

# Particularization of the sequence of spacetime/intrinsic spacetime geometries and associated sequence of theories in a metric force field to the gravitational field.

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The two stages of evolution of spacetime/intrinsic spacetime and the associated spacetime/intrinsic spacetime geometries in a long range metric force field, isolated in previous papers, are particularized to the gravitational field. The theory of relativity on flat four-dimensional spacetime ( $E^3, ct$ ) and the intrinsic theory relativity on the underlying flat two-dimensional intrinsic spacetime ( $\phi\rho, \phi c\phi t$ ), due to the presence of a metric force field, as well as the absolute intrinsic metric theory (of the metric force field) on curved ‘two-dimensional’ absolute intrinsic spacetime ( $\phi\hat{\rho}, \phi\hat{c}\phi\hat{t}$ ), which evolve at the second (and final) stage of evolution of spacetime/intrinsic spacetime in a long range metric force field, developed in the previous papers, become the theory of gravitational relativity (TGR) on the flat four-dimensional relativistic spacetime, the intrinsic theory of gravitational relativity ( $\phi$ TGR) on the underlying flat two-dimensional relativistic intrinsic spacetime and the metric theory of absolute intrinsic gravity ( $\phi$ MAG) on the curved ‘two-dimensional’ absolute intrinsic spacetime in a gravitational field. The basic aspects of these co-existing theories in every gravitational field are developed.

## 1 Spacetime/intrinsic spacetime geometries at the first and second stages of evolution of spacetime/intrinsic spacetime in a gravitational field

The geometry of Fig. 4 or Fig. 11 of [1], which evolves at the first stage of evolution of spacetime/intrinsic spacetime in a long-range metric force field and the global spacetime/intrinsic spacetime geometries of Figs. 1 and 3 and their inverses Figs. 4 and 5 of [2], which evolve at the second stage, derived in those papers, shall be adapted to the gravitational field in this section.<sup>1</sup>

Only one external gravitational field source shall be considered in this paper in order to make this first paper on application of the geometrical background within four-world picture developed in [3-6] and [7,1,2] concise, revealing only the essential features, while extension to two and larger number of external gravitational field sources shall be considered elsewhere.

Let us consider the reference spacetime/intrinsic spacetime geometry of Fig. 6 of [1] that exists in a universe assumed to be devoid of a long-range metric force field, which is now being taken to be the absence of gravitational field. Consequently there is absence of absolute intrinsic Riemannian spacetime geometry. This implies the absence of curved ‘two-dimensional’ absolute intrinsic spacetime ( $\phi\hat{\rho}, \phi\hat{c}\phi\hat{t}$ ) and its underlying flat two-dimensional relativistic intrinsic spacetime ( $\phi\rho, \phi c\phi t$ ) and flat four-dimensional relativistic spacetime ( $E^3, ct$ ) in such a universe, as is the case in Fig. 6 of [1].

Then let us introduce the absolute rest mass, to be denoted by  $\hat{M}_0$ , of a gravitational field source at a point  $\hat{S}$  in the flat ab-

solute space  $\hat{E}^3$  in Fig. 6 of [1] in our universe. The absolute rest masses  $\hat{M}_0^0, -\hat{M}_0^*$  and  $-\hat{M}_0^{0*}$  of the identical symmetry-partner gravitational field sources will be automatically introduced at the symmetry-partner points  $\hat{S}^0, \hat{S}^*$  and  $\hat{S}^{0*}$  in the assumed initially empty flat absolute spaces  $\hat{E}^{03}, -\hat{E}^{3*}$  and  $-\hat{E}^{03*}$  of the positive time-universe, negative universe and negative time-universe respectively, simultaneously with the introduction of  $\hat{M}_0$  at point  $\hat{S}$  in  $\hat{E}^3$  in the positive (or our) universe. This follows from the perfect symmetry of state among the four universes established in section 2 of [6]. The fact that  $\hat{M}_0, \hat{M}_0^0, -\hat{M}_0^*$  and  $-\hat{M}_0^{0*}$  are identical in magnitude, size and shape was also established in section 2 of [6].

As explained in section 1 of [6], the appearance of  $\hat{M}_0$  at point  $\hat{S}$  in  $\hat{E}^3$ ;  $\hat{M}_0^0$  at point  $\hat{S}^0$  in  $\hat{E}^{03}$ ;  $-\hat{M}_0^*$  at point  $\hat{S}^*$  in  $-\hat{E}^{3*}$  and  $-\hat{M}_0^{0*}$  at point  $\hat{S}^{0*}$  in  $-\hat{E}^{03*}$ , where  $\hat{S}, \hat{S}^0, \hat{S}^*$  and  $\hat{S}^{0*}$  are symmetry-partner points, will lead to the appearance of identical symmetry-partner ‘one-dimensional’ absolute intrinsic rest masses  $\phi\hat{M}_0$  in ‘one-dimensional’ absolute intrinsic space  $\phi\hat{\rho}$  directly underneath  $\hat{M}_0$  in  $\hat{E}^3$  in the positive (or our) universe; of  $\phi\hat{M}_0^0$  in  $\phi\hat{\rho}^0$  directly underneath  $\hat{M}_0^0$  in  $\hat{E}^{03}$  in the positive time-universe; of  $-\phi\hat{M}_0^*$  in  $-\phi\hat{\rho}$  directly underneath  $-\hat{M}_0^*$  in  $-\hat{E}^{3*}$  in the negative universe and of  $-\phi\hat{M}_0^{0*}$  in  $-\phi\hat{\rho}^{0*}$  directly underneath  $-\hat{M}_0^{0*}$  in  $-\hat{E}^{03*}$  in the negative time-universe, as illustrated in Fig. 1.

As explained with the transformation of Fig. 2 to Fig. 8a with respect to 3-observers in the Euclidean 3-spaces  $E'^3$  and  $-E'^{3*}$  of the positive (or our) universe and the negative universe in [5], the geometry of Fig. 1 must be replaced with that of Fig. 2 with respect to ‘3-observers’ in the absolute spaces  $\hat{E}^3$  and  $-\hat{E}^{3*}$  of our universe and the negative universe. The geometry of Fig. 2 will emerge automatically in the positive (or our) universe and the negative universe as the absolute

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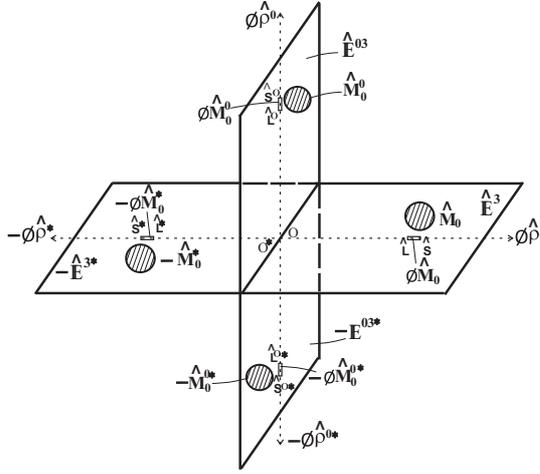


Fig. 1: The mutually orthogonal flat ‘three-dimensional’ absolute spaces and their underlying straight line ‘one-dimensional’ absolute intrinsic spaces of four symmetrical universes namely, the positive (or our) universe, the negative universe, the positive time-universe and the negative time-universe, containing the identical ‘three-dimensional’ absolute rest masses in the absolute spaces and ‘one-dimensional’ absolute intrinsic rest masses in the absolute intrinsic spaces, directly underneath the absolute rest masses in the absolute spaces, of symmetry-partner gravitational field sources at symmetry-partner points in the absolute spaces in the assumed otherwise empty universes.

rest mass  $\hat{M}_0$  of a gravitational field source is introduced at a point  $\hat{S}$  in the empty flat absolute space  $\hat{E}^3$  in our universe, which is being assumed to be initially devoid of gravitational field source. This happens by virtue of the perfect symmetry of state among the four universes.

Now the absolute intrinsic rest mass  $\phi\hat{M}_0$  will establish non-uniform absolute intrinsic static speeds  $\phi\hat{V}_S$  (isolated in section 3 of [1]) that has its maximum magnitude at point  $\hat{L}$  at the edge of  $\phi\hat{M}_0$  (point  $\hat{S}$  being at the base of  $\phi\hat{M}_0$ ) and decreases continuously to zero magnitude at point O that is far removed from point  $\hat{S}$ . The absolute intrinsic rest mass  $\phi\hat{E}/\phi\hat{c}^2$  ( $\equiv \phi\hat{M}_0$ ) in the absolute intrinsic time ‘dimension’  $\phi\hat{c}\hat{t}$  will likewise establish non-uniform absolute intrinsic static speeds  $\phi\hat{V}_S$  that has its maximum magnitude at point  $\hat{L}^0$  at the edge of  $\phi\hat{E}/\phi\hat{c}^2$  (point  $\hat{S}^0$  being at the base of  $\phi\hat{E}/\phi\hat{c}^2$ ) and decreases continuously to zero magnitude at point O that is far removed from point  $\hat{S}^0$ . (Recall from from discussion in section 3 of [5] with the aid of Figs. 9a and 9b of that paper that  $\phi\hat{E}/\phi\hat{c}^2$  in  $\phi\hat{c}\hat{t}$  possesses absolute intrinsic gravitational or absolute intrinsic inertial attributes like absolute intrinsic rest mass  $\phi\hat{M}_0^0$  in  $\phi\hat{\rho}^0$  in Fig. 1).

The absolute rest mass  $\hat{M}_0$  (assumed spherical) will establish non-uniform absolute static speed  $\hat{V}_S$  that has maximum magnitude at the surface of  $\hat{M}_0$  and decreases continuously to zero magnitude at point O along every radial direction from its centre. The ‘one-dimensional’ absolute

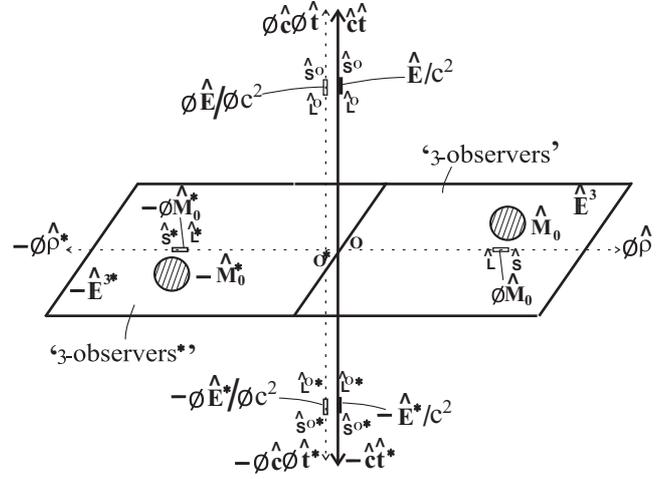


Fig. 2: The diagram of Fig. 1 in four universes naturally transforms into flat ‘four-dimensional’ absolute spacetimes and the underlying flat ‘two-dimensional’ absolute intrinsic spacetimes of the positive (or our) universe and the negative universe, containing the absolute rest masses at symmetry-partner positions in the absolute spacetimes and absolute intrinsic rest masses in the absolute intrinsic spacetimes directly underneath the absolute rest masses in absolute spacetimes, of symmetry-partner gravitational field sources in the assumed otherwise empty universes, with respect to ‘3-observers’ in the absolute spaces in our universe and the negative universe.

rest mass  $\hat{E}/\hat{c}^2$  ( $\equiv \hat{M}_0$ ) in the absolute time ‘dimension’  $\hat{c}\hat{t}$  (that possesses absolute gravitational and absolute inertial attributes like  $\hat{M}_0^0$  in  $\hat{E}^{03}$  in Fig. 1), will likewise establish non-uniform absolute static speeds  $\hat{V}_S$  along  $\hat{c}\hat{t}$ , which has maximum magnitude at point  $\hat{L}^0$  and decreases continuously to zero magnitude at point O.

The discussion in the last two paragraphs for  $(\hat{M}_0, \hat{E}/\hat{c}^2)$  in  $(\hat{E}^3, \hat{c}\hat{t})$  and its underlying  $(\phi\hat{M}_0, \phi\hat{E}/\phi\hat{c}^2)$  in  $(\phi\hat{\rho}^0, \phi\hat{c}\hat{t})$  in the positive (or our) universe, obtains for  $(-\hat{M}_0^*, -\hat{E}^{3*}/\hat{c}^2)$  in  $(-\hat{E}^{3*}, -\hat{c}\hat{t}^*)$  and its underlying  $(-\phi\hat{M}_0^*, -\phi\hat{E}^*/\phi\hat{c}^2)$  in  $(-\phi\hat{\rho}^*, -\phi\hat{c}\hat{t}^*)$  in the negative universe as well.

We shall for convenience replace the representation of the ‘three-dimensional’ absolute spaces  $\hat{E}^3$  and  $-\hat{E}^{3*}$  by horizontal plane surfaces in Fig. 2 by lines along the horizontal. We shall also revert back to the notations  $\Sigma$  and  $-\Sigma^*$  respectively for Euclidean 3-spaces in [3-6]. That is, we shall replace  $\hat{E}^3$  and  $-\hat{E}^{3*}$  that appear in Fig. 2 and in the diagrams in [7,1,2] by  $\hat{\Sigma}$  and  $-\hat{\Sigma}^*$  respectively henceforth. The assumed spherical absolute rest masses  $\hat{M}_0$  and  $-\hat{M}_0^*$ , represented by circles on  $\hat{E}^3$  and  $-\hat{E}^{3*}$  in Fig. 2, shall be represented by short line segments in  $\hat{\Sigma}$  and  $-\hat{\Sigma}^*$  respectively. These representations are dummy with no consequence on the theory being developed.

Further more, since we are now particularizing to the gravitational field, the absolute intrinsic static speed  $\phi\hat{V}_S(\phi\hat{r})$  at ‘distance’  $\phi\hat{r}$  from the base  $\hat{S}$  of  $\phi\hat{M}_0$  in Fig. 2, shall be re-

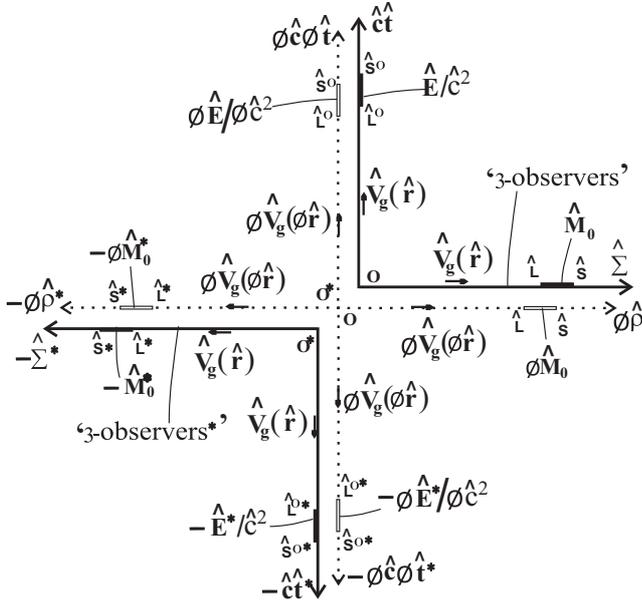


Fig. 3: The absolute rest masses of symmetry-partner gravitational field sources in flat absolute spacetimes, establish non-uniform absolute gravitational speeds in all their finite neighbourhoods in absolute spacetimes and their absolute intrinsic rest masses in the underlying absolute intrinsic spacetimes, establish non-uniform absolute intrinsic gravitational speeds in all their finite neighbourhoods in absolute intrinsic spacetimes in the positive and negative universes.

denoted by  $\phi \hat{V}_g(\phi \hat{r})$  and alternatively referred to as absolute intrinsic gravitational speed. The absolute static speed  $\hat{V}_s(\hat{r})$  at radial distance  $\hat{r}$  from the centre of  $\hat{M}_0$  shall likewise be re-denoted by  $\hat{V}_g(\hat{r})$  and alternatively referred to as absolute gravitational speed.

As follows from the discussions in the foregoing five paragraphs, Fig. 2 shall be replaced with Fig. 3, where only an absolute intrinsic gravitational speed  $\phi \hat{V}_g(\phi \hat{r})$  at an arbitrary 'distance'  $\phi \hat{r}$  along  $\phi \hat{\rho}$  from the base  $\hat{S}$  of  $\phi \hat{M}_0$  in  $\phi \hat{\rho}$  and at equal 'distance'  $\phi \hat{r}$  along  $\phi \hat{c}\hat{t}$  from the base  $\hat{S}^0$  of  $\phi \hat{E}/\phi \hat{c}^2$  in  $\phi \hat{c}\hat{t}$ , corresponding to absolute gravitational speed  $\hat{V}_g(\hat{r})$  at an arbitrary radial distance  $\hat{r}$  in the absolute space  $\hat{\Sigma}$  from the centre of  $\hat{M}_0$  in  $\hat{\Sigma}$  are shown.

The line of absolute rest mass  $\hat{M}_0$  of length  $\hat{S}\hat{L}$  in Fig. 3 is actually a spherical absolute rest mass (as being assumed) of radius  $\hat{R}_0 = \hat{S}\hat{L}$  and the segment  $\hat{S}O$  of the line of universal absolute space  $\hat{\Sigma}$  is actually a spherical region of absolute space of large radius  $\hat{S}O$  with  $\hat{M}_0$  at its centre. The 'one-dimensional' absolute intrinsic space  $\phi \hat{\rho}$  is an isotropic intrinsic dimension with respect to '3-observers' in the absolute space  $\hat{\Sigma}$ . It can be considered to lie along any of the radial direction from the centre of the spherical region of absolute space of radius  $\hat{S}O$ , as illustrated along an arbitrary radial direction in Fig. 4, with respect to '3-observers' in  $\hat{\Sigma}$ .

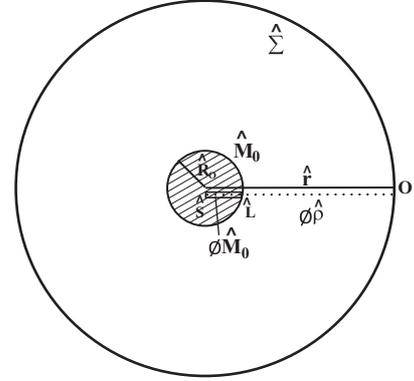


Fig. 4: The absolute rest mass of a gravitational field source at the centre of a large spherical region of the assumed otherwise empty flat universal absolute space and its 'one-dimensional' absolute intrinsic rest mass in the 'one-dimensional' universal isotropic absolute intrinsic space that can be considered to lie along any radial direction from the centre of the spherical region with respect to '3-observers' in the absolute space; where gravitational field can be considered to vanish outside the spherical region of absolute space.

The spherical region of the universal absolute space  $\hat{\Sigma}$  within the gravitational field of  $\hat{M}_0$  (assuming the gravitational field of  $\hat{M}_0$  can be considered to vanish outside this sphere), is just a portion of the vast 'three-dimensional' flat universal absolute space, which is being assumed to be devoid of the absolute rest mass of any other gravitational field source at present.

The reference spacetime/intrinsic spacetime geometry of Fig. 6 of [1] will endure for as long as a long-range absolute metric force field is absent. On the other hand, the reference geometry of Fig. 2 or 3 above, in which an absolute gravitational field source is present in absolute spacetime and an absolute intrinsic gravitational field source is present in absolute intrinsic spacetime, will endure for no moment before transforming into the geometry of Fig. 5 at the first stage of evolution of spacetime/intrinsic spacetime within the symmetry-partner gravitational fields in the positive and negative universes.

Again the line of rest mass  $M_0$  of length  $S'L$  in Fig. 5 is actually a spherical rest mass  $M_0$  (as being assumed) of radius  $R_0 (=S'L)$  and the line of proper physical Euclidean 3-space  $\Sigma'$  is actually a spherical proper physical Euclidean 3-space of large radius  $S'O$  with  $M_0$  at its centre. The one-dimensional proper intrinsic space  $\phi \rho'$  is an isotropic intrinsic dimension with respect to 3-observers in  $\Sigma'$ . It can be considered to lie along any of the radial directions of the spherical proper Euclidean 3-space  $\Sigma'$ , as illustrated along an arbitrary radial direction in Fig. 6, with respect to 3-observers in the proper Euclidean 3-space  $\Sigma'$ .

The spherical proper physical Euclidean 3-space  $\Sigma'$  of large radius  $S'O$  evolves around the rest mass  $M_0$  of the gravitational field source at its centre, where the gravitational field

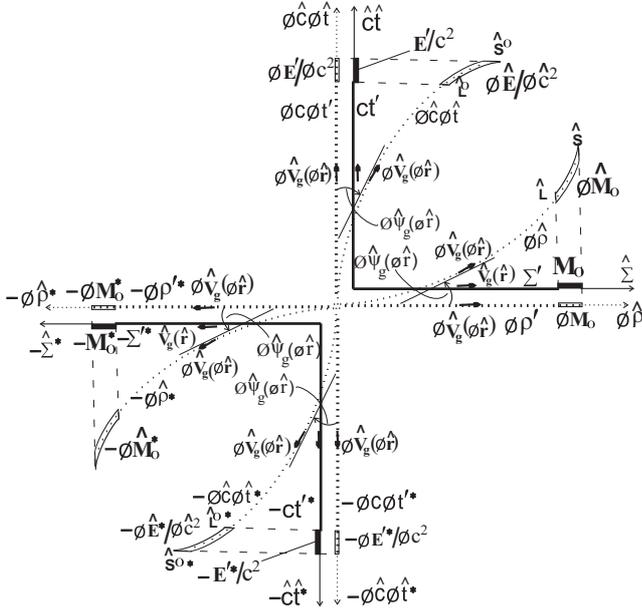


Fig. 5: The spacetime/intrinsic spacetime geometry that evolves from the reference geometry of Fig.3 at the first stage of evolution of spacetime/intrinsic spacetime in all finite neighbourhoods of symmetry-partner gravitational field sources in the assumed otherwise empty positive and negative universes.

of  $M_0$  can be taken to vanish outside  $\Sigma'$ . Since this gravitational field source is the only one in our universe, as being assumed, the region of the universal 3-space outside  $\Sigma'$  (or outside the gravitational field of  $M_0$ ) remains the flat absolute space  $\hat{\Sigma}$ .

The segment  $\hat{S}O$  of the straight line universal absolute intrinsic space  $\hat{\phi}\hat{\rho}$  along the horizontal, containing the absolute intrinsic rest mass  $\hat{\phi}\hat{M}_0$  of the gravitational field source within interval  $\hat{S}\hat{L}$  at the origin of segment  $O\hat{S}$  of  $\hat{\phi}\hat{\rho}$  in Fig. 3, becomes curved towards the vertical as a plane curve on the vertical  $(\hat{\phi}\hat{\rho}'-\hat{\phi}c\hat{t}')$ -plane, projecting straight line isotropic proper intrinsic space  $\hat{\phi}\hat{\rho}'$  along the horizontal, which is made manifest outwardly in the proper physical Euclidean 3-space  $\Sigma'$  within the gravitational field. The line of absolute intrinsic rest mass  $\hat{\phi}\hat{M}_0$  located at the origin (or base) of the curved segment  $O\hat{S}$  of  $\hat{\phi}\hat{\rho}$ , likewise 'projects' proper intrinsic rest mass  $\hat{\phi}M_0$  at the origin (or base) of the projective proper intrinsic space  $\hat{\phi}\hat{\rho}'$  along the horizontal, which is made manifest in the rest mass  $M_0$  of the gravitational field source at the centre of the spherical proper physical Euclidean 3-space  $\Sigma'$ .

The 'one-dimensional' absolute intrinsic rest mass  $\hat{\phi}\hat{M}_0$  in the straight line absolute intrinsic space  $\hat{\phi}\hat{\rho}$  along the horizontal and  $\hat{\phi}\hat{E}/\hat{\phi}\hat{c}^2$  in the straight line absolute intrinsic time 'dimension'  $\hat{\phi}\hat{c}\hat{t}$  along the vertical, of the gravitational field source in the reference geometry of Fig. 3, are indeed curved along with  $\hat{\phi}\hat{\rho}$  and  $\hat{\phi}\hat{c}\hat{t}$  at the first stage of evolution of spacetime/intrinsic spacetime in the gravitational field. However

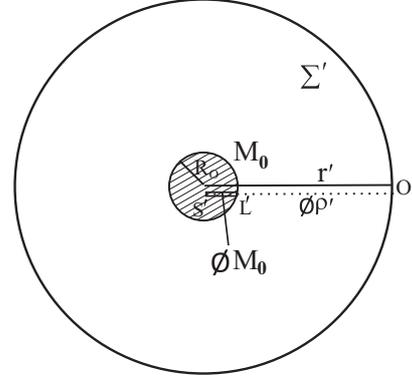


Fig. 6: The large spherical region of proper Euclidean 3-space  $\Sigma'$  containing the rest mass of the gravitational field source at its centre and its intrinsic rest mass in isotropic proper intrinsic space  $\hat{\phi}\hat{\rho}'$  that can be considered to lie along any radial direction of  $\Sigma'$  with respect to 3-observers in  $\Sigma'$ , which evolves from reference diagram of Fig. 3 at the first stage of evolution of spacetime/intrinsic spacetime in the gravitational field; where it is assumed that proper Euclidean 3-space and proper intrinsic space do not evolve outside the spherical region due to the vanishing of gravitational field.

the curvatures of  $\hat{\phi}\hat{M}_0$  within segment  $\hat{L}\hat{S}$  of the curved  $\hat{\phi}\hat{\rho}$  and the curvature of  $\hat{\phi}\hat{E}/\hat{\phi}\hat{c}^2$  within segment  $\hat{L}^0\hat{S}^0$  of the curved  $\hat{\phi}\hat{c}\hat{t}$ , shown in Fig. 5, are temporary. The final forms of the segments  $\hat{L}\hat{S}$  of the curved  $\hat{\phi}\hat{\rho}$  containing  $\hat{\phi}\hat{M}_0$  and  $\hat{L}^0\hat{S}^0$  of the curved  $\hat{\phi}\hat{c}\hat{t}$  containing  $\hat{\phi}\hat{E}/\hat{\phi}\hat{c}^2$ , shall be derived elsewhere when the need for the spacetime/intrinsic spacetime geometry at the interior of a gravitational field source arises. On the other hand, the segments  $O\hat{L}$  of the curved  $\hat{\phi}\hat{\rho}$  and  $O\hat{L}^0$  of the curved  $\hat{\phi}\hat{c}\hat{t}$  at the exterior of a gravitational field source in Fig. 5 are valid.

It is being assumed that the absolute gravitational field source  $(\hat{M}_0, \hat{E}/\hat{c}^2)$  introduced at point  $(\hat{S}, \hat{S}^0)$  in  $(\hat{\Sigma}, \hat{c}\hat{t})$  in our universe and its symmetry-partner  $(-\hat{M}_0^*, -\hat{E}^*/\hat{c}^2)$  introduced simultaneously at the symmetry-partner point  $(\hat{S}^*, \hat{S}^{0*})$  in  $(-\hat{\Sigma}^*, -\hat{c}\hat{t}^*)$  in the negative universe in Fig. 2 or 3, are the only gravitational field sources in our universe and the negative universe. Consequently only the segment  $\hat{S}O$  of curved absolute intrinsic space  $\hat{\phi}\hat{\rho}$ , its projective straight line proper intrinsic space  $\hat{\phi}\hat{\rho}'$  between points  $S'$  and  $O$  along the horizontal and the outward manifestation of  $\hat{\phi}\hat{\rho}'$  namely, the large spherical proper physical Euclidean 3-space  $\Sigma'$ , exist within the gravitational field in our universe, while the regions of the flat universal absolute spacetime  $(\hat{\Sigma}, \hat{c}\hat{t})$  underlied by flat universal absolute intrinsic spacetime  $(\hat{\phi}\hat{\rho}, \hat{\phi}\hat{c}\hat{t})$  outside the gravitational field of the introduced lone absolute gravitational field source in our universe remain unchanged. Likewise for the lone symmetry-partner absolute gravitational field source  $(-\hat{M}_0^*, -\hat{E}^*/\hat{c}^2)$  introduced at point  $(\hat{S}^*, \hat{S}^{0*})$  in  $(-\hat{\Sigma}^*, -\hat{c}\hat{t}^*)$  in Fig. 2 or 3 in the negative universe.

The segment  $O\hat{S}^0$  of the straight line universal absolute intrinsic time 'dimension'  $\hat{\phi}\hat{c}\hat{t}$  along the vertical, contain-

ing the line of absolute intrinsic rest mass  $\phi\hat{E}/\phi\hat{c}^2$  ( $\equiv \phi\hat{M}_0$ ) of the gravitational field source within interval  $\hat{L}^0\hat{S}^0$  at the origin (or base) of the segment  $O\hat{S}^0$  of  $\phi\hat{c}\phi\hat{t}$  in Fig. 3, becomes curved towards  $\phi\rho'$  along the horizontal, projecting straight line proper intrinsic time dimension  $\phi c\phi t'$  along the vertical within the gravitational field, which is made manifest outwardly in the proper physical time dimension  $ct'$  along the vertical within the gravitational field. The line of absolute intrinsic rest mass  $\phi\hat{E}/\phi\hat{c}^2$  ( $\equiv \phi\hat{M}_0$ ) of the gravitational field source at the origin (or base) of the curved segment  $O\hat{S}^0$  of  $\phi\hat{c}\phi\hat{t}$  likewise projects a line of intrinsic rest mass  $\phi E'/\phi c^2$  ( $\equiv \phi M_0$ ) at the origin (or base) of the projective proper intrinsic time dimension  $\phi c\phi t'$  along the vertical, which is made manifest in rest mass  $E'/c^2$  ( $\equiv M_0$ ) at the origin (or base) of the proper physical time dimension  $ct'$ .

The absolute intrinsic gravitational speed (an absolute intrinsic static speed)  $\phi\hat{V}_g(\phi\hat{r})$  at arbitrary 'distance'  $\phi\hat{r}$  from the base  $\hat{S}$  of  $\phi\hat{M}_0$  along the straight line absolute intrinsic space  $\phi\hat{\rho}$  in Fig. 3, is now at an arbitrary 'distance'  $\phi\hat{r}$  from the base  $\hat{S}$  of  $\phi\hat{M}_0$  along the curved  $\phi\hat{\rho}$  in Fig. 5. It invariantly projects absolute intrinsic gravitational speed  $\phi\hat{V}_g(\phi\hat{r})$  into the projective straight line proper intrinsic space  $\phi\rho'$  at the corresponding 'distance'  $\phi r'$  from the base of  $\phi M_0$  in  $\phi\rho'$ , which is made manifest outwardly in absolute gravitational speed  $\hat{V}_g(\hat{r})$  at radial distance  $r'$  from the centre of  $M_0$  in  $\Sigma'$ , with respect to 3-observers in  $\Sigma'$ . The absolute intrinsic gravitational speed  $\phi\hat{V}_g(\phi\hat{r})$  at 'distance'  $\phi\hat{r}$  from the base of  $\hat{S}^0$  of  $\phi\hat{E}/\phi\hat{c}^2$  along the curved absolute intrinsic time 'dimension'  $\phi\hat{c}\phi\hat{t}$ , likewise invariantly projects absolute intrinsic gravitational speed  $\phi\hat{V}_g(\phi\hat{r})$  into the projective straight line proper intrinsic time dimension  $\phi c\phi t'$ , which is made manifest outwardly in absolute gravitational speed  $\hat{V}_g(\hat{r})$  at 'distance'  $r'$  from the base of  $E'/c^2$  in  $ct'$ , with respect to 1-observers in  $ct'$ . The discussions for the first quadrant (or in the positive universe) in Fig. 5 in the foregoing two paragraphs and this equally obtain for the third quadrant (or in the negative universe).

The invariance of absolute intrinsic gravitational speed in the context of the theory of absolute intrinsic gravity/absolute gravity ( $\phi AG/AG$ ), (which is the theory that supports the geometry of Fig. 5), represented graphically by the invariant projection of  $\phi\hat{V}_g(\phi\hat{r})$  along the curved  $\phi\hat{\rho}$  and  $\phi\hat{c}\phi\hat{t}$  as  $\phi\hat{V}_g(\phi\hat{r})$  along the projective straight line  $\phi\rho'$  and  $\phi c\phi t'$  in Fig. 5, have been stated as invariance of absolute intrinsic static speed and absolute static speed by Eqs. (79a) and (79b) of [1], in the context of the absolute intrinsic metric phenomenon that gives rise to the geometry of Fig. 11 of [1] in the one-world picture, which corresponds to Fig. 5 here in the two-world picture. It shall be re-stated as the invariance of absolute intrinsic gravitational speed and absolute gravitational speed in the context of the theory of absolute intrinsic gravity/theory of absolute gravity ( $\phi AG/AG$ ) that gives rise to the geometry of Fig. 5 within the symmetry-partner gravitational fields in

the positive and negative universes as follows

$$\phi V'(\phi r') = \phi\hat{V}_g(\phi\hat{r}) \quad (1a)$$

and

$$V'(r') = \hat{V}_g(\hat{r}) \quad (1b)$$

It is crucial to note that the line of intrinsic rest mass  $\phi M_0$  of the gravitational field source in  $\phi\rho'$  with respect to 3-observers in  $\Sigma'$  is not the source of the non-uniform absolute intrinsic gravitational speeds  $\phi\hat{V}_g(\phi\hat{r})$  along  $\phi\rho'$  and that the three-dimensional rest mass  $M_0$  of the field source is not the source of the non-uniform absolute gravitational speeds  $\hat{V}_g(\hat{r})$  along every radial direction from its centre in  $\Sigma'$  with respect to 3-observers in  $\Sigma'$  in Fig. 5. Rather the non-uniform absolute intrinsic gravitational speeds  $\phi\hat{V}_g(\phi\hat{r})$  along  $\phi\rho'$  are the projections of the non-uniform absolute intrinsic gravitational speeds that  $\phi\hat{M}_0$  at the origin of the curved  $\phi\hat{\rho}$  establishes along the curved  $\phi\hat{\rho}$  and the non-uniform absolute gravitational speeds  $\hat{V}_g(\hat{r})$  in  $\Sigma'$  are the outward manifestations of the projective non-uniform  $\phi\hat{V}_g(\phi\hat{r})$  along  $\phi\rho'$ .

Likewise the non-uniform absolute intrinsic gravitational speeds  $\phi\hat{V}_g(\phi\hat{r})$  along the proper intrinsic time dimension  $\phi c\phi t'$  with respect to 1-observers in  $ct'$ , have not been established by  $\phi E'/\phi c^2$  in  $\phi c\phi t'$  and non-uniform absolute gravitational speeds  $\hat{V}_g(\hat{r})$  along the proper time dimension  $ct'$  with respect to 1-observers in  $ct'$  have not been established by the rest mass  $E'/c^2$  ( $\equiv M_0$ ) of the gravitational field source in  $ct'$ . Rather the non-uniform  $\phi\hat{V}_g(\phi\hat{r})$  along  $\phi c\phi t'$  are the invariant projections along the vertical of non-uniform  $\phi\hat{V}_g(\phi\hat{r})$  established along the curved  $\phi\hat{c}\phi\hat{t}$  by  $\phi\hat{E}/\phi\hat{c}^2$  ( $\equiv \phi\hat{M}_0$ ) at the origin of the curved  $\phi\hat{c}\phi\hat{t}$  and  $\hat{V}_g(\hat{r})$  along  $ct'$  are the outward manifestations of the projective  $\phi\hat{V}_g(\phi\hat{r})$  along  $\phi c\phi t'$ .

As discussed in section 2 of [1], the projective non-uniform absolute intrinsic gravitational speeds  $\phi\hat{V}_g(\phi\hat{r})$  along the proper intrinsic space  $\phi\rho'$  and along the proper intrinsic time dimension  $\phi c\phi t'$  in Fig. 5, cannot give rise to curvature of these relative intrinsic dimensions (without hat label) or produce any other effect on them. The absolute gravitational speeds  $\hat{V}_g(\hat{r})$  in the proper physical Euclidean 3-space  $\Sigma'$  can likewise not give rise to any detectable effect in  $\Sigma'$ .

Thus if the projective non-uniform absolute intrinsic gravitational speeds  $\phi\hat{V}_g(\phi\hat{r})$  along the straight line proper intrinsic spaces  $\phi\rho'$  and  $-\phi\rho'^*$  and straight line proper intrinsic time dimensions  $\phi c\phi t'$  and  $-\phi c\phi t'^*$  are all that is possible and consequently the non-uniform absolute gravitational speeds  $\hat{V}_g(\hat{r})$  in the proper physical Euclidean 3-spaces  $\Sigma'$  and  $-\Sigma'^*$  and the proper physical dimensions  $ct'$  and  $-ct'^*$  are all that is possible in Fig. 5, then the geometry of Fig. 5 will endure and evolution of spacetime/intrinsic spacetime will terminate at the first stage within a gravitational field.

However the second stage of evolution of spacetime/intrinsic spacetime within a gravitational field is immutable.

This is so because, quite apart from the projective non-uniform absolute intrinsic gravitational speeds  $\phi\hat{V}_g(\phi\hat{r})$  along the straight line  $\phi\rho'$ ,  $\phi c\phi t'$ ,  $-\phi\rho'^*$  and  $-\phi c\phi t'^*$ , the 'projective' intrinsic rest mass  $\phi M_0$  in  $\phi\rho'$ , as an intrinsic gravitational field source, establishes non-uniform proper intrinsic gravitational speeds  $\phi V'_g(\phi r')$  along  $\phi\rho'$ , whose magnitude is maximum at the edge  $L$  of  $\phi M_0$  and decreases continuously to zero at point O that is far removed from the base S of  $\phi M_0$ . The 'projective' intrinsic rest mass  $\phi E'/\phi c^2$  ( $\equiv \phi M_0$ ) in  $\phi c\phi t'$  likewise establishes non-uniform proper intrinsic gravitational speeds  $\phi V'_g(\phi r')$  along  $\phi c\phi t'$ , whose magnitude is maximum at the edge  $L^0$  of  $\phi E'/\phi c^2$  and decreases continuously to zero at point O.

The intrinsic rest mass  $-\phi M_0^*$  in  $-\phi\rho'^*$  likewise establishes non-uniform proper intrinsic gravitational speeds  $\phi V'_g(\phi r')$  along  $-\phi\rho'^*$  and  $-\phi E'^*/\phi c^2$  in  $-\phi c\phi t'^*$  establishes non-uniform proper intrinsic gravitational speeds  $\phi V'_g(\phi r')$  along  $-\phi c\phi t'^*$  in Fig. 5.

Quite apart from the non-uniform absolute gravitational speeds  $\hat{V}_g(\hat{r})$  in  $\Sigma'$  and  $ct'$  in Fig. 5, the rest mass  $M_0$  in  $\Sigma'$ , as a gravitational field source, establishes non-uniform proper gravitational speeds  $V'_g(r')$  along every radial direction from its centre in  $\Sigma'$  and the rest mass  $E'/c^2$  ( $\equiv M_0$ ) in  $ct'$  establishes non-uniform proper gravitational speeds  $V'_g(r')$  along  $ct'$ . Likewise for  $-M_0^*$  in  $-\Sigma'^*$  and  $-E'^*/c^2$  in  $-ct'^*$  in the third quadrant.

The non-uniform proper intrinsic gravitational speeds  $\phi V'_g(\phi r')$  established along the straight line  $\phi\rho'$ ,  $\phi c\phi t'$ ,  $-\phi\rho'^*$  and  $-\phi c\phi t'^*$  by the intrinsic gravitational field sources  $\phi M_0$ ,  $\phi E'/\phi c^2$ ,  $-\phi M_0^*$  and  $-\phi E'^*/\phi c^2$  respectively in these proper intrinsic dimensions, as described above, will cause  $\phi\rho'$  and  $\phi c\phi t'$  to be curved into the first quadrant and second quadrant respectively to form orthogonal curvilinear intrinsic dimensions. The curved  $\phi\rho'$  in the first quadrant will then project a straight line relativistic intrinsic space  $\phi\rho$  along the horizontal, which is made manifest in a spherical region of relativistic physical Euclidean 3-space  $\Sigma$  in the first quadrant. The curved  $\phi c\phi t'$  in the second quadrant will likewise project straight line relativistic intrinsic time dimension  $\phi c\phi t$  along the vertical, which is made manifest outwardly in relativistic physical time dimension  $ct$  along the vertical in the first quadrant.

As discussed in the process of transforming Fig. 11 of [1] into Fig. 1 of [2], Fig. 5 at the first stage of evolution of spacetime/intrinsic spacetime in a gravitational field will endure for no moment before transforming into Fig. 7 at the second stage of evolution of spacetime/intrinsic spacetime in a gravitational field.

Fig. 1 of [2] drawn within an attractive long-range metric force field has simply been adapted to the gravitational field in Fig. 7. Consequently the symmetry-partner gravitational field sources in spacetimes and symmetry-partner intrinsic gravitational field sources in intrinsic spacetimes in the positive (or our) universe and the negative universe have been integrated

into the diagram in Fig. 7. The proper intrinsic static speeds and proper static speeds denoted by  $\phi V'_{S,P}$  and  $V'_{S,P}$  in Fig. 1 of [2] have also been re-denoted by  $\phi V'_g(\phi r')$  and  $V'_g(r')$  and referred to as proper intrinsic gravitational speeds and proper gravitational speeds, as alternative names, in the case of gravitational field.

The line of relativistic mass  $M$  of length SL in Fig. 7 is actually a spherical relativistic mass  $M$  (as being assumed) of radius  $R = SL$  and the line of relativistic Euclidean 3-space  $\Sigma$  is actually a spherical relativistic Euclidean 3-space of large radius SO with  $M$  at its centre. The relativistic intrinsic space  $\phi\rho$  is an isotropic intrinsic dimension with respect to 3-observers in the relativistic Euclidean 3-space  $\Sigma$ . It can be considered to lie along any of the radial directions of the spherical relativistic Euclidean 3-space  $\Sigma$  with respect to 3-observers in  $\Sigma$ .

As illustrated in Fig. 7, the non-uniform proper intrinsic gravitational speeds  $\phi V'_g(\phi r')$  along the curved proper intrinsic space  $\phi\rho'$  are projected invariantly as non-uniform proper intrinsic gravitational speeds  $\phi V'_g(\phi r')$  along the projective straight line isotropic relativistic intrinsic space  $\phi\rho$  along the horizontal, which is made manifest in non-uniform proper gravitational velocities  $\vec{V}'_g(r')$  along every radial direction from the centre of the relativistic mass  $M$  of the gravitational field source in  $\Sigma$ . The non-uniform proper intrinsic gravitational speeds along the curved proper intrinsic time dimension  $\phi c\phi t'$ , likewise invariantly project non-uniform proper intrinsic gravitational speeds along the projective relativistic intrinsic time dimension  $\phi c\phi t$  along the vertical, which are made manifest in non-uniform proper gravitational speeds  $V'_g(r')$  along the relativistic time dimension  $ct$ .

The foregoing paragraph describes the graphical representation of the invariance of intrinsic gravitational speed and gravitational speed in the context of the theory of relative intrinsic gravity and theory of relative gravity that transform Fig. 5 into Fig. 7 at the second stage of evolution of spacetime/intrinsic spacetime in a gravitational field, expressed as follows

$$\phi V_g(\phi r) = \phi V'_g(\phi r') \quad (2a)$$

and

$$V_g(r) = V'_g(r') \quad (2b)$$

Eq. (2a) states that the non-uniform relativistic intrinsic gravitational speeds  $\phi V_g(\phi r)$  that are expected to be projected into the relativistic intrinsic space  $\phi\rho$  by the non-uniform proper intrinsic gravitational speeds  $\phi V'_g(\phi r')$  along the curved proper intrinsic space  $\phi\rho'$ , are the same as the non-uniform proper intrinsic gravitational speeds along the curved  $\phi\rho'$  and Eq. (2b) states that the non-uniform relativistic gravitational speeds  $V_g(r)$  that are expected to appear along every radial direction from the centre of the relativistic mass  $M$  of the gravitational field source in the relativistic Euclidean 3-space  $\Sigma$  in Fig. 7, are non-uniform proper gravitational speeds

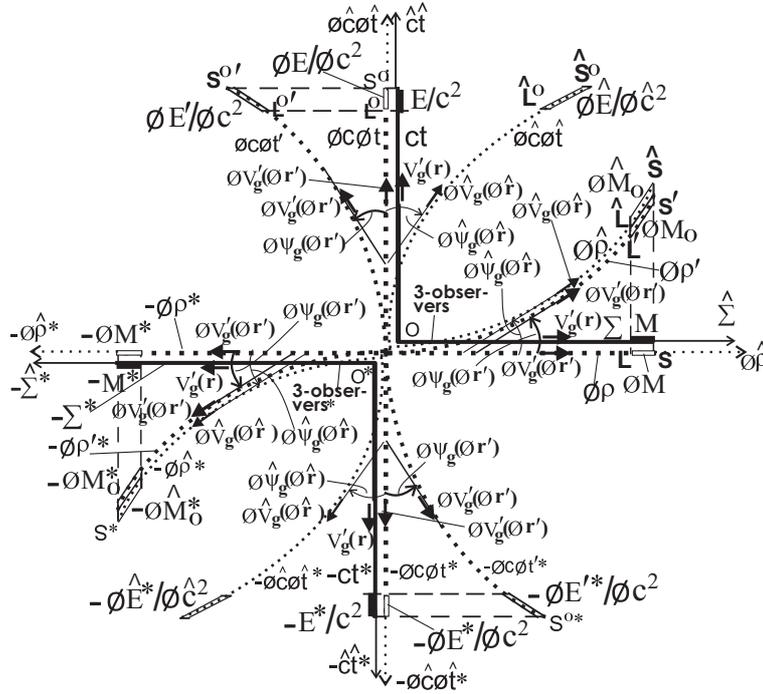


Fig. 7: The spacetime/intrinsic spacetime geometry at the second stage of evolution of spacetime/intrinsic spacetime within symmetry-partner gravitational field sources in the positive and negative universes, with respect to 3-observers in the Euclidean 3-spaces in the two universes, which evolves from the geometry of Fig. 5 at the first stage.

$V'_g(r')$ . Formal proofs of the invariance (2a) and (2b) along with those of Eqs. (1a) and (1b) shall be given elsewhere with further development.

The geometry of Fig. 7 will endure for as long as the ‘projective’ relativistic intrinsic mass  $\phi M$  does not establish non-uniform relativistic intrinsic gravitational speeds  $\phi V_g(\phi r)$  along the relativistic intrinsic space  $\phi \rho$ , which could cause the curvature of  $\phi \rho$  and as long as the ‘projective’ relativistic intrinsic mass  $\phi E/\phi c^2 (\equiv \phi M)$  does not establish non-uniform relativistic intrinsic gravitational speeds  $\phi V_g(\phi r)$  along the relativistic intrinsic time dimension  $\phi c \phi t$  along the vertical, which could cause the curvature of  $\phi c \phi t$ .

Now the relativistic mass  $M$  in the relativistic Euclidean 3-space  $\Sigma$  shall be identified as the inertial mass and passive gravitational mass, which is non-trivially related to the rest mass  $M_0$  according to a relation that shall be derived elsewhere with further development. The relativistic mass (i.e. the inertial mass or passive gravitational mass) is not a gravitational field source, the active gravitational mass  $M_{0a}$  being the source of the Newtonian gravitational field [8]. Consequently  $M$  is not a source of gravitational speed. This means that  $M$  cannot establish non-uniform relativistic gravitational velocity  $\vec{V}_g(r)$  radially from its centre in  $\Sigma$  and consequently  $\phi M$  cannot establish non-uniform relativistic intrinsic gravitational speeds  $\phi V_g(\phi r)$  along the relativistic intrinsic space  $\phi \rho$ . The relativistic mass  $E/c^2 (\equiv M)$  in the relativistic time

dimension  $ct$  cannot establish non-uniform relativistic gravitational speeds along  $ct$  and  $\phi E/\phi c^2$  cannot establish non-uniform relativistic intrinsic gravitational speeds along  $\phi c \phi t$ .

The non-existence of non-uniform relativistic intrinsic gravitational speeds  $\phi V_g(\phi r)$  along the relativistic intrinsic dimensions  $\phi \rho$ ,  $\phi c \phi t$ ,  $-\phi \rho^*$  and  $-\phi c \phi t^*$ , either by projections from the curved proper intrinsic dimensions  $\phi \rho'$ ,  $\phi c \phi t'$ ,  $-\phi \rho'^*$  and  $-\phi c \phi t'^*$  or by establishments by  $\phi M$ ,  $\phi E/\phi c^2$ ,  $-\phi M^*$  and  $-\phi E^*/\phi c^2$  as sources, as discussed in the foregoing paragraph, implies that the relativistic intrinsic spaces  $\phi \rho$  and  $-\phi \rho^*$  and relativistic intrinsic time dimensions  $\phi c \phi t$  and  $-\phi c \phi t^*$  in Fig. 7, cannot be curved. This makes the geometry of Fig. 7 to endure for as long as the symmetry-partner gravitational field sources in the positive and negative universes exist. The consequence of this is that the evolutions of spacetime/intrinsic spacetime in a gravitational field terminates at the second stage naturally. This immutable fact of nature shall become solidly established upon this initial introduction to its establishment elsewhere with further development.

The geometry of Fig. 7 is valid with respect to 3-observers in the relativistic Euclidean 3-space  $\Sigma$  and  $-\Sigma^*$  in the positive and negative universes, as indicated. It corresponds to Fig. 1 of [2]. There is a complimentary diagram to Fig. 7, which corresponds to Fig. 3 of [2], depicted in Fig. 8. Fig. 8 is valid with respect to 1-observers in the relativistic time dimensions  $ct$  and  $-ct^*$  as indicated.

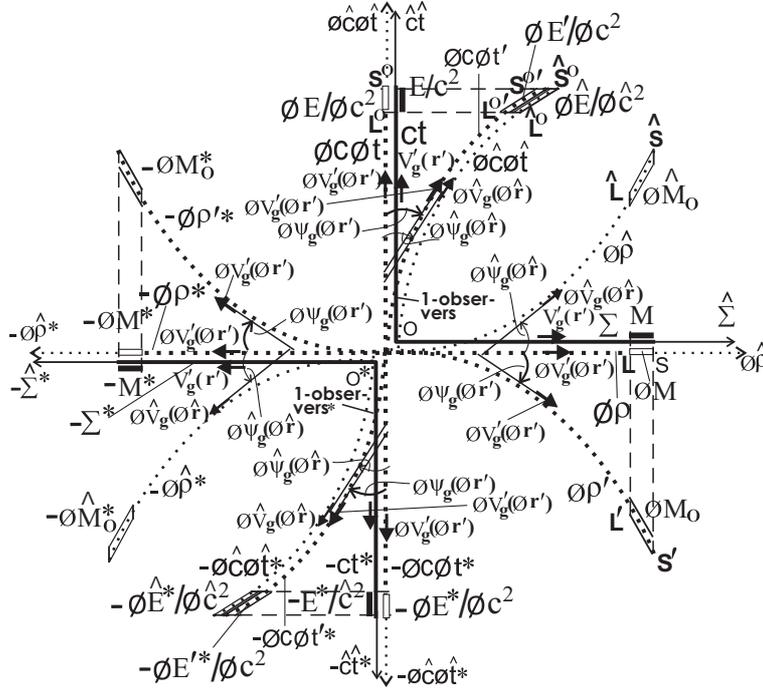


Fig. 8: The complementary diagram to Fig. 7 that is valid with respect to 1-observers in the time dimensions in the positive and negative universes.

The global spacetime/intrinsic spacetime diagram of Fig. 7 and its complimentary diagram of Fig. 8, evolve at the second stage of evolution of spacetime/intrinsic spacetime in a gravitational field. The remarkable feature of the diagrams, as has been made for Figs. 1 and 3 of [2] in a long-range metric force field in general, is that the four-dimensional relativistic metric spacetime  $(\Sigma, ct)$  in which the observers are located and its underlying two-dimensional relativistic intrinsic metric spacetime  $(\phi\rho, \phi c\phi t)$  are everywhere flat in a gravitational field. This fact has been solidly established by demonstrating local Lorentz invariance in a long-range metric force field in general in [2]. Gravitational local Lorentz invariance (GLLT) shall be established within a gravitational field shortly in this paper.

Although the extended two-dimensional proper intrinsic metric spacetimes  $(\phi\rho', \phi c\phi t')$  and  $(-\phi\rho'^*, -\phi c\phi t'^*)$  are curved in a gravitational field in Figs. 7 and 8, they are orthogonal curvilinear intrinsic dimensions. This means that they possess intrinsic Lorentzian metric tensor at every point of them with respect to 3-observers in the relativistic Euclidean 3-spaces  $\Sigma$  and  $-\Sigma^*$  and 1-observers in the relativistic time dimensions  $ct$  and  $-ct^*$ , as has been demonstrated in a long-range metric force field in general in [2].

The only curved spacetime with sub-Riemannian metric tensor in Figs. 7 and 8 at the second stage of evolution of spacetime/intrinsic spacetime in a gravitational field in our universe, so to speak, is the 'two-dimensional' absolute intrinsic metric spacetime  $(\phi\hat{\rho}, \phi\hat{c}\phi\hat{t})$  with absolute intrinsic

sub-Riemannian metric tensor with respect to 3-observers in the relativistic Euclidean 3-spaces  $\Sigma$  and  $-\Sigma^*$ , as has been established within a long-range metric force field in general in [2]. The curved  $(\phi\hat{\rho}, \phi\hat{c}\phi\hat{t})$  in Figs. 7 and 8 at the second stage of evolution of spacetime/intrinsic spacetime in a gravitational field, has been brought forward from the geometry of Fig. 5 at the first stage.

For completeness and in order to be able to derive the inverse intrinsic gravitational local Lorentz transformation (inverse  $\phi$ GLLT) and inverse GLLT, we must also draw the inverses to the global geometries of Figs. 7 and 8, shown as Figs. 9 and 10 respectively.

### 1.1 Simultaneous progression at the speed of light of the first and second stages of evolution of spacetime/intrinsic spacetime away from the location of a gravitational field source

The evolution of the extended curved 'two-dimensional' absolute intrinsic metric spacetime  $(\phi\hat{\rho}, \phi\hat{c}\phi\hat{t})$  between point  $(\hat{S}, \hat{S}^0)$  and point O in spacetime in Fig. 5, does not happen instantaneously, following the introduction of the absolute rest mass  $\hat{M}_0$  of the gravitational field source at point  $\hat{S}$  in the flat absolute space  $\hat{\Sigma}$  and the consequent introduction of the absolute rest mass  $\hat{E}/\hat{c}^2$  ( $\equiv \hat{M}_0$ ) of the gravitational field source at point  $\hat{S}^0$  in the absolute time 'dimension'  $\hat{c}\hat{t}$  in the reference geometry of Fig. 3, as would happen if gravitational effect propagated at infinite speed in spacetime. How-

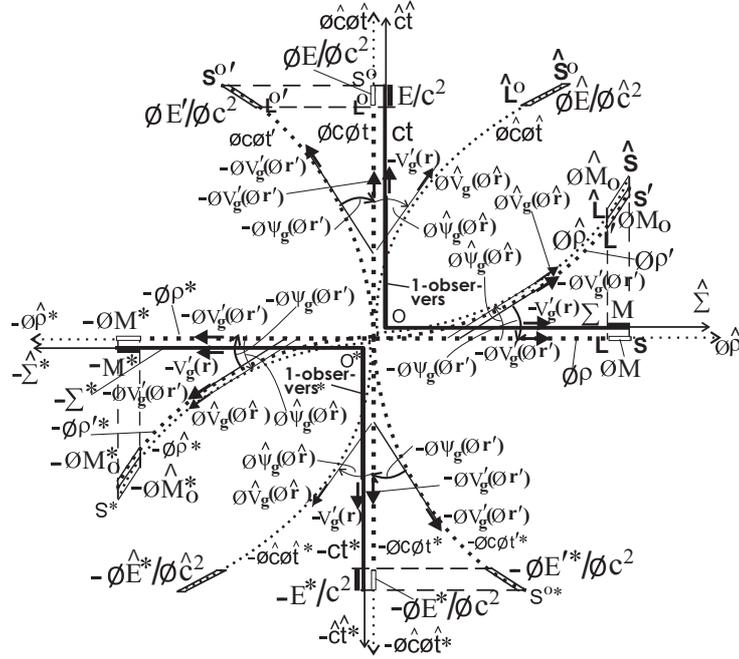


Fig. 9: The inverse to the spacetime/intrinsic spacetime geometry to Fig. 7 at the second stage of evolutions of spacetimes/intrinsic spacetimes within symmetry-partner gravitational fields in the positive and negative universes that is valid with respect to 1-observers in the relativistic time dimensions in the two universes.

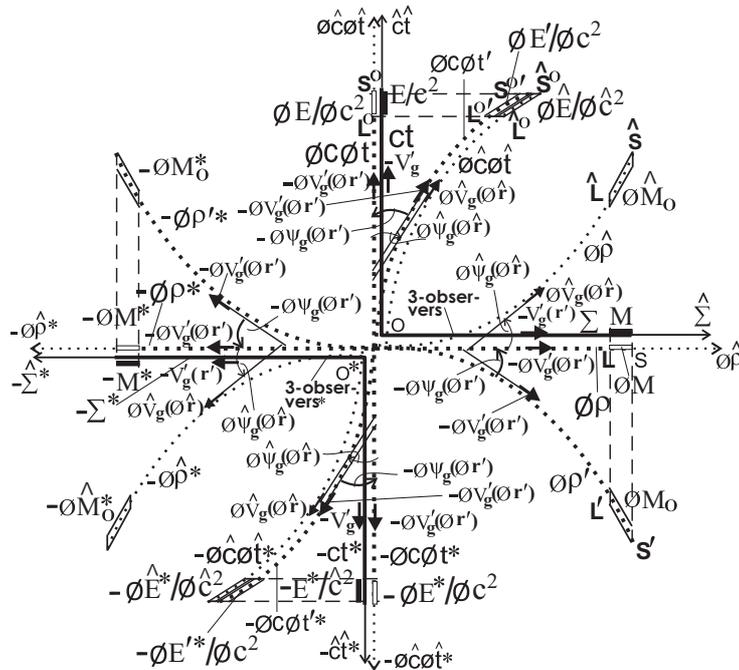


Fig. 10: The inverse to the spacetime/intrinsic spacetime geometry to Fig. 8 at the second stage of evolutions of spacetimes/intrinsic spacetimes within symmetry-partner gravitational fields in the positive and negative universe that is valid with respect to 3-observers in the relativistic Euclidean 3-spaces in the two universes.

ever the curvature of the flat  $(\phi\hat{\rho}, \phi\hat{c}\phi\hat{t})$  in Fig. 3 starts from point  $(\hat{S}, \hat{S}^0)$  at the moment  $(\hat{M}_0, \hat{E}/\hat{c}^2)$  is introduced at this point and progresses at the speed of light away from the point  $(\hat{S}, \hat{S}^0)$ , since gravitational effect propagates at the speed of light in spacetime. It therefore took a long time for curvature of  $(\phi\hat{\rho}, \phi\hat{c}\phi\hat{t})$  to propagate from point  $(\hat{S}, \hat{S}^0)$  to the distant point O.

Likewise the curvature of two-dimensional proper intrinsic metric spacetime  $(\phi\rho', \phi c\phi t')$  at the second stage of evolution of spacetime/intrinsic spacetime in a gravitational field, starts from the point  $(S', S'^0)$  where the proper gravitational field source  $(M_0, E'/c^2)$  is located in Fig. 5 and progresses at the speed of light away from this point. It therefore took a long time for curvature of  $(\phi\rho', \phi c\phi t')$  to propagate from point  $(S', S'^0)$  to the distant point O in Fig. 7.

The point to be established in this sub-section is that the curvature of the proper intrinsic metric spacetime  $(\phi\rho', \phi c\phi t')$  at the second stage of evolution of spacetime/intrinsic spacetime in a gravitational field did not start after a long time taken by curvature of the absolute intrinsic metric spacetime  $(\phi\hat{\rho}, \phi\hat{c}\phi\hat{t})$  to propagate from point  $(\hat{S}, \hat{S}^0)$  of location of the gravitational field source to the distant point O in Fig. 5, but that the curvatures of  $(\phi\hat{\rho}, \phi\hat{c}\phi\hat{t})$  and  $(\phi\rho', \phi c\phi t')$  commence simultaneously from point  $(\hat{S}, \hat{S}^0)$  at the moment  $(\hat{M}_0, \hat{E}/\hat{c}^2)$  is introduced at point  $(\hat{S}, \hat{S}^0)$  in Fig. 3 and progress simultaneously at the speed of light away from this point to simultaneously reach the distant point O after a long time following the introduction of  $(\hat{M}_0, \hat{E}/\hat{c}^2)$  at point  $(\hat{S}, \hat{S}^0)$  in Fig. 3, while the intermediate diagram of Fig. 5 did not appear. This fact is consolidated with the argument that follows.

Figure 5 represents a situation where the intrinsic rest mass  $\phi M_0$  of the gravitational field source ‘projected’ into the straight line proper intrinsic metric space  $\phi\rho'$  along the horizontal and the intrinsic rest mass  $\phi E'/\phi c^2 (\equiv \phi M_0)$  ‘projected’ into the straight line proper intrinsic metric time dimension  $\phi c\phi t'$  along the vertical, have not established non-uniform proper intrinsic gravitational speeds  $\phi V'_g(\phi r')$  along  $\phi\rho'$  and  $\phi c\phi t'$  respectively from their locations. Consequently the rest mass  $M_0$  has not established non-uniform proper gravitational speeds  $V'_g(r')$  along every radial direction from its centre in the proper physical Euclidean 3-space  $\Sigma'$  and the rest mass  $E'/c^2 (\equiv M_0)$  in the proper physical time dimension  $ct'$  has not established non-uniform proper gravitational speeds  $V'_g(r')$  along  $ct'$ .

Thus Fig. 5 represents a situation that must be described as the presence of absolute gravity and absolute intrinsic gravity, but the absence of relative gravity and relative intrinsic gravity, since only projective non-uniform absolute intrinsic gravitational speeds  $\phi\hat{V}_g(\phi\hat{r})$  are present along the straight line  $\phi\hat{\rho}$  and  $\phi\hat{c}\phi\hat{t}$  and only non-uniform absolute gravitational speeds  $\hat{V}_g(\hat{r})$  are present along every radial direction from the centre of the rest mass  $M_0$  in  $\Sigma'$  and along the proper time dimension  $ct'$  from the base of  $E'/c^2 (\equiv M_0)$  in  $ct'$  in Fig. 5. The extended flat four-dimensional proper spacetime

$(\Sigma', ct')$  and its underlying extended flat two-dimensional proper intrinsic spacetime  $(\phi\rho', \phi c\phi t')$  in Fig. 5 in a gravitational field, will endure for as long as this situation persists. The clarifications of the concepts of relative static speed and relative metric force field done in sub-section 3 of [2] are directly applicable to the concepts of relative gravitational speed and relative gravitational field being introduced in this paper.

However the situation of absence of relative gravity/relative intrinsic gravity but the presence of absolute gravity/absolute intrinsic gravity, which Fig. 5 represents, discussed in the preceding two paragraphs, is hypothetical; it does not exist in reality. This is so because as the evolution of curved ‘two-dimensional’ absolute intrinsic metric spacetime  $(\phi\hat{\rho}, \phi\hat{c}\phi\hat{t})$  starts from the point  $(\hat{S}, \hat{S}^0)$  on the flat absolute spacetime  $(\hat{\Sigma}, \hat{c}\hat{t})$  where the absolute gravitational field source  $(\hat{M}_0, \hat{E}/\hat{c}^2)$  is introduced in the reference geometry of Fig. 3 and progresses at the speed of light from this point towards point O that is far removed from the point  $(\hat{S}, \hat{S}^0)$ , as mentioned earlier, the evolution of the projective flat two-dimensional proper intrinsic metric spacetime  $(\phi\rho', \phi c\phi t')$  containing the projective intrinsic rest mass  $(\phi M_0, \phi E'/\phi c^2)$  of the gravitational field source at point  $(S', S'^0)$  at the origin (or base) of the projective flat  $(\phi\rho', \phi c\phi t')$ , as well as the evolution of the flat four-dimensional proper metric spacetime  $(\Sigma', ct')$  containing the rest mass  $(M_0, E'/c^2)$  at the point  $(S', S'^0)$ , start from this point of location of the gravitational field source and progress at the speed of light towards the distant point O, simultaneously with the progression at the speed of light of curvature of  $(\phi\hat{\rho}, \phi\hat{c}\phi\hat{t})$  from point  $(\hat{S}, \hat{S}^0)$ .

However the moment  $\phi M_0$  appears within a short segment  $\Delta\phi\rho'$  containing  $\phi M_0$  projected along the horizontal at point  $S'$  and  $\phi E'/\phi c^2 (\equiv \phi M_0)$  appears within a short segment  $\phi c\Delta\phi t'$  containing  $\phi E'/\phi c^2$  projected along the vertical at point  $S'^0$ , at the beginning of the evolution of curved absolute intrinsic spacetime  $(\phi\hat{\rho}, \phi\hat{c}\phi\hat{t})$  at the point  $(\hat{S}, \hat{S}^0)$ , the intrinsic rest mass  $\phi M_0$  contained in the projective interval  $\Delta\phi\rho'$  starts to establish non-uniform proper intrinsic gravitational speeds  $\phi V'_g(\phi r')$  within  $\Delta\phi\rho'$  located at point  $S'$  along the horizontal and  $\phi E'/\phi c^2$  contained in the projective  $\phi c\Delta\phi t'$  starts to establish non-uniform proper intrinsic gravitational speeds  $\phi V'_g(\phi r')$  within  $\phi c\Delta\phi t'$  located at point  $S'^0$  along the vertical.

Even the establishment of non-uniform proper intrinsic gravitational speeds  $\phi V'_g(\phi r')$  progresses at the speed of light from point  $S'$  along the proper intrinsic space  $\phi\rho'$  that is evolving along the horizontal and the establishment of non-uniform proper intrinsic gravitational speeds  $\phi V'_g(\phi r')$  progresses at the speed of light from point  $S'^0$  along the proper intrinsic time dimension  $\phi c\phi t'$  that is evolving along the vertical. Since the evolution of  $\phi\rho'$  also starts from point  $S'$  and progresses at the speed of light away from this point along the horizontal and the evolution of  $\phi c\phi t'$  also starts from point  $S'^0$  and progresses at the speed of light along the vertical, it

follows that at some time  $t'$  of the commencement of evolution of curved absolute intrinsic spacetime  $(\phi\hat{\rho}, \phi\hat{c}\hat{t})$ , some lengths of  $\phi\rho'$  and  $\phi c\phi t'$  have evolved along the horizontal and vertical respectively and  $\phi M_0$  at the point  $S'$  has established non-uniform proper intrinsic gravitational speeds along the whole length  $\phi\rho'$  that has evolved after the time  $t'$  and  $\phi E'/\phi c^2$  at point  $S'^0$  at the origin of the evolved  $\phi c\phi t'$  has established non-uniform proper intrinsic speeds  $\phi V'_g(\phi r')$  along the whole length of  $\phi c\phi t'$  that has evolved after the time  $t'$ .

Now the establishment of non-uniform proper intrinsic gravitational speeds  $\phi V'_g(\phi r')$  along the whole length of the evolving proper intrinsic space  $\phi\rho'$  at all times by  $\phi M_0$  located at the origin (or base)  $S'$  of the evolving  $\phi\rho'$  does not allow the evolving  $\phi\rho'$  to remain along the horizontal, but causes it to become curved anti-clockwise into the first quadrant with respect to 3-observers in  $\Sigma$  as in Fig. 7. Likewise the establishment of non-uniform proper intrinsic gravitational speeds  $\phi V'_g(\phi r')$  along the whole length of the evolving  $\phi c\phi t'$  at all times by  $\phi E'/\phi c^2$  located at the origin (or base)  $S'^0$  of the evolving  $\phi c\phi t'$ , does not allow the evolving  $\phi c\phi t'$  to remain along the vertical, but causes it to become curved anti-clockwise into the second quadrant with respect to 3-observers in  $\Sigma$  as in Fig. 7.

It follows from the foregoing paragraph that there is no time lag between the evolution of the curved 'two-dimensional' absolute intrinsic metric spacetime  $(\phi\hat{\rho}, \phi\hat{c}\hat{t})$  at the first stage of evolution of spacetime/intrinsic spacetime in a gravitational field and the evolution of the curved two-dimensional proper intrinsic metric spacetime  $(\phi\rho', \phi c\phi t')$  at the second stage. In other words, the evolutions of curved  $(\phi\hat{\rho}, \phi\hat{c}\hat{t})$  and curved  $(\phi\rho', \phi c\phi t')$  in Fig. 7 begin simultaneously from the location  $(\hat{S}, \hat{S}^0)$  of introduction of the gravitational field source

$(\hat{M}_0, \hat{E}/\hat{c}^2)$  in the reference geometry of Fig. 3 and progress simultaneously at the speed of light away from this point towards point O in that figure.

The evolutions of the projective flat two-dimensional relativistic intrinsic metric spacetime  $(\phi\rho, \phi c\phi t)$  and its outward manifestation namely, the flat four-dimensional relativistic spacetime  $(\Sigma, ct)$  in Fig. 7, start from the location  $(\hat{S}, \hat{S}^0)$  at which  $(\hat{M}_0, \hat{E}/\hat{c}^2)$  is introduced in Fig. 3 and progress at the speed of light towards the distant point O, simultaneously with the evolutions and progression at the speed of light of the curved  $(\phi\hat{\rho}, \phi\hat{c}\hat{t})$  and curved  $(\phi\rho', \phi c\phi t')$ , thereby transforming Fig. 3 to Fig. 7 after a long time, without actually passing through the intermediate Fig. 5.

Thus the evolutions of curved 'two-dimensional' absolute intrinsic metric spacetime  $(\phi\hat{\rho}, \phi\hat{c}\hat{t})$  and curved two-dimensional proper intrinsic metric spacetime  $(\phi\rho', \phi c\phi t')$  and consequently the evolutions of flat two-dimensional relativistic intrinsic metric spacetime  $(\phi\rho, \phi c\phi t)$  and its outward manifestations namely, the flat four-dimensional relativistic metric spacetime  $(\Sigma, ct)$ , have progressed simultaneously at the speed of light from the location  $(\hat{S}, \hat{S}^0)$  on the flat absolute

spacetime  $(\hat{\Sigma}, \hat{c}\hat{t})$  in Fig. 3, where the absolute rest mass  $(\hat{M}_0, \hat{E}/\hat{c}^2)$  of the gravitational field source was suddenly introduced, to some near neighbourhood of the point  $(\hat{S}, \hat{S}^0)$  after a given time  $T'$  of introduction of  $(\hat{M}_0, \hat{E}/\hat{c}^2)$  at the point  $(\hat{S}, \hat{S}^0)$ , and the effect of the gravitational field of the suddenly introduced field source can be felt on flat four-dimensional relativistic spacetime  $(\Sigma, ct)$  (that has evolved) at every point within such near neighbourhood at time  $T'$ .

On the other hand, some distant neighbourhoods of the point  $(\hat{S}, \hat{S}^0)$  have not yet experienced the evolutions of curved  $(\phi\hat{\rho}, \phi\hat{c}\hat{t})$ , curved  $(\phi\rho', \phi c\phi t')$ , flat  $(\phi\rho, \phi c\phi t)$  and flat  $(\Sigma, ct)$ , established by the suddenly introduced gravitational field source at point  $(\hat{S}, \hat{S}^0)$  at the given time  $T'$  after the sudden introduction of the gravitational field source. The flat absolute spacetime  $(\hat{\Sigma}, \hat{c}\hat{t})$  and its underlying flat absolute intrinsic spacetime  $(\phi\hat{\rho}, \phi\hat{c}\hat{t})$  of the reference geometry of Fig. 3 still exist in those distant neighbourhoods. Consequently the effect of the gravitational field of the suddenly introduced field source will not be felt in those distant neighbourhoods of point  $(\hat{S}, \hat{S}^0)$  at the given time  $T'$  after the sudden introduction of the gravitational field source at point  $(\hat{S}, \hat{S}^0)$ .

It has so far been assumed that the gravitational field source introduced at the point  $(\hat{S}, \hat{S}^0)$  in  $(\hat{\Sigma}, \hat{c}\hat{t})$  in Fig. 3 is the only one in our universe. However let us now relax this assumption and assume that other existing gravitational field sources had already established curved absolute intrinsic metric spacetime  $(\phi\hat{\rho}, \phi\hat{c}\hat{t})$ , curved two-dimensional proper intrinsic metric spacetime  $(\phi\rho', \phi c\phi t')$ , flat two-dimensional relativistic intrinsic metric spacetime  $(\phi\rho, \phi c\phi t)$  and flat four-dimensional relativistic metric spacetime  $(\Sigma, ct)$  at the distant neighbourhoods of the suddenly introduced gravitational field source at point  $(\hat{S}, \hat{S}^0)$ . Then the effect of the gravitational fields of these existing field sources will be felt on flat relativistic spacetime, while the effect of the gravitational field of the suddenly introduced field source at point  $(\hat{S}, \hat{S}^0)$  will not be felt on flat relativistic spacetime at those distant neighbourhoods of point  $(\hat{S}, \hat{S}^0)$  at the given time the  $T'$  of the sudden introduction of the gravitational field source at point  $(\hat{S}, \hat{S}^0)$ .

It follows from the foregoing that the effect of the gravitational fields of a great multitude of very distant stars and galaxies formed billions of years ago have not yet reached the earth. The foregoing also explains why if the Sun is suddenly annihilated, the earth will not be aware of this until about nine seconds required for the effect of gravity to propagated at speed of light from the location of the Sun to the surface of the earth.

Apart from the evolutions of curved absolute intrinsic metric spacetime  $(\phi\hat{\rho}, \phi\hat{c}\hat{t})$ , curved proper intrinsic metric spacetime  $(\phi\rho', \phi c\phi t')$ , flat relativistic intrinsic metric spacetime  $(\phi\rho, \phi c\phi t)$  and flat relativistic metric spacetime  $(\Sigma, ct)$ , all of which propagate simultaneously at the speed of light to distant places from the location of a suddenly introduced gravitational field source and their annihilation, which also

propagate simultaneously at the speed of light to distant places from the location of the suddenly annihilated gravitational field source, the perturbation of the source of a gravitational field, (such as will arise from sudden decrease or sudden increase of the rest mass of the gravitational field source), will lead to perturbations in curvatures of  $(\phi\hat{\rho}, \phi\hat{c}\phi\hat{t})$  and  $(\phi\rho', \phi c\phi t')$ , as well as perturbations in flat relativistic intrinsic metric spacetime  $(\phi\rho, \phi c\phi t)$  and flat relativistic metric spacetime  $(\Sigma, ct)$  established by the gravitational field source. These perturbations will also propagate simultaneously at the speed of light from the location of the perturbed gravitational field source to distant places.

Although perturbations in flat relativistic spacetime  $(\Sigma, ct)$  will be very faint at distant places from the perturbed gravitational field source, but it can be measured (as perturbations in gravitational length contractions of space intervals or objects and of gravitational time dilations of time intervals) in principle. (The concepts of gravitational length contraction and gravitational time dilation in the context of the theory of gravitational relativity (TGR) that operates on the flat relativistic spacetime  $(\Sigma, ct)$  in Fig. 7, shall be established in the next section.) However nothing in this discussion suggests that perturbations in curved  $(\phi\hat{\rho}, \phi\hat{c}\phi\hat{t})$ , curved  $(\phi\rho', \phi c\phi t')$ , flat  $(\phi\rho, \phi c\phi t)$  and flat  $(\Sigma, ct)$  propagate as waves at the speed of light. The results and conclusions reached by qualitative discussion in this sub-section shall be supported quantitatively by actual calculations elsewhere with further development.

The next step in this paper is to adapt the new results derived in section 2 of [2] from the local spacetime/intrinsic spacetime geometries of Figs. 6 – 9 of that paper, at the second stage of evolution of spacetime/intrinsic spacetime in a long-range metric force field in general, to the gravitational field. Those results are the intrinsic local Lorentz transformation ( $\phi$ LLT) in terms of proper intrinsic static speed  $\phi V'_s$ ; local Lorentz transformation (LLT) in terms of proper static speed  $V'_s$ ; intrinsic local Lorentz invariance ( $\phi$ LLI) and local Lorentz invariance (LLI) implied by  $\phi$ LLT and LLT respectively; intrinsic time dilation and intrinsic length contraction formulae in terms of proper intrinsic static speed  $\phi V'_s$  and time dilation and length contraction formulae in terms of proper static speed  $V'_s$ . This shall be accomplished in the next section.

## 2 The theory of relativity/intrinsic theory of relativity associated with the presence of relative gravitational field /relative intrinsic gravitational field at the second stage of evolution of spacetime/intrinsic spacetime in a gravitational field

As stated above, the programme of this section is to adapt the results of section 2 of [2] in a long-range metric force field in general to the gravitational field. We must simply replace the proper intrinsic static speed  $\phi V'_{S,P}$  and proper static

speed  $V'_{S,P}$  that appear in those results by the proper intrinsic gravitational speed  $\phi V'_g(\phi r')$  and proper gravitational speed  $V'_g(r')$  respectively, where  $\phi V'_g(\phi r')$  must be related to the proper intrinsic gravitational parameters and  $V'_g(r')$  must be related to the proper gravitational parameters of the external gravitational field.

It thus follows that the place to start this section is derivation of expressions for  $\phi V'_g(\phi r')$  and  $V'_g(r')$ , as well as for absolute intrinsic gravitational speed  $\phi\hat{V}_g(\phi\hat{r})$  that will appear in the absolute intrinsic metric tensor  $\phi\hat{g}_{ij}$ , absolute intrinsic Ricci tensor  $\phi\hat{R}_{ij}$  and absolute intrinsic line element of the metric theory of absolute intrinsic gravity ( $\phi$ MAG) on curved ‘two-dimensional’ absolute intrinsic metric spacetime  $(\phi\hat{\rho}, \phi\hat{c}\phi\hat{t})$  with respect to 3-observers in the relativistic Euclidean 3-space  $\Sigma$  in Fig. 7.

### 2.1 Relating gravitational speed/intrinsic gravitational speed to gravitational parameters/intrinsic gravitational parameters

The geometry of Fig. 5 is a valid geometry at the first stage of evolution of spacetime/intrinsic spacetime in a gravitational field. It does not exist in a gravitational field however because there was no time for it to be formed, since the second stage of evolution of spacetime/intrinsic spacetime commences at the same moment that the geometry of Fig. 5 at the first stage begins to evolve, thereby yielding the geometry of Fig. 7 of combined first and second stages of evolution of spacetime/intrinsic spacetime in a gravitational field as the geometry that exists in every gravitational field, as explained in sub-section 1.1.

Now let the ‘one-dimensional’ absolute intrinsic rest mass  $\phi\hat{m}_0$  of a test particle be in absolute intrinsic fall (at increasing absolute intrinsic dynamical speed  $\phi\hat{V}_d$ ) along the curved absolute intrinsic space  $\phi\hat{\rho}$  towards the absolute intrinsic rest mass  $\phi\hat{M}_0$  of the gravitational field source at the origin of the curved  $\phi\hat{\rho}$  in our universe in Fig. 5. In perfect symmetry, the symmetry-partner test particle of negative absolute intrinsic rest mass  $-\phi\hat{m}_0^*$  is in absolute intrinsic fall (at increasing absolute intrinsic dynamical speed  $\phi\hat{V}_d$ ) along the curved absolute intrinsic space  $-\phi\hat{\rho}^*$  towards the negative absolute intrinsic rest mass  $-\phi\hat{M}_0^*$  of the symmetry-partner gravitational field source at the origin of the curved  $-\phi\hat{\rho}^*$  in the negative universe. However only the absolute intrinsic gravitational fall of the absolute intrinsic rest mass  $\phi\hat{m}_0$  of the test particle in the first quadrant (or in our universe) shall be considered in the derivation hereunder, since there is no interaction between the identical geometries in the first and third quadrants (or in the positive and negative universes) in Fig. 5.

Let the absolute intrinsic rest mass  $\phi\hat{m}_0$  of the test particle in our universe possess absolute intrinsic dynamical speed  $\phi\hat{V}_d$  upon falling along the curved  $\phi\hat{\rho}$  to a position P of ‘distance’  $\phi\hat{r}$  from the base  $\hat{S}$  of  $\phi\hat{M}_0$  located at the origin of the curved  $\phi\hat{\rho}$ . It will acquire the absolute intrinsic gravitational

speed  $\phi\hat{V}_g(\phi\hat{r})$ , (which is an absolute intrinsic static speed) at the position P, of the non-uniform absolute intrinsic gravitational speeds established along the curved  $\phi\hat{\rho}$  by  $\phi\hat{M}_0$ .

Thus the absolute intrinsic rest mass  $\phi\hat{m}_0$  of the test particle will possess absolute intrinsic dynamical speed  $\phi\hat{V}_d$  and absolute intrinsic gravitational speed  $\phi\hat{V}_g(\phi\hat{r})$  it acquires at position P upon falling to this position along the curved  $\phi\hat{\rho}$ . Its absolute intrinsic total energy  $\phi\hat{U}$  at the position P is then given classically in terms of the absolute intrinsic speeds  $\phi\hat{V}_d$  and  $\phi\hat{V}_g(\phi\hat{r})$  as follows

$$\phi\hat{U} = \frac{1}{2}\phi\hat{m}_0\phi\hat{V}_d^2 - \frac{1}{2}\phi\hat{m}_0\phi\hat{V}_g(\phi\hat{r})^2 \quad (3)$$

The negative sign of the absolute intrinsic gravitational energy  $-\frac{1}{2}\phi\hat{m}_0\phi\hat{V}_g(\phi\hat{r})^2$  comes from the negativity of gravitational energy.

The expression (3) has been written by a hypothetical ‘one-dimensional’ absolute intrinsic observer (with absolute intrinsic rest mass) located at the position P along the curved  $\phi\hat{\rho}$  where Eq. (3) is written. This ‘observer’ will be referred to as the proper Riemannian observer for brevity. The absolute intrinsic 1-observers located at other positions along the curved  $\phi\hat{\rho}$  are Riemannian observers, as has been adopted in an earlier paper, see section 4 of [9].

Equation (3) takes on the classical mechanics form with respect to the proper Riemannian observer for two reasons: (i) it involves absolute (and not relative) intrinsic speeds and (ii) the curved absolute intrinsic space  $\phi\hat{\rho}$  is locally straight (or Euclidean) at point P with respect to the proper Riemannian observer. Recall from [1] that the curved ‘two-dimensional’ absolute intrinsic metric spacetime  $(\phi\hat{\rho}, \phi\hat{c}\phi\hat{t})$  in Fig. 5 possesses absolute intrinsic Lorentzian metric tensor at every point of it with respect to the proper Riemannian observer located at each point of it. The valid absolute intrinsic metric tensor on curved  $(\phi\hat{\rho}, \phi\hat{c}\phi\hat{t})$  with respect to the proper Riemannian observer is the absolute intrinsic Lorentzian metric tensor and not the absolute intrinsic sub-Riemannian metric tensor  $\phi\hat{g}_{ij}$  that is valid with respect to every 3-observer (or every Euclidean observer) in the proper Euclidean 3-space  $\Sigma'$  in Fig. 5, as developed in [1].

On the other hand, the expression (3) is given in the usual Newtonian form as follows

$$\phi\hat{U} = \frac{1}{2}\phi\hat{m}_0\phi\hat{V}_d^2 - \frac{G\phi\hat{M}_0}{\phi\hat{r}} \quad (4)$$

A comparison of Eqs. (3) and (4) gives the following expression for  $\phi\hat{V}_g(\phi\hat{r})^2$

$$\phi\hat{V}_g(\phi\hat{r})^2 = 2G\phi\hat{M}_0/\phi\hat{r} \quad (5a)$$

$$\phi\hat{V}_g(\phi\hat{r}) = -\sqrt{2G\phi\hat{M}_0/\phi\hat{r}} \quad (5b)$$

where the negative sign of the absolute intrinsic gravitational speed introduced by hand shall be discussed shortly.

The absolute intrinsic gravitational potential is dependent on the absolute intrinsic gravitational speed as follows

$$\phi\hat{\Phi}(\phi\hat{r}) = -\frac{1}{2}\phi\hat{V}_g(\phi\hat{r})^2 = -\frac{G\phi\hat{M}_0}{\phi\hat{r}} \quad (6)$$

Also because of the absolute intrinsic Lorentzian metric tensor at point P along the the curved  $\phi\hat{\rho}$  where Eqs. (3)-(6) have been written with respect to the proper Riemannian observer (at this point), the proper Riemannian observer obtains the absolute intrinsic gravitational acceleration (or absolute intrinsic gravitational) field at the point P from definition as follows

$$\phi\hat{g}(\phi\hat{r}) = \frac{1}{2} \frac{d(\phi\hat{V}_g(\phi\hat{r})^2)}{d\phi\hat{r}} = -\frac{d(\phi\hat{\Phi}(\phi\hat{r}))}{d\phi\hat{r}} = -\frac{G\phi\hat{M}_0}{\phi\hat{r}^2} \quad (7)$$

The absolute intrinsic gravitational speed  $\phi\hat{V}_g(\phi\hat{r})$  is taken to be the negative root of  $2G\phi\hat{M}_0/\phi\hat{r}$  in Eq. (5b) because of the attractive nature of the gravitational field. It is for this reason that the absolute intrinsic gravitational potential and absolute intrinsic gravitational field possess negative sign in Eqs. (6) and (7). However the negative sign of the absolute intrinsic gravitational speed shall be derived formally and shown to be the origin of the negative sign of  $\phi\hat{\Phi}(\phi\hat{r})$  and  $\phi\hat{g}$  elsewhere with further development.

By removing the symbol  $\phi$  from Eqs. (5a-b)-(7), we obtain expressions for absolute gravitational speed, absolute gravitational potential and absolute gravitational acceleration respectively as follows

$$\hat{V}_g(\hat{r})^2 = 2G\hat{M}_0/\hat{r} \quad (8a)$$

$$\hat{V}_g(\hat{r}) = -\sqrt{2G\hat{M}_0/\hat{r}} \quad (8b)$$

$$\hat{\Phi}(\hat{r}) = -\frac{1}{2}\hat{V}_g(\hat{r})^2 = -\frac{G\hat{M}_0}{\hat{r}} \quad (9)$$

$$\hat{g}(\hat{r}) = \frac{1}{2} \frac{d(\hat{V}_g(\hat{r})^2)}{d\hat{r}} = -\frac{d(\hat{\Phi}(\hat{r}))}{d\hat{r}} = -\frac{G\hat{M}_0}{\hat{r}^2} \quad (10)$$

It is crucial to note that among the absolute intrinsic gravitational speed  $\phi\hat{V}_g(\phi\hat{r})$ , absolute intrinsic gravitational potential  $\phi\hat{\Phi}(\phi\hat{r})$  and absolute intrinsic gravitational acceleration  $\phi\hat{g}(\phi\hat{r})$ , defined by Eqs. (5b), (6) and (7) at point P on the curved absolute intrinsic space  $\phi\hat{\rho}$  at ‘distance’  $\phi\hat{r}$  from the base  $\hat{S}$  of the absolute intrinsic rest mass  $\phi\hat{M}_0$  of the gravitational field source at the origin of the curved  $\phi\hat{\rho}$ , the absolute intrinsic gravitational speed is the most fundamental absolute intrinsic parameter, as can be seen directly from the dependence of  $\phi\hat{\Phi}(\phi\hat{r})$  and  $\phi\hat{g}(\phi\hat{r})$  on  $\phi\hat{V}_g(\phi\hat{r})$  in those equations.

Indeed it is by virtue of establishment of non-uniform absolute intrinsic gravitational speed (which is an absolute intrinsic static speed) along the straight line absolute intrinsic space  $\phi\hat{\rho}$  along the horizontal by  $\phi\hat{M}_0$  and along the straight

line absolute intrinsic time ‘dimension’  $\phi\hat{c}\phi\hat{t}$  by  $\phi\hat{E}/\phi\hat{c}^2$  ( $\equiv \phi\hat{M}_0$ ), in the reference geometry of Fig. 3 that curved absolute intrinsic space  $\phi\hat{\rho}$  and curved absolute intrinsic time ‘dimension’  $\phi\hat{c}\phi\hat{t}$  in Fig. 4 evolve, and not by virtue of establishment of the secondary absolute intrinsic parameters  $\phi\hat{\Phi}(\phi\hat{r})$  and  $\phi\hat{g}(\phi\hat{r})$ . Recall that absolute intrinsic static speed has been isolated as an absolute intrinsic geometrical parameter along the curved  $\phi\hat{\rho}$  and curved  $\phi\hat{c}\phi\hat{t}$  in section 2 of [1], as illustrated in Fig. 11 of that paper.

In order to derive expressions for the proper intrinsic gravitational speed  $\phi V'_g(\phi r')$ , proper intrinsic gravitational potential  $\phi\Phi'(\phi r')$  and proper intrinsic gravitational acceleration  $\phi g'(\phi r)$ , which correspond to Eqs. (5a-b), (6) and (7) for the respective absolute intrinsic parameters, and expressions for proper gravitational speed  $V'_g(r')$ , proper gravitational potential  $\Phi'(r')$  and proper gravitational acceleration  $g'(r')$ , which correspond to Eqs. (8a-b)-(10) for the respective absolute parameters, let us revisit Fig. 5 again.

It is inherently assumed that the intrinsic rest mass  $\phi M_0$  in the projective proper intrinsic space  $\phi\rho'$  along the horizontal and the intrinsic rest mass  $\phi E'/\phi c^2$  ( $\equiv \phi M_0$ ) in the projective proper intrinsic time dimension  $\phi c\phi t'$  along the vertical of the gravitational field source, have not established non-uniform proper intrinsic gravitational speeds  $\phi V'_g(\phi r')$  along  $\phi\rho'$  and  $\phi c\phi t'$  respectively in Fig. 5 and consequently the rest masses  $M_0$  in  $\Sigma'$  and  $E'/c^2$  ( $\equiv M_0$ ) in  $ct'$  have not established non-uniform proper gravitational speeds along every radial direction from the centre of  $M_0$  in  $\Sigma'$  and along  $ct'$  respectively in that figure. However in reality,  $\phi M_0$  in  $\phi\rho'$  establishes non-uniform proper intrinsic gravitational speeds  $\phi V'_g(\phi r')$  along the straight line  $\phi\rho'$ , causing  $\phi\rho'$  to become curved anti-clockwise into the first quadrant and  $\phi E'/\phi c^2$  in  $\phi c\phi t'$  establishes non-uniform proper intrinsic gravitational speeds  $\phi V'_g(\phi r')$  along the straight line  $\phi c\phi t'$ , causing  $\phi c\phi t'$  to be curved anti-clockwise into the second quadrant, with respect to 3-observers in the resulting relativistic Euclidean 3-space  $\Sigma$ , so that the curved  $\phi\rho'$  and  $\phi c\phi t'$  constitute orthogonal curvilinear intrinsic dimensions with respect to 3-observers in  $\Sigma$ , as illustrated in Fig. 7.

The absolute intrinsic rest mass  $\phi\hat{m}_0$  of the test particle that has undergone absolute intrinsic gravitational fall to ‘distance’  $\phi\hat{r}$  from the base of the absolute intrinsic rest mass  $\phi\hat{M}_0$  along the curved  $\phi\hat{\rho}$ , where it possesses absolute intrinsic dynamical speed  $\phi\hat{V}_d$  and acquires absolute intrinsic gravitational speed  $\phi\hat{V}_g(\phi\hat{r})$  in the geometry of Fig. 5, retains this situation in the geometry of Fig. 7. In other words, the absolute intrinsic rest mass  $\phi\hat{m}_0$  of the test particle at ‘distance’  $\phi\hat{r}$  from the base of  $\phi\hat{M}_0$  along the curved  $\phi\hat{\rho}$  (not shown) in Fig. 7, possesses absolute intrinsic dynamical speed  $\phi\hat{V}_d$  and acquires absolute intrinsic gravitational speed  $\phi\hat{V}_g(\phi\hat{r})$ .

The proper intrinsic rest mass  $\phi m_0$  of the test particle (not shown in Fig. 5) that possesses projective absolute intrinsic dynamical speed  $\phi\hat{V}_d$  and acquires projective absolute intrinsic gravitational speed  $\phi\hat{V}_g(\phi\hat{r})$  at ‘distance’  $\phi r'$  from

the base of  $\phi M_0$  in the straight line  $\phi\rho'$  along the horizontal (not shown) in Fig. 5, is now at ‘distance’  $\phi r'$  from the base of  $\phi M_0$  along the curved proper intrinsic metric space  $\phi\rho'$  in Fig. 7, where it still possesses absolute intrinsic dynamical speed  $\phi\hat{V}_d$  and still acquires absolute intrinsic gravitational speed  $\phi\hat{V}_g(\phi\hat{r})$ . In addition,  $\phi m_0$  also acquires proper intrinsic gravitational speed  $\phi V'_g(\phi r')$  at ‘distance’  $\phi r'$  from the base of  $\phi M_0$  along the curved  $\phi\rho'$  in Fig. 7, of the non-uniform proper intrinsic gravitational speeds that  $\phi M_0$  establishes along the curved  $\phi\rho'$ .

It shall quickly be added that as long as the intrinsic rest mass  $\phi m_0$  of a test particle is in intrinsic gravitational fall directly along the curved proper intrinsic space  $\phi\rho'$ , it is in absolute intrinsic fall at increasing absolute intrinsic dynamical speed  $\phi\hat{V}_d$ , just as the intrinsic motion of the projective intrinsic rest mass  $\phi\hat{m}_0$  directly along the straight line proper intrinsic space  $\phi\rho'$  along the horizontal in Fig. 5 is an absolute intrinsic motion.

As follows from the penultimate paragraph, the intrinsic rest mass  $\phi m_0$  of the test particle in absolute intrinsic gravitational fall to the position of ‘distance’  $\phi r'$  from the base of  $\phi M_0$  along the curved proper intrinsic metric space  $\phi\rho'$  (not shown) in Fig. 7, possesses three intrinsic speeds namely, absolute intrinsic dynamical speed  $\phi\hat{V}_d$ , absolute intrinsic gravitational speed  $\phi\hat{V}_g(\phi\hat{r})$  and proper intrinsic gravitational speed  $\phi V'_g(\phi r')$  at this location. Since expression has been derived for  $\phi\hat{V}_g(\phi\hat{r})$  earlier, our interest now is to derive expression for  $\phi V'_g(\phi r')$ .

Now the curved two-dimensional proper intrinsic metric spacetime  $(\phi\rho', \phi c\phi t')$  with orthogonal curvilinear intrinsic dimensions  $\phi\rho'$  and  $\phi c\phi t'$  possesses the Lorentzian metric tensor at every point of it with respect to the intrinsic 1-observers (with intrinsic rest masses) in  $\phi\rho'$  and 3-observers in  $\Sigma$ . Consequently the intrinsic 1-observer located at ‘distance’  $\phi r'$  from the base of  $\phi M_0$  along the curved  $\phi\rho'$  where the particle is located, will formulate theory of combined intrinsic gravity and intrinsic motion with the proper intrinsic gravitational speed  $\phi V'_g(\phi r')$  and its absolute intrinsic dynamical speed  $\phi\hat{V}_d$  along  $\phi\rho'$  at his location and write the proper intrinsic total energy of the intrinsic rest mass of the test particle as follows

$$\phi U' = \frac{1}{2}\phi m_0\phi\hat{V}_d^2 - \frac{1}{2}\phi m_0\phi V'_g(\phi r')^2 \quad (11)$$

Eq. (11) takes on take the Newtonian form in terms of the proper (or classical) intrinsic gravitational potential function as follows

$$\phi U' = \frac{1}{2}\phi m_0\phi\hat{V}_d^2 - \frac{G\phi M_0\phi m_0}{\phi r'} \quad (12)$$

A comparison of Eqs. (15) and (11) yields the following expressions for proper intrinsic gravitational speed,

$$\phi V'_g(\phi r')^2 = 2G\phi M_0/\phi r' \quad (13a)$$

$$\phi V'_g(\phi r') = -\sqrt{2G\phi M_0/\phi r'} \quad (13b)$$

The negative root of  $2G\phi M_0/\phi r'$  is chosen in the definition of  $\phi V'_g(\phi r')$ , as done in the definition of  $\phi \hat{V}_g(\phi \hat{r})$  in Eq. (5b), because of the attractive nature of the gravitational field.

The proper intrinsic gravitational potential is dependent on the proper intrinsic gravitational speed as follows

$$\phi \Phi'(\phi r') = -\frac{1}{2}\phi V'_g(\phi r')^2 = -\frac{G\phi M_0}{\phi r'} \quad (14)$$

Also because of the intrinsic Euclidean metric tensor at point P along the the curved  $\phi \rho'$  where Eqs. (11)-(14) are written with respect to an intrinsic 1-observer (at this point), this intrinsic 1-observer obtains the proper intrinsic gravitational acceleration (or proper intrinsic gravitational field) at the point P from definition in Euclidean geometry as follows

$$\phi g'(\phi r') = \frac{1}{2} \frac{d(\phi V'_g(\phi r')^2)}{d\phi r'} = -\frac{d(\phi \Phi'(\phi r'))}{d\phi r'} = -\frac{G\phi M_0}{\phi r'^2} \quad (15)$$

By removing the symbol  $\phi$  from Eqs. (13a-b)-(15) one obtains expressions for proper gravitational speed (or velocity), proper gravitational potential and proper gravitational acceleration respectively as follows

$$V'_g(r')^2 = \frac{2GM_0}{r'} \quad (16a)$$

$$\vec{V}'_g(r') = -\sqrt{\frac{2GM_0}{r'}} \frac{\vec{r}'}{r'} \quad (16b)$$

$$\Phi'(r') = -\frac{1}{2}V'_g(r')^2 = -\frac{GM_0}{r'} \quad (17)$$

and

$$\vec{g}'(r') = \frac{1}{2} \frac{d(V'_g(r')^2)}{dr'} = -\frac{d(\Phi'(r'))}{dr'} = -\frac{GM_0 \vec{r}'}{r'^3} \quad (18)$$

It is again important to note that among proper intrinsic gravitational speed  $\phi V'_g(\phi r')$ , proper intrinsic gravitational potential  $\phi \Phi'(\phi r')$  and proper intrinsic gravitational acceleration  $\phi g'(\phi r')$ , defined by Eqs. (13b), (14) and (15) at point P on the curved proper intrinsic space  $\phi \rho'$  at 'distance'  $\phi r'$  from the base  $S'$  of the intrinsic rest mass  $\phi M_0$  of the gravitational field source at the origin of the curved  $\phi \rho'$ , the proper intrinsic gravitational speed is the most fundamental, as can be seen directly from the dependence of  $\phi \Phi'(\phi r')$  and  $\phi g'(\phi r')$  on  $\phi V'_g(\phi r')$  in those equations.

Indeed it is by virtue of establishment of non-uniform proper intrinsic gravitational speeds (which is a proper intrinsic static speed) along the straight line proper intrinsic space  $\phi \rho'$  along the horizontal by  $\phi M_0$  and along the straight line proper intrinsic time 'dimension'  $\phi c\phi t'$  by  $\phi E'/\phi c^2$  ( $\equiv \phi M_0$ ) in the geometry of Fig. 5 that curved proper intrinsic space  $\phi \rho'$  and curved proper intrinsic time dimension  $\phi c\phi t'$  in Fig. 7 evolve, and not by virtue of virtue of establishment of the secondary proper intrinsic parameters  $\phi \Phi'(\phi r')$  and  $\phi g'(\phi r')$ .

## 2.2 Deriving intrinsic gravitational local Lorentz transformation and gravitational local Lorentz transformation and establishing intrinsic gravitational local Lorentz invariance and gravitational local Lorentz invariance in a gravitational field

The intrinsic local Lorentz transformation and its inverse in terms of proper intrinsic static speed  $\phi V'_s$ , as well as the intrinsic local Lorentz invariance, intrinsic time dilation and intrinsic length contraction formulae they imply, derived with the aid of the local spacetime/intrinsic spacetime geometries of Figs. 6 and 7 and their inverses of Figs. 8 and 9 of [2], on flat two-dimensional intrinsic metric spacetime within a long-range metric force field, in sub-section 2.2 of [2] and the outward manifestations of those results namely, local Lorentz transformation and its inverse in terms of proper static speed  $V'_s$ , as well as local Lorentz invariance, time dilation and length contraction formulae they imply on flat four-dimensional metric spacetime in a long-range metric force field, shall be adapted to the gravitational field in this sub-section.

The counterparts in the gravitational field of the local spacetime/intrinsic spacetime geometries of Figs. 6-9 of [2] in a long-range metric force field in general must be drawn from the global geometries of Figs. 7-10 of this paper in a gravitational field (just as Figs. 6-9 of [2] have been drawn from Figs. 1 and 3 and Figs. 4 and 5 of that paper). This is an easy task since the counterpart local geometries to be derived from Figs. 7-10 of this paper are exactly the same as Figs. 6-9 of [2], except that the proper intrinsic static speed  $\phi V'_s$  that appear in Figs. 6-9 of [2] in a long-range metric force field in general, must be replaced by proper intrinsic gravitational speed  $\phi V'_g(\phi r')$  in those figures in a gravitational field.

We shall for completeness of this paper present the counterpart in a gravitational field of Fig. 6 of [2] in a long-range metric force field in general as Fig. 11.

The local geometry of Fig. 11, drawn from the global geometry of Fig. 7, is valid with respect to 3-observers in the relativistic Euclidean 3-spaces  $\Sigma$  and  $-\Sigma^*$ . This is so because the anti-clockwise rotation of the proper intrinsic spacetime intervals  $d\phi \rho'$  and  $\phi c d\phi t'$  relative to their projective relativistic intrinsic spacetime intervals  $d\phi \rho$  and  $\phi c d\phi t$  by positive intrinsic angle in Fig. 11 is valid with respect to these 3-observers.

The partial intrinsic metric spacetime interval transformation that can be derived with respect to 3-observers in  $\Sigma$  in the first quadrant from Fig. 11, which follows from the derivation of Eq. (5) from Fig. 6 in [2], is the following

$$\left. \begin{aligned} d\phi \rho' &= d\phi \rho \sec \phi \psi_g(\phi r') - \phi c d\phi t \tan \phi \psi_g(\phi r'); \\ & \text{(w.r.t. 3 - observers in } \Sigma) \end{aligned} \right\} \quad (19)$$

The counterpart in a gravitational field to Fig. 7 of [2] in a long-range metric force field in general, is depicted in Fig. 12.

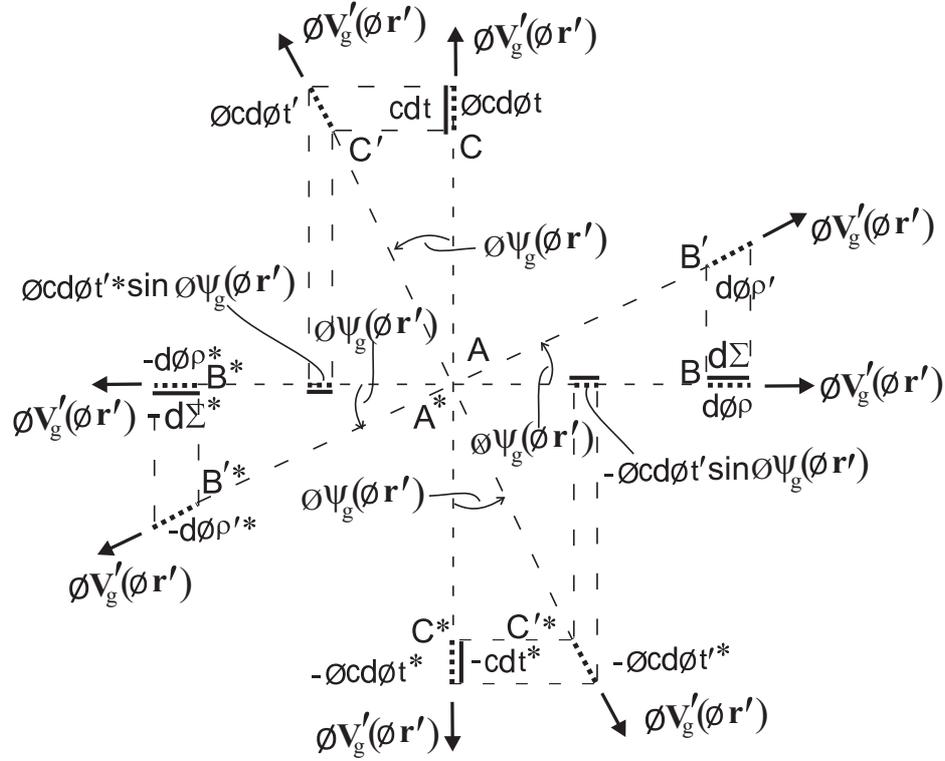


Fig. 11: Local spacetime/intrinsic spacetime geometry derived from the global geometry of Fig. 1 with respect to 3-observers in the relativistic Euclidean 3-spaces in the positive and negative universes.

The local geometry of Fig. 12, drawn from the global geometry of Fig. 8, is valid with respect to 1-observers in the relativistic time dimensions  $ct$  and  $-ct^*$ . This is so since the clockwise rotation of the proper intrinsic spacetime intervals  $d\phi\rho'$  and  $\phi cd\phi t'$  relative to their projective relativistic intrinsic spacetime intervals  $d\phi\rho$  and  $\phi cd\phi t$  by a positive intrinsic angle in Fig. 12 is valid with respect to these 1-observers.

The partial intrinsic metric spacetime interval transformation that can be derived with respect to 1-observers in  $ct$  in the first quadrant from Fig. 12, which follows from the derivation of Eq. (6) from Fig. 7 in [2], is the following

$$\left. \begin{aligned} \phi cd\phi t' &= d\phi t \sec \phi\psi_g(\phi r') - d\phi\rho \tan \phi\psi_g(\phi r'); \\ &\text{(w.r.t. 1 - observers in } ct) \end{aligned} \right\} \quad (20)$$

By collecting Eqs. (19) and (20) we obtain the full intrinsic metric spacetime interval transformation with respect to 3-observers in  $\Sigma$  and 1-observers in  $ct$  at 'distance'  $\phi r'$  along the curved proper intrinsic space  $\phi\rho'$  from the base  $S'$  of the intrinsic rest mass  $\phi M_0$  of the gravitational field source at the origin of the curved  $\phi\rho'$  in Fig. 11 and 12 as follows

$$\left. \begin{aligned} \phi cd\phi t' &= d\phi t \sec \phi\psi_g(\phi r') - d\phi\rho \tan \phi\psi_g(\phi r'); \\ &\text{(w.r.t. 1 - observers in } ct) \\ d\phi\rho' &= d\phi\rho \sec \phi\psi_g(\phi r') - \phi cd\phi t \tan \phi\psi_g(\phi r'); \\ &\text{(w.r.t. 3 - observers in } \Sigma) \end{aligned} \right\} \quad (21)$$

There is an inverse to system (21), which must be derived from the inverses to Figs. 11 and 12. The inverse to Fig. 11 is the counterpart in a gravitational field to Fig. 8 of [2] in a long range metric force field in general. It is depicted in Fig. 13.

The inverse local geometry of Fig. 13, derived from the inverse global geometry of Fig. 9, is valid with respect to 1-observers in the relativistic time dimensions  $ct$  and  $-ct^*$ . This is so since the clockwise rotation of the relativistic intrinsic spacetime intervals  $d\phi\rho$  and  $\phi cd\phi t$  relative to the proper intrinsic spacetime intervals  $d\phi\rho'$  and  $\phi cd\phi t'$  by negative intrinsic angle in Fig. 13, is equivalent to clockwise rotation of the proper intrinsic spacetime intervals  $d\phi\rho'$  and  $\phi cd\phi t'$  relative to relativistic intrinsic spacetime intervals  $d\phi\rho$  and  $\phi cd\phi t$  by positive intrinsic angle, as in Fig. 12. Consequently Figs. 12 and 13 are both valid with respect to 1-observers in the relativistic time dimensions  $ct$  and  $-ct^*$ .

The partial intrinsic metric spacetime interval transformation that can be derived with respect to 1-observers in  $ct$  in the

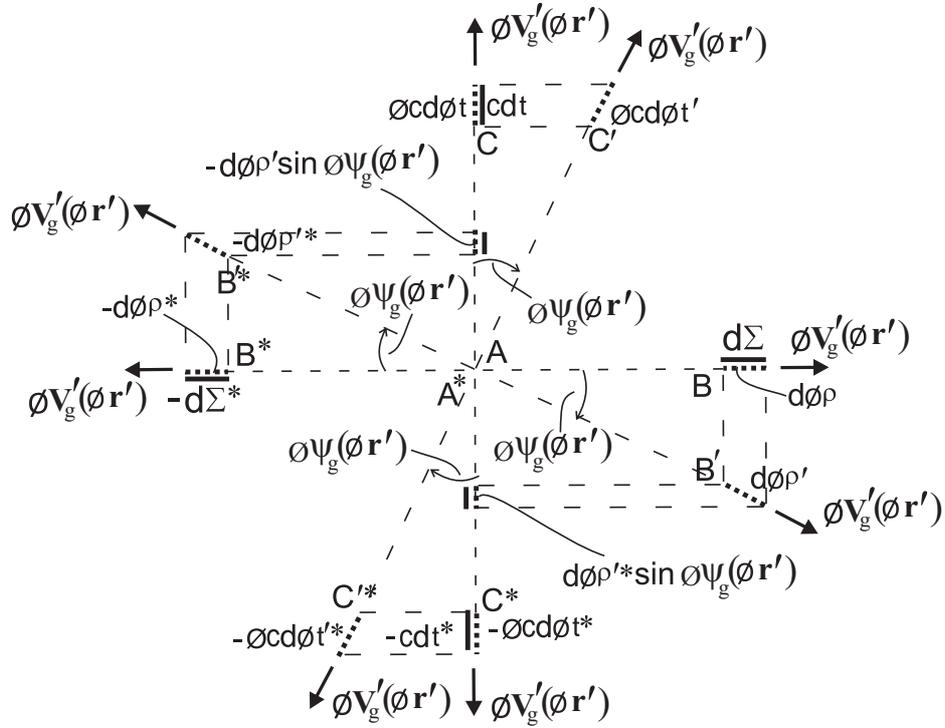


Fig. 12: Local spacetime/intrinsic spacetime geometry derived from the global geometry of Fig. 3 with respect to 1-observers in the relativistic time dimensions in the positive and negative universes; the complementary diagram to Fig. 11.

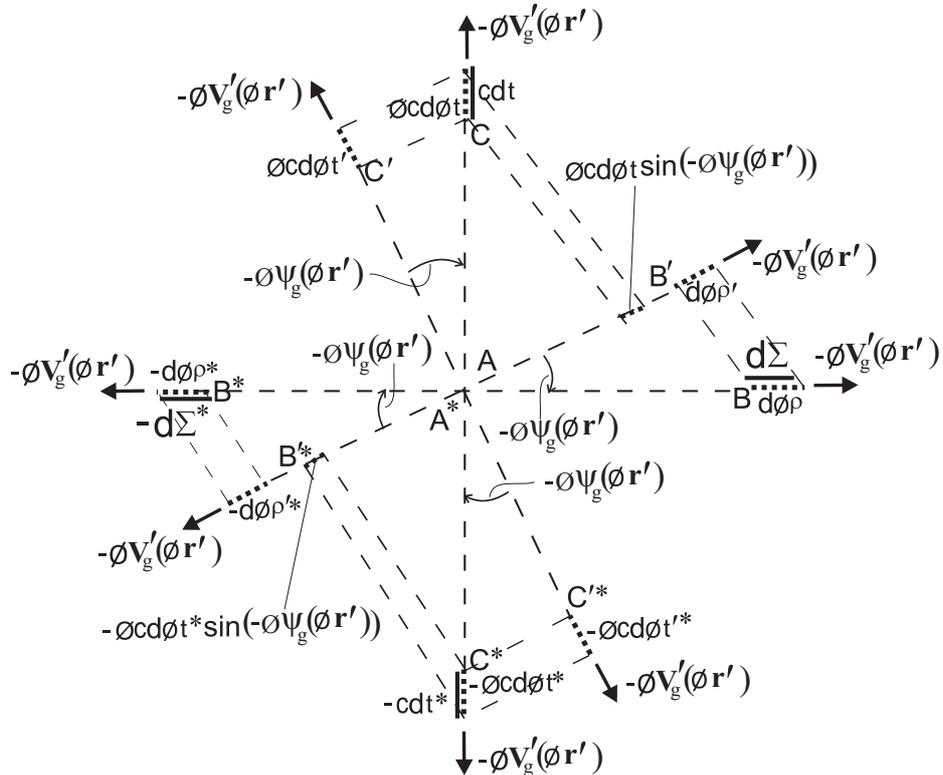


Fig. 13: The inverse to Fig. 11 with respect to 1-observers in the relativistic time dimensions in the positive and negative universes.

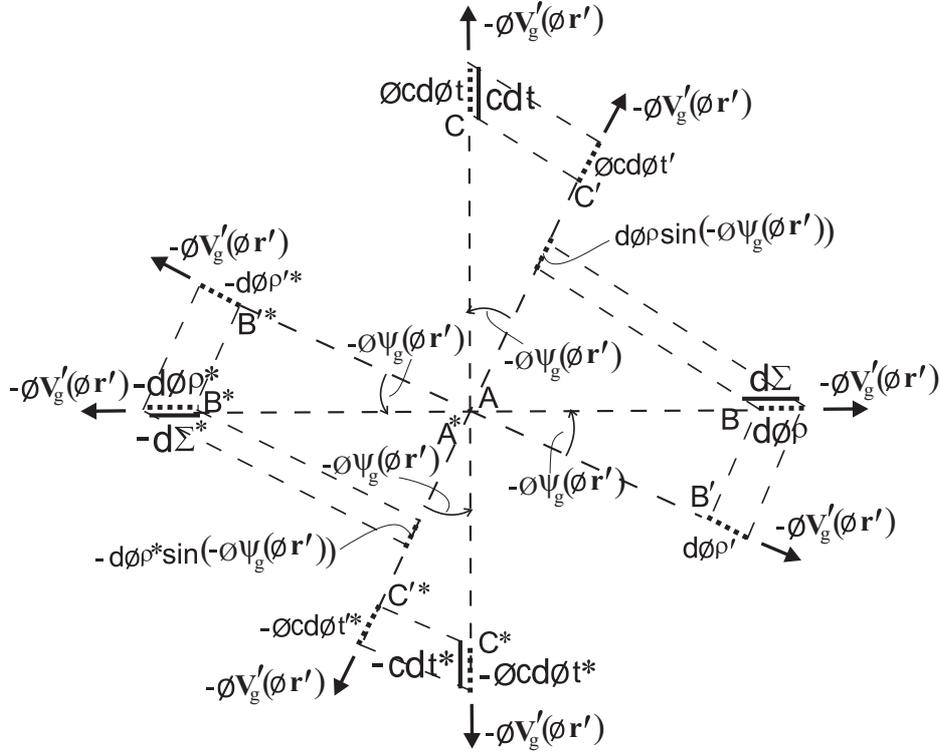


Fig. 14: The inverse to Fig. 12 with respect to 3-observers in the relativistic Euclidean 3-spaces in the positive and negative universes.

first quadrant (or in the positive universe) from Fig. 13, which follows from the derivation of Eq. (8) from Fig. 8 in [2], is the following

$$d\phi\rho = \left. \begin{aligned} d\phi\rho' \sec \phi\psi_g(\phi r') + \phi cd\phi t' \tan \phi\psi_g(\phi r'); \\ \text{(w.r.t. 1 - observers in } ct) \end{aligned} \right\} \quad (22)$$

Finally the inverse to Fig. 12 is the counterpart in a gravitational field to Fig. 9 of [2] in a long range metric force field in general. It is depicted in Fig. 14. The inverse local geometry of Fig. 14, derived from the inverse global geometry of Fig. 10, is valid with respect to 3-observers in the relativistic Euclidean 3-spaces  $\Sigma$  and  $-\Sigma^*$ . This is so because the anti-clockwise rotation of the relativistic intrinsic spacetime intervals  $d\phi\rho$  and  $\phi cd\phi t$  relative to the proper intrinsic spacetime intervals  $d\phi\rho'$  and  $\phi cd\phi t'$  by negative intrinsic angle in Fig. 14, is equivalent to anti-clockwise rotation of the proper intrinsic spacetime intervals  $d\phi\rho'$  and  $\phi cd\phi t'$  relative to relativistic intrinsic spacetime intervals  $d\phi\rho$  and  $\phi cd\phi t$  by positive intrinsic angle, as in Fig. 11. Consequently Figs. 11 and 14 are both valid with respect to 3-observers in the relativistic Euclidean 3-space  $\Sigma$  and  $-\Sigma^*$ .

The partial intrinsic metric spacetime interval transformation that can be derived with respect to 3-observers in  $\Sigma$  in the first quadrant (or in the positive universe) from Fig. 14, which follows from the derivation of Eq. (9) from Fig. 9 in [2], is the

following

$$\left. \begin{aligned} \phi cd\phi t &= \phi cd\phi t' \sec \phi\psi_g(\phi r') + d\phi\rho' \tan \phi\psi_g(\phi r'); \\ &\text{(w.r.t. 3 - observers in } \Sigma) \end{aligned} \right\} \quad (23)$$

By collecting Eqs. (22) and (23) we obtain the full inverse intrinsic spacetime interval transformation with respect to 3-observers in  $\Sigma$  and 1-observers in  $ct$  at 'distance'  $\phi r'$  along the curved proper intrinsic space  $\phi\rho'$  from the base  $S'$  of the intrinsic rest mass  $\phi M_0$  of the gravitational field source at the origin of the curved  $\phi\rho'$  in Figs. 13 and 14 as follows

$$\left. \begin{aligned} \phi cd\phi t &= \phi cd\phi t' \sec \phi\psi_g(\phi r') + d\phi\rho' \tan \phi\psi_g(\phi r'); \\ &\text{(w.r.t. 3 - observers in } \Sigma) \\ d\phi\rho &= d\phi\rho' \sec \phi\psi_g(\phi r') + \phi cd\phi t' \tan \phi\psi_g(\phi r'); \\ &\text{(w.r.t. 1 - observers in } ct) \end{aligned} \right\} \quad (24)$$

where, as follows from the derived relations (12)-(13a-b) of [2],

$$\frac{d\phi\rho}{\phi cd\phi t} = \sin \phi\psi_g(\phi r') \quad (25)$$

$$\sin \phi\psi_g(\phi r') = \frac{\phi V'_g(\phi r')}{\phi c} = \phi\beta_g(\phi r') \quad (26a)$$

$$\sec \phi\psi_g(\phi r') = \left(1 - \frac{\phi V_g'(\phi r')^2}{\phi c^2}\right)^{-1/2} = \phi\gamma_g(\phi r') \quad (26b)$$

By using Eqs. (26a) and (26b) in systems (21) and (24) we obtain the counterparts in a gravitational field to systems (14) and (15) of [2] in a general long-range metric force field respectively as follows

$$\left. \begin{aligned} d\phi t' &= \phi\gamma_g(\phi r')(d\phi t - \frac{\phi V_g'(\phi r')}{\phi c^2}d\phi\rho) \\ &\text{(w.r.t. 1 - observers in } ct) \\ d\phi\rho' &= \phi\gamma_g(\phi r')(d\phi\rho - \phi V_g'(\phi r')d\phi t) \\ &\text{(w.r.t. 3 - observers in } \Sigma) \end{aligned} \right\} \quad (27)$$

and

$$\left. \begin{aligned} d\phi t &= \phi\gamma_g(\phi r')(d\phi t' + \frac{\phi V_g'(\phi r')}{\phi c^2}d\phi\rho') \\ &\text{(w.r.t. 3 - observers in } \Sigma) \\ d\phi\rho &= \phi\gamma_g(\phi r')(d\phi\rho' + \phi V_g'(\phi r')d\phi t') \\ &\text{(w.r.t. 1 - observers in } ct) \end{aligned} \right\} \quad (28)$$

Finally by using the expression (13b) for  $\phi V_g'(\phi r')$  in Eq. (26a) and (26b) we obtain the following relations for the intrinsic angle  $\phi\psi_g(\phi r')$

$$\sin \phi\psi_g(\phi r') = \frac{\phi V_g'(\phi r')}{\phi c} = \sqrt{\frac{2G\phi M_0}{\phi r' \phi c^2}} = \phi\beta_g(\phi r') \quad (29a)$$

$$\begin{aligned} \sec \phi\psi_g(\phi r') &= \left(1 - \frac{\phi V_g'(\phi r')^2}{\phi c^2}\right)^{-1/2} \\ &= \left(1 - \frac{2G\phi M_0}{\phi r' \phi c^2}\right)^{-1/2} = \phi\gamma_g(\phi r') \end{aligned} \quad (29b)$$

Systems (21) and (24) or systems (27) and (28) are then given in terms of the intrinsic gravitational parameter  $2G\phi M_0/\phi r' \phi c^2$  respectively as follows

$$\left. \begin{aligned} d\phi t' &= \phi\gamma_g(\phi r')(d\phi t - \sqrt{\frac{2G\phi M_0}{\phi r' \phi c^4}}d\phi\rho) \\ &\text{(w.r.t. 1 - observers in } ct) \\ d\phi\rho' &= \phi\gamma_g(\phi r')(d\phi\rho - \sqrt{\frac{2G\phi M_0}{\phi r'}}d\phi t) \\ &\text{(w.r.t. 3 - observers in } \Sigma) \end{aligned} \right\} \quad (30)$$

and

$$\left. \begin{aligned} d\phi t &= \phi\gamma_g(\phi r')(d\phi t' + \sqrt{\frac{2G\phi M_0}{\phi r' \phi c^4}}d\phi\rho') \\ &\text{(w.r.t. 3 - observers in } \Sigma) \\ d\phi\rho &= \phi\gamma_g(\phi r')(d\phi\rho' + \sqrt{\frac{2G\phi M_0}{\phi r'}}d\phi t') \\ &\text{(w.r.t. 3 - observers in } ct) \end{aligned} \right\} \quad (31)$$

Systems (21) and (24), systems (27) and (28) and systems (30) and (31) are alternative forms of intrinsic gravitational local Lorentz transformation ( $\phi$ GLLT) and its inverse on flat two-dimensional intrinsic metric spacetime in every gravitational field. They are referred to as intrinsic gravitational local Lorentz transformation and its inverse, as shall be done henceforth for two reasons. First, because they are valid within an intrinsic local Lorentz frame located at arbitrary ‘distance’  $\phi r'$  along the curved proper intrinsic metric space  $\phi\rho'$  from the base of  $\phi M_0$ , which corresponds to ‘distance’  $\phi r$  along the straight line relativistic intrinsic metric space  $\phi\rho$  from the base of  $\phi M$  in Fig. 7. The proper intrinsic gravitational speed  $\phi V_g'(\phi r')$  has a constant value within this intrinsic local Lorentz frame. Secondly, because they pertain to the intrinsic theory of relativity associated with the presence of relative intrinsic gravitational field in intrinsic metric spacetime that involves proper intrinsic gravitational speed  $\phi V_g'(\phi r')$ , which is a relative intrinsic static speed, as shown explicitly by systems (27) and (28).

There is the counterpart intrinsic local Lorentz transformation ( $\phi$ LLT) and its inverse within local Lorentz frames in every gravitational field, which involves relative intrinsic dynamical speed  $\phi v$  in the context of primed intrinsic special theory of relativity ( $\phi$ SR') on flat two-dimensional proper (or primed) intrinsic metric spacetime ( $\phi\rho', \phi c\phi t'$ ) (in Fig. 5) in the assumed absence of relative gravitational field and unprimed intrinsic special theory of relativity ( $\phi$ SR) on flat two-dimensional relativistic intrinsic metric spacetime ( $\phi\rho, \phi c\phi t$ ) (in Fig. 7) in a relative gravitational field, to be derived elsewhere with further development.

Either system (21) or its inverse (24) or the more explicit forms (27) or (28) in terms of proper intrinsic gravitational speed or yet most explicit form (30) or (31) in terms of intrinsic gravitational parameters  $2G\phi M_0/\phi r'$ , leads to intrinsic gravitational local Lorentz invariance ( $\phi$ GLLI)

$$\phi c^2 d\phi t'^2 - d\phi\rho'^2 = \phi c^2 d\phi t^2 - d\phi\rho^2 \quad (32)$$

Equation (32) is referred to as intrinsic gravitational local Lorentz invariance ( $\phi$ GLLI) because it has arisen as a consequence of the intrinsic gravitational local Lorentz transformation ( $\phi$ GLLT) or its inverse. There is intrinsic local Lorentz invariance ( $\phi$ LLI) in the context of intrinsic special theory of relativity ( $\phi$ SR), within intrinsic local Lorentz frames on flat intrinsic metric spacetime in a gravitational field, which is implied by intrinsic local Lorentz transformation ( $\phi$ LLT) or its inverse in the context of  $\phi$ SR, within intrinsic local Lorentz frames in every gravitational field, to be developed elsewhere with further development.

The intrinsic gravitational local Lorentz transformation ( $\phi$ GLLT) of elementary proper intrinsic metric spacetime intervals  $d\phi\rho'$  and  $\phi c d\phi t'$  into elementary relativistic intrinsic metric spacetime intervals  $d\phi\rho$  and  $\phi c d\phi t$  of system (21), (27) or (30), is valid within intrinsic local Lorentz frame at

every point on the flat two-dimensional relativistic intrinsic metric spacetime  $(\phi\rho, \phi c\phi t)$  with respect to 3-observers in the relativistic Euclidean 3-space  $\Sigma$  and 1-observers in the relativistic time dimension  $ct$  (or with respect to 4-observers on flat four-dimensional relativistic spacetime  $(\Sigma, ct)$ ) in a gravitational field of arbitrary strength in Figs. 7 and 8. The inverse  $\phi$ GLLT of system (24), (28) or (31) is likewise valid within intrinsic local Lorentz frame at every point on the flat two-dimensional relativistic intrinsic spacetime  $(\phi\rho, \phi c\phi t)$  with respect to 4-observers on the flat relativistic metric spacetime  $(\Sigma, ct)$  in every gravitational field in Figs. 9 and 10. It then follows that the two-dimensional relativistic intrinsic metric spacetime  $(\phi\rho, \phi c\phi t)$  possesses intrinsic Lorentzian metric tensor at every point and is consequently everywhere flat in a gravitational field of arbitrary strength, as illustrated by the extended straight line  $\phi\rho$  and  $\phi c\phi t$  in Figs. 7 – 10.

Let us collect the partial intrinsic gravitational local Lorentz transformations of elementary intrinsic metric spacetime coordinate intervals with respect to 3-observers in the relativistic 3-space  $\Sigma$  in systems (21) and (24) to have as follows

$$\left. \begin{aligned} d\phi\rho' &= \sec\phi\psi_g(\phi r') (d\phi\rho - \sin\phi\psi_g(\phi r') \phi c d\phi t); \\ d\phi t &= \sec\phi\psi_g(\phi r') (d\phi t' + \frac{\sin\phi\psi_g(\phi r')}{\phi c} d\phi\rho'); \end{aligned} \right\} \quad (33)$$

(w.r.t 3-observers in  $\Sigma$ ).

Now when a hypothetical intrinsic 1-observer in the relativistic intrinsic metric space  $\phi\rho$  underlying  $\Sigma$  picks his intrinsic laboratory rod to measure the relativistic intrinsic metric space interval involved in an intrinsic event in the relativistic intrinsic metric spacetime, in the first equation of system (33), he will be able to measure the term  $d\phi\rho \sec\phi\psi_g(\phi r')$  but not the term  $-\phi c d\phi t \tan\phi\psi_g(\phi r')$  at the right-hand side of that equation with his intrinsic laboratory rod. Likewise when the hypothetical intrinsic 1-observers in  $\phi\rho$  picks his intrinsic laboratory clock to measure the intrinsic metric time interval involved in the same intrinsic event in the second equation system (33), he will be able to measure the term  $d\phi t' \sec\phi\psi_g(\phi r')$  but not the term  $(d\phi\rho'/\phi c) \sin\phi\psi_g(\phi r')$  with his intrinsic laboratory clock. By removing the terms that cannot be measured with intrinsic laboratory rod and intrinsic laboratory clock from system (35) we have

$$\left. \begin{aligned} d\phi\rho &= d\phi\rho' \cos\phi\psi_g(\phi r'); \\ d\phi t &= d\phi t' \sec\phi\psi_g(\phi r'); \end{aligned} \right\} \quad (34)$$

(w.r.t 3-observers in  $\Sigma$ ). System (34) gives the intrinsic length contraction and intrinsic time dilation formulae in terms of the intrinsic angle  $\phi\psi_g(\phi r')$  with respect to 3-observers in the relativistic Euclidean 3-space  $\Sigma$  in the context of the intrinsic theory of relativity associated with the presence of relative intrinsic gravitational field in intrinsic spacetime (with

the global geometry of Figs. 7 and 8).

System (34) is given in terms of the proper intrinsic gravitational speed by virtue of relation (25b) as follows

$$\left. \begin{aligned} d\phi\rho &= d\phi\rho' (1 - \frac{\phi V_g'(\phi r')^2}{\phi c^2})^{1/2}; \\ d\phi t &= d\phi t' (1 - \frac{\phi V_g'(\phi r')^2}{\phi c^2})^{-1/2}; \end{aligned} \right\} \quad (35)$$

(w.r.t 3-observers in  $\Sigma$ ).

And system (35) is given in terms of the intrinsic gravitational parameter  $2G\phi M_0/\phi r'$  by virtue of Eq. (29b) as follows

$$\left. \begin{aligned} d\phi\rho &= d\phi\rho' (1 - \frac{2G\phi M_0}{\phi r' \phi c^2})^{1/2}; \\ d\phi t &= d\phi t' (1 - \frac{2G\phi M_0}{\phi r' \phi c^2})^{-1/2} \end{aligned} \right\} \quad (36)$$

(w.r.t 3-observers in  $\Sigma$ ).

Now the intrinsic theory of relativity in the intrinsic metric spacetime associated with the presence of relative intrinsic gravitational field in intrinsic metric spacetime, will be made manifest in a theory of relativity in metric spacetime due to the presence of relative gravitational field in metric spacetime. Consequently the intrinsic gravitational local Lorentz transformation ( $\phi$ GLLT) of system (21) and its inverse of system (24) within an intrinsic local Lorentz frame on flat two-dimensional intrinsic metric spacetime within a gravitational field, in terms of the intrinsic angle  $\phi\psi_g(\phi r')$ , will be made manifest outwardly in gravitational local Lorentz transformation (GLLT) and its inverse within the corresponding local Lorentz frame on flat four-dimensional metric spacetime within the gravitational field. We must simply remove the symbol  $\phi$  from systems (21) and (24) to have their outward manifestations in spacetime respectively as follows

$$\left. \begin{aligned} cdt' &= cdt \sec\psi_g(r') - dr \tan\psi_g(r'); \\ &\quad (\text{w.r.t. 1 - observers in } ct); \\ dr' &= dr \sec\psi_g(r') - cdt \tan\psi_g(r'); \\ &\quad r' d\theta' = r d\theta; \quad r' \sin\theta' d\varphi' = r \sin\theta d\varphi; \\ &\quad (\text{w.r.t. 3 - observers in } \Sigma) \end{aligned} \right\} \quad (37)$$

and

$$\left. \begin{aligned} cdt &= cdt' \sec\psi_g(r') + dr' \tan\psi_g(r'); \\ &\quad (\text{w.r.t. 3 - observers in } \Sigma); \\ dr &= dr' \sec\psi_g(r') + cdt' \tan\psi_g(r'); \\ &\quad r d\theta = r' d\theta'; \quad r \sin\theta d\varphi = r' \sin\theta' d\varphi'; \\ &\quad (\text{w.r.t. 1 - observers in } ct) \end{aligned} \right\} \quad (38)$$

The appearance of the angle  $\psi_g(r')$  in system (37) and (38) conveys the impression that the coordinates  $r'$  and  $ct'$  of

the proper metric spacetime  $(\Sigma', ct') \equiv (r', r'\theta', r' \sin \theta' \varphi', ct')$  are curved with non-uniform curvature relative to dimensions  $r$  and  $ct$  respectively of the relativistic metric spacetime  $(\Sigma, ct) \equiv (r, r\theta, r \sin \theta \varphi, ct)$  in a gravitational field. It must be noted however that there is no curvature of the four-dimensional spacetime or of dimensions of the four-dimensional spacetime in the new geometrical background to the theory of relativity and gravitation within a four-world picture presented as Figs. 7 - 10 of this paper.

Only the proper intrinsic metric spacetime dimensions  $\phi\rho'$  and  $\phi c\phi t'$  are actually curved relative to their projective relativistic intrinsic metric spacetime dimensions  $\phi\rho$  and  $\phi c\phi t$  respectively in Figs. 7 and 8 and their inverses Figs. 9 and 10. The curvature of the dimensions of the physical spacetime implied by systems (37) and (38) is an intrinsic and not observable (or actual) curvature, which is what the curvatures of intrinsic spacetime in Fig. 7 - 10 represent.

Systems (37) and (38) correspond to systems (20) and (21) of [2]. The coordinates of the proper Euclidean 3-space  $\Sigma'$  represented by  $x'^1, x'^2$  and  $x'^3$  and of the relativistic Euclidean 3-space  $\Sigma$  represented by  $x^1, x^2$  and  $x^3$  in systems (20) and (21) of [2], are replaced respectively by the spherical coordinate  $r', r'\theta'$  and  $r' \sin \theta' \varphi'$  that originate from the centre of the rest mass  $M_0$  of the gravitational field source in the proper Euclidean 3-space  $\Sigma'$  (in Fig. 5), where  $M_0$  is being assumed to be spherical at present and  $r, r\theta$  and  $r \sin \theta \varphi$  that originate from the centre of the relativistic mass  $M$  of the gravitational field source in the relativistic Euclidean 3-space  $\Sigma$  (in Fig. 7), where  $M$  is also being assumed to be spherical at present.

The straight line isotropic proper intrinsic metric space  $\phi\rho'$  along the horizontal can be taken to lie along any radial direction from the centre of  $M_0$  in  $\Sigma'$  with respect to 3-observers in  $\Sigma'$  in Fig. 5 and the straight line isotropic relativistic intrinsic metric space  $\phi\rho$  along the horizontal can be taken to lie along any radial direction from the centre of  $M$  in  $\Sigma$  with respect to 3-observers in  $\Sigma$  in Fig. 7. It is for this reason that the outward manifestation in spacetime of systems (21) and (24) have taken the forms of systems (37) and (38) respectively, where the unprimed coordinates  $r\theta$  and  $r \sin \theta \varphi$  of  $\Sigma$ , along which  $\phi\rho$  does not lie, which are hence non-relativistic coordinates, transform into the corresponding proper (or primed) coordinates  $r', r'\theta'$  and  $r' \sin \theta' \varphi'$  of  $\Sigma'$  trivially as  $r'\theta' = r\theta$  and  $r' \sin \theta' \varphi' = r \sin \theta \varphi$ .

The outward manifestations on flat four-dimensional spacetime of the intrinsic gravitational local Lorentz transformation of system (27) and its inverse (28) are likewise given

respectively as follows

$$\left. \begin{aligned} dt' &= \gamma_g(r')(dt - \frac{V'_g(r')}{c^2} dr); \\ &\text{(w.r.t. 1 - observers in } ct) \\ dr' &= \gamma_g(r')(dr - V'_g(r') dt); \\ r' d\theta' &= r d\theta; \quad r' \sin \theta' d\varphi' = r \sin \theta d\varphi; \\ &\text{(w.r.t. 3 - observers in } \Sigma) \end{aligned} \right\} (39)$$

and

$$\left. \begin{aligned} dt &= \gamma_g(r')(dt' + \frac{V'_g(r')}{c^2} dr'); \\ &\text{(w.r.t. 3 - observers in } \Sigma) \\ dr &= \gamma_g(r')(dr' + V'_g(r') dt'); \\ r d\theta &= r' d\theta'; \quad r \sin \theta d\varphi = r' \sin \theta' d\varphi'; \\ &\text{(w.r.t. 1 - observers in } ct) \end{aligned} \right\} (40)$$

where

$$\gamma_g(r') = \sec \psi_g(r') = (1 - V'_g(r')^2/c^2)^{-1/2} \quad (41)$$

Systems (39) - (41) correspond to systems (22)-(24) of [2].

And the outward manifestations on flat four-dimensional spacetime of systems (30) and (31) are the following respectively

$$\left. \begin{aligned} dt' &= \gamma_g(r')(dt - \sqrt{\frac{2GM_0}{r'c^4}} dr); \\ &\text{(w.r.t. 1 - observers in } ct) \\ dr' &= \gamma_g(r')(dr - \sqrt{\frac{2GM_0}{r'}} dt); \\ r' d\theta' &= r d\theta; \quad r' \sin \theta' d\varphi' = r \sin \theta d\varphi; \\ &\text{(w.r.t. 3 - observers in } \Sigma) \end{aligned} \right\} (42)$$

and

$$\left. \begin{aligned} dt &= \gamma_g(r')(dt' + \sqrt{\frac{2GM_0}{r'c^4}} dr'); \\ &\text{(w.r.t. 3 - observers in } \Sigma) \\ dr &= \gamma_g(r')(dr' + \sqrt{\frac{2GM_0}{r'}} dt'); \\ r d\theta &= r' d\theta'; \quad r \sin \theta d\varphi = r' \sin \theta' d\varphi'; \\ &\text{(w.r.t. 1 - observers in } ct) \end{aligned} \right\} (43)$$

where

$$\gamma_g(r') = (1 - V'_g(r')^2/c^2)^{-1/2} = (1 - 2GM_0/r'c^2)^{-1/2} \quad (44)$$

Systems (37) and (38), systems (39) and (40) and systems (42) and (43) are alternative forms of gravitational local Lorentz transformation (GLLT) and its inverse on flat four-dimensional spacetime in a gravitational field of arbitrary

strength. They are called gravitational local Lorentz transformation because they involve proper gravitational speed  $V'_g(r')$  and are restricted within a local Lorentz frames located at an arbitrary radial distances  $r$  from the centre of the relativistic mass  $M$  of the gravitational field source in the relativistic Euclidean 3-space  $\Sigma$ . They pertain to the theory of relativity associated with the presence of relative gravitational field in metric spacetime. There are also local Lorentz transformation (LLT) and its inverse, involving transformations of affine spacetime coordinates and dynamical speed  $v$  of relative motion in the context of the special theory of relativity (SR) within a local Lorentz frame on the flat relativistic spacetime  $(\Sigma, ct)$  in a gravitational field, to be derived elsewhere with further development.

Either the GLLT (27), (30) or (42) or its inverse (28), (31) or (43) leads to gravitational local Lorentz invariance (GLLI),

$$\begin{aligned} c^2 dt^2 - dr^2 - r^2(d\theta^2 + \sin^2 d\varphi^2) \\ = c^2 dt'^2 - dr'^2 - r'^2(d\theta'^2 + \sin^2 d\varphi'^2) \end{aligned} \quad (45)$$

This is the outward manifestation on flat four-dimensional metric spacetime of the intrinsic gravitational local Lorentz invariance ( $\phi$ GLLI) (32) on flat two-dimensional intrinsic metric spacetime. Eq. (45) is referred to as gravitational local Lorentz invariance (GLLI) because it has arisen from gravitational Local Lorentz transformation (GLLT) or its inverse. There is also local Lorentz invariance (LLI) in the context of SR within local Lorentz frames on flat spacetime in a gravitational field to be established elsewhere.

The gravitational local Lorentz invariance (GLLI) (45) is valid at every point on four-dimensional spacetime in every gravitational field, implying flatness everywhere in a gravitational field of arbitrary strength of the four-dimensional relativistic metric spacetime  $(\Sigma, ct)$ , as deduced graphically and illustrated in Figs. 7 and 8 and their inverses Figs. 9 and 10 earlier.

Finally the intrinsic gravitational length contraction and intrinsic gravitational time dilation formulae in the context of the intrinsic theory of relativity associated with the presence of intrinsic gravitational field on flat two-dimensional intrinsic metric spacetime, presented in the alternative forms of systems (34), (35) and (36), are made manifest outwardly on flat four-dimensional metric spacetime in the context of the theory of relativity associated with the presence of gravitational field in metric spacetime respectively as follows

$$\left. \begin{aligned} dr &= dr' \cos \psi_g(r'); \quad r d\theta = r' d\theta'; \\ &\quad r \sin \theta d\varphi = r' \sin \theta' d\varphi'; \\ dt &= dt' \sec \psi_g(r') \end{aligned} \right\} \quad (46)$$

$$\left. \begin{aligned} dr &= \left(1 - \frac{V_g'(r')^2}{c^2}\right)^{1/2} dr'; \quad r d\theta = r' d\theta'; \\ &\quad r \sin \theta d\varphi = r' \sin \theta' d\varphi'; \\ dt &= \left(1 - \frac{V_g'(r')^2}{c^2}\right)^{-1/2} dt' \end{aligned} \right\} \quad (47)$$

and

$$\left. \begin{aligned} dr &= \left(1 - \frac{2GM_0}{r'c^2}\right)^{1/2} dr'; \quad r d\theta = r' d\theta'; \\ &\quad r \sin \theta d\varphi = r' \sin \theta' d\varphi'; \\ dt &= \left(1 - \frac{2GM_0}{r'c^2}\right)^{-1/2} dt' \end{aligned} \right\} \quad (48)$$

The gravitational length contraction and gravitational time dilation formulae (46) – (48) are valid with respect to 3-observers in the relativistic Euclidean 3-space  $\Sigma$  in Fig. 7.

The theory of relativity on flat four-dimensional relativistic metric spacetime  $(\Sigma, ct)$  associated with the presence of a gravitational field, within which the gravitational local Lorentz transformation (GLLT) and its inverse (37) and (38) or (39) and (40) or (42) and (43) have been derived; within which the gravitational local Lorentz invariance (45) on flat four-dimensional metric spacetime in every gravitational field has been established and within which the gravitational length contraction and gravitational time dilation of system (46), (47) or (48) has been derived, shall be referred to as the theory of gravitational relativity and given the acronym (TGR).

The TGR is the gravitational counterpart, (involving relative gravitational velocity  $\vec{V}'_g(r')$  – which is a static velocity) of the special theory of relativity (SR), (involving uniform relative dynamical velocity  $\vec{v}$ ). However while the relative dynamical velocity is spatially uniform, thereby satisfying the special principle of relativity of Einstein [10], and thus warranting the name special theory of relativity, the relative gravitational velocity (a static velocity)  $\vec{V}'_g(r')$  that appears in TGR is not spatially uniform, thereby satisfying the general principle of relativity of Einstein [11]. Thus the theory of gravitational relativity (TGR) may also be referred to as the general theory of relativity on flat spacetime, going by Einsteinian nomenclature, but we shall prefer TGR.

If we could have our way, the special theory of relativity associated with dynamical velocity would be referred to as the theory of dynamical relativity (TDR), which can then take care of the relativity of both uniform and non-uniform relative dynamical velocities. The relativity of non-uniform relative velocity motions shall be incorporated into the present theory elsewhere with further development.

The intrinsic theory of relativity on flat two-dimensional relativistic intrinsic spacetime  $(\phi\rho, \phi c\phi t)$  associated with the presence of relative intrinsic gravitational field in  $(\phi\rho, \phi c\phi t)$ , within which the intrinsic gravitational local Lorentz transformation ( $\phi$ GLLT) and its inverse of systems (21) and (24) or systems (27) and (28) or systems (30) and (31) have been derived; within which the intrinsic gravitational local Lorentz invariance ( $\phi$ GLLI) (32) has been established and within which the intrinsic gravitational length contraction and intrinsic gravitational time dilation formulae of system (34), (35) or (36) have been derived, is the intrinsic theory of gravitational relativity ( $\phi$ TGR). It is the gravitational counterpart of the intrinsic special theory of relativity ( $\phi$ SR).

The theory of gravitational relativity (TGR) on flat four-dimensional relativistic metric spacetime  $(\Sigma, ct)$  in a gravitational field of arbitrary strength in Figs. 7 and 8 and their inverses Figs. 9 and 10, is mere outward manifestation of the intrinsic theory of gravitational relativity ( $\phi$ TGR) on flat two-dimensional relativistic intrinsic metric spacetime  $(\phi\rho, \phi c\phi t)$  underlying  $(\Sigma, ct)$  in those figures. Once a result of  $\phi$ TGR has been derived on flat intrinsic spacetime  $(\phi\rho, \phi c\phi t)$ , the corresponding result of TGR on flat four-dimensional spacetime  $(\Sigma, ct)$  can be written straight away, essentially by dropping the symbol  $\phi$  from the result of  $\phi$ TGR. This procedure, which has been demonstrated above, has been demonstrated between  $\phi$ SR and SR in [3].

The “relativity” aspect of the commonly used terminology “relativity and gravitation”, when applied in the present context, refers to a theory of relativity on flat spacetime associated with the presence of gravitational field, which is the theory of gravitational relativity (TGR), while the “gravitation” aspect of the “relativity and gravitation” terminology, refers to the theory (or law) of gravity on flat four-dimensional relativistic metric spacetime  $(\Sigma, ct)$  in Fig. 7, obtained from the transformations with the aid of GLLT and its inverse (42) and (43) of the classical (or Newtonian) theory (or law) of gravity. This is analogous to the special theory of relativity and relativistic mechanics, where relativistic mechanics is classical mechanics transformed with the aid of LLT and its inverse in the context of SR.

### 2.2.1 Clarifications of the concepts of relative gravitational field, relative gravitational speed and relativity associated with relative gravitational speed (or field)

Let us for completeness and for the emphasis it deserves, adapt the clarifications of relative metric force fields, relative static speed and relativity associated with relative static speed in a long-range metric force field, done in sub-section 2.3 of [2], to relative gravitational field, relative gravitational speed and relativity associated with relative gravitational speed, referred to as theory of gravitational relativity (TGR) above hereunder.

Now the proper gravitational speed  $V'_g(r')$  that appears in the theory of gravitational relativity (TGR) found in this sub-section is a property of space, established in the proper Euclidean 3-space  $\Sigma'$  at radial distance  $r'$  along every radial direction from the centre of the rest mass  $M_0$  of a gravitational field source in  $\Sigma'$  in the geometry of Fig. 5, at the first stage of evolution of spacetime/intrinsic spacetime in a gravitational field. It transforms invariantly into proper gravitational speed  $V'_g(r')$  at the corresponding radial distance  $r$  along every radial direction in the relativistic Euclidean 3-space  $\Sigma$  from the centre of the relativistic mass  $M$  of the gravitational field source in  $\Sigma$ , in the geometry of Fig. 7, according to the yet to be proved relation (2b), at the second

stage of evolution of spacetime/intrinsic spacetime in a gravitational field.

It must be remembered that the relativistic mass  $M$  in  $\Sigma$  is not the source of the proper gravitational speed  $V'_g(r')$  in  $\Sigma$ . Rather the proper intrinsic gravitational speed  $\phi V'_g(r')$  along the curved proper intrinsic metric space  $\phi\rho'$ , which is projected invariantly as proper intrinsic gravitational speed  $\phi V'_g(\phi r')$  into the relativistic intrinsic metric space  $\phi\rho$  along the horizontal, is made manifest in proper gravitational speed  $V'_g(r')$  in  $\Sigma$  in Fig. 7. Moreover the relativistic mass  $M$  (which shall be identified as the inertial mass and passive gravitational mass ultimately), is not a gravitational field source. Hence it does not establish relativistic gravitational speed  $V'_g(r)$  in  $\Sigma$ .

Now the proper gravitational speed  $V'_g(r')$  is a property of space, established in space by the source of a gravitational field, irrespective of whether a test particle is present in space or not. A particle or object of any mass located at a point P in space where the proper gravitational speed is  $V'_g(r')$ , acquires  $V'_g(r')$  but does not move relative to any observer at this speed. If it also possesses a dynamical velocity  $\vec{v}$  relative to an observer while moving through point P, then it will be observed to move at the velocity  $\vec{v}$  only relative to the observer, despite the gravitational speed  $V'_g(r')$  it has acquired.

The gravitational speed established at a point in space cannot be observed or measured. It does not give rise to flow of space and consequently it does not give rise to translation in space of a material particle or object that acquired it, as mentioned above. Further more, the gravitational speed at a point in space is the same with respect to all observers or frames of reference. It is hence an absolute parameter from the point of view of the special (or dynamical) theory of relativity. How come then the concepts of relative gravitational speed and relativity associated with gravitational speed (or how come the theory of gravitational relativity)?

In order to answer the question ending the foregoing paragraph, let us revisit the length contraction and time dilation formulae (47) and (48). Although the proper gravitational speed  $V'_g(r')$  at a point in space cannot be observed or measured and although its square  $V'^2_g(r')$  cannot be observed or measured, the quantities  $(1 - V'^2_g(r')/c^2)^{1/2} dr'$  and  $(1 - V'^2_g(r')/c^2)^{-1/2} dt'$  can be observed and measured. This follows from the fact that  $V'^2_g(r')$  is related to the classical gravitational potential  $\Phi(r')$  as in Eq. (17). The quantity  $V'^2_g(r')$ , like the gravitational potential  $\Phi(r')$ , at a point in space, cannot be observed or measured.

As shall also be shown formally elsewhere with further development, the speed  $c$  in the factors,  $(1 - V'^2_g(r')/c^2)^{1/2}$  and  $(1 - V'^2_g(r')/c^2)^{-1/2}$ , is a gravitational speed like  $V'_g(r')$  it divides (and not the dynamical speed of light). In other words, these factors shall appear as  $(1 - V'^2_g(r')/c^2_g)^{1/2}$  and  $(1 - V'^2_g(r')/c^2_g)^{-1/2}$  with further development, where  $c_g$  is the maximum over all gravitational speeds that can be es-

established in space or that can be acquired by particles and objects, including massless gravitons, with a magnitude of  $3 \times 10^3 \text{ m/s}$ ; (the speed of light being the maximum over all dynamical speeds of particles and objects, including massless photons, with equal magnitude of  $3 \times 10^3 \text{ m/s}$ ).

Now the quantities  $(1 - \frac{V_g'(r')^2}{c_g^2})^{\frac{1}{2}} = c_g(c_g^2 - V_g'(r')^2)^{\frac{1}{2}}$  and  $(1 - \frac{V_g'(r')^2}{c_g^2})^{-\frac{1}{2}} = \frac{1}{c_g}(c_g^2 - V_g'(r')^2)^{-\frac{1}{2}}$  can be measured, since the difference  $c_g^2 - V_g'(r')^2$ , being equivalent to difference of gravitational potentials, can be measured. It then follows that the length contraction and time dilation formulae (47) and (48) can be observed and measured. Thus by allowing an event that involves proper time interval  $dt'$  and proper space intervals  $dr'$ ,  $r'd\theta'$  and  $r' \sin \theta' d\phi'$  to occur at different positions in space within a gravitational field, the observed (or relativistic) time interval  $dt$  and the observed (or relativistic) dimension of 3-space  $dr$  of the same event will vary with position in space, while the observed dimensions  $r d\theta$  and  $r \sin \theta d\phi$  of the event will be the same at all positions in space within the gravitational field, according to systems (47) and (48). The variation with the magnitude of the proper gravitational speed  $V_g'(r')$  and consequently with position in space within a gravitational field of the observed (or relativistic) time interval  $dt$  and the observed (or relativistic) interval of space  $dr$  of an event, is the concept of relativity associated with the presence of gravitational field in spacetime.

In brief, the relativity associated with proper gravitational speed in an external gravitational field (that is, the theory of gravitational relativity (TGR)) is relativity with position in space within the field (and not relativity with observer or frame of reference). Identical clocks located at different positions of different radial distances  $r$  from the centre of a gravitational field source in the relativistic Euclidean space  $\Sigma$ , which are made synchronous at an initial time, will not remain synchronous with the passage of time, by virtue of the relativity of time associated with the proper gravitational speed (or by virtue of the presence of the theory of gravitational relativity (TGR)).

Relativity of proper gravitational speed likewise refers to variation of magnitude of proper gravitational speed with position in space within a gravitational field. In other words, it refers to the fact that the proper gravitational speeds  $V_g'(r'_1)$  and  $V_g'(r'_2)$  at two positions of different radial distances  $r'_1$  and  $r'_2$  respectively from the centre of the gravitational field source have different magnitudes. It does not refer to variation of the magnitude of a gravitational speed with the observer or frame of reference. As mentioned earlier, the proper gravitational speed at a point in space is the same with respect to all observers or frames of references.

In the light of the foregoing, a relative gravitational field is the one that establishes non-zero proper gravitational speeds in space. That is, one that establishes proper gravitational speeds of different magnitudes (no matter how small in mag-

nitudes in a strict sense) at different positions in the proper Euclidean 3-space  $\Sigma'$ , which transforms invariantly as proper gravitational speeds in the relativistic Euclidean 3-space  $\Sigma$  within the gravitational field. Fig. 5 is devoid of relative gravitational speed but contains absolute gravitational speed. Hence it is a diagram in the absence of relative gravity (or absence of TGR). The possibility of the relativity of other physical parameters, such as mass, electric and magnetic fields, energy, fluxes, temperature, entropy, potentials, etc, in the sense of the variations of their observed (or relativistic) magnitudes with proper gravitational speed and consequently with position in space within a relative gravitational field, on the flat four-dimensional relativistic metric spacetime  $(E^3, ct)$  (in Fig. 7) (or in the context of TGR), shall be investigated elsewhere.

Expectedly, it will be possible to derive the transformations of physical parameters and physical constants, classical and special-relativistic non-gravitational and classical gravitational laws on flat spacetime within a gravitational field with the aid of the gravitational local Lorentz transformation (GLLT) and its inverse of systems (39) and (40), in the context of the theory of relativity associated with the presence of gravitational field in spacetime (or in the context of the theory of gravitational relativity (TGR)). This will be analogous to the Lorentz transformations of parameters and natural laws on flat spacetime in the context of the special theory of relativity.

### 3 The 'two-dimensional' metric theory of absolute intrinsic gravity on curved 'two-dimensional' absolute intrinsic spacetime

As has been shown in section 3 of the preceding paper [2], the 'two-dimensional' absolute intrinsic Riemann geometry on curved 'two-dimensional' absolute intrinsic metric spacetime  $(\phi\hat{\rho}, \phi\hat{c}\phi\hat{t})$  with respect to 3-observers in the underlying physical proper Euclidean 3-space  $E'^3$  solely in Fig. 4 or Fig. 11 of [1], at the first stage of evolution of spacetime/intrinsic spacetime in a long-range metric force field, remains unchanged on the curved  $(\phi\hat{\rho}, \phi\hat{c}\phi\hat{t})$  with respect to 3-observers in the relativistic physical Euclidean 3-space  $E^3$  in Fig. 1 of [2] at the second stage.

The foregoing implies that the absolute intrinsic line elements, (61) and (62), the implied absolute intrinsic metric tensor  $\phi\hat{g}_{ij}$  (63) and (64) and the absolute intrinsic Ricci tensor  $\phi\hat{R}_{ij}$  of Eqs. (67) and (68) of [1] on curved 'two-dimensional' absolute intrinsic metric spacetime  $(\phi\hat{\rho}, \phi\hat{c}\phi\hat{t})$  within a long-range metric force field, which are valid with respect to 3-observers in the proper physical Euclidean 3-space  $E'^3$  solely in Fig. 4 or Fig. 11 of that paper, are equally valid on curved  $(\phi\hat{\rho}, \phi\hat{c}\phi\hat{t})$  with respect to 3-observers in the relativistic Euclidean 3-space  $\Sigma$  in a gravitational field in Fig. 7 of this paper. Recall that the proper and relativistic physical Euclidean 3-spaces, denoted by  $E'^3$  and  $E^3$  respec-

tively in [7,1,2], have been re-denoted by  $\Sigma'$  and  $\Sigma$  respectively for convenience in Figs. 5 - 10 of this paper. The proper physical Euclidean 3-space was also denoted by  $\Sigma'$  in [3-6].

Let us adapt Eqs. (61) and (62), Eqs. (63) and (64) and Eqs. (67) and (68) of [1], written with respect to 3-observers in the proper Euclidean 3-space  $E'^3$  in Fig. 4 or Fig. 11 of [1], within a long-range metric force field in general in that paper, to the gravitational field with respect to 3-observers in the relativistic physical Euclidean 3-space  $\Sigma$  in Fig. 7 of this paper. We must simply replace the absolute intrinsic angle  $\phi\hat{\psi}$  and the absolute intrinsic curvature parameter  $\phi\hat{k}$  that appear in those equations and in Fig. 4 of [1] by the corresponding absolute intrinsic angle  $\phi\hat{\psi}_g(\phi\hat{r})$  and absolute intrinsic curvature parameter  $\phi\hat{k}_g(\phi\hat{r})$  in a gravitational field to have as follows

$$(d\phi\hat{s})^2 = \cos^2 \phi\hat{\psi}_g(\phi\hat{r})\phi\hat{c}^2(d\phi\hat{t})^2 - \sec^2 \phi\hat{\psi}_g(\phi\hat{r})(d\phi\hat{\rho})^2 \quad (49)$$

or

$$(d\phi\hat{s})^2 = (1 - \phi\hat{k}_g(\phi\hat{r})^2)\phi\hat{c}^2(d\phi\hat{t})^2 - \frac{(d\phi\hat{\rho})^2}{1 - \phi\hat{k}_g(\phi\hat{r})^2} \quad (50)$$

$$\phi\hat{g}_{ij} = \begin{pmatrix} \cos^2 \phi\hat{\psi}_g(\phi\hat{r}) & 0 \\ 0 & -\sec^2 \phi\hat{\psi}_g(\phi\hat{r}) \end{pmatrix} \quad (51)$$

or

$$\phi\hat{g}_{ij} = \begin{pmatrix} 1 - \phi\hat{k}_g(\phi\hat{r})^2 & 0 \\ 0 & -\frac{1}{1 - \phi\hat{k}_g(\phi\hat{r})^2} \end{pmatrix} \quad (52)$$

and

$$\phi\hat{R}_{ij} = \begin{pmatrix} -\sin^2 \phi\hat{\psi}_g(\phi\hat{r}) & 0 \\ 0 & -\tan^2 \phi\hat{\psi}_g(\phi\hat{r}) \end{pmatrix} \quad (53)$$

or

$$\phi\hat{R}_{ij} = \begin{pmatrix} -\phi\hat{k}_g(\phi\hat{r})^2 & 0 \\ 0 & -\frac{\phi\hat{k}_g(\phi\hat{r})^2}{1 - \phi\hat{k}_g(\phi\hat{r})^2} \end{pmatrix} \quad (54)$$

Then upon isolating the absolute intrinsic static speed as an absolute intrinsic geometrical parameter in section 2 of [1], the absolute intrinsic metric tensor, the absolute intrinsic Ricci tensor and the absolute intrinsic line element were re-written alternatively in terms of absolute intrinsic static speed in that section, as Eqs. (81), (82) and (83) respectively of [1]. Those equations are the following respectively in terms of absolute intrinsic gravitational speed in a gravitational field

$$(d\phi\hat{s})^2 = (1 - \frac{\phi\hat{V}_g(\phi\hat{r})^2}{\phi\hat{c}^2})\phi\hat{c}^2(d\phi\hat{t})^2 - \frac{(d\phi\hat{\rho})^2}{1 - \frac{\phi\hat{V}_g(\phi\hat{r})^2}{\phi\hat{c}^2}} \quad (55)$$

$$\phi\hat{g}_{ij} = \begin{pmatrix} 1 - \frac{\phi\hat{V}_g(\phi\hat{r})^2}{\phi\hat{c}^2} & 0 \\ 0 & -\frac{1}{1 - \frac{\phi\hat{V}_g(\phi\hat{r})^2}{\phi\hat{c}^2}} \end{pmatrix} \quad (56)$$

and

$$\phi\hat{R}_{ij} = \begin{pmatrix} -\frac{\phi\hat{V}_g(\phi\hat{r})^2}{\phi\hat{c}^2} & 0 \\ 0 & -\frac{\phi\hat{V}_g(\phi\hat{r})^2/\phi\hat{c}^2}{1 - \frac{\phi\hat{V}_g(\phi\hat{r})^2}{\phi\hat{c}^2}} \end{pmatrix} \quad (57)$$

Let us then apply the expressions (5a) or (5b), derived for  $\phi\hat{V}_g(\phi\hat{r})$  earlier in this paper in Eqs. (55) - (57) to obtain  $(d\phi\hat{s})^2$ ,  $\phi\hat{g}_{ij}$  and  $\phi\hat{R}_{ij}$  explicitly in terms of absolute intrinsic gravitational parameters as follows

$$(d\phi\hat{s})^2 = (1 - \frac{2G\phi\hat{M}_0}{\phi\hat{r}\phi\hat{c}^2})\phi\hat{c}^2(d\phi\hat{t})^2 - \frac{(d\phi\hat{\rho})^2}{1 - \frac{2G\phi\hat{M}_0}{\phi\hat{r}\phi\hat{c}^2}} \quad (58)$$

$$\phi\hat{g}_{ij} = \begin{pmatrix} 1 - \frac{2G\phi\hat{M}_0}{\phi\hat{r}\phi\hat{c}^2} & 0 \\ 0 & -\frac{1}{1 - \frac{2G\phi\hat{M}_0}{\phi\hat{r}\phi\hat{c}^2}} \end{pmatrix} \quad (59)$$

and

$$\phi\hat{R}_{ij} = \begin{pmatrix} -\frac{2G\phi\hat{M}_0}{\phi\hat{r}\phi\hat{c}^2} & 0 \\ 0 & -\frac{2G\phi\hat{M}_0/\phi\hat{r}\phi\hat{c}^2}{1 - \frac{2G\phi\hat{M}_0}{\phi\hat{r}\phi\hat{c}^2}} \end{pmatrix} \quad (60)$$

Although Eqs. (49)-(57) are important in their own right, the forms (58)-(60), given explicitly in terms of the absolute intrinsic parameters  $2G\phi\hat{M}_0/\phi\hat{r}$ , are the final forms and the forms that shall be found most useful in the metric theory of absolute intrinsic gravity, with the acronym  $\phi$ MAG, on the curved 'two-dimensional' absolute intrinsic metric spacetime  $(\phi\hat{\rho}, \phi\hat{c}\phi\hat{t})$ , which is valid with respect to all 3-observers in the underlying relativistic Euclidean 3-space  $\Sigma$  in Fig. 7, at the second stage of evolution of spacetime/intrinsic spacetime in a gravitational field.

The relationships among  $\phi\hat{\psi}_g(\phi\hat{r})$ ,  $\phi\hat{k}_g(\phi\hat{r})$ ,  $\phi\hat{V}_g(\phi\hat{r})$  and  $2G\phi\hat{M}_0/\phi\hat{r}$  that follow from Eqs. (49)-(60) shall be expressed linearly as follows

$$\sin^2 \phi\hat{\psi}_g(\phi\hat{r}) = \phi\hat{k}_g^2(\phi\hat{r}) = \frac{\phi\hat{V}_g(\phi\hat{r})^2}{\phi\hat{c}_g^2} = \frac{2G\phi\hat{M}_0}{\phi\hat{r}\phi\hat{c}^2} \quad (61)$$

The approach applied in the derivation of Eqs. (49)-(60) in [1], based on the results of the graphical analysis of absolute intrinsic Riemannian metric space  $\hat{M}^3$  in [7], must be

described as graphical approach, as is obvious. However a pair of absolute intrinsic tensor equations was derived from the graphical analysis of the absolute intrinsic Riemannian metric space  $\hat{M}^3$ , which were adapted to the curved ‘two-dimensional’ absolute intrinsic metric spacetime  $(\phi\hat{\rho}, \phi\hat{c}\phi\hat{t})$  in Fig. 4 of [1] and re-written as Eqs. (34) and (38) of [1]. They are given in terms of starred absolute intrinsic metric tensor  $\phi\hat{g}_{ij}^*$  and starred absolute intrinsic Ricci tensor  $\phi\hat{R}_{ij}^*$  as follows

$$\phi\hat{g}_{ij}^* - \phi\hat{R}_{ij}^* = \delta_{ij} \quad (\phi\text{LEI}) \quad (62)$$

$$\phi\hat{R}_{ij}^* - \phi\hat{k}_g(\phi\hat{r})^2 \phi\hat{g}_{ij}^* = 0 \quad (63)$$

where the absolute intrinsic curvature parameter  $\phi\hat{k}_g(\phi\hat{r})$  in a gravitational field has been used in Eq. (63).

Eq. (62) is a tensorial statement of intrinsic local Euclidean invariance ( $\phi\text{LEI}$ ) on the curved ‘two-dimensional intrinsic metric spacetime  $(\phi\hat{\rho}, \phi\hat{c}\phi\hat{t})$ ’ partially with respect to 3-observers in the proper Euclidean 3-space  $\Sigma'$  and partially with respect to 1-observers in the proper time dimension  $ct'$  in Fig. 4 of [1], at the first stage of evolution of spacetime/intrinsic spacetime in a long range metric force field. Eqs. (62) and (63) are equally valid on the curved  $(\phi\hat{\rho}, \phi\hat{c}\phi\hat{t})$  partially with respect to 3-observers in the relativistic Euclidean 3-space  $\Sigma$  and partially with respect to 1-observers in the relativistic time dimension  $ct$  in Fig. 7 of this paper, at the second stage of evolution of spacetime/intrinsic spacetime in a gravitational field.

Equations (62) and (63) are amenable to simultaneous algebraic solution, giving the following

$$\begin{aligned} \phi\hat{g}_{ij}^* &= (1 - \phi\hat{k}_g(\phi\hat{r})^2)\delta_{ij} \\ &= \begin{pmatrix} 1 - \phi\hat{k}_g(\phi\hat{r})^2 & 0 \\ 0 & 1 - \phi\hat{k}_g(\phi\hat{r})^2 \end{pmatrix} \quad (64) \end{aligned}$$

$$\begin{aligned} \phi\hat{R}_{ij}^* &= -\frac{\phi\hat{k}_g(\phi\hat{r})^2\delta_{ij}}{1 - \phi\hat{k}_g(\phi\hat{r})^2} \\ &= \begin{pmatrix} -\frac{\phi\hat{k}_g(\phi\hat{r})^2}{1 - \phi\hat{k}_g(\phi\hat{r})^2} & 0 \\ 0 & -\frac{\phi\hat{k}_g(\phi\hat{r})^2}{1 - \phi\hat{k}_g(\phi\hat{r})^2} \end{pmatrix} \quad (65) \end{aligned}$$

The validity of the starred absolute intrinsic tensors  $\phi\hat{g}_{ij}^*$  and  $\phi\hat{R}_{ij}^*$  on curved  $(\phi\hat{\rho}, \phi\hat{c}\phi\hat{t})$  partially with respect to 3-observers in  $\Sigma$  and partially with respect to 1-observers in  $ct$  in Fig. 7, implies that the components  $\phi\hat{g}_{00}^*$  and  $\phi\hat{R}_{00}^*$  are valid with respect to 1-observers in  $ct$ , while the components  $\phi\hat{g}_{11}^*$  and  $\phi\hat{R}_{11}^*$  are valid with respect to 3-observers in  $\Sigma$ .

Having obtained Eqs. (64) and (65), then the derived relationships among the components of the starred absolute intrinsic metric tensor  $\phi\hat{g}_{ij}^*$  and the absolute intrinsic metric tensor without star label  $\phi\hat{g}_{ij}$ , presented as Eqs. (65a) and (65b)

of [1], must be used to convert  $\phi\hat{g}_{ij}^*$  to  $\phi\hat{g}_{ij}$ . Those relationships shall be re-written here as follows

$$\phi\hat{g}_{00} = 1/\phi\hat{g}_{00}^*; \phi\hat{g}_{11} = -\phi\hat{g}_{11}^*; \phi\hat{g}_{ij} = \phi\hat{g}_{ij} = 0; i \neq j \quad (64a)$$

$$\phi\hat{g}_{11} = -1/\phi\hat{g}_{00} \quad (64b)$$

The starred absolute intrinsic metric tensor  $\phi\hat{g}_{ij}^*$  of Eq. (62) transforms to the following absolute intrinsic metric tensor without star label  $\phi\hat{g}_{ij}$  by virtue of system (64a)

$$\phi\hat{g}_{ij} = \begin{pmatrix} 1 - \phi\hat{k}_g(\phi\hat{r})^2 & 0 \\ 0 & -\frac{1}{1 - \phi\hat{k}_g(\phi\hat{r})^2} \end{pmatrix} \quad (67)$$

The absolute intrinsic metric tensor without star label is valid with respect to 3-observers in the relativistic physical Euclidean 3-space  $\Sigma$  solely in Fig. 7.

While intrinsic local Euclidean invariance ( $\phi\text{LEI}$ ) expressed by Eq. (60) obtains on the curved ‘two-dimensional’ absolute intrinsic metric spacetime  $(\phi\hat{\rho}, \phi\hat{c}\phi\hat{t})$  with respect to 3-observers in the relativistic Euclidean 3-space  $\Sigma$  and 1-observers in the relativistic time dimension  $ct$  in Fig. 7, it is intrinsic local Lorentz invariance that obtains on the curved  $(\phi\hat{\rho}, \phi\hat{c}\phi\hat{t})$  with respect to 3-observers in the Euclidean 3-space  $\Sigma$  solely in that figure, as robustly demonstrated in [1] with respect to 3-observers in proper Euclidean 3-space  $\Sigma'$  solely in Fig. 4 of that paper. Thus in order to obtain an absolute intrinsic Ricci tensor without star label  $\phi\hat{R}_{ij}$ , which is valid with respect to 3-observers in  $\Sigma$  solely, like  $\phi\hat{g}_{ij}$ , we must apply intrinsic local Lorentz invariance on  $(\phi\hat{\rho}, \phi\hat{c}\phi\hat{t})$ , which is given as follows by simply replacing the Euclidean metric tensor  $\delta_{ij}$  by the Lorentzian metric tensor  $\eta_{ij}$  in Eq. (60)

$$\phi\hat{g}_{ij} - \phi\hat{R}_{ij} = \eta_{ij} \quad (\phi\text{LLI}) \quad (68)$$

The absolute intrinsic Ricci tensor without star label  $\phi\hat{R}_{ij}$  that follows from Eqs. (65) and (66) is the following

$$\phi\hat{R}_{ij} = \begin{pmatrix} -\phi\hat{k}_g(\phi\hat{r})^2 & 0 \\ 0 & -\frac{\phi\hat{k}_g(\phi\hat{r})^2}{1 - \phi\hat{k}_g(\phi\hat{r})^2} \end{pmatrix} \quad (69)$$

Equations (65) and (67) obtained by solving the pair of absolute intrinsic tensor equations (60) and (61) by following the steps from those equations to Eq. (67) are the same as Eqs. (57) and (59) of [1], derived graphically (by actually drawing the spacetime/intrinsic spacetime diagrams and obtaining intrinsic coordinate projections) in [1]. By applying the chain of relations (59) to Eqs. (65) and (67), one obtains Eqs. (49)-(60) again in the absolute intrinsic covariant tensor approach to the ‘two-dimensional’ metric theory of absolute intrinsic gravity ( $\phi\text{MAG}$ ), involving the solution of the pair of absolute intrinsic tensor equations (62) and (63).

The approach to the derivation of the absolute intrinsic line element (50), the absolute intrinsic metric tensor (59) and absolute intrinsic Ricci tensor (60), of the metric theory of absolute intrinsic gravity ( $\phi$ MAG), on curved ‘two-dimensional’ absolute intrinsic metric spacetime  $(\phi\hat{\rho}, \phi\hat{c}\phi\hat{t})$ , with respect to 3-observers in the relativistic physical Euclidean 3-space  $\Sigma$  in Fig. 7, by solving the pair of absolute intrinsic covariant tensor equations (62) and (63) and following the steps from those equations to Eq. (69), along with the chain of relations (61), must be described as absolute intrinsic covariant tensor approach to  $\phi$ MAG, as mentioned at the end of the foregoing paragraph. Although the absolute intrinsic tensor approach has been isolated from the graphical/analytical approach (within which Eqs. (62) and (63) were derived in [7,1]), the absolute intrinsic tensorial approach is a valid approach, but which cannot completely stand on its own, since the chain of relations (61) and the fact of intrinsic local Lorentz invariance ( $\phi$ LLI) expressed by Eq. (68), which are used in the tensorial approach have been derived within the graphical approach.

The absolute intrinsic metric tensor  $\phi\hat{g}_{ij}$  of Eq. (59) shall find useful application in formulating absolute intrinsic law of gravity and absolute intrinsic non-gravitational laws on the curved ‘two-dimensional’ absolute intrinsic metric spacetime  $(\phi\hat{\rho}, \phi\hat{c}\phi\hat{t})$  with respect to 3-observers in the relativistic Euclidean 3-space  $\Sigma$  in Fig. 7 among other useful purposes, elsewhere with further development.

Finally the extension of the results derived within a singular gravitational field in this section to the situations of presence of two, three and larger number of gravitational field sources, whose relativistic (or inertial) masses are arbitrarily scattered in the relativistic Euclidean 3-space  $\Sigma$ , is straight forward, by virtue of the theory of superposition of two, three and larger number of curved absolute intrinsic metric spacetimes (or absolute intrinsic Riemannian metric spacetimes) developed in [7,1]. However we shall not go into those in this paper.

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