Two Charged Scalar Field-Based Mass Generation Mechanisms

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Abstract

Despite consistency of the Higgs mechanism, experimental data have not revealed existence of the Higgs particle. Moreover, the Higgs mechanism explains why photon is massless, while another experimental data reveal very small but detectable photon mass. In this manner the crucial problem is to combine abstract ideas of the Standard Model with the verified experimental data to obtain constructive physical picture.

In this paper we discuss two alternative consistent mass generation mechanisms which are based on charged scalar field and the $O(2)$ symmetric Higgs potential. Both the mechanisms for abelian fields of the Standard Model lead to nonzero photon mass, but predict distinguishable mass of the new neutral scalar boson. Both the models are similar to the Higgs mechanism. The scenarios base on existence of a new scalar neutral boson χ and an auxiliary scalar neutral field φ which can be interpreted as a dilaton. In the first model a new scalar particle is massive, and the value of its mass can be estimated by the present day experimental limits on the photon mass. In the second model dilaton is massless and a new scalar particle has a mass which can be determined only by experimental data. The mass of a photon in this model does not depend on the mass of a Higgs-like particle.

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1 Introduction

In modern physics, from both theoretical and experimental points of view, the most fruitful theory is the Standard Model of particles and fundamental interactions. Its various extensions and modifications create the natural possibilities for new physics. Albeit, there is still the problematic presence of what can be called non-physical mathematics, i.e. the part of the Standard Model which is still unverified positively by experiments. This part of particle physics can be treated, however, as invisible or undetectable Nature. To such a collection the Higgs particle belongs, what is confirmed by negative verification of its hypothetical existence with the rigorous data of experimental particle physics. In fact, the great success of this commonly accepted and plausible physical theory is the constructive mass generation mechanism predicting the masses of particles, and in itself leading to existence of the hypothetical Higgs boson.

However, in spite of a number of the successes of the Standard Model, which is application the idea of spontaneous symmetry breaking to beautiful explain why the $W^{\pm}$ and Z^0 bosons are massive while the photon is massless [1], the acceptable mass generation mechanism leaves a lot to be desired from the experimental point of view. Namely, this explanation demands the existence of at least one additional particle, and since 1964, when the mass generation mechanism based on spontaneous symmetry breakdown [2] was proposed, this existence of the Higgs boson has never been satisfactory confirmed by experimental data. Despite this mass generation mechanism leads to probably the most beautiful picture of modern physics, it possesses manifestly certain singular points that, in the light of experimental data, are inadequate and result in inconsistency of the theory. One of the gross point is the question about the mass of photon. Despite the electroweak mass generation mechanism is plausible if photon is really massless, the experimental data (See e.g. the fundamental papers [3, 4, 5]) say something different about this property of a photon. It looks like that a photon is rather massive particle, and its mass is very small but detectable. In the light of this fact the present mass generation mechanism of the Standard Model looks like implausibly.

In this paper I propose the new mass generation mechanism for abelian fields of the Standard Model. This algorithm can be straightforwardly generalized to the most general Yang–Mills theories. The our proposal, however, differs from the usual mass generation mechanism by involving of the mass of photon. I express our calculations by using of the Planck units, what creates natural scenario for new physics at the Planck scale. The content of the paper is as follows. The Section 2 discusses concisely the mass generation mechanism for abelian sector of the Standard Model based on the Higgs potential and charged scalar field. In the Section 3 I present the construction of the alternative mass generation mechanism to abelian fields within the Standard Model. This model is based also on charged scalar field, but on the Higgs potential is modified. Both the models are manifestly invariant with respect to action of $O(2)$ symmetry group. Finally, in the Section 4 the results of whole paper are concisely summarized.

2 The Higgs Potential

The constructive approach to the mass generation mechanism of elementary particles within the Standard Model, well-known as the Higgs mechanism (For detailed discussion see e.g. the Ref. [11]), is based on the Higgs potential and, in fact, lays the foundations of the particle physics. In this paper I shall focus our attention on the generation of mass to the abelian gauge field A_μ, i.e. photon, which interacts with a charged scalar field Φ. This is the theory of a $U(1)$ gauge field coupled to a charged spinless boson, which is usually called scalar quantum electrodynamics (See e.g. the Refs. [12]). In such a particular situation the algorithm of the spontaneous symmetry breaking is realized via so called the Abelian Higgs mechanism.

It must be emphasized that both scalar electrodynamics as well as the Higgs potential possess a number of essential applications. The first context is the pioneering Ginzburg–Landau model of superconductor [13], which formulates superconductivity as a charged Bose–Einstein condensate. In this model a complex order field is applied for description of fluctuations in the order parameter by adding a gradient to the Gorter–Casimir two-fluid model of superconductors, what lead them to the theory of superconductivity near the critical temperature. As was shown by Abrikosov in 1957 [14] and Nielsen and Olesen in 1973 [15], for the $2+1$-dimensional situation in the Ginzburg–Landau theory there are vortices carrying magnetic flux.

The contexts strictly related to numerous applications in particle physics, including charged scalar field, have been recently studied by a number of authors (See e.g. the papers in the Ref. [16]). Phenomenological context, in the borderland of cosmology and particle physics, has been discussed recently by Arbuzov, Glinka, and Pervushin [17].

Let us present the consistent approach to the mass generation mechanism, which is based on the Higgs potential. The Standard Model is described in the Minkowski space-time by the action of the form

$$S = \frac{1}{c}\int d^4x\mathscr{L}, \tag{1}$$

in which $\mathscr{L}$ is the total Lagrangian density of the theory

$$\mathscr{L} = \mathscr{L}_0 + \mathscr{L}_I, \tag{2}$$

where $\mathscr{L}_0$ is the Lagrangian density of free fields, and $\mathscr{L}_I$ is an interaction Lagrangian density of the theory that, expressed in the Planck units, have the following form[1]

$$\mathscr{L}_0 = E_P\ell_P\partial_\mu\Phi^\dagger\partial^\mu\Phi - \frac{1}{\ell_P^3}V(|\Phi|) - \frac{1}{4\mu_0}F_{\mu\nu}F^{\mu\nu}, \tag{3}$$

$$\mathscr{L}_I = -j^\mu A_\mu + \frac{e^2}{M_P\ell_P}\Phi^\dagger\Phi A_\mu A^\mu. \tag{4}$$

Here the field $\Phi(x)$ is a complex (charged) scalar field, and $\Phi^\dagger(x)$ is the complex conjugate of $\Phi(x)$

$$\Phi = \frac{\varphi_1(x) + i\varphi_2(x)}{\sqrt{2}}, \tag{5}$$

$$\Phi^\dagger = \frac{\varphi_1(x) - i\varphi_2(x)}{\sqrt{2}}, \tag{6}$$

having physical dimension L^{-1}, characterized by a mass m and a *dimensionless* coupling constant g, $V(|\Phi|)$ is an effective $O(2)$-symmetric potential of such a scalar field in the standard form of the Higgs potential,

$$V(|\Phi|) = \ell_P^2\frac{m^2c^2}{M_P}|\Phi|^2 + gE_P\ell_P^4|\Phi|^4, \tag{7}$$

where $|\Phi|^2 = \Phi^\dagger\Phi$. The Lagrangian (2) is invariant with respect of the action of the $U(1)$ symmetry group transformations

$$\Phi' = \exp(-i\theta)\Phi, \tag{8}$$

$$\Phi^{\dagger\prime} = \exp(i\theta)\Phi^\dagger, \tag{9}$$

$$A_\mu{}' = A_\mu + \frac{\hbar}{e}\partial_\mu\theta, \tag{10}$$

$$A^{\mu\prime} = A^\mu - \frac{\hbar}{e}\partial^\mu\theta, \tag{11}$$

[1] I use the Planck units immediately as the parameters which allow to express the theory in the correct dimensional form. When one applies the standard unit system of particle physics, then $\ell_P = 1$, $E_P = 1$, $M_P = 1$ etc.

where $\theta(x)$ is a local phase. The conserved Noether current j_μ is

$$j^\mu = iec\left(\Phi^\dagger\partial^\mu\Phi - \left(\partial^\mu\Phi^\dagger\right)\Phi\right). \tag{12}$$

The electromagnetic interaction is described by the abelian field - chargeless Maxwell electromagnetic four-potential A_μ which in standard theory is massless, which strength tensor is $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$. I use the standard conventions $A_\mu = \eta_{\mu\nu}A^\nu$, $\partial_\mu = \eta_{\mu\nu}\partial^\nu$, where $\eta_{\mu\nu} = \mathrm{diag}[-1,1,1,1]$ is the metric of the Minkowski space-time.

As it is commonly accepted in particle physics, I take into account the following vacuum expectation values of the real scalar fields

$$\langle 0|\varphi_1(x)|0\rangle = \varphi_0, \tag{13}$$
$$\langle 0|\varphi_2(x)|0\rangle = 0, \tag{14}$$

where φ_0 is a real constant, and the vacuum state $|0\rangle$ is treated as belonging to the static Fock space of the theory. The scalar fields that provoke masses have the following decomposition in terms of the Fourier harmonics

$$\varphi_1(x) = \varphi_0 + \chi(x), \tag{15}$$
$$\varphi_2(x) = \varphi(x), \tag{16}$$

where the Higgs boson $\chi(x)$, and the auxiliary scalar field φ have identically vanishing vacuum expectation values

$$\langle 0|\chi(x)|0\rangle = 0, \tag{17}$$
$$\langle 0|\varphi(x)|0\rangle = 0. \tag{18}$$

By this reason the vacuum expectation value of the Higgs potential can be derived straightforwardly as

$$\langle 0|V(|\Phi|)|0\rangle = \ell_P^2\frac{m^2c^2}{2M_P}\varphi_0^2 + \ell_P^4E_P\frac{g}{4}\varphi_0^4 \equiv \mathscr{E}(\varphi_0), \tag{19}$$

and extremal values of this energy are established by vanishing of corresponding force

$$-\frac{d\mathscr{E}(\varphi_0)}{d\varphi_0} = -\left(\ell_P^2\frac{m^2c^2}{M_P}\varphi_0 + gE_P\ell_P^4\varphi_0^3\right) = 0, \tag{20}$$

which leads to two solutions

$$\varphi_0 = 0, \tag{21}$$
$$\varphi_0^2 = -\frac{1}{g}\frac{m^2}{M_P^2}\frac{1}{\ell_P^2}. \tag{22}$$

The values of the energy in these points are respectively

$$\mathscr{E}(\varphi_0 = 0) = 0 \quad , \quad \mathscr{E}\left(\varphi_0^2 = -\frac{1}{g}\frac{m^2}{M_P^2}\frac{1}{\ell_P^2}\right) = -\frac{E_P}{4g}\frac{m^4}{M_P^4}. \tag{23}$$

The first solution is trivial and does not give a contribution to the theory. The second one is essential for the theory if and only if there are satisfied the following stability conditions

$$g > 0 \quad , \quad m^2 = -m_0^2 \quad , \quad m_0^2 > 0, \tag{24}$$

which mean that the scalar Higgs particle must be a *tachyon*.

With using of all the definitions presented above the total Lagrangian density (2) can be rewritten in the following form

$$\mathscr{L} = \mathscr{L}_0 + \mathscr{L}_\chi + \mathscr{L}_\varphi + \mathscr{L}_A + \mathscr{L}_{\chi\varphi A} \tag{25}$$

where the parts of the Lagrangian describing: constant contribution $\mathscr{L}_0$, free χ real massive scalar field $\mathscr{L}_\chi$, free φ real massive scalar field $\mathscr{L}_\varphi$, free massive photon $\mathscr{L}_A$ are

$$\mathscr{L}_0 = \left(\frac{m_0^2c^2}{2M_P\ell_P} - \frac{g}{4}E_P\ell_P\varphi_0^2\right)\varphi_0^2, \tag{26}$$

$$\mathscr{L}_\chi = \frac{E_P\ell_P}{2}\partial_\mu\chi\partial^\mu\chi + \varphi_0\left(\frac{m_0^2c^2}{M_P\ell_P} - gE_P\ell_P\varphi_0^2\right)\chi \tag{27}$$

$$+ \left(\frac{m_0^2c^2}{2M_P\ell_P} - \frac{3}{2}gE_P\ell_P\varphi_0^2\right)\chi^2 - gE_P\ell_P\varphi_0\chi^3 - \frac{g}{4}E_P\ell_P\chi^4, \tag{28}$$

$$\mathscr{L}_\varphi = \frac{E_P\ell_P}{2}\partial_\mu\varphi\partial^\mu\varphi + \left(\frac{m_0^2c^2}{2M_P\ell_P} - \frac{g}{2}E_P\ell_P\varphi_0^2\right)\varphi^2 - \frac{g}{4}E_P\ell_P\varphi^4, \tag{29}$$

$$\mathscr{L}_A = -\frac{1}{4\mu_0}F_{\mu\nu}F^{\mu\nu} + \frac{e^2\varphi_0^2}{2M_P\ell_P}A_\mu A^\mu, \tag{30}$$

while the effective interaction Lagrangian $\mathscr{L}_{\chi\varphi A}$ has the form

$$\mathscr{L}_{\chi\varphi A} = -gE_P\ell_P\varphi_0\chi\varphi^2 - \frac{g}{2}E_P\ell_P\chi^2\varphi^2 + \frac{e^2}{2M_P\ell_P}\eta_{\mu\nu}\chi^2A^\mu A^\nu \tag{31}$$

$$+ \frac{e^2}{2M_P\ell_P}\eta_{\mu\nu}\varphi^2A^\mu A^\nu + \frac{e^2\varphi_0}{M_P\ell_P}\eta_{\mu\nu}\chi A^\mu A^\nu \tag{32}$$

$$+ ec\eta_{\mu\nu}\chi\partial^\mu\varphi A^\nu - ec\eta_{\mu\nu}\varphi\partial^\mu\chi A^\nu. \tag{33}$$

The quadratic in the fields and the derivatives part of the total Lagrangian density (25)

$$\begin{aligned}\mathscr{L}_Q &= -\frac{1}{4\mu_0}F_{\mu\nu}F^{\mu\nu}+\frac{e^2m_A^2}{2M_P^3\ell_P^3}A_\mu A^\mu+\frac{E_P\ell_P}{2}\partial_\mu\chi\partial^\mu\chi-\frac{m_\chi^2c^2}{2M_P\ell_P}\chi^2 &(34)\\ &+ \frac{E_P\ell_P}{2}\partial_\mu\varphi\partial^\mu\varphi-\frac{m_\varphi^2c^2}{2M_P\ell_P}\varphi^2, &(35)\end{aligned}$$

allows to establish the masses of the particles

$$\begin{aligned} m_A &= M_P\ell_P\varphi_0, &(36)\\ m_\varphi &= \sqrt{gm_A^2-m_0^2}, &(37)\\ m_\chi &= \sqrt{3gm_A^2-m_0^2}. &(38)\end{aligned}$$

In this manner one receives

$$\varphi_0=\frac{m_A}{M_P}\frac{1}{\ell_P}=\frac{m_Ac}{\hbar}. \tag{39}$$

Applying the formulas (36), (37), and (38) one can express the coupling constant g by the masses of the particles

$$g=\frac{m_\chi^2-m_\varphi^2}{2m_A^2}. \tag{40}$$

Because physical meaning have the only masses which are real numbers one obtains the system of inequalities

$$\left\{\begin{array}{l} gm_A^2-m_0^2\geqslant 0,\\ 3gm_A^2-m_0^2\geqslant 0,\end{array}\right. \tag{41}$$

which lead to the lower bound for the coupling constant

$$g\geqslant\frac{m_0^2}{m_A^2}, \tag{42}$$

which applied to the basic relation (40) gives the upper bound for the mass of the charged scalar field Φ

$$m_0\leqslant\sqrt{\frac{m_\chi^2-m_\varphi^2}{2}}. \tag{43}$$

In the light of the Markov hypothesis $m_0 \leqslant M_P$ one obtains the condition for the squared mass difference $\Delta m^2_{\chi\varphi} = m^2_\chi - m^2_\varphi$

$$\Delta m^2_{\chi\varphi} \leqslant 2M_P^2. \tag{44}$$

The masses derived in the maximal point $\varphi_0^2 = \dfrac{1}{g}\dfrac{m_0^2}{M_P^2}\dfrac{1}{\ell_P^2}$ have the values

$$\begin{aligned} m_A &= \frac{m_0}{\sqrt{g}}, & (45)\\ m_\varphi &= 0, & (46)\\ m_\chi &= \sqrt{2}m_0, & (47)\end{aligned}$$

and possess natural interpretation as the ground state masses of the elementary particles – the photon $A_\mu(x)$, the auxiliary scalar field $\varphi(x)$, the $\chi(x)$ boson. In this manner in the model presented above the scalar auxiliary field $\varphi(x)$ is massless, whereas both the Higgs boson $\chi(x)$ and the photon A^μ are massive are their masses are determined by the free parameter m_0. For the trivial solution $\varphi_0 = 0$ one obtains

$$\begin{aligned} m_A &= 0, & (48)\\ m_\varphi &= im_0, & (49)\\ m_\chi &= im_0, & (50)\end{aligned}$$

what means that in such a situation the photon is massless, but both the scalar fields as tachyons are nonphysical.

The Euler–Lagrange equations of motion following from the Lagrangian (25) are easy to direct derivation. The equation of motion for photon is

$$\Box A^\mu - \partial^\mu(\partial_\nu A^\nu) + \frac{\mu_0 e^2 m_A^2}{M_P^3\ell_P^3}A^\mu = \mu_0 j^\mu - \frac{\mu_0 e^2}{M_P\ell_P}\left(\chi^2 + \varphi^2 + \frac{2m_A}{M_P\ell_P}\chi\right)A^\mu, \tag{51}$$

where $\Box = \partial^\nu\partial_\nu$ and j^μ is the conserved Noether current

$$j^\mu = ec\left(\varphi\partial^\mu\chi - \chi\partial^\mu\varphi\right). \tag{52}$$

The equation for motion for the auxiliary scalar field is

$$\begin{aligned} \Box\varphi + \frac{m_\varphi^2}{M_P^2\ell_P^2}\varphi + g\varphi^3 &= -2\frac{gm_A}{M_P\ell_P}\chi\varphi - g\chi^2\varphi + \frac{e^2}{M_P\ell_P}\varphi A^\mu A_\mu & (53)\\ &- 2\frac{e}{\hbar}\partial^\mu\chi A_\mu - \frac{e}{\hbar}\chi\partial^\mu A_\mu, & (54)\end{aligned}$$

and the equation for motion for the χ boson is

$$\Box\chi+\frac{m_\chi^2}{M_P^2\ell_P^2}\chi+3\frac{gm_A}{M_P\ell_P}\chi^2+g\chi^3 = -\frac{m_\varphi^2}{M_P^2\ell_P^2}\varphi_0-\frac{gm_A}{M_P\ell_P}\varphi^2-g\chi\varphi^2 \tag{55}$$

$$+\frac{e^2}{M_P\ell_P}\chi A^\mu A_\mu+\frac{e^2m_A}{M_P^2\ell_P^2}A^\mu A_\mu \tag{56}$$

$$+2\frac{e}{\hbar}\partial^\mu\varphi A_\mu+\frac{e}{\hbar}\chi\partial^\mu A_\mu. \tag{57}$$

It can bee seen by straightforward and easy calculation that if one takes into consideration the Markov hypothesis [18], stating that a mass of any elementary particle is not more than the Planck mass, applied to the mass of the charged scalar field m_0

$$m_0\leqslant M_P, \tag{58}$$

then of obtains the upper bound for the initial datum φ_0

$$\varphi_0\leqslant\frac{1}{\sqrt{g}}\frac{1}{\ell_P}. \tag{59}$$

Recently Glinka (See Chapter 5 of the Ref. [19]) has been presented certain deductions for the Higgs–Hubble inflaton, i.e. the particular case of the Higgs scalar field obeying the Hubble law, which gives the most adequate inflationary cosmology to the Planck scale. In such a situation the initial datum of the scalar field is

$$\varphi_0=\frac{2}{3}\sqrt{\pi}\frac{1}{\ell_P}. \tag{60}$$

Applying this value to the inequality (59) one receives the upper bound to the coupling constant in such a Planck-scale situation

$$g\leqslant\frac{9}{4\pi}\approx 0.716, \tag{61}$$

and applying the maximal initial datum value $\varphi_0^2=\frac{1}{g}\frac{m_0^2}{M_P^2}\frac{1}{\ell_P^2}$ to the equation (60) one receives

$$m_0=\frac{2}{3}\sqrt{g\pi}M_P, \tag{62}$$

or equivalently

$$g=\frac{9}{4\pi}\left(\frac{m_0}{M_P}\right)^2. \tag{63}$$

Using of this relation together with the formula (40) one receives

$$m_0 = \sqrt{2\pi\Delta m_{\chi\varphi}^2 \frac{M_P}{3m_A}}, \tag{64}$$

where $\Delta m_{\chi\varphi}^2 = m_\chi^2 - m_\varphi^2$. This formula allows to deduce that

$$\Delta m_{\chi\varphi}^2 = \frac{9}{2\pi}\left(\frac{m_0 m_A}{M_P}\right)^2, \tag{65}$$

so that involving the bound (44) one obtains the inequality

$$m_0 m_A \leqslant \frac{2}{3}\sqrt{\pi} M_P^2. \tag{66}$$

Another consequence of the Higgs–Hubble initial datum (60) are the masses of the particles. In the maximum of the Higgs potential they have the following values

$$m_A = \frac{2}{3}\sqrt{\pi} M_P, \tag{67}$$

$$m_\varphi = 0, \tag{68}$$

$$m_\chi = \sqrt{2g}\, m_A. \tag{69}$$

By using of the inequalities (42) and (61) one can deduce easy the upper bound

$$m_0 \leqslant \frac{3}{2\sqrt{\pi}} m_A, \tag{70}$$

which after application of the formulas (62) and (36) leads to the lower bound for the initial datum φ_0

$$\varphi_0 \geqslant \frac{4\pi}{9}\sqrt{g}\,\ell_P. \tag{71}$$

The scenario presented in this section is in itself a certain algorithm for generation of the masses of the abelian field A^μ within the Standard Model. This mechanism involves two neutral scalar fields - the massive χ boson, and the auxiliary scalar field φ which is massless at the maximum of the vacuum expectation value of the Higgs potential. The photon A^μ is always massive in this scenario. This mechanism can be straightforwardly generalized to the case of non-abelian gauge fields.

3 The Modified Higgs Potential

In the present section I shall take into account the alternative mass generation mechanism for abelian fields within the Standard Model, which is based on the Higgs potential modified due to the constant shift $|\Phi_0|$ of the variable $|\Phi|$, where Φ is the charged scalar field considered in the previous section. This mechanism can be also straightforwardly generalized to the situations involving non-abelian gauge field theories.

I shall base the mass generation mechanism on the following modification of the standard Higgs potential of the scalar field $|\Phi|$

$$V(|\Phi|) = \ell_P^2 \frac{m^2c^2}{M_P}\left(|\Phi| - |\Phi_0|\right)^2 + gE_P\ell_P^4\left(|\Phi| - |\Phi_0|\right)^4, \tag{72}$$

that essentially is the standard scalar field potential, discussed in the previous section, rotated around the point $|\Phi| = 0$. Such a potential (72) still is invariant under action of $O(2)$ group. The shift-field Φ_0 can be linked to the zero Fourier harmonic of the charged scalar field Φ. Let us presume that there is the decomposition

$$\sigma(x) := |\Phi(x)| \equiv \sqrt{\frac{\varphi_1^2(x) + \varphi_2^2(x)}{2}} = \frac{\sigma_0 + \chi(x)}{\sqrt{2}}. \tag{73}$$

I shall assume that χ is the new neutral scalar boson

$$\langle 0|\chi(x)|0\rangle = 0, \tag{74}$$

so that the vacuum expectation value of the field σ is exactly the zeroth Fourier harmonic of the real scalar field $|\Phi(x)|$

$$\langle 0|\sigma(x)|0\rangle = \frac{\sigma_0}{\sqrt{2}}. \tag{75}$$

In this manner if one takes into account the relation

$$\langle 0|\Phi|0\rangle = \Phi_0, \tag{76}$$

then one obtains the identification

$$\frac{\sigma_0}{\sqrt{2}} \equiv |\Phi_0|. \tag{77}$$

Consequently the modified Higgs potential (72) is expressed solely via the neutral χ-boson

$$V(|\Phi|) = V(\chi) = \ell_P^2 \frac{m_\chi^2 c^2}{2M_P}\chi^2 + g\frac{E_P\ell_P^4}{4}\chi^4, \tag{78}$$

and the mass of the χ-boson is the mass m of the charged scalar field Φ

$$m_\chi = m. \tag{79}$$

Charged scalar field $\Phi(x)$ can be rewritten in the polar decomposition

$$\Phi(x) = \sigma(x)\exp\{i\theta(x)\}, \tag{80}$$

where its local phase $\theta(x)$ is

$$\theta(x) = \arg\Phi(x) = \arctan\frac{\Im\Phi(x)}{\Re\Phi(x)} = \arctan\frac{\varphi_2(x)}{\varphi_1(x)} + 2\pi n, \tag{81}$$

where $n \in \mathbb{Z}$ and according to the decomposition (73) the neutral scalar fields φ_1 and φ_2 satisfy the following constraint

$$\varphi_1^2(x) + \varphi_2^2(x) = \sigma_0^2 + 2\sigma_0\chi(x) + \chi^2(x), \tag{82}$$

which means that the dependence of the neutral scalar fields φ_1 and φ_2 on the χ-boson is not unambiguous.

In the most general situation the local phase can be also expressed by nonlinear dependence on wave four-vector $k_\mu(x)$

$$\theta(x) = \theta(x^\mu) = \alpha\int^{x^\mu} dx'^\mu k_\mu(x'), \tag{83}$$

where $x^\mu = [ct, \vec{x}]$ is a position four-vector, and $k^\mu = \eta^{\mu\nu}k_\nu$ is given by

$$k^\mu(x) = \left[\frac{1}{c}\omega(\vec{k}(x)), \vec{k}(x)\right], \tag{84}$$

where $\omega(\vec{k})$ is an oscillation frequency. The length of the wave vector $\vec{k}$ is linked to wavelength $\lambda(x)$

$$|\vec{k}| = k(x) = \frac{2\pi}{\lambda(x)}, \tag{85}$$

of the wave identified with the charged scalar field $\Phi(x)$. The momentum of the particle identified with the charged scalar field $\Phi(x)$ shall be $\vec{p} = \hbar\vec{k}$. In the formula (83) α is certain dimensionless parameter, which I shall to establish straightforwardly below. In this manner one can connect the formulas (81) and (83) and obtain the ambiguous expressions for the nautral scalar fields φ_1 and φ_2

$$\varphi_1(x) = \frac{\sigma_0+\chi(x)}{\sqrt{1+\tan\left(\alpha\int^{x^\mu}dx'^\mu k_\mu(x')-2\pi n\right)}}, \tag{86}$$

$$\varphi_2(x) = (\sigma_0+\chi(x))\frac{\tan\left(\alpha\int^{x^\mu}dx'^\mu k_\mu(x')-2\pi n\right)}{\sqrt{1+\tan\left(\alpha\int^{x^\mu}dx'^\mu k_\mu(x')-2\pi n\right)}}. \tag{87}$$

The problem is, however, to establish the wave four-vector $k_\mu(x)$. Note that to the case of the Minkowski space-time one has the identity

$$\int^{x^\mu}dx'^\mu k_\mu(x') = \int^{x_\mu}dx'_\mu k^\mu(x'), \tag{88}$$

which does not hold to the general case of space-times characterized by non-flat metric tensors $g^{\mu\nu}(x)$.

Let us consider the total Lagrangian of the scalar quantum electrodynamics

$$\mathscr{L} = E_P\ell_P\partial_\mu\Phi^\dagger\partial^\mu\Phi - \frac{1}{\ell_P^3}V(|\Phi|) - \frac{1}{4\mu_0}F_{\mu\nu}F^{\mu\nu} - j^\mu A_\mu + \frac{e^2}{M_P\ell_P}|\Phi|^2A_\mu A^\mu, \tag{89}$$

and express this theory in terms of the χ-boson following from the decomposition (73), and the wave four-vector $k_\mu(x)$ following from the formula (83).

By straightforward calculation one can find that the following relations between derivatives of the complex scalar field $\Phi(x)$ and the real scalar field $\sigma(x) = |\Phi(x)|$ hold

$$\partial^\mu\Phi = \exp\left\{i\alpha\int^{x^\mu}dx'^\mu k_\mu(x')\right\}\left[\partial^\mu\sigma(x)+i\alpha\sigma(x)k^\mu(x)\right], \tag{90}$$

$$\partial_\mu\Phi^\dagger = \exp\left\{-i\alpha\int^{x^\mu}dx'^\mu k_\mu(x')\right\}\left[\partial_\mu\sigma(x)-i\alpha\sigma(x)k_\mu(x)\right], \tag{91}$$

so that the kinetic Lagrangian of the charged scalar field is

$$\partial_\mu\Phi^\dagger\partial^\mu\Phi = \partial_\mu\sigma\partial^\mu\sigma+\alpha^2\sigma^2k_\mu k^\mu = \tag{92}$$

$$= \frac{1}{2}\partial_\mu\chi\partial^\mu\chi+\frac{\alpha^2\sigma_0^2}{2}k_\mu k^\mu+\alpha^2\sigma_0\chi k_\mu k^\mu+\frac{\alpha^2}{2}\chi^2k_\mu k^\mu. \tag{93}$$

Similarly one can determine the current

$$j^\mu = -2ec\alpha\sigma^2 k^\mu = -\alpha ec\left(\sigma_0^2 k^\mu + 2\sigma_0\chi k^\mu + \chi^2 k^\mu\right), \tag{94}$$

and the term

$$|\Phi|^2 A_\mu A^\mu = \frac{\sigma_0^2}{2} A_\mu A^\mu + \sigma_0 \chi A_\mu A^\mu + \frac{1}{2}\chi^2 A_\mu A^\mu. \tag{95}$$

Collecting all together one receives the total Lagrangian (89)

$$\begin{aligned}
\mathscr{L} \;=\; & \frac{E_P\ell_P}{2}\partial_\mu\chi\partial^\mu\chi + \alpha^2\sigma_0^2\frac{E_P\ell_P}{2}k_\mu k^\mu + \alpha^2\sigma_0 E_P\ell_P\chi k_\mu k^\mu + \alpha^2\frac{E_P\ell_P}{2}\chi^2 k_\mu k^\mu \\
- \; & \frac{m_\chi^2 c^2}{2M_P\ell_P}\chi^2 - g\frac{E_P\ell_P}{4}\chi^4 - \frac{1}{4\mu_0}F_{\mu\nu}F^{\mu\nu} + ec\alpha\sigma_0^2 k^\mu A_\mu + 2ec\alpha\sigma_0\chi k^\mu A_\mu \\
+ \; & ec\alpha\chi^2 k^\mu A_\mu + \frac{e^2\sigma_0^2}{2M_P\ell_P}A_\mu A^\mu + \frac{e^2\sigma_0}{M_P\ell_P}\chi A_\mu A^\mu + \frac{e^2}{2M_P\ell_P}\chi^2 A_\mu A^\mu.
\end{aligned} \tag{96}$$

The Lagrangian (96) describes the classical theory of three interacting fields – the photon A_μ, the neutral scalar χ, and the non-dynamical massive vector field k_μ. On the one hand, this theory is rather complicated if there is no any restriction for the relation between the wave four-vector k_μ and the electromagnetic four-potential A_μ. On the other hand, such a relation is necessary because of the non-kinetic nature of the gauge field k_μ. Anyway, the choice of the suitable restraint is, in fact, the choice of the appropriate gauge and straightforwardly imparts the physical meaning to the wave four-vector k_μ.

Let us consider first the case of the generic gauge

$$k_\mu = \frac{ec}{E_P\ell_P}A_\mu = \frac{e}{\hbar}A_\mu. \tag{97}$$

In such a situation the classical field theory (96) simplifies to the theory of interacting photon and neutral scalar field

$$\mathscr{L} = \frac{E_P\ell_P}{2}\partial_\mu\chi\partial^\mu\chi - \frac{m_\chi^2 c^2}{2M_P\ell_P}\chi^2 - g\frac{E_P\ell_P}{4}\chi^4 \tag{98}$$

$$- \frac{1}{4\mu_0}F_{\mu\nu}F^{\mu\nu} + \frac{e^2\sigma_0^2}{2M_P\ell_P}(\alpha+1)^2 A_\mu A^\mu \tag{99}$$

$$+ \frac{e^2\sigma_0}{M_P\ell_P}(\alpha+1)^2\chi A_\mu A^\mu \tag{100}$$

$$+ \frac{e^2}{2M_P\ell_P}(\alpha+1)^2\chi^2 A_\mu A^\mu. \tag{101}$$

Writing out the part of the Lagrangian which is quadratic in fields and its derivatives in the form

$$\mathscr{L}_Q = \frac{E_P \ell_P}{2} \partial_\mu \chi \partial^\mu \chi - \frac{m_\chi^2 c^2}{2 M_P \ell_P} \chi^2 - \frac{1}{4\mu_0} F_{\mu\nu} F^{\mu\nu} + \frac{e^2 m_A^2}{2 M_P^3 \ell_P^3} A_\mu A^\mu, \tag{102}$$

one receives immediately the mass of the photon

$$m_A = M_P \ell_P \sigma_0 |\alpha + 1|. \tag{103}$$

The mass (103) of photon does not vanish if $\alpha \neq -1$. This mass formula is completely different from the analogical relation obtained in the previous section in the framework applying the Higgs potential. The crucial and most important difference is that in the present scenario the mass of photon is independent on the mass of the scalar neutral boson of the theory. The mass of χ-boson is a free parameter and can be established by analysis of suitable experimental data. Meanwhile, the mass of photon is solely determined only by two independent constants, the initial datum σ_0 and the parameter α. The parameter α shall be established below, while the initial datum σ_0 possesses the freedom of choice. One can study the concrete physical situations which predict the appropriate values of this number. For example one can put *ad hoc* $\sigma_0 = \frac{2}{3}\sqrt{\pi}\frac{1}{\ell_P}$ and study the Higgs–Hubble inflaton [19] context of the classical field theory which I presented in this section. Recall that nonzero value of the mass of a photon in deeply rooted in interpretation of the experimental data. Such a belief was deduced in early 1970s by A. Mazer, C. Imbert, and S. Huard [3] by observations of total internal reflection to test the Goos–Hänchen effect [4] of the beam shift. Soon after this essential analysis L. De Broglie and J.-P. Vigier [5] discussed this contradiction in the context of the quantum theory of radiation and generalized it to the quantum theory of massive spin-1 photons. Soon after this paper G.J. Troup et al [6] suggested this proposal as the untenable. Albeit, still the question of nonzero mass of a photon is one of the most important and actual problem of physics. One of the best examples is is the analysis due to R. Lakes [7], in which the photon mass is analyzed in frames of the Maxwell–Proca equations. Moreover, the possibility of nonzero mass of a photon has been studied also by Bass and Schrödinger [8] and Feynman [9]. Georgi, Ginsparg, and Glashow [10] have suggested nonzero photon mass to justify cosmic microwave background radiation. Another important point is that since at least three decades there is no success in verification of existence of the Higgs boson arising within the framework of the Standard Model. Such a state of affairs suggests that the Higgs boson is non-existent. In this manner the model presented in this section is consistent with both these empirical facts.

The problem is to establish the value of the parameter α. Let us employ for this the Euler–Lagrange equations of motion arising from the Lagrangian (96). The equations of motion for the neutral χ boson are

$$\Box\chi + g\chi^3 + \frac{m_\chi^2}{M_P^2\ell_P^2}\chi = \left(\alpha k_\mu + \frac{e}{\hbar}A_\mu\right)\left(\alpha k^\mu + \frac{e}{\hbar}A^\mu\right)(\sigma_0+\chi), \tag{104}$$

while the photon A_μ obeys the equations of motion

$$\Box A^\mu - \partial^\mu(\partial_\nu A^\nu) + \frac{\mu_0 e^2\sigma_0^2}{M_P\ell_P}A^\mu = \mu_0 j^\mu - \frac{\mu_0 e^2}{M_P\ell_P}\left(\chi^2 + 2\sigma_0\chi\right)A^\mu, \tag{105}$$

and the non-dynamical vector field k_μ satisfies the equations of motion

$$(\sigma_0+\chi)^2\left(\frac{1}{2}\alpha k_\mu + \frac{e}{\hbar}A_\mu\right) = 0. \tag{106}$$

The equations (106) are non-dynamical, and by this reason are the constraints. They can be solved straightforwardly with the result

$$\alpha k_\mu = -2\frac{e}{\hbar}A_\mu, \tag{107}$$

what suggests that $\alpha = -2$. Involving this solution into the equations of motion (104) one receives

$$\Box\chi + \frac{m_\chi^2}{M_P^2\ell_P^2}\chi + g\chi^3 = \frac{e^2\sigma_0}{\hbar^2}A_\mu A^\mu + \frac{e^2}{\hbar^2}\chi A_\mu A^\mu, \tag{108}$$

while the conserved current (94) simplifies to

$$j^\mu = \frac{2e^2}{M_P\ell_P}\left(\sigma_0^2 + 2\sigma_0\chi + \chi^2\right)A^\mu, \tag{109}$$

so that the equations of motion for a photon (105) become

$$\Box A^\mu - \partial^\mu(\partial_\nu A^\nu) - \frac{\mu_0 e^2\sigma_0^2}{M_P\ell_P}A^\mu = \frac{2\mu_0 e^2}{M_P\ell_P}\left(\chi^2 + \sigma_0\chi\right)A^\mu. \tag{110}$$

Applying, however, the gauge (107) to the total Lagrangian (96) one receives

$$\begin{aligned}\mathscr{L} &= \frac{E_P\ell_P}{2}\partial_\mu\chi\partial^\mu\chi - \frac{m_\chi^2c^2}{2M_P\ell_P}\chi^2 - g\frac{E_P\ell_P}{4}\chi^4 \\ &- \frac{1}{4\mu_0}F_{\mu\nu}F^{\mu\nu} + \frac{e^2\sigma_0^2}{2M_P\ell_P}A_\mu A^\mu + \frac{e^2\sigma_0}{M_P\ell_P}\chi A_\mu A^\mu + \frac{e^2}{2M_P\ell_P}\chi^2 A_\mu A^\mu,\end{aligned} \tag{111}$$

what means that the mass of a photon is

$$m_A = M_P\ell_P\sigma_0, \tag{112}$$

and exactly corresponds to $\alpha = -2$ in the formula (103). The photon mass formula (112) is principally the same as the photon mass formula (36) obtained in the previous section. In this manner in the present model the free parameter σ_0 can be also expressed via the photon mass only

$$\sigma_0 = \frac{m_A}{M_P\ell_P} = \frac{m_A c}{\hbar}, \tag{113}$$

and all deductions related to the Higgs–Hubble inflaton, performed in the previous section, are analogical. The difference between the present and the previous model of mass generation mechanism is, however, essential because of in the present situation the mass of the χ boson remains a free parameter which can be determined by experimental data, and this parameter is independent on the photon mass.

Interestingly, in the present mass generation mechanism the neutral scalar fields which create the charged scalar field have nontrivial decomposition

$$\varphi_1(x) = \frac{\sigma_0 + \chi(x)}{\sqrt{1 - \tan\left(2\frac{e}{\hbar}\int^{x^\mu} dx'^\mu A_\mu(x') + 2\pi n\right)}}, \tag{114}$$

$$\varphi_2(x) = -(\sigma_0 + \chi(x))\frac{\tan\left(2\frac{e}{\hbar}\int^{x^\mu} dx'^\mu A_\mu(x') + 2\pi n\right)}{\sqrt{1 - \tan\left(2\frac{e}{\hbar}\int^{x^\mu} dx'^\mu A_\mu(x') + 2\pi n\right)}}. \tag{115}$$

It is evident that the charged scalar field becomes neutral if and only if

$$\int^{x^\mu} dx'^\mu A_\mu(x') = \left(\frac{p}{2} - n\right)\frac{\pi\hbar}{e} = \left(\frac{p}{2} - n\right)\frac{h}{2e} \quad , \quad p, n \in \mathbb{Z} \tag{116}$$

where the photon A_μ satisfies the system of equations (108) and (110).

The photon states defined by the condition (116) are nontrivial. In fact the non-triviality can be seen straightforwardly from the equations of motion (108) and (110). The equations (108) can be rewritten in the following form

$$A_\mu A^\mu = \frac{\hbar^2}{e^2}\frac{\Box\chi + \dfrac{m_\chi^2}{M_P^2\ell_P^2}\chi + g\chi^3}{\sigma_0+\chi}, \tag{117}$$

and solved immediately as $A_\mu = \left[\dfrac{\varphi(x)}{c}, \mathbf{A}(x)\right]$, where $\varphi(x)$ is the electric potential

$$\varphi(x) = \frac{\hbar c}{e}\sqrt{\frac{\Box\chi + \dfrac{m_\chi^2}{M_P^2\ell_P^2}\chi + g\chi^3}{\sigma_0+\chi} - \left(\frac{e\mathbf{A}}{\hbar}\right)^2}, \tag{118}$$

and $\mathbf{A}$ is the magnetic potential. In this manner the condition (116) says that

$$c\int dt\sqrt{\frac{\Box\chi + \dfrac{m_\chi^2}{M_P^2\ell_P^2}\chi + g\chi^3}{\sigma_0+\chi} - \left(\frac{e\mathbf{A}}{\hbar}\right)^2} + \frac{e}{\hbar}\int d\mathbf{x}\mathbf{A} = \left(\frac{p}{2}-n\right)\pi. \tag{119}$$

It is visible that the Aharonov–Bohm effect [20] is the particular case within this situation. In this manner in the present scenario the Goldstone field $\varphi_2(x)$ vanishes if there is the generalized Aharonov–Bohm effect for the photon. It must be emphasized also that the χ boson and the magnetic potential $\mathbf{A}$ are not arbitrary, but satisfies also the equations of motion (110).

4 Summary

In this paper I have considered two models of mass generation mechanism based on the Higgs mechanism, which is accepted but empirically half-true algorithm for explanation of the masses of particles within the Standard Model. Both the models were based on the concept of charged scalar field Φ. I have considered the case of abelian gauge fields, i.e. photons.

The first model was strictly based on the Higgs potential and lead us to consistent and constructive explanation of the photon mass and the neutral scalar field χ which breaks the symmetry spontaneously. In this model there is dependence between the photon mass and the χ boson mass.

The second model was based on the Higgs potential modified by a constant shift in the variable $|\Phi|$. I have employed also the idea of the wave four-vector k_μ which allowed to see the structure of the theory. The concept of the χ boson was also determined in an another way. This model resulted in the formula for the photon mass, which is practically the same as the formula obtained in the first model. However, in the modified model there is no dependence between the χ boson mass and the photon mass, and the mass of the neutral scalar field remains a free parameter which can be established by experimental data.

Both the models present certain scenarios for consistent explanation of the photon mass. Moreover, both the models present different point of view on the idea of the Higgs boson. I shall to present next results of the models in our next topical papers.

Both the models are seemingly very similar to the standard Higgs mechanism. However, in the Higgs mechanism the photon is explained as massless, and moreover the Higgs field is charged four-real-component complex spinor. In both the scenarios presented in this paper I have no considered *ad hoc* that there is the Higgs field. The scenarios have based on existence of a new scalar neutral boson χ, a Higgs-like particle, and an auxiliary scalar neutral field φ which can be interpreted as a dilaton. The phenomenological status of both the new particle and a dilaton is not clear presently. The second model is much more prospective from this point of view, because of dilaton is massless there and a new scalar particle which is a free parameter and has a mass which can be determined only by experimental data. Moreover, in the second model the mass of a photon does not depend on the mass of a Higgs-like particle, while in the first model the photon mass strongly depends on the mass of χ-boson. In frames of the Standard Model estimation of the mass of a photon is senseless, because by the Higgs mechanism photon is massless in this theory. In the first model of this paper a new scalar particle is massive, and the value of its mass can be estimated by the present day experimental limits on the photon mass.

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