On the quantities of energy and momentum in contemporary physics

Peter Šujak
Prague, Czech Republic
peter.sujak@email.cz

(July 15, 2011)

Abstract

This paper discusses the meaning and role of the quantities of energy and momentum in the definition relations of relativistic, quantum and classical mechanics with focusing on kinetic and total relativistic energy, on the definition of the de Broglie momentum hypothesis and using momentum and energy in Schrodinger, Klein-Gordon and Dirac equation.

The relationships of quantum physics for energy $E=h\nu$ and momentum $p=h/\lambda$ of a photon and the relationship of relativistic mechanics for total energy $E_t=mc^2=(m_o^2v^2c^2/(1-v^2/c^2)+m_o^2c^4)^{1/2}$ are the basic relationships of contemporary physics. From quadratic form of the total energy relation $E_t^2= m^2c^4 = m_o^2c^4 +m^2v^2c^2= E_o^2+E_k^2$ we can deduce that kinetic so added energy to rest energy $E_o=m_oc^2$ is $E_k=mvc$. This relation we can also derive from total relativistic energy

$$E_k=(m^2c^4-m_o^2c^4)^{1/2}=m_oc^2(1/(1-v^2/c^2)-1)^{1/2}=m_oc^2(c^2/(c^2-v^2)-1)^{1/2}=m_oc^2(v^2/c^2-v^2)^{1/2}=$$
$$=m_ovc(c^2/(c^2-v^2))^{1/2}= m_ovc/(1-v^2/c^2)^{1/2} = mvc=pc \quad \text{so} \quad E_k =mvc=pc= mc^2.\, v/c\ .$$

In quantum mechanics (QM) so as in relativistic mechanics (RM) for kinetic energy we write directly $E_k=mc^2-m_oc^2=E_t-E_o$ so $E_k=mc^2-m_oc^2=mvc=pc$ in contrast to writing for total energy in RM where we write the square root of the sum of squares $E_t= (m^2v^2c^2+m_o^2c^4)^{1/2}=(E_k^2+ E_o^2)^{1/2}$. This way we get for the ratio of kinetic energy to momentum relation $E_k/p=mvc/mv=c$ and for the ratio of total energy to momentum $E_t/p = mc^2/mv =c^2/v$ or $E_t/pc= mcc/mvc= c/v$. Consequently as a speed v approaches the speed of light c $v\rightarrow c$ so momentum multiplied by c approaches total energy $pc\rightarrow E_t$.

RM establishes the different definition of kinetic energy $E_k=mc^2-m_oc^2=mvc$ from classical mechanics (CM) $E_k =½.mv^2$. In RM the relation of classical kinetic energy is subsequently seen as an approximation of the relativistic kinetic energy relation and the classical kinetic energy relation can be found by expanding the relativistic relation into Taylor series $E_k=mc^2-m_oc^2=m_oc^2(1/(1-v^2/c^2)^{1/2} -1)=(1+½.m_ov^2 +3/8.m_ov^2/ c^2..-1) \approx ½.m_ov^2$. By this expansion we change the definition status of kinetic energy from a linear functionality in RM at a speed approaches the speed of light into a quadratic functionality in CM at a speed much slower than the light's speed in the vacuum.

The relation for relativistic kinetic energy $E_k=mc^2-m_oc^2 =mvc$ however has a general force and is active at all speeds thus as well as at speeds much slower than the speed of light.

The relation of photon's energy $E=h\nu$ and relativistic energy $E_t=(m^2v^2c^2 + m_o^2c^4)^{1/2}$ are tied together by the interaction of photons with particles such as the photoelectric effect, the scattering effect or electron –positron pair production (EPP). For EPP we write the relation between photon's energy and kinetic energy of an electron and a positron $h\nu-h\nu_o = =½.m_ev^2+½.m_pv^2$ and for the electron portion we can write $½.h\nu-½.h\nu_o =½.m_ev^2$. The frequency ν_o is the minimal frequency of photon's energy necessary for EPP and equals to internal energy of an electron $h\nu_o = hc/\lambda_o =m_oc^2$. This frequency ν_o corresponds to the

Compton wavelength of an electron $\lambda_o = h/m_oc$ and thus we can reasonably suppose that λ_o corresponds to the maximum radius $l_o=h/m_oc^2$ of created electrons .The energy balance relation for EPP is based on the photoelectric effect explanation where difference between incident energy of photons $h\nu$ and binding energy of electrons $h\nu_o$ equals to kinetic energy of electrons $\Delta E=h\nu-h\nu_o=½.m_ov^2$ emitted out of atoms. The classical mechanics relationship for electron's kinetic energy $E_k =½.m_ov^2$ in the photoelectric effect relation $\Delta E=h\nu-h\nu_o=½.m_ov^2$ is however approximative and according to RM we can write the relation $h\nu-h\nu_o=hc/\lambda-hc/\lambda_o =$ $=mc^2-m_oc^2= mvc=pc$. This relation is in an accordance with the relativistic understanding of energy $E=pc$ since dividing $hc/\lambda-hc/\lambda_o=mvc$ by c we get the relation for momentum at the photoelectric effect $h/\lambda-h/\lambda_o=h\nu/c-h\nu_o/c= mv =mc-m_oc=p_t-p_o= p$. This momentum relation has the same form in CM and RM and in contrast to the energy relation there is none two-faced writing or approximation of its right side hand. Following this momentum relation we can came to an idea that in the same way as we in RM consider on kinetic, total and internal energy we can consider on momentum p, total momentum p_t and internal rest momentum p_o . The internal rest momentum p_o we can imagine as rotation or as in QM installed the angular momentum spin.

Thus we can talk about an Einstein's definition of energy (EDE) where momentum multiplied by c equals energy $E=pc$. In EDE total momentum of a photon and particle $p_t=h/\lambda=h\nu/c=mc$ multiplied by c gives total energy $E_t=p_t.c=hc/\lambda=h\nu c/c =mc.c$. Total photon energy begins from zero energy then zero mass then from the zero frequency $\nu=0$ and infinity wavelength $\lambda=\infty$ and particle energy begins from rest energy then rest mass $E_o= m_oc.c =p_o.c= hc/\lambda_o = h\nu_oc/c$ where λ_o and ν_o are the wavelength and frequency of photon's energy needed for EPP .In EDE change of momentum $p=mv=mcv/c=(h/\lambda-h/\lambda_o)=h(\lambda_o-\lambda/\lambda\lambda_o)= h/\lambda .(\lambda_o-\lambda/\lambda_o)= h/\lambda.v/c$ multiplied by c gives change in energy $E_k=p.c=mv.c=h \nu-h \nu_o =(h/\lambda-h/\lambda_o).c=h(\lambda_o-\lambda/\lambda\lambda_o).c$. The relationship $E/p=c$ in EDE is however valid just for the ratio of corresponding quantities that is for total quantities $E_t/p_t= h\nu/h\lambda^{-1}=\nu\lambda=mc^2/mc=c$ for the change in quantities $E_k/p=mvc/mv=$ $(h/\lambda-h/\lambda_o).c /(h/\lambda-h/\lambda_o) =c$ and for rest quantities $E_o/p_o= h\nu_oc/h\lambda_o^{-1}=c$. The ratio for non corresponding quantities is $E_t/p=mc^2/mv=h\nu/(h/\lambda-h/\lambda_o) =hc\lambda^{-1}/h(\lambda_o-\lambda/\lambda\lambda_o)=c\lambda_o/(\lambda_o-\lambda)=$ $=c.c/v=c^2/v$ and also $E_k / E_t =mvc/mcc=v/c$ and $p/p_t=mv/mc=v/c$. The substantial relation valid in RM and QM $E_t/p = c^2/v$ resulting also from $p^2c^2 = m^2c^4 - m_o^2c^4 = m^2v^2c^2$ then $pc=mvc$ then $pc= mc^2v/c=E_tv/c$ then $E_t/p = c^2/v$ is interpreted that as the speed $v\to c$ then momentum multiplied c approach to total energy $pc\to E_t$ .Then we can interpret the relation $E_t/pc = mcc/mvc=p_tc/pc=c/v$ so that for $v\to c$ consequently $pc\to E_t$ since kinetic energy approach to total energy $mvc\to mcc$ and since momentum approach total momentum $mv\to mc$ so $p\to p_t$. The same way as in the relation $E_k=mc^2-m_oc^2=mvc$ for $v\to c$ rest energy becomes negligible and kinetic energy approach total energy $mvc\to mcc$ so in a relation $p=mc-m_oc=mv$ rest momentum becomes negligible and momentum approach total momentum $mv\to mc$.

In RM and QM we talk about total energy of a particle $E_t =mc^2$ and about total energy of a photon $E_t = h\nu=hc/\lambda=mc^2$ and just as is necessarily to accent that at the relationship for photon's momentum $p=h/\lambda$ we talk about photon's total momentum .Total energy and momentum of a photon as well as the mass equivalent and frequency of a photon are running from the zero values and a wavelength from an infinite value.Total relativistic energy for a particle is running from rest energy $m_oc^2=h\nu_o=hc/\lambda_o$ by adding kinetic energy .Thus if we want formulate the energy relation of a particle similarly to the photon's relation $E_t=h\nu =hc/\lambda= mc^2$ then kinetic energy is $mc^2-m_oc^2=mvc=h\nu-h\nu_o=hc/\lambda- hc/\lambda_o =hc(\lambda_o-\lambda/\lambda\lambda_o)=pc$ and for momentum thus we get $mc-m_oc=mv=h\nu/c-h\nu_o/c=h/\lambda-h/\lambda_o=h(\lambda_o-\lambda/\lambda\lambda_o)=h/\lambda .(\lambda_o-\lambda/\lambda_o)= h/\lambda.v/c=mcv/c= p$.

Thus we can believe that if we wish to transfer the photon's relation $E=h\nu$ and $p=h/\lambda$ onto an electron and if we will consider as momentum of particle $h/\lambda-h/\lambda_o=mv$ just as we take kinetic energy $mc^2 - m_oc^2 = h\nu - h\nu_o= hc/\lambda - hc/\lambda_o$ and consider as total momentum of a particle

$h/\lambda=mc$ just as total energy $hc/\lambda=mc^2$ then we can obtain the non-controversial ratio of energy and momentum $E/p=E_t/p=mc^2/mv=h\nu/(h/\lambda-h/\lambda_o)=hc\lambda^{-1}/h(\lambda_o-\lambda/\lambda\lambda_o)=c\lambda_o/(\lambda_o-\lambda)=c.c/v=c^2/v$ and also the ratio $E/p=E_k/p=(mc^2-m_oc^2)/mv=mvc/mv=(h\nu-h\nu_o)/(h/\lambda-h/\lambda_o)=(h/\lambda-h/\lambda_o).c/(h/\lambda-h/\lambda_o)=c$. Subsequently we can believe that is not valid the ratio $E/p=E_t/p=mc^2/mv\neq h\nu/h\lambda^{-1}$ and accurate are the ratios $E/p=E_t/p=mc^2/mv=h\nu/(h/\lambda-h/\lambda_o)=c^2/v$ and $E/p=E_t/p_t=mc^2/mc=h\nu/h\lambda^{-1}=c$

De Broglie introduced the presumptions, that the photon's relationships can be transferred onto a particle as $p=h/\lambda=mv$ and $E=h\nu=mc^2$ where rest energy of a particle is associated with the frequency $h\nu_o=m_oc^2$ and particle momentum is associated with a particle's wavelength $\lambda=h/mv$ that value is infinite $\lambda=\infty$ for zero momentum $mv=0$. From this two presumption de Broglie came to two different ratios of energy to momentum $E/p=h\nu/h\lambda^{-1}=hc\lambda/h\lambda=\lambda\nu=c$ and concurrently $E/p=mc^2/mv=c^2/v$. This paradox de Broglie worked out by the phase velocity $w=c^2/v$ and the group velocity of a particle. Consequently a phase velocity is always higher than the speed of light c when at a low speed v the phase velocity approach to infinity and at the speed near c the phase velocity approach to c. The entire taking–over the de Broglie formalism of the wave property of matter by QM leads to the wave function or the wave probability of particle propagation. Up to this day the meaning and role of the wave function in QM is unexplained. The discrepancy is clearly seen if we into the relation $E=mc^2=h\nu=hc/\lambda$ use $\lambda=h/mv$ and then $E=mc^2=hc/\lambda=hcmv/h=mvc$ so $E=mc^2=mvc$ what is valid only when $c=v$ thus when total energy equals to kinetic energy what as well as is valid in QM for a free particle. The discrepancy $c=c^2/v$ so $c=v$ results from the simultaneous seeming validity $E/p=c$ and $E/p=c^2/v$ instead of the factual validity $E_k/p=E_t/p_t=c$ and $E_t/p=c^2/v$ .Thus this discrepancy absents for the ratio of corresponding quantities $E_t/p_t=mc^2/mc=h\nu/h\lambda^{-1}=hc\lambda/h\lambda=\lambda.\nu=c$ and $E_k/p=mvc/mv=p.c/p=c.h\lambda^{-1}.(\lambda_o-\lambda/\lambda_o)/h\lambda^{-1}.(\lambda_o-\lambda/\lambda_o)=c$ for $p=h/\lambda.(\lambda_o-\lambda/\lambda_o)=mv$. The discrepancy disappear also as for $p_t=h/\lambda=mc$ so $\lambda=h/mc$ we get $E=mc^2=h\nu=hc/\lambda=hcmc/h=mc^2$ and so $mc^2=mc^2$. Thus we see that the ratio of energy and momentum is $E/p=h\nu/h\lambda^{-1}=mc^2/mv=c^2/v$ or $E/p=(h\nu-h\nu_o)/mv=(mc^2-m_oc^2)/mv=mvc/mv=c$ according as we take energy from $\nu=0$ so $E=h\nu(0)=mc^2$ or from $\nu=\nu_o$ so $\Delta E=h\nu-h\nu_o=mvc$.

Based upon the foregoing consideration in this paper we can come to the conviction that for transferring the photon's relation of momentum $p=h/\lambda$ and energy $E=h\nu$ onto an electron it is necessary transfer correctly the quantities of total, added and rest energy and momentum as well as transfer the dynamism of increasing in the spatial energy concentration expressed in a raising photon frequency with a shortening of the photon's dimension expressed in shortening of the photon wavelength. Thus in the same way consider about shortening in the electron dimension with increasing in electron's energy. The principle of the spatial shrinking of mass-energy with increasing of mass-energy we can regard as the universal principle of the nature. So as at the photon so as in nuclear physics so as in physics of the universe the greater accumulation of mass represents greater energy and leads to its less spatial localization .About this principle in fact predicate also the relativistic relationships on the length contraction and increase in effective mass with increasing energy as a result of an increasing speed. If we consider the relations of increasing in mass $m=m_o/(1-v^2/c^2)^{1/2}$ and the length contraction $l=l_o(1-v^2/c^2)^{1/2}$ than we can write $(1-v^2/c^2)=m_o^2/m^2=l^2/l_o^2$ or rewrite it to the relation $m_o^2c^2/m^2(c^2-v^2)=l^2c^2/l_o^2(c^2-v^2)$ or $m_o^2l_o^2c^2/(c^2-v^2)=m^2l^2c^2/(c^2-v^2)$.Thus for any difference (c^2-v^2) products $m_o^2l_o^2c^2=m^2c^2l^2=h^2$ remains constant. Then against the calibration basis $h^2=m_o^2l_o^2c^2$ or $h^2/l_o^2=m_o^2c^2$ we can express the total value as $h^2/l^2=m^2c^2$ and added or change in the value as $h^2/l^2-h^2/l_o^2=m^2c^2-m_o^2c^2$.

Consequently for an electron with increasing in its energy we have to consider about decrease in its radius from the rest value l_o at rest energy needed for EPP so $m_oc^2=h\nu_o=hc/l_o$ so $m_oc=h/l_o$ so from Compton wavelength $l_o=h/m_oc$ in compliance with the length contraction

and increase in effective mass . Then for speed approaching c energy approach infinity and radius approach zero and thus the speed of particle can not equals c since its radius would be zero. For the great concentrations of matter so great concentrations of energy we nowadays accept as a natural that the dimensions approach zero e.g. for the black holes but this natural way we do not consider the great changes of the speed ,energies an the potentials of particles in micro-world. Physics up to present-day has not arrived to the concrete value of an electron's dimension and at many publication (also in CODATA) for slow electrons great radius and for fast electrons short radius of an electron is stated and so we may believe in the relation between the changing of an electron's dimension with its energy . Thus we may consider that if we use rest mass in relation $h/\lambda=mc$ we get the Compton wavelength of an electron that is rest diameter of free electron $\lambda_o=l_o=h/m_oc=2.43\times10^{-12}$ so as for a free proton we get $\lambda_o=l_o=h/m_oc=1.32\times10^{-15}$. So if in QM rest energy of the particle is associated with the frequency $h\nu_o=m_oc^2$ then we can reasonably suppose that this rest energy is associated also with the dimension $m_oc^2=h\nu_o=hc/l_o$. This frequency $\nu_o=m_oc^2/h$ thus corresponds to the Compton wavelength of an electron $\lambda_o =h/m_oc^2$ thus to the ultimate diameter $l_o=h/m_oc^2$ of a created electron in EPP. This way the transfer of the photon's momentum relation $p=h/\lambda$ onto a particle represents the same way as for a photon a change in a particle dimension from the Compton wavelength value $h/l_o=m_oc$ following changing in momentum with increasing in the speed v as $h/l-h/l_o=h(l_o-l/ll_o)=h/l.(l_o-l/l_o) = mv =mc.v/c$.
According to the presented convince of changing in the electron's dimension with changing in electron's energy we can believe that the experiments demonstrating the wave property of electrons can be explained by real changing in the dimensions of electrons. In this meaning we can think upon the Bragg's x-rays interference law $n\lambda= 2d\sin\theta$ so the light interference as the interference of photons firmly linked with the photon's wavelength λ so with photon's dimension. Subsequently we can believe that experiments for electrons presented as support of the de Broglie wave hypothesis as the Davisson-Germer's experiment where for an explanation the relation $n\lambda= d\sin\theta$ is active can be interpreted as the real change of an electron's dimension with a change in its energy.
Thus we can reasonably suppose that so as for the change in momentum of a photon so for the change in electron's momentum from the rest state we can write $h/\lambda_1-h/\lambda_2=h/l-h/l_o=mv$ where l_o is the real dimension of an electron that becomes shorter as its energy increase . Kinetic energy of an electron so we get as $E_k=½.mv^2=p^2/2m_o=h^2/2m_o\nabla^2=h^2/2m_ol^2-h^2/2m_ol_o^2=$ $=h^2/2m_o.(l_o^2-l^2/l^2l_o^2)$. Moreover if we consider the relativistic relations of the length contraction $l=l_o(1-v^2/c^2)^{1/2}$ so $(1-v^2/c^2)= l^2/l_o^2$ so $v^2/c^2=l_o^2-l^2/l_o^2$ and increase in mass $m= m_o/(1-v^2/c^2)^{1/2}$ we can write electron's kinetic energy as $h^2/2m_o(l_o^2-l^2/l^2l_o^2)= h^2/2m_ol^2.(l_o^2-l^2/l_o^2)= h^2/2m_ol^2.v^2/c^2$ and for $h= m_ol_oc$ we get

$$h^2/2m_ol^2.v^2/c^2 = m_o^2l_o^2c^2/2m_ol^2.v^2/c^2= m_o^2c^2/2m_o.(1-v^2/c^2).v^2/c^2=m_o^2v^2/2m_o(1-v^2/c^2)= m^2v^2/2m_o= =p^2/2m_o =1/2\ mv^2.$$

So if we directly input relativistic momentum into the classical kinetic energy form $E_k=½.mv^2=p^2/2m_o=m_o^2v^2/2m_o(1-v^2/c^2)$ then from classical kinetic energy we get classical relativistic kinetic energy. As in RM is valid $m^2v^2=m^2c^2- m_o^2c^2$ then just as is valid $m^2v^2/2m_o=m^2c^2/2m_o- m_o^2c^2/2m_o$ then total energy we can write

$$m^2v^2/2m_o + m_o^2c^2/2m_o = m_o^2v^2/2m_o(1-v^2/c^2) + m_o^2c^2/2m_o = m_o^2c^2/2m_o(v^2/(c^2-v^2)+1) = =m_o^2c^2/2m_o(1-v^2/c^2) =m^2c^2 /2m_o \quad \text{so} \quad m^2/m^2_o =1/(1-v^2/c^2) \text{ then } m/m_o =1/(1-v^2/c^2)^{1/2} .$$

So we can believe that the relation $E_k=m_o^2v^2/2m_o(1-v^2/c^2)=p^2/2m_o=m^2c^2/2m_o-m_o^2c^2/2m_o$ has to be perceived as classical kinetic energy and consider the relation $\mathbf{E}_k=2mE_k=m^2v^2=p^2$ as classical relativistic kinetic energy (CRKE).
In RM we derive the equation of total relativistic energy from the relation $m= m_o/(1-v^2/c^2)^{1/2}$ and bring it to the square $m^2c^2 = m^2v^2 + m_o^2c^2$ we multiply it by c^2 and after applying the square root we get $E= mc^2 =(m^2v^2c^2+ m_o^2c^4)^{1/2}$. Then we can reasonably believe that the step of multiplying by c^2 is physically unfounded and is intentional in order to ensure the dimension of energy after resulting square root and just this way we determine energy like momentum multiplied by c.
Afterwards according to transfer of the relationships of photon's momentum onto a particle presented in this paper the relation for CRKE $\mathbf{E}_k=m^2c^2 - m_o^2c^2 =m^2v^2 = m^2c^2v^2/c^2$ harmonize with relation $\mathbf{E}_k =h^2/l^2-h^2/l_o^2=h^2/(l_o^2-l^2/l^2l_o^2)= h^2/l^2.v^2/c^2$ as well as with relation for frequency expression $\mathbf{E}_k=h^2\nu^2/c^2- h^2\nu_o^2/ c^2 =h^2(\nu^2-\nu_o^2)/c^2= h^2\nu^2/c^2.v^2/c^2$ where l_o is the rest dimension and ν_o the rest spin frequency of a particle at rest mass m_o jointed in the relation for the Compton wavelength $h/l_o= m_oc= h\nu_o/c$. So for clasical relativistic kinetic energy we can write the relation $\mathbf{E}_k =2mE_k= h^2/\lambda^2- h^2\lambda_o^2= h^2/l^2- h^2/l_o^2=h^2(\nu^2-\nu_o^2)/c^2 = m^2c^2 - m_o^2c^2 =m^2v^2= p^2$ and for momentum we can write $h/\lambda-h/ \lambda_o= h/l-h/l_o=h(\nu - \nu_o)/c = mc -m_oc =mv=p$. Multiplying the last relation by c $hc/\lambda-hc/ \lambda_o= hc/l-hc/l_o=h(\nu-\nu_o)=mc^2-m_oc^2=mvc$ we get the photoelectric effect explanation by Einstein and total relativistic energy $mc^2 =m_oc^2+mvc$ so EDE definition of energy .
Then for added so classical relativistic kinetic energy we can write the relation

$$\mathbf{E}_k=2mE_k=h^2/l^2-h^2/l_o^2=h^2\nabla_o^2=h^2/_ol^2.(l_o^2-l^2/l_o^2)=h^2/_ol^2.v^2/c^2=h^2{}_o\nu^2/c^2.v^2/c^2=m^2c^2v^2/c^2=m^2v^2$$

Classical limit $E_k =½.m_o^2v^2/m_o = p^2/2m_o =½.m_ov^2$ means added so kinetic energy compared from the values l_o ,ν_o and from rest energy $m_o^2c^2$. Added energy so kinetic energy writed as $h^2/\nabla_o^2=h^2/_ol^2.v^2/c^2= h^2\partial_o\nu^2/c^2 \partial t^2= h^2{}_o\nu^2/c^2.v^2/c^2$ runs for an electron from the rest energy $m_o^2c^2$ and to this energy linked rest values l_o ,ν_o are jointed in relation $h^2/l_o^2=h^2\nu_o^2/c^2= m_o^2c^2$ where symbols ∇_o, $_ol$ and $_o\nu$ means that l, ν, $\nabla_o^2=(1/l^2-/l_o^2)$ and $\partial_o\nu^2/\partial t^2=(\nu^2-\nu_o^2)$ runs from l_o ,ν_o. The difference expressed by l or ν equals zero $h^2\nabla_o^2- h^2\partial_o\nu^2/c^2 \partial t^2 =h^2/_ol^2.v^2/c^2-h^2{}_o\nu^2/c^2.v^2/c^2=0$. If we write the relation $h^2/\nabla^2= h^2\partial^2/c^2 \partial t^2= h^2/ l(\infty)^2 = h^2\nu(0)^2/c^2$ and we mean that l and ν runs from $l_o = \infty$ a $\nu_o =0$ so we talk about total energies $h^2/\nabla^2- h^2\partial^2/c^2 \partial t^2= h^2\nu^2/c^2 -h^2/l^2 = 0$ where both terms increases from a zero and up to the own values $m^2{}_oc^2= h^2/l_o^2=h^2\nu_o^2/c^2$ this increase means rest energy of the particle so for instance energy of the photon needed for EPP .
Total energy $\mathbf{E}_t$ of particle is

$$h^2\nabla^2=h^2\nabla(\infty)^2=h^2/l(\infty)^2=h^2\nabla_o^2+h^2/l_o^2=h^2/_ol^2.v^2/c^2+ m_o^2c^2=m^2c^2v^2/c^2+ m_o^2c^2= m^2v^2+ m_o^2c^2= m^2c^2$$

and for frequencies expression

$$h^2\partial^2/c^2 \partial t^2=h^2\nu(0)^2/c^2=h^2\partial_o\nu^2/c^2\partial t^2+h^2\nu_o^2/c^2=h^2{}_o\nu^2/c^2.v^2/c^2+ m_o^2c^2=m^2c^2v^2/c^2+ m_o^2c^2= m^2c^2$$

where l_o,ν_o, m_o are the Compton values and symbols $l(0)$, $\nu(0)$ means that l ,ν runs from zero. Thus we can see that for added values $h^2\partial_o\nu^2/c^2\partial t^2-h^2\nabla_o^2 = h^2{}_o\nu^2/c^2v^2/c^2- h^2/_ol^2.v^2/c^2 = =m^2c^2v^2/c^2-m^2c^2v^2/c^2\neq m_o^2c^2$ as well for total value $h^2\nabla^2- h^2\partial^2/c^2 \partial t^2= h^2\nu^2/c^2 -h^2/l^2 \neq m_o^2c^2$.
Then in the classical relativistic energy definition for the particle total energy we can write

$$h^2\nabla^2-h^2\partial^2/c^2 \partial t^2=(h^2\partial_o\nu^2/c^2 \partial t^2+m_o^2c^2)-(h^2\nabla_o^2+m_o^2c^2)=(h^2\nu^2/c^2v^2/c^2+ m_o^2c^2)-(h^2/l^2.v^2/c^2+ m_o^2c^2)=$$
$$=(m^2c^2v^2/c^2+m_o^2c^2)-(m^2c^2v^2/c^2 + m_o^2c^2)=(m^2v^2+ m^2{}_oc^2)-(m^2v^2+ m_o^2c^2)=m^2c^2-m^2c^2=0 .$$

For the difference of total and added energy we can write $h^2\partial^2/c^2\partial t^2$ - $h^2\nabla_o^2$= $h^2\partial_o v^2/c^2\,\partial t^2$+ +$m_o c^2$-$h^2\nabla_o^2$=$h^2\nu(0)^2/c^2$- $h^2/_o l^2.v^2/c^2$=$m_o c^2$ that is the same as m^2c^2-m^2v^2= m^2c^2- $m^2c^2v^2/c^2$=$m_o c^2$ so $\mathbf{E}_t$-$\mathbf{E}_k$=$\mathbf{E}_o$ so $\mathbf{E}_t$-p^2=$\mathbf{E}_o$ so p_t^2-p^2=p_o^2. We can write the relation $\mathbf{E}_t$–p^2=$h^2\nu(0)^2/c^2$-$h^2/_o l^2.v^2/c^2$= = $h^2{}_o\nu^2/c^2v^2/c^2$+ $m_o c^2$- $h^2/_o l^2.v^2/c^2$=$\hbar^2\omega^2/c^2$-$\hbar^2k^2$= $m_o c^2$ where ν and l are connected at continuous function ν.l=c and ω ,k are connected at discontinuous function $\omega/k=c^2/v$(resulting from $E_t/p=c^2/v$ or $E_t^2=p^2c^4/v^2$) as a consequence that from $\omega = 0$ up to $\omega = \omega_o$ according to the de Broglie anticipation we can not link ω to any value of a k since with an increasing of kinetic energy k starts from k = 0 when ω starts from $\omega = \omega_o$.

In QM the wave function Ψ by providing ratio c^2/v =νλ so $\nu^2=1/\lambda^2.c^4/v^2$ so $\nu^2/c^2 =1/\lambda^2.c^2/v^2$ so $\nu^2v^2/c^4 =1/\lambda^2$ thus Ψ ensures that writing for ν.l= ω/k=c valid without the wave function $h^2\nu^2/c^2.v^2/c^2$ -h^2/l^2= $m^2c^2v^2/c^2$- m^2c^2 = - $m_o^2c^2$ or $h^2\nu^2/c^2$ -$h^2/l^2\ v^2/c^2$= m^2c^2- $m^2c^2v^2/c^2$= m_o^2c equals non valide writing $\hbar^2\omega^2/c^2$-$\hbar^2k^2$= $h^2\nu^2/c^2$- h^2/l^2= $h^2\partial^2/c^2\partial t^2$ -$h^2\nabla^2\neq m_o^2c^2$. So for $\omega/k= c^2/v$ using the wave function we can write $h^2\,\partial^2/\,c^2\partial t^2$- Ψ $h^2/\nabla^2\Psi$ = - $m_o^2c^2\Psi$ or without wave function $\hbar^2k^2$-$\hbar^2\omega^2/c^2$= h^2/l^2- $h^2\nu^2/c^2.v^2/c^2$ = $m_o^2c^2$ what is writing of the Klein-Gordon (K-G) equation .Thus the Klein –Gordon equation without wave function Ψ we can write in the form ± $h^2\nu^2/c^2.(v^2/c^2$ or 1)±h^2/l^2.(1 or v^2/c^2) = ±o $m_o^2c^2$ according to our request start from $\nu=\nu_o$ and $l_o = \infty$ (De Broglie anticipation) then $m^2{}_oc^2$ = - $m_o^2c^2$ or ν =0 and $l=l_o$ then $m_o^2c^2$= $m_o^2c^2$ and if we start from ν =0 and l=∞ or $\nu = \nu_o$ and $l=l_o$ then $m^2{}_oc^2$ =0.
Thus we can believe that if ν and l alternatively t and x runs from mutually corresponding value than the d'Alembertian is always zero □ =0. The K-G equation □Ψ = -($m_o^2c^2/\hbar^2$)Ψ we can write if ω and k alternatively t and x do not runs from the mutually corresponding value and than the value of energy expressed by ω and k alternatively t and x are mutually shifted with the constant $m_o^2c^2$. Using the wave function Ψ we perform correction v^2/c^2 whereby we subtract or the correction c^2/v^2 whereby we add value $m_o^2c^2$ to value expressed by ω and k alternatively t and x .
Similarly we can believe that momentum of a particle we can write p_t- p= p_o so mc-mv= m_oc so $h\partial/c\partial t$ - $h\nabla_o$ - m_oc=0 or $h\partial/c\partial t$ - $h\nabla$ = ±o m_oc where x and t runs from the diverse value and with the wave function we can write $h\partial/c\partial t\,\Psi$ ± $h\nabla\Psi$ ±o $m_oc\Psi$ =0 what represents Dirac equation. In matrices writing of Dirac equation $h\partial/c\partial t\,\Psi$ + $\boldsymbol{\alpha}$ $h\nabla\Psi$ + $\boldsymbol{\beta}$ $m_oc\Psi$ =0 the wave function provides shift over m_oc one or both of the terms h/∇, $h\partial/\partial t$ to $h\nabla_o$, $h\partial\nu_o/c\,\partial t$ or reverse and matrix $\boldsymbol{\alpha}$ provides relevant sign + or - and matrix β provides relevant sign for m_oc to – or + or m_oc =0 .
From the foregoing reasoning in this paper we can come to belief that momentum of photon represents the relation h/λ=hν/c=mc where λ runs from infinity ν,m runs from zero, momentum of a particle represents the relation $h/l-h/l_o=h/l.v/c=h(\nu-\nu_o)/c$= hν/c.v/c=mcv/c=mv where l,ν ,m runs from ν_o,l_o a m_o when h/λ_o= $h\nu_o/c$ =m_oc, total energy of photon represents the relation $h^2/\lambda^2=h^2\nu^2/c^2=m^2c^2$ where λ runs from infinity and ν,m runs from zero, total energy of particle represents $h^2/l^2.v^2/c^2+h^2/l_o^2=h^2\nu^2/c^2v^2/c^2+h^2\nu_o^2/c^2=m^2c^2v^2/c^2+ m_o^2c^2=m^2v^2+ m_o^2c^2$ where l, ν ,m runs from ν_o , l_o a m_o .
As relation $\mathbf{E}_t$ =2mE_t =$h^2\nu^2/c^2$ represents photon's energy for the photoelectric effect we have to write relation $h^2\nu^2/c^2$- $h^2\nu_o^2/c^2$ =m^2c^2 - $m_o^2c^2$ =m^2v^2 what for momentum equals the relation hν/c- $h\nu_o/c$ = mc -m_oc = mv. The last relation multiplied by c in EDE where E=pc represents Einstein's writing for energy balance at the photoelectric effect hν-$h\nu_o$=mc^2-m_oc^2 =mvc .This writing we can get also multiplying by c the Dirac equation $h\partial/c\partial t$ - $h\nabla$ = ±o m_oc (the equation for momentum) what results in $h\partial/\partial t$ -$hc\nabla$-$m_oc.c$=hν(0)-(hc/l-hc/l_o)-m_oc^2= mc^2- mvc - m_oc^2= 0 .

Millikan who for many years disagreed with Einstein's understanding of the photoelectric effect at his experiments found out the proportional increase in kinetic energy of electrons released from a metal surface with the linear frequency increase of photons striking on that surface. Thus we can believe that the Millikan's experiments could be interpreted so that with linear increase in the frequency of photons their energy increase quadratically and this energy equals to the quadratic increase in kinetic energy of electrons

$$\mathbf{E}_k=2mE_k=h^2/l^2- h^2/l_o^2=h^2/l^2.v^2/c^2=h^2(\nu^2-\nu_o^2)/c^2=h^2\nu^2/c^2.v^2/c^2 =m^2c^2 - m_o^2c^2 = m^2c^2v^2/c^2 =m^2v^2$$

what in classical limit is seen as $E=m^2v^2/2m_o =½.m_ov^2$. The same way in classical physics with a linear increase in the speed of an electron we get quadratic increase in its energy .
On the bases of an observation of quadratic changes at hydrogen atomic line emission spectra formulated in the Rydberg formula $1/\lambda=R_H(1/n_1^2-1/n_2^2)$ atomic physics come to solution that the differences $hc/\lambda=R_y(1/n_1^2-1/n_2^2)$ represents the transition between different energy levels of an atom and that for energy levels compared to the maximum energy level we can write $E_n= E_h/n^2=R_y/n^2 =hc/n^2\lambda_H = h\nu_h/n^2$ .This quadratic changes in energy levels of atoms QM explained (neglecting changes level of a proton, 1836 times greater in mass than an electron) by quadratic changes in electron's energy of atoms expressed in the stationary Schrodinger equation (SchrE) $-h^2\nabla^2\Psi =2m(E-V)\Psi$ where the classic kinetic energy relation $½. mv^2= p^2/2m_o= h^2/2m_o\lambda^2$ and de Broglie hypothesis $p=h/\lambda=mv$ are used. Obviously the same conditions are valid also for absorption spectra so that a quadratic change in the wavelength $1/\lambda^2$ or frequency ν^2/c^2 of incident photons give rise to the quadratic change in electron's energy of atom $h^2/2m_o\lambda^2=p^2/2m_o= ½.mv^2$. But absorption spectra we can see like a first stage of the photoelectric effect. So quadratic changes in energy of incident photons equal to a quadratic changes in electron's energy before its emission out of an atom. So we can consider as unreasonable to change the relation of classic kinetic energy in the photoelectric effect equation $h\nu- h\nu_o =½.mv^2$ into $h\nu- h\nu_o = mc^2-m_oc^2=mvc$ by reason of a conservation the linearity of the equation. Also by the same reason we can see as unreasonable to declare the SchrE as non-relativistic so require to change SchrE for a free particle $ih\boldsymbol{\partial}\varphi/\boldsymbol{\partial}t = h^2/2m_o\nabla^2\varphi$ i.e. $h\partial_o\nu/\partial t= h\nu- h\nu_o= h^2\nabla_o^2/2m_o=p^2/2m_o = ½.mv^2$ into the linearized term $h\partial/c\partial t\,\Psi= h\nabla\Psi$ (of momentum!) what results in the Dirac equation. On the contrary we can believe that $h\nu - h\nu_o$ or $h\boldsymbol{\partial}\varphi/\boldsymbol{\partial}t$ do not represents a change in energy but the change in momentum multiplied by c $E=pc$ .Thus we can believe that a change in energy at the photoelectric effect so as a change in energy at SchrE represents $h^2\nu^2/c^2-h^2\nu_o^2/c^2= h^2\boldsymbol{\partial}^2/c^2\boldsymbol{\partial}t^2=m^2c^2- m_o^2c^2 =m^2v^2$ in the classical limit this we can write as $h^2\nu^2/2m_oc^2-h^2\nu_o^2/2m_oc^2=h^2\boldsymbol{\partial}^2/2m_oc^2\boldsymbol{\partial}t^2= m^2c^2/2m_o- m_o^2c^2/2m_o = m^2v^2/2m_o$. Consequently in this paper we believe that if we want to talk about energy than we have to change the left hand side of the photoelectric effect equation just same as the left hand side of the ShrE from $h\boldsymbol{\partial}_o\nu/\boldsymbol{\partial}t= h\nu- h\nu_o$ representing $E=pc$ into $h^2\boldsymbol{\partial}^2/c^2\boldsymbol{\partial}t^2= h^2\nu^2/2m_oc^2- h^2\nu_o^2/2m_oc^2$ resp. $2m(E_1-E_0) = h^2\nu^2/c^2 - h^2\nu_o^2/c^2$ but after that we are talking about the K-G equation for energy $h^2\boldsymbol{\partial}^2/c^2\boldsymbol{\partial}t^2 -h^2\nabla^2= \pm o\ m_o^2c^2$ so $m^2c^2-m^2v^2- m_o^2c^2 =0$. If we want to persist in the energy definition by EDE so $E=pc$ and we want to change right side hand of the SchrE than we get the Dirac equation for momentum $h\partial/c\partial t\,\Psi + \boldsymbol{\alpha}h\nabla\Psi +\boldsymbol{\beta}\, m_oc\Psi =0$ so $h\partial/c\partial t - h\nabla = \pm o\ m_oc$ so $h\boldsymbol{\partial}\nu(0)/c\boldsymbol{\partial}t - h\nabla_o- m_oc =0$ so $mc-mv- m_oc =0$ that after multiplying by c represents equation for energy in system of EDE so $mc^2-mvc-m_oc^2=0$ or $mc^2=mvc+m_oc^2$.
Consequently a change in energy of a free particle out of the potential forces by changing in particle's speed we can write $2m(E_1-E_o) = h^2\nabla_o^2 = p^2$ so $m^2c^2 - m_o^2c^2 = m^2v^2 = h^2\nabla_o^2= p^2$ or in form $h^2\nabla_o^2 -2m(E_1-E_o) = 0$ .This equation is valid in QM only for a particle influenced by the potential forces and for free particle we write the SchrE as $-h^2\nabla^2\Psi = 2mE\Psi$ which solution is classical kinetic energy and therefore in QM for the free particle total energy equals to kinetic energy. So we can believe that already the basic presumption of QM associating momentum of particle as $p=h/\lambda=mv$ (where $\lambda =\infty$ for $mv=0$) leads to incompatibility of QM and RM .
Thus we can believe, that associating momentum of the particle with $p=h/l-h/l_o=mv$ (where l

is dimension of particle) we can write for a particle influenced by the potential forces so as for a free particle equation $h^2\nabla_o^2 - 2m(E_1 - E_o) = m^2v^2 - (m^2c^2 - m_o^2c^2) = 0$ and this way we can talk about the compatibility of QM and RM.

References

(1)L.V. de Broglie, On the Theory of Quanta, (1925)
(2)C.Davisson,L.H.Germer, Diffraction of electrons by a crystal of nickel, (1927)
(3) CODATA Recommended Values
(4)N.Bohr, On the Constitution of Atoms and Molecules, (1913)
(5)P.A.M.Dirac, The quantum theory of the electron, (1928)
(6)R.A.Millikan, A direct photoelectric determination of Plansks "h", (1916).
(7)E.Schrodinger, An undulatory theory of the mechanics of atoms and molecules, (1926)
(8)A.S. Davidov, Quantum Mechanics,(1973)
(9) The Feynman Lectures on Physics, Vol 1.2.3., (1964)