The atoms of space

Bichara Sahely
422 Half Moon Drive, Half Moon Heights, St. Peters, St. Kitts
doctor@sahely.com

Abstract

In this brief note, it will be shown that space has hidden properties normally attributed to elementary particles, such as mass and charge. We will also elucidate the thermodynamic properties of these atoms of space, and in so doing stumble upon a potentially simple and elegant quantum field theory of gravity. We have only demanded consistency of the equations of geometry, special relativity, quantum mechanics, gravity, electrostatics and thermodynamics, and only evoked three correspondence principles/postulates: the particle-wave, holographic and fractal (scale relativity) principles. We will also reinterpret Einstein's special theory of relativity, solve the mystery of the double slit experiment, muse on the physical nature of dark energy, and throw some light on the different approaches to constructing a theory of fundamental interactions.

September 11, 2011

Introduction

Over the past 400 years, since Newton to our present day, we have been trying to understand the nature of our physical reality. We have made some assumptions/postulates along the way, and have discovered via the formalism of mathematics several successful theories like Newton's theory of universal Gravitation, the laws of electrostatics, thermodynamics, special relativity, quantum mechanics and quantum field theory. If we ask why they are successful, we will discover that this is due to some underlying symmetry. Some of these symmetries stem from local and global gauge invariance. Also in quantum field theories, we have to use renormalization procedures to get rid of the infinities. Lately we have also discovered that our universe is made up predominantly of dark energy, for which we have no explanation. Moreover, we are unable to unite quantum theory and gravity in one simple framework, and have to invent structures like strings, extra dimensions and super symmetry to make such a consistent theory realizable. Instead of creating epicycles on epicycles of postulates and assumptions, is there another avenue that one can pursue that even a high school student can understand? What has been attempted here is to see how far one can go by looking at the static/invariant solutions of all our successful theories, from geometry of circles to black holes thermodynamics, and see what these are "telling us." Is there a solution to these equations that is complete and consistent, and if so, how can this solution be represented? The aim of this program is not to discover new equations, but to discover a hidden pattern within these equations that may throw some light on the foundational issues of quantum mechanics, special relativity, string theory and even loop quantum gravity.

Although it has always been suspected from dimensional analysis that the Planck regime is where the fundamental insights lie, no assumption is made a priori in this regard. What will be done is as follows: We will first evoke de Broglie's wave-particle duality, and represent the rest energy of a particle by a a photon rotating in a circle. We will then equate the centripetal acceleration of this photon to the gravitational field of the point particle. By doing so, we will discover that this will only admit one unique set of solutions, the traditional Planck quantities. Instead of taking these Planck particles as mathematical constructs, we will discover that if in addition to mass, there is also a Planck charge associated with each Planck particle, then these mathematical objects may represent real physical objects which we can use to model the dynamics of the real world around us. In this light, we can model these physical objects as black holes with charge and mass, and we will discover that they are none other than stable extremal Reissner–Nordström black holes.

Then we will proceed to use some to the fundamental relationships of black hole physics to show that these are actually the fundamental degrees of freedom of space, and provide a representation of the mass and charge in terms of spin states, thus providing a physical basis for the wave function in quantum mechanics, and spin networks in loop quantum gravity. And finally, we are left asking the question, if these atoms have energy, and are not gravitationally and electromagnetically visible, have we also stumbled on the elementary particles of dark energy?

The particle-light wave duality

Let m = mass of a point particle at rest, and c = the speed of light. Applying Einstein's equation, the rest energy E_{particle} of the point particle is

$$E_{\text{particle}} = \text{mc}^2. \tag{1}$$

Also the energy E_{wave} of a quantum of light in terms of its frequency f and Planck constant h is E_{wave} = hf, which can also be expressed in terms of the reduced Planck constant $\hbar$ and angular frequency ω as

$$E_{\text{wave}} = \hbar\omega. \tag{2}$$

Using the deBroglie's wave-particle duality, and letting $E_{\text{particle}} = E_{\text{wave}}$, and solving for ω, we get

$$\omega = \frac{\text{mc}^2}{\hbar}. \tag{3}$$

Let us now represent the rest energy E_{particle} of the point mass m, with the energy E_{wave} of the quanta of light revolving in a circle of radius r, speed v = c, and angular velocity ω as shown in Figure 1.

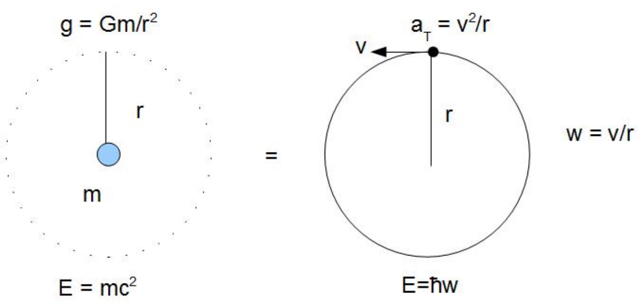

Figure 1

The angular velocity ω is related to the tangential speed v and the radius r of the circle by

$$\omega = \frac{v}{r}. \tag{4}$$

The centripetal acceleration a is

$$a = \frac{v^2}{r}. \tag{5}$$

To determine what r is for the circle, let v = c the speed of light, equate (3) and (4), and solve for r. We get

$$r = \frac{v}{\omega} = \frac{c}{\omega} = \frac{c}{\frac{\text{mc}^2}{\hbar}} = \frac{\hbar}{\text{mc}}. \tag{6}$$

which is also the reduced Compton wavelength for the particle.

The centripetal acceleration a in terms of the mass of the particle m is determined by substituting (6) into (5) and equating v = c:

$$a = \frac{v^2}{r} = \frac{c^2}{r} = \frac{c^2}{\frac{\hbar}{\text{mc}}} = \frac{\text{mc}^3}{\hbar}. \tag{7}$$

If, as shown in Figure 1, we equate the gravity field strength g at the distance r for the point particle with the centripetal acceleration a of light, then according to Newton's law of Universal gravitation, g = $\frac{\text{Gm}}{r^2} = \frac{\text{mc}^3}{\hbar}$. Solving for r,

$$r = \left(\frac{\hbar G}{c^3}\right)^{1/2}, \tag{8}$$

which is also the Planck length, L_P. By equating (6) with (8), $r = \frac{\hbar}{mc} = \left(\frac{\hbar G}{c^3}\right)^{1/2}$, we can solve for m:

$$m = \left(\frac{\hbar c}{G}\right)^{1/2}, \tag{9}$$

which is also the Planck mass, M_p. Finally, for the circle, $\omega = 2\pi f$, where f = frequency or cycles per unit time, f = 1/t, which is the time for one cycle. Solving for t, and using (3) for ω and (9) for m, we get

$$t = \frac{(2\pi)}{\omega} = 2\pi\frac{\hbar}{mc^2} = 2\pi\left(\frac{\hbar}{c^2}\right)\left(\frac{1}{m}\right) = 2\pi\left(\frac{\hbar}{c^2}\right)\left(\frac{G}{\hbar c}\right)^{1/2} = 2\pi\left(\frac{\hbar G}{c^5}\right)^{1/2}, \tag{10}$$

which is also the traditional Planck time x 2π, which we now call t_p. If the momentum of the light in this frame is p, then we can find the angular momentum J of this light wave by using the identity E = pc, (1) and (6). This results in

$$J = pr = \left(\frac{E}{c}\right)r = \left(\frac{mc^2}{c}\right)r = (mc)\left(\frac{\hbar}{mc}\right) = \hbar. \tag{11}$$

which implies that the angular momentum of this light representation is $\hbar$.

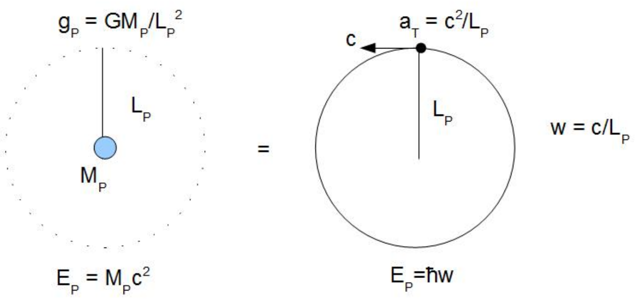

Figure 2

Thus by evoking 2 equivalence principles, that of 1) rest energy = quantum of energy and 2) the gravitational field = centripetal acceleration of light in the light-frame, we have stumbled upon the zero-point quantum of energy of the gravitational field, $E_p = M_p c^2 = \left(\frac{\hbar c}{G}\right)^{1/2} c^2 = \left(\frac{\hbar c^5}{G}\right)^{1/2}$ (Figure 2). At this point, I will postulate that we have also stumbled upon the quantum of space, or the atom of space, whose mass is M_p, whose radius is L_p, an invariant length scale and which also possesses an intrinsic clock whose invariant time period is t_p and whose energy is E_p and angular momentum is J_p. These are summarized below.

$$M_p = \left(\frac{\hbar c}{G}\right)^{1/2};\ L_p = \left(\frac{\hbar G}{c^3}\right)^{1/2};\ t_p = 2\pi\left(\frac{\hbar G}{c^5}\right)^{1/2};\ E_p = \left(\frac{\hbar c^5}{G}\right)^{1/2};\ J_p = \hbar$$

This picture is incomplete for two reasons: 1) if space has mass, and hence inertia, why do we not perceive it? and 2) if space has mass and mass is attracted gravitationally to each other, why does not space collapse on itself? A hint on how to proceed, is to look at the Schwarzschild radius r_s of the M_p which is $r_s = 2\frac{GM_p}{c^2} = 2\frac{G}{c^2}\left(\frac{\hbar c}{G}\right)^{1/2} = 2\left(\frac{\hbar G}{c^3}\right)^{1/2} = 2L_p$. Thus the event horizon of the atom of space, where information of the space is stored, is the surface of a sphere of radius $2L_p$. For an atom of space to have an event horizon of sphere of radius L_p and not $2L_p$, I postulate that the atom of space in addition to M_p, should have another quantity which should be the Planck charge Q_p. As is known, from the Reissner–Nordström metric corresponding to the gravitational field of a charged, non-rotating, spherically symmetric body of mass M, r_Q is a length-scale corresponding to the electrical charge Q of the mass, and is

$$r_Q = \left(Q^2 \frac{G}{4\pi\epsilon_0 c^4}\right)^{1/2}. \tag{12}$$

Letting $r_Q = L_p$, $\left(Q^2 \frac{G}{4\pi\epsilon_0 c^4}\right)^{1/2} = \left(\frac{\hbar G}{c^3}\right)^{1/2}$, and solving for Q, one gets

$$Q = (4\pi\epsilon_0 c\hbar)^{1/2}, \tag{13}$$

which is none other than the Planck charge Q_p, which was postulated above.

So if an atom of space has, in addition to mass, also a charge, we have in one swoop solved both problems mentioned above. Since gravity is always attractive and identical atoms would have identical charge, and both the strength of the gravitation field and electrical field decay with the inverse square distance, then the net force between 2 atoms has to always be zero, hence there would be no net force causing attraction or repulsion of the atoms of space. Equating both the Newton's force and the Coulomb's force gives F = $\frac{GM_p^2}{r^2} = \frac{\left(\frac{1}{4\pi\epsilon_0}\right)Q_p^2}{r^2}$, where r is the distance between both atoms, solving for Q_p, one gets $Q_{p=}(4\pi\epsilon_0 c\hbar)^{1/2}$, which is the same as (13) above. Note also that mass and charge are inter-convertible and are related by

$$GM_p^2 = \left(\frac{1}{4\pi\epsilon_0}\right)Q_p^2. \tag{14}$$

Since the gravitational field is cancelled by the electric field for all distances, this may explain why space is thought not to have any mass or charge. However, by postulating an atom of space has an invariable mass-scale, charge-scale, length-scale and time-scale, then space provides the yardstick, by which all of the physical quantities in nature can be measured. This thus begs the question and asks which units are more fundamental, $\hbar$, c, G and ϵ, or M_p, Q_p, L_p, and t_p. Please note that the Planck quantities were derived from the empirical physical constants. If however the Planck quantities are postulated to be more fundamental, then the empirical physical constants are macroscopical consequences of these microscopic degrees of freedom.

The particle-black hole duality

Since at distance L_p, the surface of the atom is equivalent to the surface of a non-rotating charged black hole with mass M_p and charge Q_p, we will use the established results from black hole thermodynamics to determine the maximum information that can be stored in the atom of space and also its fundamental degrees of freedom. We need to find a measure in 2 dimensions that can be used as a gauge or yardstick in 3 dimensions. The holographic principle guides us by relating the scale of the surface of the event horizon $4\pi L_p^2$ with the scale of the area corresponding to the particle-light correspondence $A_p = \pi L_p^2$. For the event horizon (surface), then the maximum number of bits of information that can be stored is

$$N_p = \frac{A_{\text{surface}}}{A_p} = \frac{4\pi L_p^2}{\pi L_p^2} = 4. \tag{15}$$

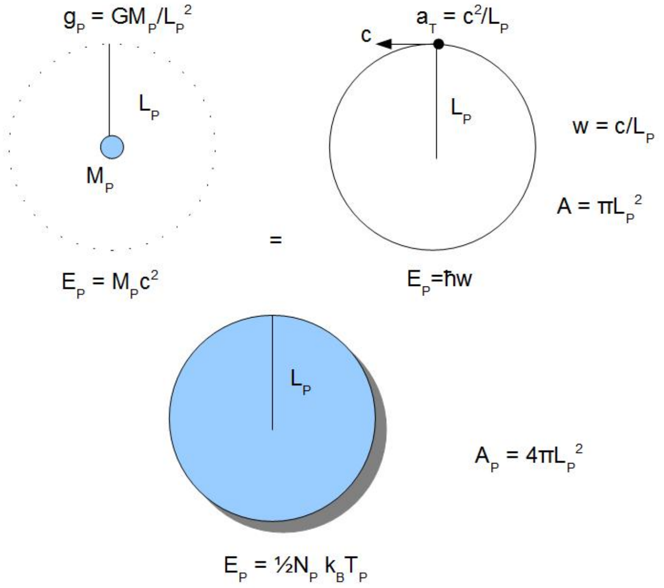

Figure 3

According to the equipartition energy theorem, the energy for one degree of freedom $\frac{1}{2}\ k_B$T, where k_B is Boltzmann constant and T is the temperature of the surface, so in this case the total energy of the atom E would be

$$E = \frac{1}{2} N_p k_B T = \frac{1}{2} 4 k_B T = 2 k_B T . \tag{16}$$

Equating this to the energy of the rest mass of the particle (see Figure 3 above), we have

$$M_p c^2 = 2 k_B T_p . \tag{17}$$

Solving for T_p,

$$T_p = M_p \frac{c^2}{2 k_B} = \frac{\left[\left(\frac{\hbar c}{G}\right)^{1/2} c^2\right]}{2 k_B} = \frac{1}{2}\left(\hbar \frac{c^5}{G k_B^2}\right)^{1/2} . \tag{18}$$

From the particle perspective, the surface gravity,

$$g_p = \frac{\mathrm{GM}_p}{L_p^2} = \frac{\mathrm{GM}_p}{\frac{\hbar \mathrm{G}}{c^3}} = \frac{M_p c^3}{\hbar} . \tag{19}$$

Inserting (17) into (19), then $g_p = \frac{2 k_B T_p c}{\hbar}$. Thus

$$T_p = \frac{1}{2}\left(\frac{\hbar}{\mathrm{c}k_B}\right) g_p , \tag{20}$$

which is none other that Unruh temperature T_{Unruh} x π. (Please note that T_{Unruh} would have been derived if in (15) $A_p = L_p^2$, instead of $\pi \mathrm{L}_p^2$. The latter was chosen so that N = 4 and not 4π.)

At this point, we would like to understand why an atom of space is able to store four bits of information, and to identify the identity of each bit. But before we do so, let us confirm that the atom is actually the smallest unit of entropy.

By using the Bekenstein-Hawking entropy formula for a black hole, $S_{\text{BH}} = \frac{1}{4} k_B (\frac{A_{\text{surface}}}{A_p})$, we discover by using (15), that

$$S_p = k_B . \tag{21}$$

To elucidate the number of degrees of freedom each atom of space represents, we have to now consider a collection of such atoms.

The thermodynamics of a collection of atoms of space

Let us now consider a collection of such N atoms of space, confined to a large volume V whose radius L >> L_p . We will model this collection as an ideal gas whose density is $\rho = NM_P / V$, and whose pressure $P = Nk_B T / V$, where T is the temperature of this collection of atoms. We will now invoke another principle, the fractal or scale relativity principle, as per Notalle, which equates the temperature T_p of the surface of an atom with the temperature T of the collection of atoms. (Figure 4)

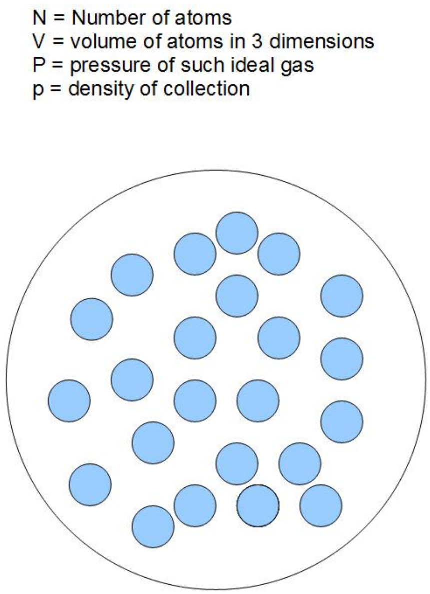

Figure 4

To determine the speed v at which information can propagate through this collection of atoms, we can model this propagation to that of sound in air and deduce the adiabatic index γ from the equation

$$v = \left(\gamma \frac{P}{\rho}\right)^{\frac{1}{2}} = \left(\gamma \frac{\left(\frac{N k_B T_p}{V}\right)}{\left(N \frac{M_P}{V}\right)}\right)^{\frac{1}{2}} = \left(\gamma \frac{k_B T_p}{M_P}\right)^{\frac{1}{2}} \tag{22}$$

Substituting (17) into (22), this implies,

$$v = \left(\gamma \frac{1}{2} \frac{M_p c^2}{M_P}\right)^{\frac{1}{2}} = c\left(\frac{\gamma}{2}\right)^{1/2} . \tag{23}$$

If we equate the speed of propagation of information to the speed of light c in a vacuum, then this implies

$$\gamma = 2 . \tag{24}$$

To determine the number of degrees of freedom f each atom of space would have, we use the formula $\gamma = \frac{(f+2)}{2}$, implying

$$f = 2\gamma - 2 = 2(2) - 2 = 2 . \tag{25}$$

Thus an atom of space represents one unit of entropy, having 2 degrees of freedom coded for by 4 bits of information!! We can thus model this atom of space, by representing its mass and charge as orthogonal spin $\frac{1}{2}$ states each with J = $\pm\frac{1}{2}\hbar$. This can be modelled as shown in Figure 5. Therefore we end up with four degenerate states: ↑↑, ↑↓, ↓↓, ↓↑, hence the ability to store 4 bits of information.

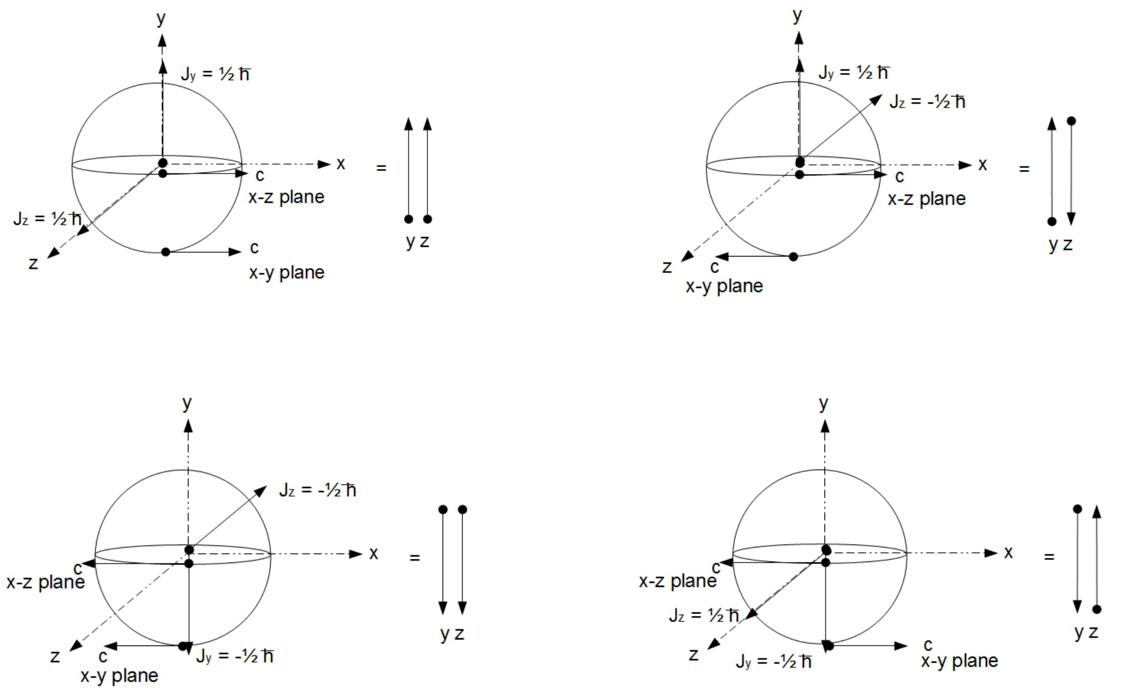

Figure 5

The 2 degrees of freedom are the 2 orthogonal $\frac{1}{2}$ spin states representing the two types of charges (gravitational charge M_p, and the electrical charge Q_p). To reiterate, since these atoms of space have a unique mass-scale, charge-scale, length-scale and time-scale, these plank particles provides the yardstick, by which all of the physical quantities in nature can be measured. These atoms are equivalent to extremal RN black holes and since they do not produce any Hawking radiation, their stability is guaranteed!!

Einstein's theories of relativity revisited

This model of space is still consistent with the conclusions of special relativity. The postulate that the speed of light is constant in a vacuum finds a natural interpretation, as the speed of information transfer (such as light) from a source to detector is independent of the velocity of the source and that of the detector, as the speed only depends on the physical properties of the atoms of space. I conjecture that it is the flipping of the spin states that is responsible for the traveling disturbance in space, as space now acts as the ultimate digital storage and transportation medium. Even more so, gravity as an entropic force a la Verlinde or thermodynamic state becomes more credible in its reformulation.

The double slit experiment revisited

Let us now model the movement of a particle of mass m with velocity v in a vacuum containing the collection of atoms of space. Since v is always less that c = speed of light, this implies that the speed of disturbance will always be greater than the speed of the particle.

This then throws the double-slit experiment into a new light, as it is only the propagation of the disturbances in the vacuum of the atoms of space that interferes with itself (just like water waves interfering with itself), and thus guides the trailing particle after it passes the slit to its final location on the screen; hence the pilot wave a la Bohm's interpretation of quantum mechanic. Hence quantum mechanics has now acquired it correct interpretation. It is truly remarkable that from a thermodynamics perspective, we have identified the quantum of space, and also solved the mystery of the double-slit experiment at the same time.

Triality and the emerging worldview

Thus we see that from the empirical observations of Newton's law of gravitation, Coulomb's law of electrostatics, and the maximum speed of light and the quantum of energy, we have discovered the atoms of space by demanding consistency of the equations at both the quantum, classical (relativistic) level and also at the statistical (thermodynamics) level. The connection between these levels is coded in the wave-particle principle, and also the holographic principle, respectively. In this sense, our description of space is complete.

This triality between particle framework - wave/light framework - holographic/thermodynamic framework, can be viewed as dealing with 0-dim (0-brane), 1-dim (1-brane = string), and 2-dim objects (2-brane = membrane) points of view, respectively, all equivalent or dual representations or codifications of information of any physical system. String theory uses the light frame perspective and M-theory uses a thermodynamics perspective. I conjecture that Matrix theory uses the 0-brane perspective. Also, quantum loop gravity provides the simple framework (using spin networks) to represent the degrees of freedom of space. If these atoms of space represent the background of space, then we see that both background-dependent and background independent approaches are equivalent, because although the atoms of space have mass and charge, they cancel each other out and appear not to exist. Hence we have something and nothing at the some time. Was this what gauge theory and renormalization theory trying to tell us? In a way, we have discovered the Rosetta stone of physics, the missing link, or as Edward Witten has suggested, the fundamental degrees of freedom for a theory of fundamental interactions or fundamental causation. It would be interesting to see how far this new emerging worldview can go to providing a complete and consistent quantum field theory of gravity or quantum gravity. And finally, we are left asking the question, if these atoms have energy, and are not gravitationally and electromagnetically visible, have we also stumbled on the elementary particles of dark energy?

Conclusion

We have arrived at this model of space by only demanding consistency of the equations/relationships in geometry, special relativity, quantum mechanics, gravity, electrostatics and thermodynamics and invoking only three correspondence principles: the wave-particle, holographic and fractal (scale relativity) principles, nothing more or nothing less; hence paving the way for a potentially simple and elegant theory of quantum gravity.

References

Schumm, B.A., 2004. Deep Down Things: The Breathtaking Beauty of Particle Physics, The Johns Hopkins University Press.
Penrose, R., 2007. The Road to Reality: A Complete Guide to the Laws of the Universe, Vintage.
http://www.wikipedia.org/