\sqrt{1-\frac{v^2}{u^2}}}$ for $-u < v < u$ and where $\vec{u}$ is an invariant and isotropic velocity.

Finally, I obtain theoretically Lorentz transformations with a limit velocity u.

Nothing in the above theorem points towards the velocity of light and this fact has an important implication in the sequel.

Invariance of Lorentz transformations with scale changing[4]

Consider for a while, we plunge in a new environment where time scale is no longer t but $\bar{t} = kt$ and where space scale is $\bar{x}^i = k'x^i$ where $k,\ k'$ are a positive numbers.

So for velocities $$\bar{v}^i = \frac{d\bar{x}^i}{d\bar{t}} = \frac{k'}{k}\frac{dx^i}{dt} = \frac{k'}{k}v^i \quad \text{or} \quad \bar{v} = \frac{k'}{k}v$$

and for x' and t'

$$x' = \gamma_u(x - vt) = \gamma_{\bar{u}}\left(\frac{\bar{x}}{k'} - \frac{\bar{v}\bar{t}}{k'}\right) \quad \Leftrightarrow \quad \bar{x}' = \gamma_{\bar{u}}(\bar{x} - \bar{v}\bar{t}\,)$$

$$\gamma_u = \frac{1}{\sqrt{1-\frac{v^2}{u^2}}} = \frac{1}{\sqrt{1-\frac{\bar{v}^2}{\bar{u}^2}}} = \gamma_{\bar{u}} \quad \text{where} \quad \bar{v} = \frac{k'}{k}v \quad \text{and} \quad \bar{u} = \frac{k'}{k}u$$

$$t' = \gamma_u\left(t - \frac{vx}{u^2}\right) = \gamma_{\bar{u}}\left(\frac{\bar{t}}{k} - \frac{\bar{v}\bar{x}}{k\bar{u}^2}\right) \quad \Leftrightarrow \quad \bar{t}' = \gamma_{\bar{u}}\left(\bar{t} - \frac{\bar{v}\bar{x}}{\bar{u}^2}\right)$$

Also $y' = y \quad \Leftrightarrow \quad \bar{y}' = \bar{y}$ and $z' = z \quad \Leftrightarrow \quad \bar{z}' = \bar{z}$

So $$\left\{\begin{array}{l} t' = \gamma_u\left(t - \frac{vx}{u^2}\right) \\ x' = \gamma_u(x - vt) \\ y' = y \\ z' = z \end{array}\right\} \Leftrightarrow \left\{\begin{array}{l} \bar{t}' = \gamma_{\bar{u}}\left(\bar{t} - \frac{\bar{v}\bar{x}}{\bar{u}^2}\right) \\ \bar{x}' = \gamma_{\bar{u}}(\bar{x} - \bar{v}\bar{t}) \\ \bar{y}' = \bar{y} \\ \bar{z}' = \bar{z} \end{array}\right\}$$

From this equivalence, the limit velocity $\bar{u}$ in bared environment is seen as limit velocity u from unbarred one. It depends then in which environment the observer resides.
I show then that the limit speed u depends on the environment from which observer notice things, a very important fact to keep in mind.

Conclusion : "**Lorentz transformations are true at all scales**"

Now that for certain, Lorentz transformations are shape invariant with scale changing, I have to extend the relativity principle and state:

"**All lows of nature are the same in all inertial reference frames and in all environments**"

So, that in the empty space as particular environment special relativity is firmly proved with light speed c, then inside any environment $\bar{u} = c$ in accordance with my extended principle of relativity.

[4] This part I worked out recently in 11 October 2011

$$\left\{\begin{array}{l} \bar{t}' = \gamma_c\left(\bar{t} - \frac{\bar{v}\bar{x}}{c^2}\right) \\ \bar{x}' = \gamma_c(\bar{x} - \bar{v}\bar{t}) \\ \bar{y}' = \bar{y} \\ \bar{z}' = \bar{z} \end{array}\right\}$$ for any observer totally inside an environment.

$$\left\{\begin{array}{l} t' = \gamma_u\left(t - \frac{vx}{u^2}\right) \\ x' = \gamma_u(x - vt) \\ y' = y \\ z' = z \end{array}\right\}$$ as seen by an observer from outside this environment.

So the velocity of light as limit speed inside an environment emerges not from the preceding theorem but from the extended principle of relativity and from the fact that relativity in empty space with light velocity as limit speed is firmly proved.

Restatement of the second principle of relativity

" Inside any environment there is a limit speed which is the light velocity c "

NB : It is the refraction index $n = \frac{c}{u} = \frac{k'}{k}$ which define the scales inside an environment.

Concrete example of environment[5]

There is one thing physicists never paid attention to, which is!
How can light velocity change **abruptly** from c to $u < c$ passing from empty space into a medium of refraction index $n = \frac{c}{u} > 1$?
In my opinion, this is simply impossible!
By continuity light must preserve its velocity during the crossing, and must have also c inside the medium it crosses.
It is just because we measure light velocity from outside medium, that we find a value $u < c$, but inside the medium light speed remains c as I said.
So, for same space scale ($k' = 1$) and some distance L traveled by light inside medium,

$L = ut$ from outside and in our time scale t, u stands for light speed as measured by us.
$L = c\bar{t}$ from inside and in medium time scale $\bar{t}$.

Then $L = ut = c\bar{t}$ so $\bar{t} = \frac{u}{c}t = \frac{t}{n}$ where n stand of course for medium refraction index, and again time scale is in this particular case $k = \frac{1}{n}$.

Cerenkov effect comes to help and reinforce my belief. Indeed electron speed v_e as measured from outside the medium (water) corresponds to $\bar{v}_e = nv_e = c\frac{v_e}{u} > c$ for $\frac{v_e}{u} > 1$ as known from outside.
Electron can't then violate special relativity inside medium, and has to slow down its velocity by radiating to acquire a velocity inside medium less than c.

[5] The hypothesis of light velocity continuity and the role of the refraction index I expressed since 1997

It seems then, that inside a medium of refraction index $n > 1$, time suffers a sort of **contraction!**, its period is less than its one's in the empty space.

OPERA[6] experiment [1]

Let me bring this little contribution if I am allowed to.
There was no violation of relativity by neutrinos in OPERA experiment, no one has exceeded speed of light inside medium of course, It is just because of time dilatation[7] ($k > 1$) [2] inside the medium crossed by neutrinos and because of measurement carried out from outside, that it appeared to outsiders like there was a sort of special relativity violation.
To be correct we have to compare things in the same environment.

In the OPERA experiment, let v_n be the neutrino velocity as measured from outside and $\bar{v}_n$ its velocity inside medium.
I suppose for simplicity that the ratio k is constant [8] everywhere in the medium and $k' = 1$.

$$v_n = k\bar{v}_n = \frac{u}{c}\bar{v}_n = \frac{\bar{v}_n}{c}u < u \quad \text{for} \quad \frac{\bar{v}_n}{c} < 1 \text{ in virtue of special relativity inside medium.}$$

$u = kc > c$ stands for the speed of light - crossing medium if ever possible - as measured from outside medium.

The possibility for neutrino to exceed the speed of light ($v_n > c$) **as seen from outside** is not forbidden then, because the inequality $v_n > c$ compare two velocities one from outside and the other from inside, then $v_n > c$ is not a violation of relativity principle at all.

It was still $v_n < u$ or $\bar{v}_n < c$ during neutrinos' trip [9], so there was no violation of relativity by neutrinos, reason why they needed no loss of energy by radiating or bremsstrahllung and most of them reached Gran Sasso.

References

[1] T.Adam et al. Measurement of the neutrino velocity with the OPERA detector in the CNGS beam, arXiv:1109.4897v1 [hep-ex]

[2] J. Manuel Garcia-Islas Avery simple solution to OPERA neutrino velocity problem arXiv:1110.5866v1 [physics.gen-ph]

[6] Experiment called OPERA (Oscillation Project with Emulsion-tRacking Apparatus), http://arxiv.org/abs/1109.4897v1

[7] The cause of this time dilatation may be earth gravitation (the most probable) or other unknown phenomena.

[8] In fact, the ratio k is a function of point on the path covered by neutrino and a precise calculation must be done.

[9] Indeed we have to compare speeds in the same environment as it is for v_n and u from outside and for $\bar{v}_n$ and c from inside.