The mass gap problem in the Wu gauge model framework

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Abstract: We demonstrate that Wu's version of the quantum chromodynamics (QCD) predicts mass gap ($\Delta > 0$) for the compact simple gauge group SU (3). This provides a solution to the second part of the Yang-Mills problem.

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Contents

1. Introduction

The laws of quantum physics are for the world of elementary particles what Newton's laws of classical mechanics are for the macroscopic world. Almost half a century ago, Yang and Mills introduced a remarkable new framework to describe elementary particles by using geometrical structures. Since 1954, the Yang-Mills theory [1] has been the foundation of contemporary elementary particle theory. Although the predictions of the Yang-Mills theory have been tested in many experiments [13], its mathematical foundations remain unclear. The success of the Yang-Mills theory in describing the strong interactions between elementary particles depends on a subtle quantum-mechanical property, termed the *mass gap*: quantum particles have positive masses even though classical waves travel at the speed of light. The mass gap has been discovered experimentaly and confirmed through computer simulations, but is yet to be understood theoretically [2], [3], [4]. The theoretical foundations of the Yang-Mills theory and the mass gap may require the introduction of new fundamental ideas in physics and mathematics [1]. As explained by Arthur Jaffe and Edward Witten in the Caly Mathematical Institute (CMI) problem description [2], the Yang-Mills theory is a generalization of Maxwell's theory of electromagnetism, in which the basic dynamical variable is a connection on a G-bundle over four-dimensional space-time.
The Yang-Mills version is the key ingredient in the Standard Model of elementary particles and their interactions. A solution to the Yang-Mills problem, therefore, would both place the Yang-Mills theory on a firm mathematical footing and demonstrate a key feature of the physics of strong interactions.

The foundation of the Yang-Mills theory and the mass gap problem could be formulated as follows: Prove that, for any compact simple gauge group G, there exists a quantum Yang-Mills theory of R^4, and that this theory predicts a mass gap $\Delta > 0$. Or, more explicitly, given a simple Lie group G (e.g. SU (2) or SU (3)), show that

A) *there exists a full renormalized quantum version of Yang-Mills theory on R^4 based on this group;*

B) *there is a number $\Delta > 0$ such that every state in the theory (except the vacuum) has energy at least Δ. In other words, there are no massless particles predicted by the theory (except the vacuum state)*

Assuming that the quantum chromodynamics (QCD) is the valid theory[1], the first part of CMI problem description, the problem of the foundation of the Yang-Mills theory, is a technical rather than a fundamental physical problem. The goal of this paper is to investigate the possibility of a solution to the second part of the official (CMI) problem description, namely the mass gap problem, by utilizing Wu's formulation of QCD ([5], [6], [7], [8], [9], [10], [11],[12]).

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[1] Note: Lattice QCD is a non-perturbative approach to solving the QCD theory [27], [28]. Analytic solutions in low-energy QCD are hard due to the nonlinear nature of the strong force. This formulation of QCD in discrete space-time introduces a momentum cut off at the order 1/s, where s is the lattice spacing [29], [30].

2. The physics of the mass gap problem

In the standard QCD model, processes that probe the *short-distance* structure of hadrons predict that quarks inside the hadrons interact weakly [13], [14], [15]. Since coupling, g, is small, the classical QCD analysis is a good first approximation for these processes of interaction [13], [14], [15]. However, for Yang-Mills theories in general, it is a general requirement of the renormalization group equations of the quantum field that coupling, g_s increases in reverse law to the hadrons' momentum transfer, until momentum transfer becomes equal to the vector boson. Since spontaneous symmetry-breaking via the Higgs mechanism to give the gluons mass is not present in QCD ([16], [17], [18], [19], [20], [21], [22], [23]), QCD contains no mechanism to stop the increase of coupling, g_s. In consequence, quantum effects become more and more dominant as distances increase. Analysis of the behavior of QCD at *long distances*, which includes deriving the hadrons spectrum, requires foundation of the full Yang-Mills quantum theory. This analysis is proving to be very difficult ([2], [3], [4]).

The mass gap problem can be expressed as follows:

Why are nuclear forces short-range? Why there are no massless gauge particles, even though current experiments suggest that gluons have no mass?

This contradiction is the physical basis of the mass gap problem. More explicitly, let us denote this energy minimum (vacuum energy) of Yang-Mills Hamiltonian H' by E_m, and let us denote the lowest energy state by φ_m. By shifting original Hamiltonian H by $-E_m$, the new Hamiltonian $H' = H - E_m$ has its minimum at $E = 0$ (massless state) and the first state φ_m is the vacuum vector. We now notice that in space-time the spectrum of Hamiltonian is not supported in region $\left(0,\Delta\right)$, with $\Delta > 0$.

3. The mass gap in the framework of Wu's QCD

As we have seen, the Higgs mechanism ([16], [17], [18], [19], [20], [21], [22], [23]) is excluded in the QCD theory (gluons are massless because the QCD Lagrangian has no spinless fields and, therefore, no obvious possibility for spontaneous symmetry-breaking). It follows that the only possible mechanism that can introduce the mass term in the gluon field while remaining consistent with the occurrence of mass gap is the Wu mass generator mechanism ([5], [6], [7], [8], [9], [10],[11], [12]). The Wu mass generator mechanism introduces mass terms without violating the SU (3) gauge symmetry.

The quark field is denoted as

$$\psi_a(x), (a = 1,2,3) \tag{1}$$

where a is color index and the flavor index is omitted.

Ψ is defined as follows:

$$\psi = \begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \end{pmatrix} \tag{2}$$

All ψ_α form the fundamental representative space of SU (3) c.

Starting with Wu's gauge model framework as described [5], [6], [7], [8], [9], [10], [11] and [12], we introduce two gluon fields $(A^a_\mu, A^a_{2\mu})$ simultaneously. The Lagrangian of the model is:

$$\begin{aligned} \Im = &-\bar{\psi}[\gamma^5(\partial_\mu - igA_{1\mu}) + m]\psi - \frac{1}{4K}Tr(A_1^{\mu\nu}A_{1\mu\nu}) \\ &-\frac{1}{4K}Tr(A_2^{\mu\nu}A_{2\mu\nu}) - \frac{m^2}{2K}Tr[(\cos aA_1^\mu \sin aA_2^\mu)(\cos aA_{1\mu} + \sin aA_{2\mu})] \end{aligned} \tag{3}$$

Where

$$A_{1\mu\nu} = \partial_\mu A_{1\nu} - \partial_\nu A_{1\mu} - ig[A_{1\mu}, A_{1\nu}],\ A_{2\mu\nu} = \partial_\mu A_{2\nu} - \partial_\nu A_{2\mu} + igta[A_{2\mu}, A_{2\nu}] \tag{4}$$

This Lagrangian can be proved to have strict SU (3) c gauge symmetry. Since A_μ and $A_{2\mu}$ are not eigenvectors of mass matrix, we can apply the following transformations:

$$\begin{aligned} G_{1\mu} &= \cos aA_{1\mu} + \sin aA_{2\mu} \\ G_{2\mu} &= -\sin aA_{1\mu} + \cos aA_{2\mu} \end{aligned} \tag{5}$$

After these, the Lagrangian given by equation (3) changes into

$$\begin{aligned} \Im = &-\bar{\psi}[\gamma^5(\partial_\mu - ig\cos aG_{1\mu} + ig\sin aG_{2\mu}) + m]\psi - \frac{1}{4}G_{10}^{\mu\nu}G_{10\mu\nu}^i \\ &-\frac{1}{4}G_{20}^{\mu\nu}G_{20\mu\nu}^i - \frac{m^2}{2}G_1^{i\mu}G_{1\mu}^i + \Im_{gI} \end{aligned} \tag{6}$$

Where $\quad G_{m0\mu\nu} = \partial_\mu G^i{}_{m\nu} - \partial_\nu G^i{}_{m\mu}, (m = 1,2), G_{m\mu} = G^i_{m\mu}\lambda_i / 2$ (7)

The Lagrangian $\Im_{gI}$ only contains interaction terms of gauge fields. $\Im_{gI}$ includes a massive gluon field, G_1, with mass m, and a massless gluon field, G_2.

Since there exist two sets of gluons, there may exist three sets of glueballs in mass spectrum, (g_1g_1), (g_1g_2) and (g_2g_2), with the same spin-parity but different masses (see [12]).

In Quantum Field Theory (QFT) the number of particles is not constant, due to the particles' creation and annihilation. Therefore, while we can define the mass-state of the system as the mass-state of all particles or equivalent, we cannot define the mass-state of a specific particle. In Wu's version of QCD, nevertheless, the mass-state of the system has non-zero mass. A possible interpretation of the existence of both massive and massless gluon states is that Wu's mass generator mechanism introduces the phenomenon of mass gap.

This is achieved in the following steps: First, in the Wu QCD version the non-zero gluon mass corresponds to the massive vector propagator [24] and the zero gluon mass corresponds to the massless vector propagator of the short-distance QCD [13], [14], [15]. For the given wave vectors k_1 and k_2 the propagators of massless and massive gluons are given by:

$$\Delta^{\alpha\beta}_{2\mu\nu}(k_2) = -i\delta^{\alpha\beta}\left(\frac{g_{2\mu\nu}}{k_2^2 + i\varepsilon}\right) \tag{8}$$

$$\Delta^{\alpha\beta}_{1\mu\nu}(k_1) = \frac{-i\delta^{\alpha\beta}}{k_1^2 + M^2 - i\varepsilon}\left(g_{1\mu\nu} + \frac{k_{1\mu}k_{1\nu}}{M^2}\right) \tag{9}$$

where [2] $1\mu \neq 2\mu, 1\nu \neq 2\nu$, (10a)

$$g_{1\mu\nu} = (e_{1\mu}, e_{1\nu}),\ g_{2\mu\nu} = (e_{2\mu}, e_{2\nu}), \tag{10b}$$

$$\frac{\partial}{\partial x^{1\mu}} = e_{1\mu}, \qquad \frac{\partial}{\partial x^{2\mu}} = e_{2\mu} \tag{10c}$$

$g_{1\mu\nu}$, $g_{2\mu\nu}$ are metrics of the space-time regions and $e_{2\mu}$, $e_{2\mu}$ are vectors of the basis induced by the selection of local coordinates $x^{1\mu}$, $x^{2\mu}$ on the neighborhood of the given points.

Furthermore, due to equation (10a), the wave vectors $k_{1\mu}$ and $k_{2\mu}$ lie in different regions in the momentum space in their own right.

Here, we express the meet of the propagators $\Delta^{\alpha\beta}_{1\mu\nu}(k_1)$ and $\Delta^{\alpha\beta}_{2\mu\nu}(k_2)$ in the momentum space, in terms of its hypersurface of the present ([25]) at the origin of the propagators paths as follows

[2] Note: In Wu's version of QCD, the letters $1\mu, 1\nu$ and $2\mu, 2\nu$ denote the different space-time indices of the massive and massless propagators.

$$\delta_{\alpha\beta}^{1\mu\nu}\Delta_{1\mu\nu}^{\alpha\beta}(k_1)\wedge\delta_{\alpha\beta}^{2\mu\nu}\Delta_{2\mu\nu}^{\alpha\beta}(k_2)=\liminf_{(k_1,k_2)\to S'}\{\min_{i\varepsilon\to 0}(\Delta_1(k_1),\Delta_2(k_2))\} \tag{11a}$$

where

$$\delta_{\alpha\beta}^{1\mu\nu}\Delta_{1\mu\nu}^{\alpha\beta}(k_1)=\Delta_1(k_1)=\frac{1}{k_1^2+m^2-i\varepsilon}, \tag{11b}$$

$$\delta_{\alpha\beta}^{2\mu\nu}\Delta_{2\mu\nu}^{\alpha\beta}(k_2)=\Delta_2(k_2)=\frac{1}{k_2^2-i\varepsilon} \tag{11c}$$

are functions in the momentum space. The subset $S'=S\setminus r_1$ of the hypersurface of the present is disconnected, since it is the disjoint union of the two propagator's half-planes of the present that corresponds to the different 1μ and 2μ indices.

$$U_1^+\bigcap U_2^-=\varnothing \quad \text{and} \qquad S'=S\setminus r_1=U_1^+\bigcup U_2^-=(-\infty,+\infty)\setminus\{0\}, \tag{11d}$$

$$U_1^+=S'\bigcap\{(k_{11},k_{12})\in R^2: ak_{11}+bk_{12}+c>0\}=(0,+\infty), \tag{11e}$$

$$U_1^-=S'\bigcap\{(k_{21},k_{22})\in R^2: ak_{21}+bk_{22}+c<0\}=(-\infty,0) \tag{11f}$$

the $r_1: ak_x+bk_y+c=0$ is line such that the points (k_{11},k_{12}) and (k_{21},k_{22}) of S lie on different sides of r_1 and U_1^+,U_2^- are the propagator's half-planes of the present.

Substituting equations (11d), (11c), (11d) to equation (11a), we calculate the meet of the $\Delta_{1\mu\nu}^{\alpha\beta}(k_1)$ and $\Delta_{2\mu\nu}^{\alpha\beta}(k_2)$ propagator in the momentum space

$$\delta_{\alpha\beta}^{1\mu\nu}\Delta_{1\mu\nu}^{\alpha\beta}(k_1)\wedge\delta_{\alpha\beta}^{2\mu\nu}\Delta_{2\mu\nu}^{\alpha\beta}(k_2)=0 \tag{11g}$$

Equation (11g) shows that the meet of the propagators $\Delta_{1\mu\nu}^{\alpha\beta}(k_1)$ and $\Delta_{2\mu\nu}^{\alpha\beta}(k_2)$ is zero, because the propagator's half-planes of the present are disjoint in the momentum space. After some calculations, the Fourier transformation to the position space of equation (11g) is given by

$$\delta^{(4)}(x_2-y_2)\delta_{\alpha\beta}^{1\mu\nu}\Delta_{1\mu\nu}^{\alpha\beta}(x_1-y_1)\wedge\delta^{(4)}(x_1-y_1)\delta_{\alpha\beta}^{2\mu\nu}\Delta_{2\mu\nu}^{\alpha\beta}(x_2-y_2)=0 \tag{12}$$

where

$$\delta^{(4)}(x_1-y_1)=\int_{-\infty}^{+\infty}\frac{d^4k_1}{(2\pi)^4}e^{ik_1(x_1-y_1)}, \qquad \delta^{(4)}(x_2-y_2)=\int_{-\infty}^{+\infty}\frac{d^4k_2}{(2\pi)^4}e^{ik_2(x_2-y_2)} \tag{13}$$

and

$$\Delta^{\alpha\beta}_{2\mu\nu}(x_2-y_2)=\lim_{\varepsilon\to 0^+}\int_{-\infty}^{+\infty}\frac{d^4k_2}{(2\pi)^4}\frac{e^{ik_2(x_2-y_2)}}{k_2^2+i\varepsilon}g_{2\mu\nu}(-i\delta^{\alpha\beta}) \quad , \tag{14}$$

$$\Delta^{\alpha\beta}_{1\mu\nu}(x_1-y_1)=\lim_{\varepsilon\to 0^+}\int_{-\infty}^{+\infty}\frac{d^4k_1}{(2\pi)^4}\frac{e^{ik_1(x_1-y_1)}}{k_1^2+M^2-i\varepsilon}(-i\delta^{\alpha\beta})\left(g_{1\mu\nu}+\frac{k_{1\mu}k_{1\nu}}{M^2}\right) \tag{15}$$

$\delta^{(4)}(x_1-y_1)$, $\delta^{(4)}(x_2-y_2)$ are the Dirac delta functions and $\Delta^{\alpha\beta}_{1\mu\nu}(x_1-y_1)$, $\Delta^{\alpha\beta}_{2\mu\nu}(x_2-y_2)$ are the two gluon propagators in the position space.

The integral of equation (12) over d^4x_1 , d^4x_2 is given as follows:

$$\iint\delta^{(4)}(x_2-y_2)\delta^{1\mu\nu}_{\alpha\beta}\Delta^{\alpha\beta}_{1\mu\nu}(x_1-y_1)d^4x_2d^4x_1\wedge\iint\delta^{(4)}(x_1-y_1)\delta^{2\mu\nu}_{\alpha\beta}\Delta^{\alpha\beta}_{2\mu\nu}(x_2-y_2)d^4x_1d^4x_2=0 \tag{16}$$

Here, we integrate equation (16) in the following steps. First, by fixed y_1, y_2, the equation (16) becomes

$$\int\delta^{1\mu\nu}_{\alpha\beta}\Delta^{\alpha\beta}_{1\mu\nu}(x_1-b_1)d^4x_1\wedge\int\delta^{2\mu\nu}_{\alpha\beta}\Delta^{\alpha\beta}_{2\mu\nu}(x_2-b_2)d^4x_2=0 \tag{17}$$

Next, by considering an insulating color source, in Feynman gauge, the propagators magnitudes are constant in the position space. Therefore the integral of the equation (12) over d^4x_1, d^4x_2, gives

$$\delta^{1\mu\nu}_{\alpha\beta}\Delta^{\alpha\beta}_{1\mu\nu}(x_1-b_1)V_4^{(1)}\wedge\delta^{2\mu\nu}_{\alpha\beta}\Delta^{\alpha\beta}_{2\mu\nu}(x_2-b_2)V_4^{(2)}=0 \tag{18}$$

where

$$V_4^{(1)}=\int d^3x_1\int dt1 \ , \ V_4^{(2)}=\int d^3x_2\int dt1 \tag{19}$$

$V_4^{(1)}$, $V_4^{(2)}$ are the space-time volumes.

Equation (18) is written in style of (11g) as follows

$$\delta^{1\mu\nu}_{\alpha\beta}\Delta^{\alpha\beta}_{1\mu\nu}(x_1)\wedge\delta^{2\mu\nu}_{\alpha\beta}\Delta^{\alpha\beta}_{2\mu\nu}(x_2)=\liminf_{(x_1,x_2)\to\tilde{S}'}\{\min(\Delta_1(x_1),\Delta_2(x_2))\}=0 \tag{20a}$$

where

$$\delta^{1\mu\nu}_{\alpha\beta}\Delta^{\alpha\beta}_{1\mu\nu}(x_1)=\Delta_1(x_1)=\frac{e^{-mx_1}}{x_1}, \tag{20b}$$

$$\delta^{2\mu\nu}_{\alpha\beta}\Delta^{\alpha\beta}_{2\mu\nu}(x_2)=\Delta_2(x_2)=\frac{1}{x_2} \tag{20c}$$

are functions in the position space. The subset $\tilde{S}'=\tilde{S}\setminus r_2$ of the hypersurface of the present is disconnected, since it is the disjoint union of the two propagator's half-planes of the present that corresponds to the different 1μ and 2μ indices.

$$\tilde{U}_1^{+}\bigcap\tilde{U}_2^{-}=\varnothing \quad \text{and} \qquad \tilde{S}'=\tilde{S}\setminus r_2=\tilde{U}_1^{+}\bigcup\tilde{U}_2^{-}=(-\infty,+\infty)\setminus\{0\} \tag{20d}$$

$$\tilde{U}_1^{+}=\tilde{S}'\bigcap\{(x_{11},x_{12})\in R^2:ax_{11}+bx_{12}+c>0\}=(0,+\infty), \tag{20e}$$

$$\tilde{U}_1^{-}=\tilde{S}'\bigcap\{(x_{21},x_{22})\in R^2:ax_{21}+bx_{22}+c<0\}=(-\infty,0) \tag{20f}$$

the $r_2:ax+by+c=0$ is line such that the points (x_{11},x_{12}) and (x_{21},x_{22}) of $\tilde{S}$ lie on different sides of r_2 and $\tilde{U}_1^{+},\tilde{U}_2^{-}$ are the propagator's half-planes of the present.

Equation (20a) shows that the propagator's half-planes of the present are disjoint in the position space, because the meet of $\Delta^{\alpha\beta}_{1\mu\nu}(x_1)$ and $\Delta^{\alpha\beta}_{2\mu\nu}(x_2)$ is zero. $\Delta^{\alpha\beta}_{1\mu\nu}(x_1-b_1)$ and $\Delta^{\alpha\beta}_{2\mu\nu}(x_2-b_2)$ lie outside each other's light cone [25]; these propagators, therefore, are not casually connected [26]. Propagators $\Delta^{\alpha\beta}_{1\mu\nu}(x_1-b_1)$ and $\Delta^{\alpha\beta}_{2\mu\nu}(x_2-b_2)$ are separated from each other in a (space-like) direction of distance

$$S^2=|\eta_{1\mu\nu}x^{1\mu}x^{1\nu}-\eta_{2\mu\nu}x^{2\mu}x^{2\nu}| \tag{20g}$$

where $\eta_{1\mu\nu}$, $\eta_{2\mu\nu}$ are the metrics of the disjoint space-time regions. The arbitrary space-like region S^2 corresponds to an arbitrary energy scale ($m^2c^4=\hbar^2/S^2$). m is the mass of the particle that is constrained to the above space-like portion of space-time [37], [38]. This particle of mass m either travels forward in time with imaginary mass or travels backward in time with real mass [37],[38].

Next, a basic principle in nuclear physics which states that the combined operations of charge conjugation (C), time reversal (T), space inversion (P) in any order is an exact symmetry of the strong force [31], [32]. The CPT symmetry is conserved only if our theory respects the Lorentz invariant and the microcasuality principle [33], [34].

Here, the observations of particles that are constrained to the space-like portion of space-time, without violating the microcasuality principle are allowed by the following postulate:

The particle with space-like four-momentum that travels backward in time is equivalent to the mass gap (Δ) that is observed at the rest frame.

The gap in the particle's space-like four-momentum that travels backward in time is equivalent to the superluminal particle that travels forward in time and is observed at the laboratory frame.

The postulate predicts the observations of either superluminal particles that travel forward in time [39], [40] or a mass gap (Δ) at the rest frame [1], [2].

From the above, equations (11g) and (20a) predict that the Wu Hamiltonian is not supported in an arbitrary space-like region S_{Δ}^{2} that corresponds to an arbitrary energy scale ($\Delta^2 c^4 = \hbar^2 / S_{\Delta}^2$). Therefore, the Wu Hamiltonian in space-time is bounded by Δ.

Finally, in Wu's version of QCD, the vacuum state has zero energy (*E=0*), corresponding to G_2 (massless gluon). The excitation state above the vacuum energy has non-zero energy E_m, corresponding to G_1 (massive gluon). The latter predicts that the Wu Hamiltonian is not supported in the region $\left(0,\Delta\right)$ with $\Delta > 0$ and is bounded below by Δ close to the gluon mass.

4. Discussion

Hadron colliders, such as the Tevatron, can place the strongest limits on the new colored particles (squarks and gluinos in supersymmetry or KK excitations of quarks and gluons in models with universal extra dimensions [35–36]. Typically, limits of∼ 200 GeV are obtained for new colored particles. Here, the predicted gluon mass is close to the mass gap state Δ, thus gluon mass is not subject to the Hadron collider's limits [35-36].

In the Lorentz gauge, the free Lagrangian of Wu's QCD field theory (equation (6), with $\Im_{gI} = 0$) yields to the following field equations:

$$\frac{1}{c^2}\frac{\partial^2 G_{1\mu}}{\partial t^2} = \nabla^2 G_{1\mu}(r) - \frac{1}{L^2} G_{1\mu}(r),$$
$$\frac{1}{c^2}\frac{\partial^2 G_{2\mu}}{\partial t^2} = \nabla^2 G_{2\mu}(r) \tag{21}$$

L the length of the system given by

$$L = \frac{\hbar}{mc} \tag{22}$$

Where $\hbar$ the Planck constant, c is the speed of light, m the mass of the system. If the source is a point color at the origin, only the time-components $C_{02} = \Phi_2$, $C_{01} = \Phi_1$ are nonvanishing. The short-distance solution of the eq (21) that corresponds to the massless gluons is given as follows:

$$G_{20}(r) = \frac{a_s}{4\pi |r|} \tag{23}$$

where a_s is the quark-gluon coupling.

Gluons that respect the solution (23) are mediators of the long-range nuclear force at high energy scale. The long-distance solution of the eq (21) corresponds to the massive gluons and is given as follows:

$$G_{10}(r) = \frac{a_s}{4\pi |r|} e^{-|r|/L} \tag{24}$$

Gluons that respect the solution (24) are mediators of the short-range nuclear force at low energy scale. Here, the massive and massless gluons lie outside each other's light cone, because of eq (20a). The term (22) in the eq (21) and (24) reveals as a mass gap. Supposing that the range of the strong force is about the radius of the proton, equation (22) predicts mass gap of about 1GeV.
We find that, in Wu's version of QCD, free system's energies are indeed bounded in space-time. Consequently, in space-time, the Wu Yang-Mills Hamiltonian H_{wu} has spectrum bounded below. Let us denote this energy minimum of the Wu Yang-Mills Hamiltonian H_{wu} by E_m, and let us denote the lowest energy state by φ_m. By shifting the original Hamiltonian H by $-E_m$, the new Hamiltonian $H_{wu} = H - E_m$ has its minimum at $E = 0$. The first state φ_m is the vacuum vector. We now notice that, in space-time, the spectrum of the Wu Hamiltonian is not supported in region $(0, \Delta)$, with $\Delta > 0$.

According to the Wu Yang-Mills theory, therefore, over compact gauge groups SU (3) there is always a vacuum vector φ_m and a mass gap Δ. Wu's mass generator mechanism can always be applied to any Yang–Mills type of action. Thus, any Yang–Mills theory accounts for a mass gap.

5. Conclusion

We have shown that the Wu gauge model can introduce mass terms while being consistent with the existing mass gap Δ. In this model the mass gap is introduced by Wu's mass generator mechanism.

Since the gluon field G_1 is massive, whereas the gluon field G_2 is massless, we conclude that system's free energies in Wu's version of QCD are bounded in space-time.

We also notice that, in space-time, the spectrum of the Wu Hamiltonian is not supported in region $(0,\Delta)$ with $\Delta > 0$. Wu's mass generator mechanism can always be applied to any Yang–Mills type of action. Therefore any Yang–Mills theory accounts for a mass gap.

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