**A Note on the Mass-Energy Relation**

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The famous equation that relates the mass with the energy can be deduced without using the special relativity of Einstein; however, the relation obtained is slightly different.

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The famous equation $E = mc^2$ can be deduced without using the special relativity (SR) of Einstein, such as in [1], or from this other simpler form: when an atom absorbs a photon, the energy is converted into matter, that is, into mass. Thus, an atom at rest of mass $m_0$ recoils with a speed $v$ when it absorbs a photon of an energy $E$ that corresponds to a mass $\mu$. The momentum of the photon would be $p = F\tau = F\lambda/c = W/c = E/c$, where $F$ is the force exerted by the photon, $\tau = \lambda/c$ the duration of the event, $\lambda$ the wavelength, $c$ the speed of the light in the vacuum and $W = F\lambda$ the work done by the photon (the energy $E$ is converted into the work $W$ during the event). (Note that as $E = hf$ and $c = \lambda f$, then $p = E/c = hf/\lambda f = h/\lambda$, where $h$ is the Planck's constant and $f$ the frequency; and also that $\tau = \lambda/c = \lambda/\lambda f = 1/f$). From the conservation of the momentum, $(p_1 + p_2)_{final} = (p_1 + p_2)_{initial}$, where the subscript $1$ is for the atom and the $2$ for the photon; we would have that $mv + 0 = 0 + E/c$, or $mv = E/c = (E/c^2)c = \mu c$, where $m$ is the moving mass of the atom and $\mu = E/c^2 = hf/c^2$ the so-called "effective mass" of the photon. From the conservation of the energy, $(E_1 + E_2)_{final} = (E_1 + E_2)_{initial}$, we would have that $E_a + 0 = E_{0a} + \mu c^2$, $E_a - E_{0a} = \mu c^2$, and as $\mu = m - m_0$, then $E_a = mc^2$, $E_{0a} = m_0c^2$ and $T_a = \mu c^2$, where $E_a$, $E_{0a}$ and $T_a$ are, respectively, the total, rest and kinetic energies of the atom. If we do $m = \gamma m_0$, then $\gamma m_0 = m = m_0 + \mu$, $(\gamma - 1)m_0 = \mu$, $(\gamma - 1)m_0c = \mu c = mv = \gamma m_0 v$, $(\gamma - 1)c = \gamma v$ and $\gamma = (1 - v/c)^{-1}$. Therefore, for a body of rest and moving masses $m_0$ and $m$ its energy would be $E = mc^2 = \gamma m_0c^2 = (1 - v/c)^{-1}m_0c^2$, and for $v << c$, $E \approx m_0c^2 + m_0vc + m_0v^2$, which is a balanced expression but erroneous. In the SR, it is $\gamma = (1 - v^2/c^2)^{-1/2}$ and $E = mc^2 = \gamma m_0c^2 = (1 - v^2/c^2)^{-1/2}m_0c^2$, and for $v^2 << c^2$, $E \approx m_0c^2 + (1/2)m_0v^2$, which is correct because $(1/2)m_0v^2$ is the Newton's kinetic energy. From the absorption process, we cannot obtain the correct value for the gamma factor; we need the SR. (Note that in both cases it is $0 \leq v < c$ since $v = c$ implies $\gamma = \infty$). In short, we have deduced the mass-energy relation without using the SR; however, the relation obtained is slightly different.

[1] An Elementary Derivation of E = mc$^2$. This handout is based on the treatment given on pages 283 to 286 of the book, Einstein's Theory of Relativity, by Max Born, Dover Publications, New York (1965).
http://www.personal.psu.edu/pjm11/Einstein.doc