Author name

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Title

Can electromagnetic scalar waves be radiated by a metal sphere?

Abstract

There is a lot of chattering on the Internet about Tesla waves, vacuum energy, scalar waves and so on. Professor Meyl says he has a complete theory, experimental evidence and apparatus on these waves. In a theoretical paper Van Vlaenderen introduced a generalization of classical electrodynamics for the prediction of scalar field effects. It is said the Monstein has demonstrated the physical existence of such scalar waves. NASA in a report seems to consider such waves as a promising item to be studied. Some other papers appeared in arXiv.
I've already showed that such waves are a consequence of "generalized" Maxwell fields which simply mean space time analytic functions not limited by the Lorenz gauge condition, but accepted instead in a wide sense.
In this paper I remember my ideas on these waves, together with my doubts about their physical existence. In fact, the deduction of the scalar waves equations, together with their physical interpretation, in my opinion demonstrates nothing about the physical existence of scalar waves.
I discuss the experiment of Monstein, and suggest some other experiment.
Obviously I think that the lack of demonstration of the existence doesn't mean the demonstration of inexistence.

1-Forewords: scalar waves, Tesla waves and other things

There is a lot of chattering on the Internet about Tesla waves, vacuum energy, scalar waves and so on. Professor Meyl says he has a complete theory [1], experimental evidence and apparatus on these waves. In a theoretical paper Van Vlaenderen [2] introduced a generalization of classical electrodynamics for the prediction of scalar field effects. It is said the Monstein and Wesley [3] have demonstrated the physical existence of such scalar waves. NASA in a report [4] seems to consider such waves as a promising item to be studied. Some other papers appeared in arXiv on this issue [5], [6], [7].

I've already showed [8] that such waves are a consequence of "generalized" Maxwell fields which simply mean space time analytic functions not limited by the Lorenz gauge condition, but accepted instead in a wide sense. At the same time from the physical point of view this has questionable and "strange" consequences, ie possibly charges and currents in empty space (as if empty space behaves like a plasma).

In this paper I try to present my ideas on these waves (however limited to the electrical ones), together with my doubts about their physical existence. Obviously I think that the lack of demonstration of the existence doesn't mean the demonstration of inexistence.

This work is organized as follows.

Paragraph 2 summarizes how "generalized Maxwell equations" predict electromagnetic scalar waves.

In Paragraph 3 I show the equations pertinent to the radiation by a metal sphere and discusses some physical peculiarities of these equations.

In Paragraph 4 I briefly remember the Monstein experiment.

Paragraph 5 discusses what is true amazing reality of this phenomenon that is a wave of longitudinal electric field (E_z wave) and a wave of charge (ρ wave) exanging energy in free space.

Paragraph 6 proposes a replica of the Monstein experiment in anechoic chamber.

Appendices 1 and 2 are dedicated to some boring calculations.

In Appendix 3 I summarize my thoughts on this topic (scalar waves) over the years.

In Appendix 4 I show that these (hypothetical) scalar waves transmitted by a metal sphere are no more that a sub case of electromagnetic waves predicted by the generalized Maxwell equations.

Note that in most cases I express my ideas with the language of Clifford algebra applied to electromagnetic and analytic functions. (For a good recent résumé on this issue, see [8]).

However I will translate the main results in the usual 3D vector calculus.

2-Scalar waves and generalized Maxwell equations

I refer here to a complex Clifford algebra based on $1, i, j, T$. For details and symbols refer to [8].

I assume for Maxwell equations the analyticity in a "wide sense".

In conditions of maximum generality the analyticity condition $\partial^* F = 0$ applied to a 8 components F results in terms which are all interpretable in electromagnetic sense.

The resolution of these equations may seem a rather complicated problem, it is already difficult to solve the Maxwell equations without currents or fields derived from charges and currents that generate them. In fact, paradoxically, the opposite is true: that is easy to produce solutions in abundance in fully automatic mode. How? The premise holds, that if any "thing" A is harmonic ($\partial\partial^* A = 0$), then ∂A is analytic $\partial^*(\partial A) = 0$.

So the thing A , if harmonic, can be anything: a scalar $\varphi(z,t)$, or $\varphi(x,y,z,t)$, a thing with indices, such that $e^{i(\omega t - kz)}$ which, if $\omega = k$, is harmonic, and so on. To be relativistic, you may take the "thing" A as a four vector, or you may define it as a part of a four vector.

We can say that A playing the role of potential, “generalized potential” of the field, no longer obeying the Lorenz gauge condition but the more "simple" condition:

$$\square A = \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} - \frac{\partial^2}{\partial \tau^2}\right) A = \partial^* \partial A = 0$$

All which follows is automatic.
Differentiating, we get a field $F = \partial A$ that provides a structure of electric fields, magnetic fields, charges and currents, interpreted every time, and who satisfies the Maxwell equations in the sense that these currents generate these fields according to Maxwell's equations.
(At least this formally. It should of course be met physical condition, which make physically acceptable those current and those fields).
We can say that these currents generate these fields according to Maxwell's equations , but at this point would be equally justified in saying that these fields generate the current ... and then in general we tend to say that those fields and currents <u>are self-sustaining</u> .
I think it is important to note that these "generalized" equations to 8 components are in a certain sense <u>not different</u> from the usual Maxwell equations. Are the usual Maxwell's equations and more precisely <u>are the usual Maxwell equations in the presence of charges and currents</u>. The only (….. so to speak) difference is that the charges and currents that appear in the equations are no longer <u>assigned</u> outside (and then they will derive these fields), but appear automatically as part of the solution.
In other words, the electromagnetic fields $\vec{E}, \vec{H}$ that the equations provide are the fields that were (are) drawn from the usual Maxwell equations in the presence of charges and currents <u>if you were able to generate this distributions of charges and currents.</u>
So we must consider from time to time if charges and currents correspond to physically realizable situations. That is what happens with these “scalar longitudinal waves”.

Just to start with an example, take in general a physical quantity:

$$U = (U_1 + iU_2 + jU_3 + TU_4)$$

The analyticity condition for it leads to

$$\begin{array}{ll}
1 & \dfrac{\partial U_1}{\partial x} - \dfrac{\partial U_2}{\partial y} - \dfrac{\partial U_3}{\partial z} + \dfrac{\partial U_4}{\partial \tau} = 0 \\
i & \dfrac{\partial U_2}{\partial x} + \dfrac{\partial U_1}{\partial y} = 0 \\
j & \dfrac{\partial U_3}{\partial x} + \dfrac{\partial U_1}{\partial z} = 0 \\
T & \dfrac{\partial U_4}{\partial x} + \dfrac{\partial U_1}{\partial \tau} = 0 \\
ij & \dfrac{\partial U_3}{\partial y} - \dfrac{\partial U_2}{\partial z} = 0 \\
iT & \dfrac{\partial U_4}{\partial y} - \dfrac{\partial U_2}{\partial \tau} = 0 \\
jT & \dfrac{\partial U_4}{\partial z} - \dfrac{\partial U_3}{\partial \tau} = 0
\end{array}$$

We can see for example that if $U_4 = 0$, these conditions <u>require</u> that (U_1, U_2, U_3) are independent of time. For the remaining components (U_1, U_2, U_3) we can see that ... they mean the cancellation of the rotor and divergence for the conjugate $(U_1, -U_2, -U_3)$.

(Note that this immediately generalizes what happens for 2D analytic functions).
In the general case the equations mean, for the conjugate $(U_1, -U_2, -U_3, -U_4)$, putting $\vec{v} = (U_1, -U_2, -U_3)$ and $v_4 = -U_4$:

$$rot\ \vec{v} = 0$$

$$div\,\vec{v} + \frac{\partial v_4}{\partial \tau} = 0$$

$$\frac{\partial \vec{v}}{\partial \tau} + grad\ v_4 = 0$$

ie the equations <u>of motion of a irrotational, compressible fluid</u>, in 3 dimensions (if $v_4 = 0$, even stationary).

Let's go now to the electromagnetic situation.
If we introduce a quantity F electromagnetic field [8]:

$$F = \left(E_x + iE_y + jE_z - TH_\tau\right) + Tji(H_x + iH_y + jH_z + TE_\tau)$$

the analyticity of F means

$$\partial^* F = 0$$

ie:

$$\left(\frac{\partial}{\partial x} + i\frac{\partial}{\partial y} + j\frac{\partial}{\partial z} + T\frac{\partial}{\partial \tau}\right)\left[\left(E_x + iE_y + jE_z - TH_\tau\right) + Tji\left(H_x + iH_y + jH_z + TE_\tau\right)\right] = 0$$

Developing and then separating the "parts" 1, i, j, etc, we have 8 scalar equations:

$$1 \qquad \frac{\partial E_x}{\partial x} - \frac{\partial E_y}{\partial y} - \frac{\partial E_z}{\partial z} - \frac{\partial H_\tau}{\partial \tau} = 0$$

$$i \qquad \frac{\partial E_x}{\partial y} + \frac{\partial E_y}{\partial x} + \frac{\partial H_z}{\partial \tau} - \frac{\partial E_\tau}{\partial z} = 0$$

$$j \qquad \qquad \text{etc.}$$

With 6 components we have Maxwell's equations in empty space. With non-zero H_τ and E_τ other terms appear, related to density of electric and magnetic charges and currents.
You can see that the term H_τ is enough to give Maxwell's equations with charges and currents.
Consider for example the first equation written. Moving on as usual to the conjugate $\left(E_x - iE_y - jE_z + TH_\tau\right)$ and then placing $\rho = -\frac{\partial H_\tau}{\partial \tau}$ we see that it looks like:

$$div\,\vec{E} = \rho$$

In this particular situation we are facing now, the electromagnetic field is only "electric" and reduces to $F = \left(E_x + iE_y + jE_z - TH_\tau\right)$.
For the conjugate $\left(E_x - iE_y - jE_z + TH_\tau\right)$ putting $\vec{E} = (E_x, -E_y, -E_z)$, we have the "generalized" Maxwell equations:

$$\begin{cases} rot\vec{E} = 0 \\ div\vec{E} + \dfrac{\partial H_\tau}{\partial \tau} = 0 \\ \dfrac{\partial \vec{E}}{\partial \tau} + gradH_\tau = 0 \end{cases}$$

Note that instead of taking the conjugate components, an alternative deduction with Clifford algebra is possible (see Appendices 1, 2).
Let's go now to this particular case, ie (hypothetical) scalar waves radiated by a metal sphere.

3-Discussion

Deduce the (hypothetical) scalar waves radiated by a metal sphere as generalized Maxwell field (ie: an analytic function I'll name U).
Refer [8] to the Cauchy Riemann operator ∂^* in spherical coordinates

$$\partial^* = \frac{z}{r}\frac{\partial}{\partial r} + \frac{z}{r^2}\Gamma^* + T\frac{\partial}{\partial t}$$

where of course $z = x + iy + jz$.
Take $\Phi = \frac{1}{r} sin\,(kr - \omega t)$, harmonic, and so $U = \partial\Phi$ as analytic. We get:

$$U = \partial\Phi = \frac{z^*}{r}\frac{\partial\Phi}{\partial r} - T\frac{\partial\Phi}{\partial t}$$

$$\frac{\partial\Phi}{\partial r} = \frac{k}{r}cos(kr - \omega t) - \frac{1}{r^2}sin\,(kr - \omega t)$$

$$\frac{\partial\Phi}{\partial t} = -\frac{\omega}{r}cos(kr - \omega t)$$

$$U = \partial\Phi = \frac{z^*}{r}(\frac{k}{r}cos(kr - \omega t) - \frac{1}{r^2}sin\,(kr - \omega t)) + T\frac{\omega}{r}cos(kr - \omega t)$$

We can directly check that $\partial^* U = \frac{z}{r}\frac{\partial U}{\partial r} + \frac{z}{r^2}\Gamma^* U + T\frac{\partial U}{\partial t} = 0$.
We get step by step:

$$\frac{z}{r}\frac{\partial U}{\partial r} = -\frac{k}{r^2}cos(kr - \omega t) - \frac{k^2}{r}sin(kr - \omega t) + \frac{2}{r^3}si\,n(kr - \omega t) - \frac{k}{r^2}cos(kr - \omega t) + T\omega\frac{z^*}{r}(-\frac{1}{r^2}cos(kr - \omega t) - \frac{k}{r}sin(kr - \omega t)$$

and then:

$$T\frac{\partial U}{\partial t} = T\frac{z^*}{r}\frac{k\omega}{r}sin(kr - \omega t) + T\frac{z^*}{r}\frac{\omega}{r^2}cos(kr - \omega t) + \frac{\omega^2}{r}sin(kr - \omega t)$$

We need also $\Gamma^*\frac{z^*}{r} = 2\frac{z^*}{r}$

$$\frac{z}{r^2}\Gamma^* U = \frac{2}{r}(\frac{k}{r}cos(kr - \omega t) - \frac{1}{r^2}sin\,(kr - \omega t))$$

so finally:

$$\partial^* U = -\frac{k}{r^2}cos(kr - \omega t) - \frac{k^2}{r}sin(kr - \omega t) + \frac{2}{r^3}si\,n(kr - \omega t) - \frac{k}{r^2}cos(kr - \omega t) + T\omega\frac{z^*}{r}(-\frac{1}{r^2}cos(kr - \omega t) - \frac{k}{r}sin(kr - \omega t) + T\frac{z^*}{r}\frac{k\omega}{r}sin(kr - \omega t) + T\frac{z^*}{r}\frac{\omega}{r^2}cos(kr - \omega t) + \frac{\omega^2}{r}sin(kr - \omega t) + \frac{2}{r}(\frac{k}{r}cos(kr - \omega t) - \frac{1}{r^2}sin\,(kr - \omega t))$$

$$\partial^* U = 0 \qquad QED$$

This field is radiated by a sphere of radius R.
What do I mean?
The equipotential surfaces of this field are spherical surfaces. The spherical surface at $r = R$ has a potential $\Phi = \frac{1}{R}sin\,(kR - \omega t)$.

Suppose it constructed of metal and connected with a potential $\Phi = \frac{1}{R} sin\,(kR - \omega t)$.
We can say, as Maxwell said (Maxwell [9] Chap. XI, "Theory of electric images and electric inversion", obviously modified for the present circumstances):
"So if we keep the metallic shell in connection with this potential $\Phi = \frac{1}{R} sin\,(kR - \omega t)$, outside it will remain the same as before. For the surface of the sphere still remains at the same potential as before, and no changes has be made in the exterior electrification".

So we may assume that the spherical antenna with potential $\Phi = \frac{1}{R} sin\,(kR - \omega t)$ is the origin of the (analytic) scalar field $U = \frac{z^*}{r}(\frac{k}{r} cos(kr - \omega t) - \frac{1}{r^2} sin\,(kr - \omega t)) + T\frac{\omega}{r} cos(kr - \omega t)$.
What is a such field?
Summarize.
We have shown that the field $U = \frac{z^*}{r}(\frac{k}{r} cos(kr - \omega t) - \frac{1}{r^2} sin\,(kr - \omega t)) + T\frac{\omega}{r} cos(kr - \omega t)$ is analytic. Then we have shown that the conjugate obeys the equations

$$(1) \qquad \begin{cases} rot\vec{E} = 0 \\ div\vec{E} + \dfrac{\partial H_\tau}{\partial \tau} = 0 \\ \dfrac{\partial \vec{E}}{\partial \tau} + gradH_\tau = 0 \end{cases}$$

From the conjugate $(U)^*$ the explicit expression for $\vec{E}$ is:

$$(2) \qquad \vec{E} = \hat{r}(\frac{k}{r} cos(kr - \omega t) - \frac{1}{r^2} sin\,(kr - \omega t))$$

where $\hat{r}$ is the unit vector $\hat{r}$ along $\vec{r}$.
The explicit expression for H_τ is:

$$(3) \qquad H_\tau = -\frac{\omega}{r} cos(kr - \omega t)$$

If we take for granted that the meaning of $div\vec{E}$ is ρ, which means the Gauss theorem, from $div\vec{E} + \frac{\partial H_\tau}{\partial \tau} = 0$ we deduce that the medium exhibits a charge $\rho = -\frac{\partial H_\tau}{\partial \tau}$.

$$(4) \qquad \rho = -\frac{\partial H_\tau}{\partial \tau}$$

In the same time if we interpret the term $\frac{\partial \vec{E}}{\partial \tau}$ as a current $\vec{J}$ ie $-\frac{\partial \vec{E}}{\partial \tau} = \vec{J}$ (as usual, and in proper units), we get from (1)

$$(5) \qquad \vec{J} = gradH_\tau$$

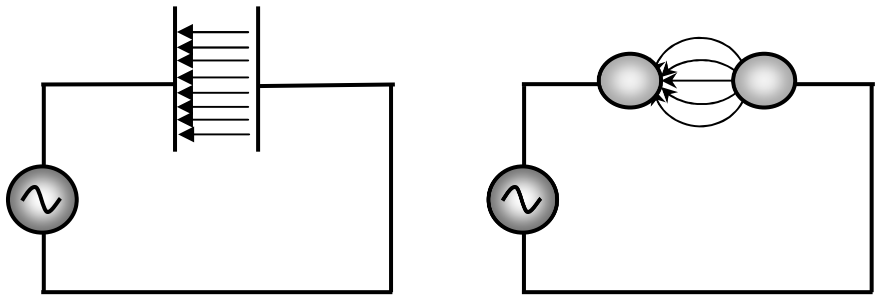
But H_τ is harmonic, being a part of an analytic function, which means $\nabla(\nabla H_\tau) - \frac{\partial}{\partial \tau}(\frac{\partial H_\tau}{\partial \tau}) = 0$, so we get from (4), (5):

$$(6) \qquad div\,\vec{J} + \frac{\partial \rho}{\partial \tau} = 0$$

which is the conservation of charge.
You can compare now with what is asserted in [2], [3], [5], [6], [7]. Some (or many) results coincides, even if with different approach and interpretations. For example, (2) coincides with Monstein Wesley [3], formula (5). Equations (1) coincide with Van Vlaenderen [2], (18), (19), (20) in free space ($-H_\tau$ being the Van Vlaenderen S).

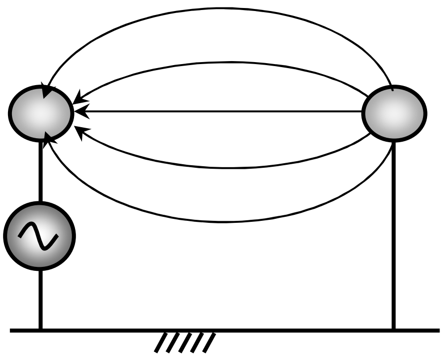
4-Experimental verification

Of course a place in which we have a longitudinal electric field together with an oscillating charge <u>seems to be</u>, as Monstein and Wesley say, "a condenser" [3]. However we are accustomed to think in term of Maxwell displacement current, and not scalar waves.



In [3] the Authors say:
"It is well known that energy can be transmitted from one plate of a parallel plate condenser to the other. Thus, it is trivially obvious that energy is transmitted in the direction of an electric $\vec{E}$ field across the condenser. Since normally the plates are much closer than a wavelength apart, this is usually assumed to be no proof of longitudinal electrodynamic waves. Yet the theory presented above is quite independent of the size of the wavelength; so the flow of energy across an ordinary condenser does, in fact, demonstrate the existence of longitudinal electrodynamic waves. One of us (Monstein) extended the distance between two parallel plates of a parallel plate condenser from near to more than a wavelength and continued to register a flow of energy from one plate to the other, as expected from the theory for longitudinal electrodynamic waves".
A "demonstration of the longitudinality of the observed waves" then follows.

But a doubt arises, ie have we made up nothing more than a big capacitor?



To be sure, we must go over, to the true amazing reality of this phenomenon.

5-The sound of space

What is the true amazing reality of this phenomenon?
It is shown by the simple generalized Maxwell equations (only $E_z \neq 0,\ H_\tau \neq 0$):

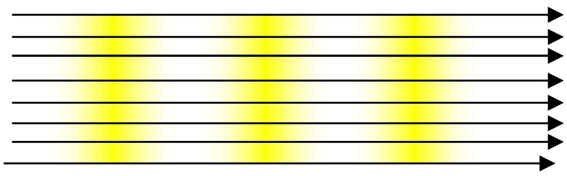
$$\begin{cases} \dfrac{\partial E_z}{\partial z} + \dfrac{\partial H_\tau}{\partial ct} = 0 \\ \dfrac{\partial H_\tau}{\partial z} + \dfrac{\partial E_z}{\partial ct} = 0 \end{cases}$$

Compare with some other system which gives rise of waves and/or oscillations, example a transmission line, sound [10] etc.

$$\begin{cases} \dfrac{\partial v}{\partial z} + L\dfrac{\partial i}{\partial t} = 0 \\ \dfrac{\partial i}{\partial z} + C\dfrac{\partial v}{\partial t} = 0 \end{cases}$$

$$\begin{cases} \rho_0\dfrac{\partial u}{\partial z} + \dfrac{\partial \rho}{\partial t} = 0 \\ c^2\dfrac{\partial \rho}{\partial z} + \rho_0\dfrac{\partial u}{\partial t} = 0 \end{cases}$$

In any cases equations are two coupled first-order ordinary differential equations, which may be solved simultaneously to find v, i or E_z, H_τ or ρ, u.
In any case to the second-order they give the wave equation.
In any case the energy carried by the wave converts back and forth between potential energy and kinetic energy.
Sound is a good example of such a longitudinal wave.
Matter in the medium is periodically displaced by a sound wave, and thus oscillates.
The air molecules are pushed back and forth in the same direction that the wave propagates.
The wave causes the air molecules to move horizontally, at the same time they are being stretched and compressed. This allows you to see the zones of compression and rarefaction (stretching) that are steadily progressing to the right.



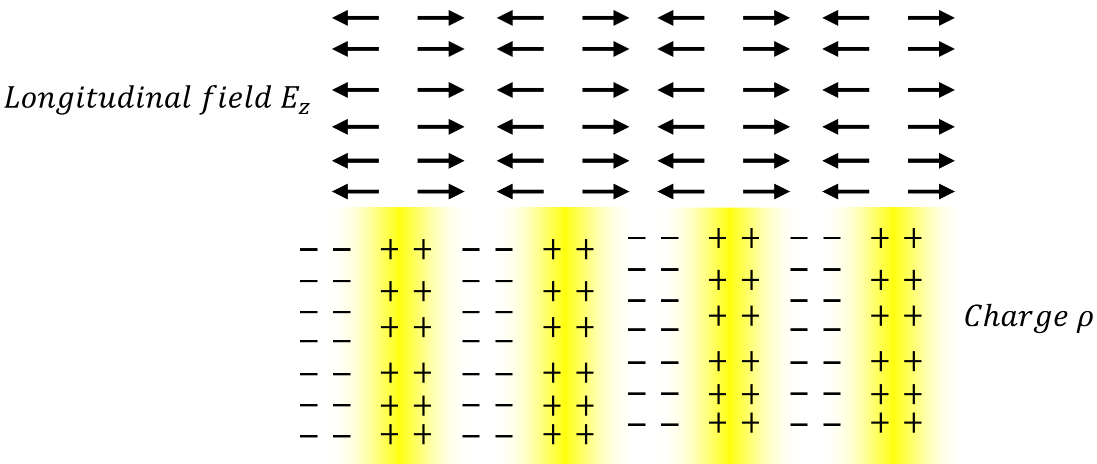
The energy carried by the sound wave converts back and forth between the potential energy of the extra compression of the air (in case of longitudinal waves) and the kinetic energy of the oscillations of the molecules.
Coming back to our electromagnetic case, a simultaneous solution of the above mentioned differential equations is (check):

$$\begin{cases} \dfrac{\partial E_z}{\partial z} + \dfrac{\partial H_\tau}{\partial ct} = 0 \\ \dfrac{\partial H_\tau}{\partial z} + \dfrac{\partial E_z}{\partial ct} = 0 \end{cases} \quad \rightarrow \quad E_z = \sin(kz - \omega t)\,; \quad H_\tau = \sin(kz - \omega t)\,; \quad k = \frac{\omega}{c}$$

(See also [8], Exercise 5).
If we take for granted that the meaning of $div\vec{E}$ is ρ, which means the Gauss theorem, from $div\vec{E} + \frac{\partial H_\tau}{\partial \tau} = 0$ we deduce that the medium exhibits a charge $\rho = -\frac{\partial H_\tau}{\partial \tau}$. So, summing up, we have two waves, a wave of longitudinal electric field (E_z wave) and a wave of charge (ρ wave):

$$\begin{cases} E_z = \sin(kz - \omega t) \\ \rho = kcos(kz - \omega t) \end{cases}$$

Zeroes of electric field correspond to maxima (or minima) of charge. The electric field E_z goes (as expected) from positive ρ to negative ρ.



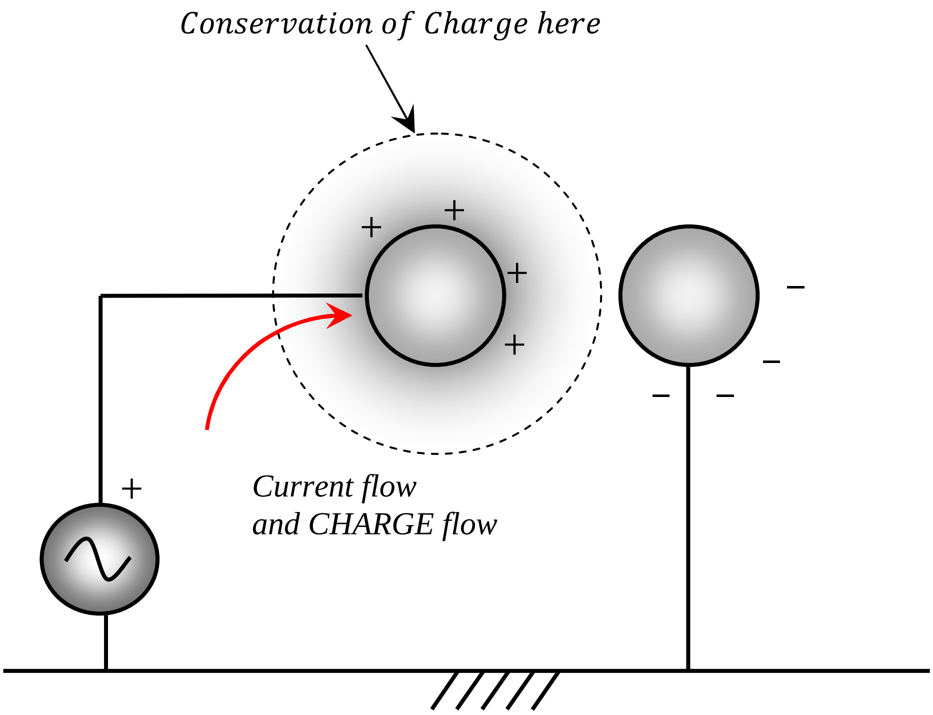
More, the relation that follows

$$(E_z)^2 + \frac{1}{k^2}\rho^2 = sin^2\varphi + cos^2\varphi = const = 1$$

tells us (in due units) that the energy carried by the wave is constant, changing from “potential” to “kinetic” and vice versa.
Note also that the E_z and ρ waves we are facing now <u>are not attended by any magnetic field</u>.
Summarizing, we see a very peculiar phenomenon.
The situation is completely different from what happens between plates of a parallel plate condenser, despite to what Monstein says [3] ie:
“the flow of energy across an ordinary condenser does, in fact, demonstrate the existence of longitudinal electrodynamic waves”.
This is not true.
Despite the appearance, the presence of the wave E_z (and $\frac{\partial E_z}{\partial t}$) means “displacement current”, doesn’t mean “longitudinal wave”.
What do I mean?
Consider that, as we have noted, the E_z (and $\frac{\partial E_z}{\partial t}$) waves <u>are not attended by any magnetic field</u>.
Conversely (Tamm, [11]):
“In other words, we shall assume that in the magnetic respect *the displacement currents* are equivalent to the conduction currents, ie *induce a magnetic field according to the same laws as the conduction currents do*”.

Also says Tamm:
"We shall note in conclusion that from a modern viewpoint (unlike the original notions of J. Maxwell who was the first to establish the existence of displacement currents and who gave them this name) *conduction currents* on one hand and *displacement currents* in a vacuum on the other, notwithstanding the similarity of their names, are in essence *absolutely different physical concepts*. Their *only* common feature is that they induce a magnetic field in an identical manner (omissis). The most significant distinction is that conduction currents correspond to the motion of electric charges whereas a "pure" displacement current - *a displacement current in a vacuum – corresponds only to a change in the intensity of the electric field and is not attended by any motion of electric charges* or other particles of a substance".
(The italic is in the Tamm book).

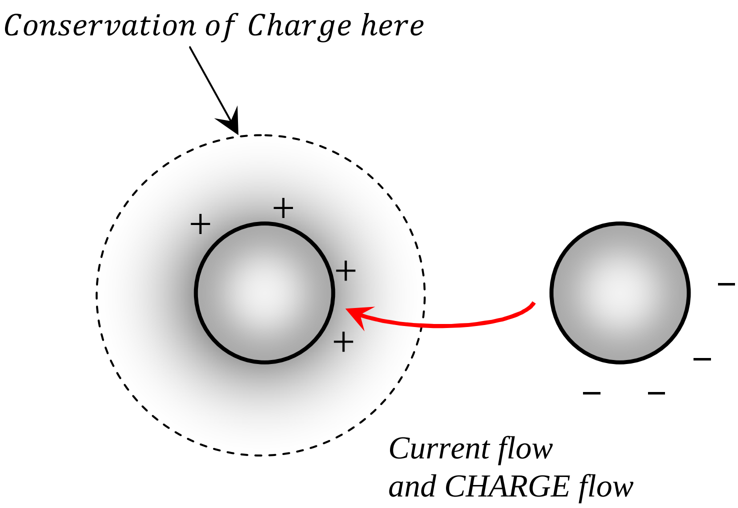
If we think about, the situation is quite clear. The difference is the current flowing through the capacitor and the circuit. Consider a sphere with varying potential $\Phi \propto \frac{1}{r} sin\omega t$. When the potential changes (ex. with respect to the earth) charges must go somewhere, or must came from somewhere. In figure I show a growing potential. Charge comes from the left. No charge at all flows between the two spheres, but only displacement current.



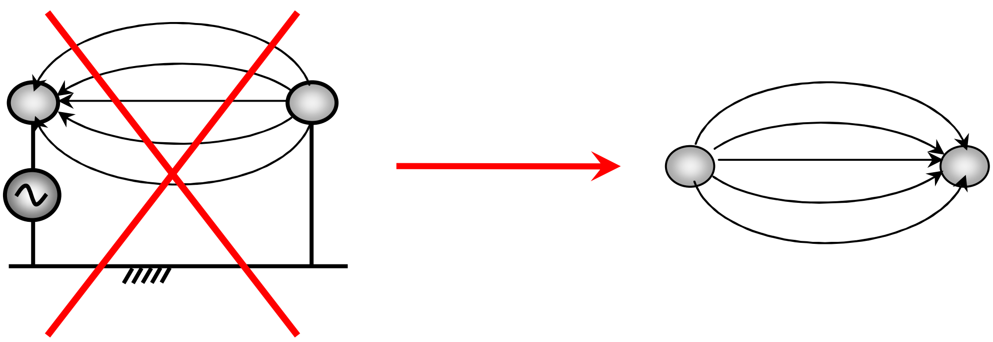
In other words, in order to assure the conservation of charge, a current must flow.
In figure the current which assures the charge conservation is depicted in red.

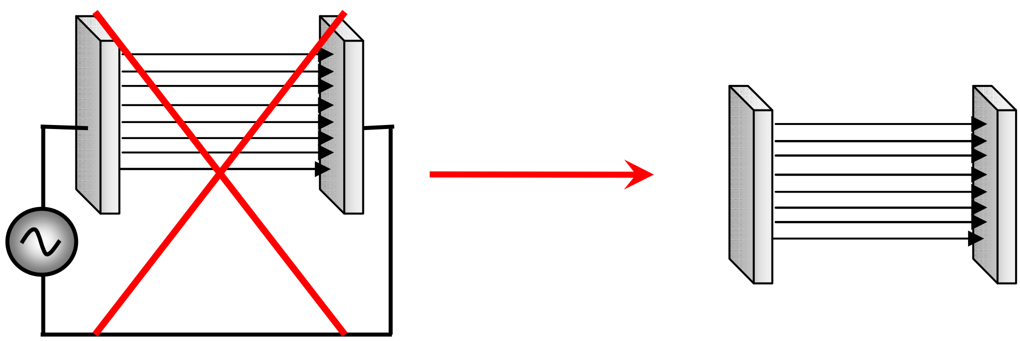
Consider now the supposed scalar field we previously described, and a sphere with varying potential $\Phi \propto \frac{1}{r} sin\omega t$. When the potential changes (ex. with respect to the earth) charges again must go somewhere, or must came from somewhere.

In figure I show a growing potential.
Again in order to assure the conservation of charge, a current must flow.
The current which assures the charge conservation is depicted in red.
Charge comes from the right. Charge flow in empty space and comes from space.
All this could appear quite strange, but is what the equations describe (…… if it exists).

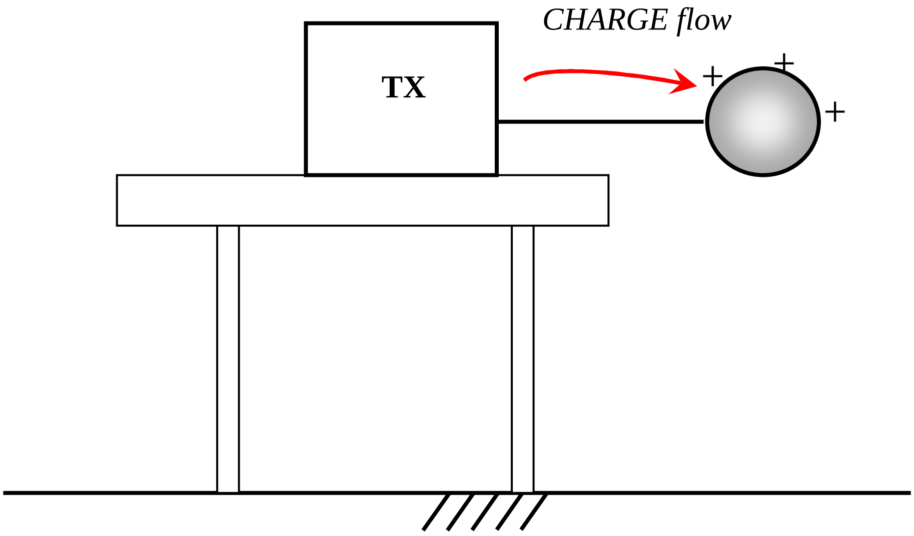


So in my opinion we must avoid experiments involving closed circuits, but we have only to consider spheres or parallel plates in free space. Two very indicative tests (with spheres or with flat plates) would be the absence of magnetic field and the lack of closed circuits. Another crucial test would be not only the presence of longitudinal lines of force E_z, but also waves of electric charge between transmitter and receiver.

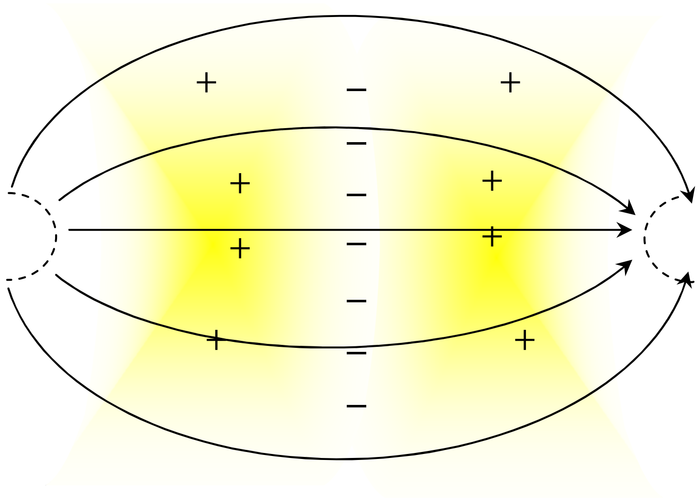




It is not so easy to have a sphere both in free space and with varying potential $\Phi \propto \frac{1}{r} sin\omega t$.
It is quite easy to avoid ground connection, we can realize an experiment so to say "airborne", ie transmitter supplied by a battery and insulated from ground.
The same for receiver.
But the doubt remains, if electric charge is exchanged with transmitter, not with space.



A very convincing experiment would be measure both a wave of longitudinal electric field (E_z wave) and a wave of charge (ρ wave) exanging energy in free space, and possibly self – sustaining. This "oscillation of space" could be very difficult to realize (admitted that it were possible) but would be of course very convincing. This really would be a new phenomenon, a new astonishing phenomenon. Oscillations should be sustained at both ends by appropriate boundary conditions (not a metallic surface, which doesn't reflect the perpendicular E field).



Of course, Monstein and Wesley [12] say:
“Since the project was a ’side effect’ of our daily engineering work, there was and is no regular budget available for further detailed investigations. (….) Unfortunately, it was not possible to use a controlled environment in the form of an outdoor antenna range. (….)The character of the waves was demonstrated to be longitudinal by introducing a polarizer between sender and receiving power meter. (….) We also know that further investigations are necessary, and we hope that other, better situated investigators will improve the experimental setup and will repeat our observations”.

I agree that further investigations are necessary.

6- Replica of the Monstein experiment in a controlled environment

Of course, the experiment I mentioned earlier is probably a difficult experiment.
More, as I said in Paragraph 1, unfortunately from the physical point of view we are faced with strange consequences, ie we must justify charges and currents in empty space (as if empty space behaves like a plasma).
We first examine how to overcome these conceptual difficulties.
Wanting a) to believe in longitudinal waves, b) to give a physical interpretation and c) avoid the "plasma", I think we are faced with two alternative interpretations, both very questionable indeed.
Alternative 1):
$(-\frac{\partial H_\tau}{\partial \tau})$ is NOT electric charge. It’s a source (or sink) of lines of force of $\vec{E}$, which do not corresponds to electric charge. But then what? It is, as the formula $div\vec{E} = -\frac{\partial H_\tau}{\partial \tau}$ says .... simply a point in which $div\vec{E}$ is non-zero.
$(-\frac{\partial H_\tau}{\partial \tau})$ obeys a conservation law, as the formulas say.
(Besides, this would pair with the other ”strange” fact that his “current” is a strange current, does NOT generate the magnetic field).
This, I believe, is tantamount to a modification of Gauss' law as hypothesized by some authors such as Van Vlaenderen. Also, I think, re-awaken certain ideas of Faraday about lines of force with respect to charges.
What propagates are ….. compressions and rarefactions of $(-\frac{\partial H_\tau}{\partial \tau})$. No more the need to have “plasma”.
Alternative 2):
the electric charges never exist as physical entity, but charges are, in a vacuum, points with $div\vec{E} = -\frac{\partial H_\tau}{\partial \tau}$ different from zero. I.e. the charge ρ isn’t a physical entity, it is precisely … $(-\frac{\partial H_\tau}{\partial \tau})$.
The Gauss law continues to hold. The existence, experimental, of electric charge (ie electron) would correspond to a region free of singularities which is a source of lines of force.
What propagates, again, are compressions and rarefactions of $(-\frac{\partial H_\tau}{\partial \tau})$. No more the need to have “plasma”.
After all, strange as it may seem, Einstein hated the electrons as singularities and said: "I will never stop until I’ll able to demonstrate the presence of a region of space WITHOUT SINGULARITIES, and on which the surface integral of $div\vec{E}$ is not zero”.
Anyway, do we have some experiment that can prove the existence of $div\vec{E}$ waves in vacuum, <u>independent of what $div\vec{E}$ means</u>, and without using a difficult experiment which I mentioned earlier?
Yes we have (I think).

To illustrate my idea, I resort to an elegant argument with which Sommerfeld [13] demonstrates the impossibility of the existence of longitudinal waves in a vacuum.
So says Sommerfeld ([13], pag. 34, with slightly modified symbols):
“We now turn to equation:

$$(a) \qquad \left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}-\frac{\partial^2}{\partial \tau^2}\right)\vec{E}=0$$

with the auxiliary condition already made use of

$$(b) \qquad div\vec{E}=0$$

We seek, in particular, solutions of (a) which are independent of y and z. For purely periodic time dependence these represent monochromatic plane waves which advance along the x-axis. We shall show that they are necessarily transverse. In view of the assumed independence of y and z the function Eq. (b) reduces to

$$\frac{\partial E_x}{\partial x}=0$$

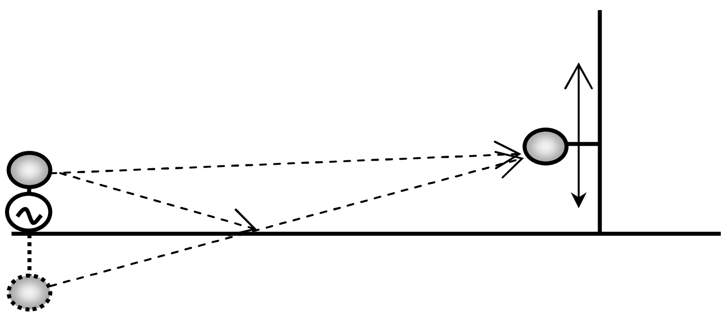
Equation (a) yields accordingly:

$$\frac{\partial^2 E_x}{\partial \tau^2}=0$$

E_x would thus be a linear function of t, which is inconsistent with the periodic dependence on t. Hence $E_x=0$”.
Now reverse the reasoning. Suppose we can show, in vacuum, the existence of a *longitudinal* wave $E_x=sin(kx-\omega t)$, solution of Eq. (a) independent of y and z.
Equation (b) no more holds, but instead:

$$\frac{\partial E_x}{\partial x}=kcos(kx-\omega t)$$

Hence $div\vec{E}$ is is not zero, it’s equal to *something*.
So finally the existence of a longitudinal plane wave $E_x=sin(kx-\omega t)$ indirectly shows (<u>independent of what $div\vec{E}$ means</u>) that
-surely there is $div\vec{E}\neq 0$ *in vacuum*;
-surely there are *waves of $div\vec{E}$ which advance along the x-axis.*
But do we have some experiment that can prove the existence of a longitudinal travelling wave $E_x=sin(kx-\omega t)$? I try to propose some tentative hypotheses(see figure).



The experiment I propose is practically a Monstein experiment replica, in controlled environment (anechoic chamber). The clue of experiment is what they name, in radar technique, “lobing”.
The ground, instead of to be avoided, should have reflection coefficient 1.
The lobing ie interference in the receiving antenna assures that the wave, both the direct and reflected ray, has a phase shift depending of the path length ie assures a space dependence $E_x = sin(kx - \omega t)$

This is of course only a preliminary definition of the experimental configuration.
All parameters must be defined to ensure a variety of circumstances.
Some examples.
We need to prevent other elements involved in the transmission - reception, other than spheres.
For example, transmitter and receiver must be probably connected to the two spheres by means of coaxial cables with the shield connected to ground. This in order to avoid, as much as possible, they act as antennas.
You must put some device to verify that the field is truly longitudinal.
You have to size the parameters (in particular the physical dimensions and frequency used) to ensure that the transmission takes place in far field and not in near field. Should be avoided a near field coupling that could lead to erroneous interpretations.
Must be measured with certainty and precision "minima" of lobing. Et cetera.

7-Conclusion

The deduction I’ve made here for the scalar waves equations, together with its physical interpretation, in my opinion demonstrates <u>nothing</u> about the physical existence of scalar waves.
The same holds for other works on this issue.
Much more work is needed.
Decisive would be an experiment in which a resonance is measured, in free space, sustained at both ends by appropriate boundary conditions.
However, this is probably difficult, or perhaps impossible. A possible alternative is a replica of the Monstein experiment in anechoic chamber.

8-Acknowledgements

I would like to thank my friend Dr. Alberto Bicci for his precious suggestions and criticisms on this subject.

Appendix 1

As I have pointed out in [14], the analyticity of F can be written:

$$\partial^* \vec{F} = 0$$

where:

$$\vec{F} = F\hat{\imath}$$

who has the physical components of $\vec{E}, \vec{H}$.

The *y* and *z* components (and *t* components if any) are not the same as in F, but the same with change of sign ie the conjugate components.

(Really instead of $\vec{E}, \vec{H}$ we must have "time-like bivectors" in Space Time Algebra (Hestenes, [15]), so we should consider $F\hat{\imath}\hat{T}$, not $F\hat{\imath}$, but for the present scope $F\hat{\imath}$ is enough).

Write also:

$$\vec{\partial}\vec{F} = 0$$

where:

$$\vec{\partial} = \hat{\imath}\partial^*$$

It is now immediate and very smart from $\vec{\partial}\vec{F} = 0$ to derive the "generalized" Maxwell's equations with div and rot.

Explicitly $\vec{\partial}\vec{F} = 0$ gives:

$$\vec{\partial} = \left(\frac{\partial}{\partial x}\hat{\imath} + \frac{\partial}{\partial y}\hat{\jmath} + \frac{\partial}{\partial z}\hat{k} + \frac{\partial}{\partial \tau}\hat{T}\right)$$

$$\vec{F} = \vec{E} - \hat{T}H_\tau$$

$$\vec{\partial}\vec{F} = \left(\frac{\partial}{\partial x}\hat{\imath} + \frac{\partial}{\partial y}\hat{\jmath} + \frac{\partial}{\partial z}\hat{k} + \frac{\partial}{\partial \tau}\hat{T}\right)\left(\vec{E} - \hat{T}H_\tau\right) = 0$$

By separating the index $\hat{T}$ we get

$$\begin{cases} \left(\dfrac{\partial}{\partial x}\hat{\imath} + \dfrac{\partial}{\partial y}\hat{\jmath} + \dfrac{\partial}{\partial z}\hat{k}\right)\vec{E} + \dfrac{\partial H_\tau}{\partial \tau} = 0 \\ \left(\dfrac{\partial}{\partial x}\hat{\imath} + \dfrac{\partial}{\partial y}\hat{\jmath} + \dfrac{\partial}{\partial z}\hat{k}\right)H_\tau + \dfrac{\partial \vec{E}}{\partial \tau} = 0 \end{cases}$$

Now remembering the property:

$$\vec{a}\vec{b} = \vec{a}\cdot\vec{b} + \vec{a}\wedge\vec{b} = \vec{a}\cdot\vec{b} + (\hat{\imath}\hat{\jmath}\hat{k})\vec{a}\times\vec{b}$$

we get

$$\begin{cases} \left(\dfrac{\partial}{\partial x}\hat{\imath} + \dfrac{\partial}{\partial y}\hat{\jmath} + \dfrac{\partial}{\partial z}\hat{k}\right)\vec{E} + \dfrac{\partial H_\tau}{\partial \tau} = div\vec{E} + \left(\hat{\imath}\hat{\jmath}\hat{k}\right)rot\vec{E} + \dfrac{\partial H_\tau}{\partial \tau} = 0 \\ grad H_\tau + \dfrac{\partial \vec{E}}{\partial \tau} = 0 \end{cases}$$

and by separating indices 1 and $\left(\hat{\imath}\hat{\jmath}\hat{k}\right)$ in the first equation, finally we have:

$$\begin{cases} rot\vec{E} = 0 \\ div\vec{E} + \dfrac{\partial H_\tau}{\partial \tau} = 0 \\ \dfrac{\partial \vec{E}}{\partial \tau} + gradH_\tau = 0 \end{cases}$$

QED.

Appendix 2

In this Appendix we deduce the 3D vector expression of

1) fields as a function of the potentials;
2) generalized Maxwell equations.

Let $A = (A_1 + iA_2 + jA_3 - T\varphi)$ (this gives only electric charges. For magnetic ones, a further ij term is needed).
In ref. [8], [14] it is showed that the "imaginary" i, j, T are no more that the bivectors $\hat{\imath}\hat{\jmath}$ $\hat{\imath}\hat{k}$ $\hat{\imath}\hat{T}$ formed by $\hat{\imath}$ $\hat{\jmath}$ $\hat{k}$ $\hat{T}$, the anticommuting unit vectors of the spacetime with signature (+++-).
So $A = (A_1 + iA_2 + jA_3 - T\varphi) = \hat{\imath}(\vec{A} - \hat{T}\varphi)$, and $\partial = \frac{\partial}{\partial x} - i\frac{\partial}{\partial y} - j\frac{\partial}{\partial z} - T\frac{\partial}{\partial \tau} = \left(\vec{\partial} + \frac{\partial}{\partial \tau}\hat{T}\right)\hat{\imath}$.

This allows to write ∂A in this way:

$$\partial A = \left(\vec{\partial} + \frac{\partial}{\partial \tau}\hat{T}\right)(\vec{A} - \hat{T}\varphi)$$

Developing with the property

$$\vec{a}\vec{b} = \vec{a}\cdot\vec{b} + \vec{a}\wedge\vec{b} = \vec{a}\cdot\vec{b} + (\hat{\imath}\hat{\jmath}\hat{k})\vec{a}\times\vec{b}$$

we get $\vec{\partial}\vec{A} = div\vec{A} + (\hat{\imath}\hat{\jmath}\hat{k})rot\vec{A}$ and then:

$$\partial A = div\vec{A} + Tji(rot\vec{A})\hat{T} - \vec{\partial}\,\varphi\hat{T} - \frac{\partial \vec{A}}{\partial \tau}\hat{T} + \frac{\partial \varphi}{\partial \tau}$$

Write the first member $F\hat{\imath}\hat{T} = \left[(\vec{E} - \hat{T}H_\tau) + Tji\vec{H}\right]\hat{T}$ and by comparing we get

$$\begin{cases} \vec{E} = -\dfrac{\partial \vec{A}}{\partial \tau} - \vec{\partial}\varphi \\ \vec{H} = rot\vec{A} \\ H_\tau = div\vec{A} + \dfrac{\partial \varphi}{\partial \tau} \end{cases}$$

These are the usual expressions of fields as a function of the potentials, plus an extra – term H_τ.
The Lorenz gauge $div\vec{A} + \frac{\partial \varphi}{\partial \tau} = 0$ makes H_τ to disappear.

Now consider that if A is harmonic ($\partial\partial^* A = \partial^*\partial A = 0$), then ∂A is analytic $\partial^*(\partial A) = 0$.
We may rewrite "A is harmonic" in this way:

$$\partial^*\partial A = \hat{\imath}\left(\vec{\partial} + \frac{\partial}{\partial \tau}\hat{T}\right)\left(\vec{\partial} + \frac{\partial}{\partial \tau}\hat{T}\right)(\vec{A} - \hat{T}\varphi) = 0$$

which shows that $F\hat{\imath}\hat{T}$ is analytic.

But $F\hat{\imath}\hat{T} = \left[\left(\vec{E} - \hat{T}H_\tau\right) + Tji\vec{H}\right]\hat{T}$ so we can develop $\hat{\imath}\left(\vec{\partial} + \frac{\partial}{\partial\tau}\hat{T}\right)\left[\left(\vec{E} - \hat{T}H_\tau\right) + Tji\vec{H}\right]\hat{T}$=0 obtaining the condition:

$$\begin{aligned}\left(\vec{\partial} + \frac{\partial}{\partial\tau}\hat{T}\right)&\left[\left(\vec{E} - \hat{T}H_\tau\right) + Tji\vec{H}\right]\\ &= div\vec{E} + \left(\hat{\imath}\hat{\jmath}\hat{k}\right)rot\vec{E} \ - \vec{\partial}\hat{T}H_\tau - Tji(div\vec{H} + \left(\hat{\imath}\hat{\jmath}\hat{k}\right)rot\vec{H}\ ) + \frac{\partial}{\partial\tau}\hat{T}\vec{E} + \frac{\partial}{\partial\tau}H_\tau\\ &+ \frac{\partial}{\partial\tau}\hat{T}Tji\vec{H} = 0\end{aligned}$$

Separating by parts we get:

$$\begin{cases} rot\vec{E} = -\dfrac{\partial\vec{H}}{\partial\tau} \\ rot\vec{H} = \dfrac{\partial\vec{E}}{\partial\tau} + \vec{\partial}H_\tau \\ div\vec{E} = -\dfrac{\partial H_\tau}{\partial\tau} \\ div\vec{H} = 0 \end{cases}$$

These are the usual Maxwell equations, but the term H_τ gives rise to charges and currents.
Last it's easy to verify what was asserted, ie if $F = \left(E_x + iE_y + jE_z - TH_\tau\right) + Tji\left(H_x + iH_y + jH_z\right)$ is analytic, taking the conjugate components we obtain the Maxwell equations.

We may reverse, starting from $F\hat{\imath}\hat{T} = \left[\left(\vec{E} - \hat{T}H_\tau\right) + Tji\vec{H}\right]\hat{T}$ analytic. Write explicitly the physical components:

$$F\hat{\imath}\hat{T} = \left[\left(\hat{\imath}E_x + \hat{\jmath}E_y + \hat{k}E_z - \hat{T}H_\tau\right) + Tji\left(\hat{\imath}H_x + \hat{\jmath}H_y + \hat{k}H_z\right)\right]\hat{T}$$

Rewrite as

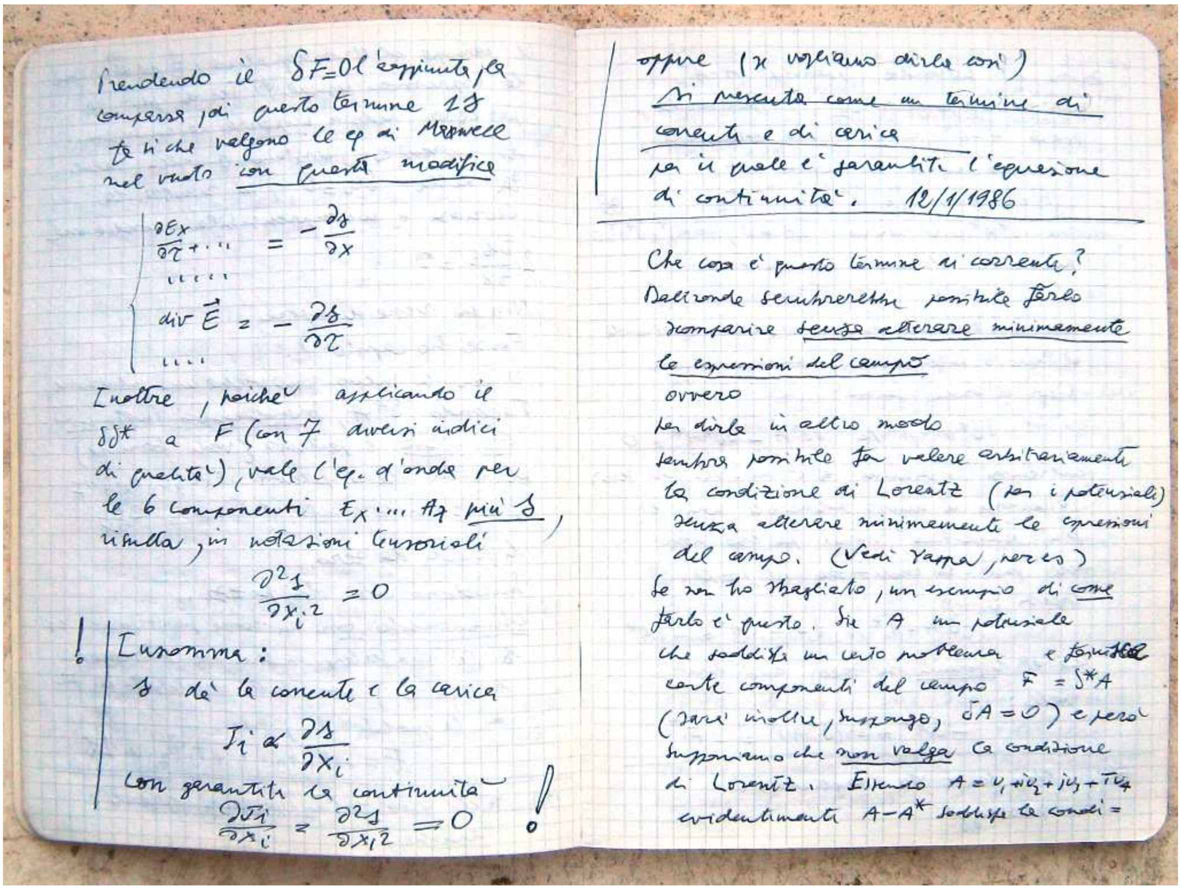
$$F\hat{\imath}\hat{T} = \left[\left(E_x - iE_y - jE_z + TH_\tau\right) + Tji\left(H_x - iH_y - jH_z\right)\right]\hat{\imath}\hat{T}$$

which shows that $F = \left(E_x - iE_y - jE_z + TH_\tau\right) + Tji\left(H_x - iH_y - jH_z\right)$ is analytic, QED.

Appendix 3

In this appendix I summarize my thoughts on this topic over the years.
If I remember correctly, my first contact with generalized Maxwell's equations, and possible charges and currents in the vacuum, was in the '80.
At that time I was very concerned (among other things) on Clifford algebra and its possible impact on mathematics, electromagnetism and quantum mechanics. Unfortunately (or fortunately) I was not aware of the work of Hestenes, and even the work of Doran and colleagues at Cambridge. For this reason I was tinkering with my own notations and symbols (which I still use).
I was very impressed to find that the Maxwell equations in vacuum coincide with the Cauchy Riemann conditions or with the analyticity of an even number F to 6 components.
The analytic field F could be constructed using the derivative of a four-vector. Rather, staying in the field of pure and simple mathematics, an even number F can be obtained from the derivative of another even number, but this generally gives rise to an F with 8 components.

I noticed that the special case of reduction to 6 components was produced by the Lorenz condition. I noticed that an additional component, which at that time gave him the name s , would have produced densities of charges and currents in vacuum and, more, obeying the continuity equation.
I was very surprised (find an exclamation point in my notes at the time), but obviously I thought that they should not give any credit at all, as long as there had been experimental confirmation.



Prendendo il $\delta F=0$ l'aggiunta, la comparsa poi questo termine $1s$ fa sì che valgono le eq di Maxwell nel vuoto con questa modifica

$$\begin{cases} \frac{\partial E_x}{\partial \tau}+\dots = -\frac{\partial s}{\partial x} \\ \dots\dots \\ \mathrm{div}\,\vec{E} = -\frac{\partial s}{\partial \tau} \\ \dots \end{cases}$$

Inoltre, poiché applicando il $\delta\delta^*$ a F (con 7 diversi indici di qualità), vale l'eq. d'onda per le 6 componenti $E_x \dots H_z$ più s, risulta, in notazioni tensoriali

$$\frac{\partial^2 s}{\partial x_i^2} = 0$$

! Insomma:
s dà la corrente e la carica

$$J_i \propto \frac{\partial s}{\partial x_i}$$

con garantita la continuità

$$\frac{\partial J_i}{\partial x_i} = \frac{\partial^2 s}{\partial x_i^2} = 0 \quad !$$

oppure (se vogliamo dirla così)
si presenta come un termine di corrente e di carica
per il quale è garantita l'equazione di continuità. 12/1/1986

Che cosa è questo termine di corrente?
D'altronde sembrerebbe possibile farlo scomparire senza alterare minimamente le espressioni del campo
ovvero
per dirla in altro modo
sembra possibile far valere arbitrariamente la condizione di Lorentz (per i potenziali) senza alterare minimamente le espressioni del campo. (Vedi [illegible])
Se non ho sbagliato, un esempio di come farlo è questo. Sia A un potenziale che soddisfa un certo problema e fornisca certe componenti del campo $F = \delta^* A$ (sarà inoltre, suppongo, $\delta A = 0$) e però supponiamo che non valga la condizione di Lorentz. Essendo $A = v_1 + iv_2 + jv_3 + Tv_4$ evidentemente $A - A^*$ soddisfa la condi=

In later years I came across an endless series of "generalized" electromagnetic fields ie, with possible charges and currents in the vacuum (see [8]).
Also gradually changed my notation for the field F.
I do not remember well because I decided the notation $F = \left(E_x + iE_y + jE_z - TH_\tau\right) + Tji(H_x + iH_y + jH_z + TE_\tau)$ with those symbols and signs for H_τ, E_τ, but this was probably the reason (or even this one): I was thinking about Maxwell's equations (with 8 components) in comparison with the Dirac equation.
In the case of neutrino correspondence led PSI $(\psi_1, \psi_2, \psi_3, \psi_4)$ to compare with the 8 components electromagnetic field F written in this way [8]:

$$F = (E_t + jE_l) + Tji(H_t + jH_l)$$

$$E_t = E_x + iE_y$$
$$E_l = E_z + iE_\tau$$
$$H_t = H_x + iH_y$$
$$H_l = H_z + iH_\tau$$

The field F, fully developed, leads to $F = \left(E_x + iE_y + jE_z - TH_\tau\right) + Tji(H_x + iH_y + jH_z + TE_\tau)$
From that moment on I was left with this notation.

The article by Van Vlaenderen.
I had available (I speak of 2003, 2004) a version of Van Vlaenderen article, and there was a long email exchange with Alberto Bicci. I recovered a few fragments remaining from my old computer, but unfortunately I lost almost everything.
In these discussions, I remember only in part, remember that Alberto asked “but who the hell is H_τ! and what is its physical meaning?”. Then we discussed already on the gauge transformations, of which he was objections.
For my part, I remember that all things ended there. I said “well, here's another guy that talks about it, and that also says he knew almost everything on how the Poynting vector is written, energy and so on. I do not venture to give an opinion and I do not want”.
Precisely here the reason (apart from laziness).
I said: the Clifford algebra gives me so simply and clearly the possible existence of a complete F field not to 6 but 8 components, with two additional components H_τ and E_τ.
Their effect is obvious: electric and magnetic charges and currents in a vacuum, scalar waves and so on.
I'm happy and it’s enough.
However electromagnetism taught me that you have to put in place the physical dimensions, and particularly in the energy relations quantities such as $B_\tau = \mu H_\tau$, $D_\tau = \varepsilon E_\tau$ are involved.
I have no idea if and how they are now defined and what are the values μ, ε to be introduced.
I need experimental data. It is useless to put me to introduce new symbols if I did not experimental evidence.
So, I repeat, for me things ended there.

The Monstein paper.
At one point I came across on the Internet in Monstein paper.
I said, “very interesting: there's a guy, Monstein, which tells me that he took two balls, has removed up to 1 km and with a polarizer he has verified the existence and transmission of scalar waves”.
But perhaps he has created a large capacitor?
Quickly I wrote to Monstein and told him with more gracefully as possible some of my doubts.
However in the end my doubts remained.
In conclusion, after all this I had concluded: “The scalar waves are a physical phenomenon that is, or would be, really fascinating. But, until proven otherwise, my impression is that scalar waves are a hoax”.
The NASA report [4] prompted me to revisit the subject.
Quote.
“Scalar waves is the name of a phenomena associated originally with the research of Dr. Nikola Tesla and other pioneering researchers of advanced electric topics. Research into and the recognition of the importance of scalar waves is now significantly growing worldwide”. (!!!)
More:
“The traditional teaching on electric waves is actually on *transverse* electric waves in which the wave vibration is transverse to the direction of wave propagation. However, contemporary experimenters have found it possible to reproduce the remarkable experiments and results described by Dr. Nikola Tesla more than a century ago on *longitudinal* electric waves (in which the wave vibration is in the same direction as wave propagation, provided the experimental apparatus is built exactly according to the principles that Tesla described)”. (!!!)
So, I said to myself, it's time to decide if they really exist or not scalar waves.

Appendix 4

The generalized Maxwell equations in spherical coordinates have the rich variety of solutions [8]:

$$\begin{cases} F_1 = (j_l\psi_l^m - Tj_{l+1}\dfrac{z}{r}\psi_l^m i)e^{i\omega_0 t} \\ F_2 = (T\dfrac{z}{r}\psi_l^m j_{l+1}T + \psi_l^m T j_l i)e^{i\omega_0 t} \\ F_3 = (Tj_{l+1}\dfrac{z}{r}\psi_l^m T - Tij_l\psi_l^m)e^{-i\omega_0 t} \\ F_4 = (+j_l\psi_l^m + T\dfrac{z}{r}\psi_l^m j_{l+1} i)e^{-i\omega_0 t} \end{cases}$$

Here $z = x\ + iy + jz$, the j_l, j_{l+1} are the spherical Bessel functions j (the same solutions holds for the n_l, n_{l+1} instead of j_l, j_{l+1}) and the ψ_l^m are the angular part of the spherical monogenics $\Psi_l^m = r^l\psi_l^m(\theta,\varphi)$.
All solutions are normalized to $\omega \equiv k = 1$. (Refer to [8] for details).
Compare to U with the same normalization $\omega \equiv k = 1$:

$$U = \frac{z^*}{r}(\frac{1}{r}cos(r-t) - \frac{1}{r^2}sin\,(r-t)) + T\frac{1}{r}cos(r-t)$$

Of course, being U a very simple spherical symmetric analytic function, it must be deduced as a sub-case.
Try to demonstrate it.
For maximum spherical symmetry $l = 0$ the ψ_l^m reduces to 1 and:

$$j_l = j_0(r) = \frac{sinr}{r}$$

$$j_{l+1} = j_1(r) = (\frac{sinr}{r^2} - \frac{cosr}{r})$$

Substituting, we get the four analytic fields:

$$\begin{cases} F_1 = (\dfrac{sinr}{r} - (\dfrac{sinr}{r^2} - \dfrac{cosr}{r})\dfrac{z^*}{r}Ti)e^{i\omega_0 t} \\ F_2 = (\dfrac{z^*}{r}(\dfrac{sinr}{r^2} - \dfrac{cosr}{r}) + \dfrac{sinr}{r}Ti)e^{i\omega_0 t} \\ F_3 = ((\dfrac{sinr}{r^2} - \dfrac{cosr}{r})\dfrac{z^*}{r} - Ti\dfrac{sinr}{r})e^{-i\omega_0 t} \\ F_4 = (\dfrac{sinr}{r} + \dfrac{z^*}{r}(\dfrac{sinr}{r^2} - \dfrac{cosr}{r})Ti)e^{-i\omega_0 t} \end{cases}$$

Developing the $e^{\mp i\omega_0 t}$ in F_2 and F_3 we get

$$\begin{cases} F_2 = \dfrac{z^*}{r}\left(\dfrac{sinrcost}{r^2} - \dfrac{cosrcost}{r}\right) + \dfrac{sinrcost}{r}Ti + \dfrac{z^*}{r}\left(\dfrac{sinrsint}{r^2} - \dfrac{cosrsint}{r}\right)i - \dfrac{sinr}{r}Tsint \\ F_3 = \left(\dfrac{sinrcost}{r^2} - \dfrac{cosrcost}{r}\right)\dfrac{z^*}{r} - Ti\dfrac{sinrcost}{r} - \left(\dfrac{sinrsint}{r^2} - \dfrac{cosrsint}{r}\right)\dfrac{z^*}{r}\ i - T\dfrac{sinr}{r}sint \end{cases}$$

If you want, these are exotic "scalar waves" ie analytic fields $F = \left(E_x + iE_y + jE_z - TH_\tau\right) + Tji(H_x + iH_y + jH_z + TE_\tau)$ with terms H_τ and E_τ giving rise to electric and magnetic charges and currents.

Who is (F_2+F_3)? With (F_2+F_3) magnetic charge disappear and we have another exotic "scalar field":

$$\frac{1}{2}(F_2+F_3)=\left(\frac{sinrcost}{r^2}-\frac{cosrcost}{r}\right)\frac{z^*}{r}-T\frac{sinrsint}{r}$$

which is a radial <u>standing</u> electric wave, together with a term H_τ giving rise to a charge $\rho=-\frac{\partial H_\tau}{\partial \tau}$.
But we need a <u>travelling</u> (radiated) wave $U=\frac{z^*}{r}(\frac{1}{r}cos(r-t)-\frac{1}{r^2}sin\,(r-t))+T\frac{1}{r}cos(r-t)$.
Let's start from scratch taking the corresponding four analytic solutions with the n_l, n_{l+1} instead of j_l, j_{l+1}:

$$n_0(r)=-\frac{cosr}{r}$$
$$n_1(r)=-\frac{cosr}{r^2}-\frac{sinr}{r}$$

We obtain four analytic fields I name N_1 and N_2, N_3, N_4.
Example look here N_3:

$$N_3=-\left(\frac{cosrcost}{r^2}+\frac{sinrcost}{r}\right)\frac{z^*}{r}+Ti\frac{cosrcost}{r}+\left(\frac{cosrsint}{r^2}+\frac{sinrsint}{r}\right)\frac{z^*}{r}\,i+T\frac{cosr}{r}sint$$

Compute now:

$$F_3=\left(\frac{sinrcost}{r^2}-\frac{cosrcost}{r}\right)\frac{z^*}{r}-Ti\frac{sinrcost}{r}-\left(\frac{sinrsint}{r^2}-\frac{cosrsint}{r}\right)\frac{z^*}{r}\,i-T\frac{sinr}{r}sint$$
$$N_3i=-\left(\frac{cosrcost}{r^2}+\frac{sinrcost}{r}\right)\frac{z^*}{r}i-T\frac{cosrcost}{r}-\left(\frac{cosrsint}{r^2}+\frac{sinrsint}{r}\right)\frac{z^*}{r}+Ti\frac{cosr}{r}sint$$

Now we need

$$sin(r-t)=sinrcost-cosrsint$$
$$cos(r-t)=cosrcost+sinrsint$$

and after boring calculations we get

$$F_3+N_3i=\left(\frac{\sin(r-t)}{r^2}-\frac{\cos(r-t)}{r}\right)\frac{z^*}{r}-T\frac{\cos\,(r-t)}{r}-Ti\frac{\sin\,(r-t)}{r}-(\frac{\cos(r-t)}{r^2}$$
$$+\frac{\sin(r-t)}{r})\frac{z^*}{r}\,i$$

This finally is a travelling wave, with the desired term $\left(\frac{\sin(r-t)}{r^2}-\frac{\cos(r-t)}{r}\right)\frac{z^*}{r}-T\frac{\cos\,(r-t)}{r}$.
Unfortunately we have also a term E_τ giving rise to a magnetic charge.
But performing the same trick with F_2 and N_2 we get:

$$F_2=\frac{z^*}{r}\left(\frac{sinrcost}{r^2}-\frac{cosrcost}{r}\right)+\frac{sinrcost}{r}Ti+\frac{z^*}{r}\left(\frac{sinrsint}{r^2}-\frac{cosrsint}{r}\right)i-\frac{sinr}{r}Tsint$$
$$-N_2i=\frac{z^*}{r}\left(\frac{cosrcost}{r^2}+\frac{sinrcost}{r}\right)i-\frac{cosrcost}{r}T-\frac{z^*}{r}\left(\frac{cosrsint}{r^2}+\frac{sinrsint}{r}\right)-\frac{cosr}{r}Tisint$$

and:

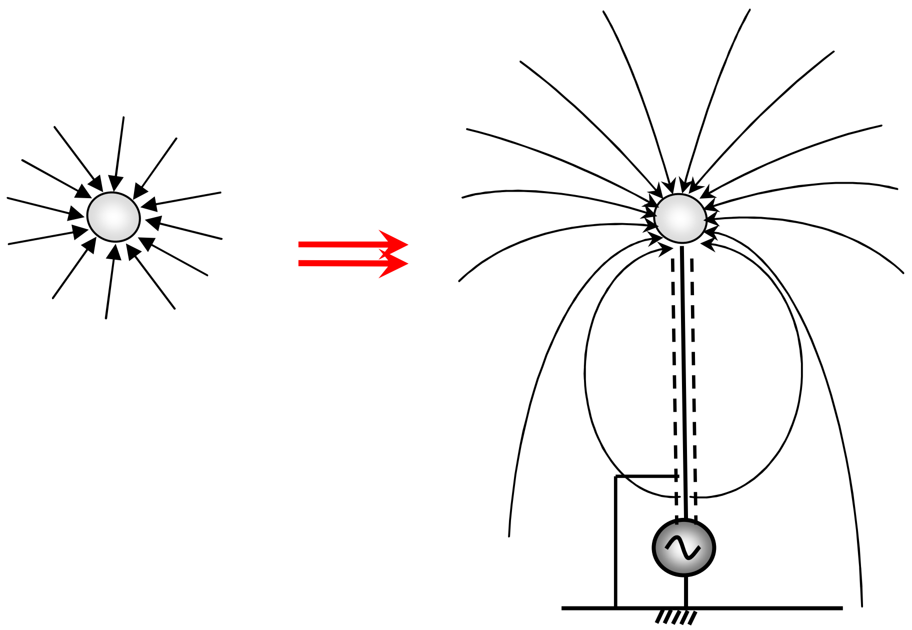
$$F_2 - N_2 i = \left(\frac{\sin(r-t)}{r^2} - \frac{\cos(r-t)}{r}\right)\frac{z^*}{r} - T\frac{\cos\,(r-t)}{r} + Ti\frac{\sin\,(r-t)}{r} + (\frac{\cos(r-t)}{r^2} + \frac{\sin(r-t)}{r})\frac{z^*}{r}\,i$$

Summing up, magnetic charge disappear and finally we get

$$U = -\frac{(F_3 + N_3 i) + (F_2 - N_2 i)}{2} = \frac{z^*}{r}\left(\frac{1}{r}cos(r-t) - \frac{1}{r^2}sin(r-t)\right) + T\frac{1}{r}cos(r-t)$$

QED.

Obviously, all this holds in conditions of maximum symmetry.
However if for example a transmitter (and a receiver) are connected to two spheres by means of coaxial cables with the shield connected to ground (this in order to avoid, as much as possible, they act as antennas) then the lines of force will be distorted.



If the field is partially distorted by neighboring objects, more of these analytic “multipoles” are needed to describe the scalar field.

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