NTEP: Chapter 9. Physical vacuum and its effect on elementary particles and their interactions

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The first purpose of this chapter is to describe the influence of physical vacuum (PV) on the characteristics of particles (e.g. electron) and their interactions. Our second purpose is to show that all methods of calculation of this influence, which are used in quantum field theory (QFT), can be represented in electromagnetic (EM) form within the framework of nonlinear theory of elementary particles (NTEP).

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1.0. The introduction of physical vacuum in physics

A physical vacuum is a key issue of fundamental physics. The existence of vacuum is in many respects responsible for the properties of elementary particles and their interactions in quantum field theory and needed for the description of such phenomena as radiation effects in quantum electrodynamics (the Lamb shift, the anomalous magnetic moments of the electron and muon, etc), colour confinement in quantum chromodynamics, spontaneous symmetry breaking in the physics of electroweak interactions, the massiveness of W- and Z-bozons, etc.

1.1. The second quantization and physical vacuum

The application of quantization principle of discrete particles to the continuous fields is conditionally called second quantization. Because of the second quantization the wave functions of elementary particles become the operators, which are subordinated to the specified quantum conditions - commutation relations. This relates both to the classical wave functions of electromagnetic field and to the wave functions of Dirac's electron and other particles.

The procedure of second quantization confirms the existence of the lowest level of the energy state of particles, in which there are no real particles. This state is called physical vacuum (PV). It is considered that PV consists of the so-called virtual particles. Nevertheless, it turned out that the virtual particles interact with the real particles. Generally this interaction is conditionally called the polarization of vacuum. The experimental data, obtained even for the free particles, cannot be explained without taking into account this interaction.

The method of second quantization of the amplitudes due to its expansion in a Fourier integral (Medvedev and Shirkov, 1987), which was developed by Dirac (Dirac, 1927b) in application to the electromagnetic field and by Jordan (Jordan, 1927) and Jordan and Klein (Jordan and Klein, 1927) in application to the field of electrons developed into a common theory of an arbitrary free quantum field.

According to this approach a quantum (or quantized) field is a sort of synthesis of the concepts of a classical field of the electromagnetic type and of a probability field of quantum mechanics. According to the present understanding, it is the most fundamental and universal form of matter, underlying all specific manifestations of matter.

It should be noted that this hybrid of classical field and field of probability is strange already because of their different dimensionality. The explanation of this strangeness is given by NTEP. In NTEP classical field and field of probability do not come out as the synthesis of different fields, but as equal concepts. As we showed (see (Kyriakos, 2010a,b)), the field of probability is the successful mathematical interpretation of the normalized nonlinear electromagnetic field. This restores the unity of the picture of physical fields without any stretch.

On the other hand, if we speak about the fundamental field, i.e. physical vacuum, between it and the real field (especially particles) there is no equality . Fundamental field is the basis, on which the elementary particles appear. The opposite assertion is not correct: particles are not the basis of existence of fundamental field. Actually, in 1927 (Medvedev and Shirkov, 1987) Dirac subjected the variables describing a field to second quantization (Dirac, 1927b), but six years later he raised a decisive objection that Heisenberg and Pauli "regard the field itself as a dynamical system amenable to Hamiltonian treatment so that the usual methods of Hamiltonian quantum mechanics may be applied. There are serious objections to these views.... We cannot... suppose the field to be a dynamical system on the same footing as the particles . . . The field should appear in the theory as something more elementary and fundamental" (Dirac, 1932).

The influence of physical vacuum on the particles was also predicted for the first time by P. Dirac. In his Solvay report in 1933, Dirac (Dirac, 1934b) stated that external charges should polarize the vacuum in his theory, with the result that the electric charges which are normally observable for the electron, the proton, and other electrified particles are not the charges which are actually carried by these particles and which figure in the fundamental equations; they are instead smaller. A calculation which he carried out on this new physical effect reduced to a logarithmically divergent integral, whose cutoff at momenta of the order of 100 mc (corresponding to the classical radius of an electron; here m is electron mass and c is light velocity) yields a "radiation correction" to the charge of an electron, which reduced it by a fraction of about 1/137. Calculations on the "field" self-energy of a photon also led to an infinite result and again violated gauge invariance.

As early as the middle of 1930s, there were suggestions (Weisskopf, 1936; Euler, 1936) that the infinities in higher orders for the observable effects were traces of these fundamental ultraviolet divergences and that they could be eliminated by subtracting from the infinite quantity for a bound electron the corresponding infinite quantity for a free electron (Kramers, 1938; Stuckelberg, 1935, 1938). This was the basic idea of the renormalization method.

2.0. The mathematical description of physical vacuum

2.1. The introduction of physical vacuum for free fields

In order to fix the state of a particle (Medvedev and Shirkov, 1987), it is necessary in quantum mechanics to specify the values of a complete set of commuting operators. For a free particle, it is convenient to choose the three components of the momentum $\vec{p}$ (or wave vector $\vec{k}$) and the projection s of the spin l_s onto some direction. The state of a single free particle is thus characterized completely by specifying the six numbers m, l_s, p_x, p_y, p_z, s, (for charged particles, one adds some other quantum numbers, which we denote by the single letter t).

A straightforward extension of these arguments to a system of *n* particles would result in the use of *n* sextets, one for each particle. In 1927 Dirac suggested that the state of an ensemble of *n* identical particles be characterized not by the state of each particle but by the number of particles, $n_{\vec{p},s,t}$ - occupation numbers - in each of the one-particle states (Dirac, 1927b). The "interpretation" of a wave function gives us not simply the expected numbers of particles, but the probability for any given distribution of particles among different states. This probability is actually the square of the modulus of the normalized solution of the wave equation.

2.1.1. The introduction of creation and annihilation operators

In the occupation-number representation, a state $\left|n_{\vec{p},s,t}\right\rangle$, is written as the result $\left|n_{\vec{p},s,t}\right\rangle = \left(n_{\vec{p},s,t}\right)^{-1/2}\left[a^{+}\left(\vec{p},s,t\right)\right]^{n_{\vec{p},s,t}}\left|0\right\rangle$ of an action on a *vacuum state* (i.e., a state in which there are no particles at all) of the *creation operators* $a^{+}(\vec{\mathrm{p}},s,t)$. The creation operators a^{+} and their Hermitian-conjugate *annihilation operators* a^{-}, were introduced by Dirac in the same paper (Dirac , 1927b).

The creation and annihilation operators $a^{\pm}$ describe particles with definite momentum and spin values. To take the local properties into account, we need to put the $a^{\pm}$ in the coordinate representation. As transformation functions it is convenient to use the classical solutions of the equations of motion of Euler-Lagrange of a suitable free field.

It is possible to show (Landau and Lifshitz, 1975; Levich et al, 1973; Martynenko, 2001) that electromagnetic field can be conditionally represented in the form of the superposition of some harmonic oscillators (see in detail (Kyriakos, 2010c)).

$$\varepsilon = \sum_{k}\left(\frac{\left|\dot{\vec{A}}_{\vec{k}}\right|^{2}}{8\pi\ c^{2}} + \frac{\vec{k}^{\,2}\left|\vec{A}_{\vec{k}}\right|^{2}}{8\pi}\right), \tag{3.5.4}$$

where ε is the energy and $\vec{A}$ is the vector potential of EM field. Here the total energy of the EM field can be represented as the sum of energies of harmonic oscillators.

$$\varepsilon = \frac{1}{2m}\sum_{k}\left(p_{k}^{2} + m^{2}\omega_{k}^{2}q_{k}^{2}\right), \tag{3.5.5}$$

In this sense, the first term in (3.5.4) is a kinetic electromagnetic energy, and the second term is a potential energy. Thus, the EM field in space without charges can be considered as the sum of independent harmonic oscillators with all possible values of the wave vector $\vec{k}$. Note that the transition from (3.5.4) to (3.5.5) is clearly artificial assumption. In other words, this is not proof; this is only hypothesis.

Let us examine how on this basis are introduced the creation and annihilation (destruction) operators of particles, which lead to the appearance of the concept of physical vacuum.

The one-dimensional harmonic oscillator (Feynman, 1972) has a Hamiltonian of the form

$$\hat{\mathrm{H}} = \frac{1}{2m}p^{2} + \frac{m\omega^{2}}{2}x^{2}, \tag{9.2.1}$$

where *x* and *p* are the position and momentum operators for the particle and satisfy

$$\left[x, p\right] = i\hbar, \tag{9.2.2}$$

Lets find the eigenvalues and eigenstates of $\hat{\mathrm{H}}$. We define

$$a^{-} = \frac{1}{\sqrt{2}}\left(\sqrt{\frac{m\omega}{\hbar}}x + i\frac{1}{\sqrt{m\omega\hbar}}p\right), \tag{9.2.3}$$

Because *x* and *p* are Hermitian it follows that

$$a^{+} = \frac{1}{\sqrt{2}}\left(\sqrt{\frac{m\omega}{\hbar}}x - i\frac{1}{\sqrt{m\omega\hbar}}p\right), \tag{9.2.4}$$

2.1.2. Properties of creation and annihilation operators

From (9.2.2) we obtain

$$\left[a^{-}, a^{+}\right] = 1, \tag{9.2.5}$$

where a^+ denotes the Hermitian conjugate of a^-, and [*A*, *B*] is the commutator *AB* - *BA*.

To find the the eigenvalues and eigenstates of $\hat{\mathrm{H}}$, the problem is to find the eigenvalues of the Hermitian operator a^+, a^-, and to relate the eigenvectors. We may then construct the eigenstatcs of a^+, a^- as follows. First we find a state $|0\rangle$ such that

$$a^-|0\rangle = 0\,, \tag{9.2.6}$$

Then we define $|1\rangle = a^+|0\rangle$; $|2\rangle = \frac{1}{\sqrt{2}}a^+|1\rangle = \frac{1}{\sqrt{2}}\left(a^+\right)^2|0\rangle$; …, and in general

$$|n\rangle = \frac{1}{\sqrt{n!}}\left(a^+\right)^n|0\rangle\,, \tag{9.2.7}$$

(Note that we could have included arbitrary phase factors in the definition of $|n\rangle$; for simplicity we make them unity.) With this definition, the $|n\rangle$ are orthonormal and satisfy

$$a^+|n\rangle = \sqrt{n+1}|n+1\rangle\,,\quad a^-|n\rangle = \sqrt{n}|n-1\rangle\,,\quad a^+, a^-|n\rangle = n|n\rangle\,, \tag{9.2.8}$$

The operators a^- and a^+ are called "raising" and "lowering" operators, respectively, because they raise and lower the eigenvalue of a^+, a^-. In later applications a^+, a^- will be interpreted as the observable representing the number of particles of a certain kind, in which case a^- and a^+ are called "creation" and "annihilation" (destruction) operators, or "emission" and "absorption" operators.

Two first equations from (9.2.8) may be alternatively expressed in terms of matrix elements:

$$\langle m|a^+|n\rangle = \sqrt{n+1}\,\delta_{m,n+1}\,,\quad \langle m|a^-|n\rangle = \sqrt{n}\,\delta_{m,n-1}\,, \tag{9.2..9).}$$

Equation (9.2.6) through the last equation from (9.2.8) form the answer to the problem of finding the the eigenvalues and eigenstates of $\hat{\mathrm{H}}$.

Expressing x and p in terms of a^- and a^+, we have

$$x = \sqrt{\frac{\hbar}{m\omega}}\,\frac{a^- + a^+}{\sqrt{2}}\,,\quad p = \sqrt{m\omega\hbar}\,\frac{a^- - a^+}{\sqrt{2}}\,, \tag{9.2.10}$$

We get, for the Hamiltonian,

$$\hat{\mathrm{H}} = \frac{\hbar\omega}{2}\left(a^+a^- + a^-a^+\right) = \hbar\omega\left(a^+a^- + \frac{1}{2}\right), \tag{9.2.11}$$

Thus, the eigenstates of $\hat{\mathrm{H}}$ are those of a^+a^-. Now we can apply the results of (9.2.6) – (9.2.8), obtaining the eigenstates $|0\rangle, |1\rangle, |2\rangle$, … that satisfy

$$\hat{\mathrm{H}}|n\rangle = \left(n + \frac{1}{2}\right)\hbar\omega|n\rangle\,, \tag{9.2.12}$$

The energy levels are thus $\varepsilon_n = \left(n + \frac{1}{2}\right)\hbar\omega$. The eigenstates themselves are given by equations (9.2.6) and (9.2.7).

The eigenstates themselves are given by (9.2.6) and (9.2.7). We can easily obtain the wave functions $\varphi_n(x) = \langle x|n\rangle$ as follows: from (9.2.3) and (9.2.6).

$$a^-|0\rangle = \sqrt{\frac{m\omega}{2\hbar}}\left(x + \frac{i}{m\omega}\,p\right)|0\rangle = 0\,, \tag{9.2.13}$$

Applying $\langle x|$, and noticing that $\langle x|p|\varphi\rangle = -i\hbar\left(d\langle x|\varphi\rangle/dx\right)$, we get

$$\sqrt{\frac{m\omega}{2\hbar}}\left(x+\frac{\hbar}{m\omega}\frac{d}{dx}\right)\langle x|0\rangle=0\,, \tag{9.2.14}$$

(where x is now a number, rather than an operator.) Equation (9.2.14) is merely Eq. (9.2.6) *in coordinate representation*, in which it takes the form of a differential equation. Solving it, we get $\langle x|0\rangle = Ae^{-m\omega x^2/2\hbar}$, where A is a constant.

Within the framework of electromagnetic theory the entire procedure of second quantization, described above, appears much more sequentially and can be without change transferred to NTEP. Usually in our days the quantum electrodynamics (QED) and generally the quantum field theory (QFT) is constructed with the aid of the potentials. In this case the procedure of second quantization is based on the following mathematical apparatus.

2.2. Second quantization of electromagnetic field

2.2.1. Second quantization with use of vector potential

The Lagrangian and Hamiltonian in QFT (Schiff, 1955) is most conveniently expressed in terms of the potentials $\vec{A}, \varphi$ that are partially defined by

$$\vec{\mathrm{E}} = -\frac{1}{c}\frac{\partial\vec{A}}{\partial t} - grad\varphi = -4\pi\, c\vec{P},\ \vec{\mathrm{H}} = rot\ \vec{A}\,, \tag{9.2.15}$$

where $\vec{P} = \frac{1}{4\pi\, c}\left(\frac{1}{c}\frac{\partial\vec{A}}{\partial t} + grad\varphi\right)$ is the momentum canonically conjugate to $\vec{A}$. As is known, this does not specify the potentials completely, since gauge transformations of the potentials can still be made without altering the electric and magnetic field strengths computed from (9.2.15). The momentum canonically conjugate to φ vanishes identically, since φ does not appear in the Lagrangian density. The Hamiltonian is then

$$\hat{\mathrm{H}} = \int\left[2\pi\, c^2\vec{P}^2 + \frac{1}{8\pi}\left(rot\ \vec{A}\right)^2\right]d\tau\,, \tag{9.2.16}$$

and φ has disappeared. This is in agreement with the usual expression $\varepsilon = (1/8\pi)\int\left(\vec{\mathrm{E}}^2 + \vec{\mathrm{H}}^2\right)d\tau$ for the total energy in the electromagnetic field.

The commutation relations between the field variables become

$$\begin{aligned}&\left[A_s(\vec{r},t),\, A_{s'}(\vec{r}',t)\right] = \left[P_s(\vec{r},t),\, P_{s'}(\vec{r}',t)\right] = 0,\\ &\left[A_s(\vec{r},t),\ P_{s'}(\vec{r}',t)\right] = i\hbar\delta_{ss'}(\vec{r}-\vec{r}')\end{aligned}\,, \tag{9.2.17}$$

The equation of motion for a typical component of A_s and P_s are

$$i\hbar\dot{A}_s(\vec{r},t) = \left[A_s(\vec{r},t),\, \hat{\mathrm{H}}\right], \tag{9.2.18}$$

$$i\hbar\dot{P}_s(\vec{r},t) = \left[P_s(\vec{r},t),\, \hat{\mathrm{H}}\right], \tag{9.2.19}$$

where each of the indices s, s' can be x, y, or z.

Thus if we define (9.2.15), the quantum equations of motion for $\vec{A}$ and $\vec{P}$ agree with the three of Maxwell's equations except the equation $\mathrm{div}\vec{\mathrm{E}} = 0$, which corresponds to $\mathrm{div}\vec{P} = 0$. This Maxwell's equations must be imposed as a supplementary condition, as in the classical case. If we set $\mathrm{div}\vec{P}$ equal to zero at a particular time, it is always zero since its time derivative is zero. Equation (9.2.9) then shows that the time derivative of $\mathrm{div}\vec{A}$ is always zero, or that div A is a constant in time. It is convenient to restrict the choice of gauge so that $\mathrm{div}\,\vec{A}$ is zero everywhere at a particular time, in which case we see that it is zero at all space-time points. It is apparent, however, that the introduction of the supplementary condition is inconsistent with the

commutation relations (9.2.17). For example, the commutator bracket of A_s and $\operatorname{div}\vec{P}=0$ should be zero, since $\operatorname{div}\vec{P}=0$ is zero, but is computed from (9.2.17) to be

$$\left[A_s(\vec{r},t),\ \operatorname{div}'\vec{P}(\vec{r}',t)\right]=i\hbar\frac{\partial}{\partial s'}\delta(\vec{r}-\vec{r}'), \tag{9.2.20}$$

It is not surprising that this inconsistency should arise, since (9.2.17) imply that there are three independent pairs of canonical variables, whereas the restrictions $\operatorname{div}\vec{P}=0$ and $\operatorname{div}\vec{A}=0$ cause only two of these pairs to be linearly independent. We should therefore modify the commutation relations so that they are consistent with the supplementary condition.

It turns out that the commutator brackets of $\vec{A}(\vec{r},t)$ and $\vec{P}(\vec{r},t)$ do not vanish when $\vec{r}-\vec{r}'$ is finite. This would appear at first to contradict the physical principle that there can be no interference between measurements performed at different places and the same time. However, the vector potential $\vec{A}$ is not in itself a physical quantity; only the electric and magnetic fields are directly measurable.

As we repeatedly noted, the use of potentials not only is not the unique way of constructing the quantum field theory, but leads to numerous difficulties. The use of field strengths is not only possible and convenient, but makes it possible to avoid these difficulties.

2.2.2. Second quantization with use of field strengths

We will now show with the help of (9.2.17) that (Schiff, 1955) the commutation relations of $\vec{\mathrm{E}}$ and $\vec{\mathrm{H}}$ have the required infinitesimal character and are, moreover, consistent with the supplementary condition $\operatorname{div}\vec{\mathrm{E}}=0$. It can also be shown that the same results are obtained by starting with the modified canonical commutation relations.

The electric and magnetic fields are defined by the equations

$$\vec{\mathrm{E}}=-4\pi c\vec{P},\ \vec{\mathrm{H}}=\operatorname{rot}\vec{A}, \tag{9.2.10}$$

where the commutation relations for A and P are assumed to have the form (9.2.17). We see at once that

$$\left[\mathrm{E}_s(\vec{\mathrm{r}},\mathrm{t}),\ \mathrm{E}_{s'}(\vec{\mathrm{r}}',\mathrm{t})\right]=\left[\mathrm{H}_s(\vec{\mathrm{r}},\mathrm{t}),\ \mathrm{H}_{s'}(\vec{\mathrm{r}}',\mathrm{t})\right]=0, \tag{9.2.21}$$

where each of the indices s,s' can be x,y or z. The commutator bracket for typical parallel components of E and H is:

$$\left[\mathrm{E}_x(\vec{\mathrm{r}},\mathrm{t}),\ \mathrm{H}_x(\vec{\mathrm{r}}',\mathrm{t}')\right]=0, \tag{9.2.22}$$

For typical perpendicular components of E and H, we obtain:

$$\left[\mathrm{E}_x(\vec{\mathrm{r}},\mathrm{t}),\ \mathrm{H}_y(\vec{\mathrm{r}}',\mathrm{t}')\right]=4\pi ci\hbar\frac{\partial}{\partial \mathrm{z}'}\delta(\vec{\mathrm{r}}-\vec{\mathrm{r}}'), \tag{9.2.22'}$$

Other relations similar to (9.2.13) are obtained by cyclic permutation of x,y,z.

Let us examine how the operators of creation and annihilation of particles are introduced in EM theory and what physical sense do they make there. The Hamiltonian equation of EM theory is:

$$\hat{\mathrm{H}}=\frac{1}{8\pi}\int\left(\vec{\mathrm{E}}^2+\vec{\mathrm{H}}^2\right)d\tau, \tag{9.2.23}$$

As would be expected, the field commutation relations (9.2.21), (9.2.22), and (9.2.22'), together with the Hamiltonian (9.2.23), can be used in place of the canonical formalism originally developed in terms of $\vec{A}$ and $\vec{P}$.

The first two of Maxwell's equations then follow as special cases of the general equation of motion. The equation of motion for any quantum dynamical variable *F* is obtained if the Poisson bracket is replaced by the commutator bracket divided by $i\hbar$:

$$\dot{F} = \frac{1}{i\hbar}\left[F,\ \hat{\hat{\mathrm{H}}}\right], \tag{9.2.24}$$

where we write $\hat{\hat{\mathrm{H}}}$ as the volume integral of a Hamiltonian density $\hat{\mathrm{H}}$

$$\hat{\hat{\mathrm{H}}} = \int \hat{\mathrm{H}}\, d\tau\,, \tag{9.2.25}$$

The commutator bracket can be evaluated with the help of the quantum conditions for the canonical EM field variables (9.2.21)-(9.2.2'):

$$\begin{aligned} i\hbar\dot{\mathrm{E}}_x &= \left[\mathrm{E}_x, \hat{\hat{\mathrm{H}}}\right] = \frac{1}{8\pi}\int\left[\mathrm{E}_x,\ \left(\mathrm{H}_y'^2 + \mathrm{H}_z'^2\right)\right]d\tau' = i\hbar\left(rot\,\vec{\mathrm{H}}\right)_x \\ i\hbar\dot{\mathrm{H}}_x &= \left[\mathrm{H}_x, \hat{\hat{\mathrm{H}}}\right] = \frac{1}{8\pi}\int\left[\mathrm{H}_x,\ \left(\mathrm{E}_y'^2 + \mathrm{E}_z'^2\right)\right]d\tau' = i\hbar\left(rot\,\vec{\mathrm{E}}\right)_x \end{aligned}, \tag{9.2.26}$$

The Hamiltonian density $\hat{\mathrm{H}}$ of EM theory

$$\hat{\mathrm{H}} = \frac{1}{8\pi}\left(\vec{\mathrm{E}}^2 + \vec{\mathrm{H}}^2\right), \tag{9.2.27}$$

can be expressed also through various forms of the electromagnetic theory, which are used in NTEP. In particular, using the 6-vectors of EM field, it is possible to write down the Hamiltonian density in the following form:

$$\hat{\mathrm{H}} = \frac{1}{8\pi}\hat{F}^+\hat{F} = \frac{1}{8\pi}\left(\vec{\mathrm{E}} - i\vec{\mathrm{H}}\right)\left(\vec{\mathrm{E}} + i\vec{\mathrm{H}}\right), \tag{9.2.28}$$

where $\hat{F}^+ = \vec{\mathrm{E}} - i\vec{\mathrm{H}}$ is the Hermitian-conjugate 6-vector of EM field. In the case, using a wave function of EM field in the form (see (Kyriakos, 2009a))

$$\Phi = \begin{pmatrix} \mathrm{E}_x \\ \mathrm{E}_z \\ i\mathrm{H}_x \\ i\mathrm{H}_z \end{pmatrix},\quad \Phi^+ = \begin{pmatrix} \mathrm{E}_x & \mathrm{E}_z & -i\mathrm{H}_x & -i\mathrm{H}_z \end{pmatrix}, \tag{2.2.8}$$

it is possible to write down a Hamiltonian density in the following form:

$$\hat{\mathrm{H}} = \frac{1}{8\pi}\Phi^+\Phi\,, \tag{9.2.29}$$

If we consider the quantization of field, then it is necessary to replace the integral in the equation (9.2.23) by the sum.

In the case of normalization of the wave function by maximum field according to M. Born (or by other equivalent form through the mass of particle or its energy), it is possible to obtain a dimensionless Hamiltonian. In this case the operator of the number of particles is given by

$$\hat{\hat{\mathrm{H}}} = \int \Phi^+\Phi\, d\tau = \sum_{\vec{\mathrm{k}}} \hat{a}^+_{\vec{\mathrm{k}}}\hat{a}^-_{\vec{\mathrm{k}}} = \hat{N}\,, \tag{9.2.30}$$

The expressions (9.2.23) and (9.2.27), (9.2.28), (9.2.29) relate to the electromagnetic wave, which consists of many photons. This corresponds to the fact that we used commutators (9.2.21) - (9.2.13), which are characteristic for the bosons.

Let us examine how the operators of second quantization act on the state vectors Φ. Let as denote Φ_0 as the state vector of field without the particles, i.e., the vector of vacuum state. From above follows that the state vector Φ_0 satisfies the condition $\hat{a}^-_{\vec{\mathrm{k}}}\Phi_0 = 0$ for any values of wave vector $\vec{\mathrm{k}}$.

State vector $\hat{a}^{+}_{\vec{k}}\Phi_0 \equiv \Phi_1$ describes a single-particle state, i.e. the state of field, in which there is one boson in the state $\vec{k}$. Let us note the equality $(\Phi_0)^{+}\hat{a}^{+}_{\vec{k}} = 0$ and the condition of normalization of the vector of vacuum state $(\Phi_0)^{+}\Phi_0 = 1$. Multiparticle states can be obtained, if we act on the state vector by the operator of particle production the necessary number of times. For example, state vector $(\hat{a}^{+}_{\vec{k}})^n(\hat{a}^{+}_{\vec{k}'})^m\Phi_0$ describes the state of field with n bosons in the state $\vec{k}$ and m bosons in the state $\vec{k}'$.

Analysis show (Schiff, 1955) that the energy and momentum of each plane wave are quantized in units of $\hbar kc$ for the energy and $\hbar\vec{k}$ for the momentum.

We will therefore require an expression for $\Phi(\vec{r},t)$ in the plane wave representation that is specified by the eigenvalues $n_{\vec{k}}$ of the operators $\hat{N}$. A typical wave functional for this representation can be written as $\Phi(\vec{r},t) = \Phi(...n_{\vec{k}}...)$, which describes a state of the electromagnetic field in which there are $n_{\vec{k}}$ light quanta with momentum $\hbar\vec{k}$ and polarization $\vec{\varepsilon}_{\vec{k}}$ (which are the unit vectors).

Expansions for $\vec{E}$ and $\vec{H}$ in terms of the amplitudes $a_{\vec{k}}$ of plane waves can be found without difficulty:

$$\begin{aligned}\vec{E}(\vec{r},t) &= \sum_{\vec{k}} ik\vec{\varepsilon}_{\vec{k}}\left[a_{\vec{k}}\, e^{i(\vec{k}\cdot\vec{r}-\omega t)} - a^{*}_{\vec{k}}\, e^{-i(\vec{k}\cdot\vec{r}-\omega t)}\right]\\ \vec{H}(\vec{r},t) &= \sum_{\vec{k}} i(\vec{k}\times\vec{\varepsilon}_{\vec{k}})\left[a_{\vec{k}}\, e^{i(\vec{k}\cdot\vec{r}-\omega t)} - a^{*}_{\vec{k}}\, e^{-i(\vec{k}\cdot\vec{r}-\omega t)}\right],\end{aligned} \tag{9.2.31}$$

We then see from (9.2.28) that the operators $a_{\vec{k}}$ and $a^{*}_{\vec{k}}$ have the properties

$$\begin{aligned}a_{\vec{k}}\psi(...n_{\vec{k}}...) &= n^{1/2}_{\vec{k}}\psi(...n_{\vec{k}}+1...)\\ a^{*}_{\vec{k}}\psi(...n_{\vec{k}}...) &= (n_{\vec{k}}+1)^{1/2}\psi(...n_{\vec{k}}+1...),\end{aligned} \tag{9.2.32}$$

It follows from the structure of (9.2.31) that and $a^{*}_{\vec{k}}$ are, annihilation and creation operators $a^{-}_{\vec{k}}$ and $a^{+}_{\vec{k}}$, respectively, for a light quantum in the state $\{\vec{k},\varepsilon_{\vec{k}}\}$. Thus the Hamiltonian would give rise to the emission and absorption of light quanta.

2.3. Second quantization of the fermion fields

Let us pause briefly at the results of second quantization in the case of the fermions.

In this case the fundamental aspect of the matter remains without change. At the same time the concrete formulas will change.

For the bosons the operators of creation and annihilation satisfy the commutation relations:

$$\left[\hat{a}^{+}_{\vec{k}},\hat{a}^{+}_{\vec{k}'}\right] = 0,\quad \left[\hat{a}^{-}_{\vec{k}},\hat{a}^{-}_{\vec{k}'}\right] = 0,\quad \left[\hat{a}^{-}_{\vec{k}},\hat{a}^{+}_{\vec{k}'}\right] = \delta(\vec{k}-\vec{k}'), \tag{9.2.33}$$

where $\delta(\vec{k})$ is Dirac's delta function and the brackets, as usual, indicate a commutator, i.e. $[b,c] = bc - cb$.

For fermions the operators of creation and annihilation satisfy the commutation relations of another type:

$$\left\{\hat{a}^{+}_{\vec{k}},\hat{a}^{+}_{\vec{k}'}\right\} = 0,\quad \left\{\hat{a}^{-}_{\vec{k}},\hat{a}^{-}_{\vec{k}'}\right\} = 0,\quad \left\{\hat{a}^{-}_{\vec{k}},\hat{a}^{+}_{\vec{k}'}\right\} = \delta(\vec{k}-\vec{k}'), \tag{9.2.34}$$

where the braces indicate an anti-commutator, i.e. $\{b,c\} = bc + cb$. Now the wave functions are antisymmetric. In this case the occupation numbers can be only equal to 0 or 1. All the other formulas remain valid. The rules of commutation for $\hat{\psi}$-operators now take the form:

$$\hat{\psi}^+(\xi')\hat{\psi}(\xi)+\hat{\psi}(\xi)\hat{\psi}^+(\xi')=\delta(\xi-\xi'), \qquad (9.2.35)$$

The antisymmetry and other special features of the quantum description of fermions in comparison with the bosons are connected, as we noted (see (Kyriakos, 2010a,b)) with the fact that bosons are formed by full wave period, and fermions are formed by the half-period of wave.

3.0. The consideration of influence of physical vacuum on free electron

3.1. The quantum-mechanical description

At present the experiments actually confirmed that there is a special medium - physical vacuum (PV), which influences the characteristics of free particles, and also their interaction with each other.

The initial equations of free particles describe the particles as if they were in empty space, without the influence of PV. Such particles are called "bare" particles, and their characteristics can be named "bare" characteristics. The particles can influence the physical vacuum. In turn this produces a change in the characteristics of free particles, and also a change in their interaction with each other.

In QED the free electron in the empty space is described by Dirac's equation

$$\left[\left(\hat{\alpha}_o\hat{\varepsilon}+c\hat{\vec{\alpha}}\,\hat{\vec{p}}\right)+\hat{\beta}\,m_e c^2\right]\psi=0, \qquad (9.3.1)$$

It has only one free parameter: a certain constant m. It is assumed that this constant is a theoretical mass of electron in an empty space. Therefore this mass is called the mass of "bare" electron. In the calculations as mass the experimental mass of electron is used, i.e. a mass, which corresponds to the electron in PV.

Since in the Dirac equation there is no a size of electron, in QED the electron is considered as point, i.e., one that has zero size. Therefore, the solution for the stationary field of electron here coincides with the formula of Coulomb and has the form e/r^2 for the force, and $1/r$ for the energy. These formulas with $r\to 0$ give the infinite values of force and energy (and, therefore, of mass) of the electron.

The problem of infinities in quantum field theory (Weinberg, 1995) was apparently first noted in the papers of Heisenberg and Pauli (Heisenberg and Pauli, 1929, 1930). Soon after, the presence of infinities was confirmed in calculations of the electromagnetic self-energy of a bound electron by Oppenheimer, and of a free electron by Ivar Waller. They used ordinary second-order perturbation theory, with an intermediate state consisting of an electron and a photon: for instance, the shift of the energy ΔE_n of an electron in the n^{th} energy level of hydrogen is given by

$$\Delta E_n=\sum_{m,\lambda}\int d^3k\,\frac{\left|\left\langle m;\vec{k},h\left|\mathrm{H}'\right|\vec{n}\right\rangle\right|^2}{E_n-E_m-\left|\vec{k}\right|c}, \qquad (9.3.2)$$

where the sums and integral are over all intermediate electron states *m,* photon helicities h, and photon momenta $\vec{k}$, and H' *is* the term in the Hamiltonian representing the interaction of radiation and electrons. This calculation gave a self-energy that is formally infinite; further: if this infinity is removed by discarding all intermediate states with photon wave numbers greater than $1/a$, then the self-energy behaves like $1/a^2$ as $a\to 0$. Infinities of this sort are often called ultraviolet divergences, because they arise from intermediate states containing particles of very short wavelength.

These calculations treated the electron according to the rules of the original Dirac theory, without filled negative-electron states. A few years later Weisskopf (Weisskopf, 1939) repeated the calculation of the electron self-mass in the new "hole" Dirac theory, with all negative-energy states full. In this case another term appears in second-order perturbation theory, which in a non-hole-theory language can be described as arising from processes in which the electron in its final

state first appears out of the vacuum together with a photon and a positron which then annihilate along with the initial electron.

Initially Weisskopf found a $1/a^2$ dependence on the photon wave-number cutoff $1/a$. The same calculation was being carried out (at the suggestion of Bohr) at that time by Carlson and Furry. After seeing Weisskopfs results, Furry realized that while Weisskopf had included an electrostatic term that he and Carlson had neglected, Weisskopf had made a new mistake in the calculation of the magnetic self-energy. Alter hearing from Furry and correcting his own error, Weisskopf found that the $1/a^2$ terms in the total mass shift cancelled! However, despite this cancellation, an infinity remained: with a wave-number cutoff $1/a$, the self-mass was found to be:

$$m_{em} = \frac{3\alpha}{2\pi} m \ln\left(\frac{\hbar}{mca}\right), \tag{9.3.3}$$

The weakening of the cut-off dependence, to $\ln a$ as compared with the classical $1/a$ or the early quantum $1/a^2$, was mildly encouraging at the time and turned out to be of great importance later, in the development of renormalization theory.

An infinity of quite a different kind (Weinberg, 1995) was encountered in 1933, apparently first by Dirac (Dirac, 1933). He considered the effect of an external static nearly uniform charge density $\rho_e(x)$ on the vacuum, i.e., on the negative-energy electrons in the filled energy levels of hole theory, The Coulomb interaction between $\rho_e(x)$ and the charge density of the negative-energy electrons produces a 'vacuum polarization,' with induced charge density

$$\delta\rho_e = A\rho_e + B\left(\frac{\hbar}{mc}\right)^2 \nabla^2\rho_e + \ldots\ , \tag{9.3.4}$$

The constant *B is* finite, and of order α . On the other hand, *A* is logarithmically divergent, of order $\alpha \ln \alpha$, where $1/a$ is the wave-number cutoff.

It is interesting that the results of the calculations of the effects of scattering of photons on the electrons (i.e. interaction of electron and photon) without the consideration of PV (but using the real mass of electron), give the final results of the interaction cross-section, which contain as the coefficient a classical radius of electron $r_0 = e^2/m_e c^2$ (Thomson cross-section and others) In QED it is usual to assume that this coefficient does not refer to the dimensions of electron. In framework of NTEP the results are other.

3.2. The description in framework of NTEP

The equation for “bare” electron is derived here on the basis of the postulate of nonlinearity. According to the derivation logic and in accordance with the analysis of the electron equation solution (see (Kyriakos, 2010b)) the “bare” electron is the ring field, whose radius is equal to the Compton wavelength of free electron r_C . The electron equation in this case takes the form:

$$\frac{1}{c}\frac{\partial\psi}{\partial t} + \vec{\alpha}\cdot\vec{\nabla}\psi + i\frac{1}{r_C}\psi = 0\,, \tag{9.3.5}$$

Substituting $r_C = \hbar/mc$ in (4.5.2), it is not difficult to see that (4.5.2) is Dirac's electron equation

$$\frac{1}{c}\frac{\partial\psi}{\partial t} + \vec{\alpha}\cdot\vec{\nabla}\psi + i\frac{mc}{\hbar}\psi = 0\,, \tag{9.3.6}$$

which in this record *does not contain* a size of electron. Nevertheless, the Dirac electron equation and the equation of electron in NTEP coincide completely. Obviously, their solutions coincide. But we have different interpretation of these solutions in QED and NTEP. Actually, in the first case the size is infinite, and in the second case it has a finite value. Why this is possible?

As we know, in the case of QED the renormalization procedure is used for obtaining of the final result. In NTEP this result is obtained without the additional procedures by direct solution of problem. The solutions of nonlinear equations in the spherical approximation (Kyriakos, 2011) show that in the nonlinear theory there are two types of solution - point and nonpoint. Moreover,

the second type of the solution appears as a result of self-action of the electron fields. This self-action can be described as some “self-polarization” of the electron fields. But this is not a polarization of physical vacuum.

4.0. Influence of vacuum polarization on the electron characteristics

4.1. Peculiarities of renormalization procedure in quantum field theory

The polarization of the physical vacuum and the renormalization procedure are considered in a number of modern books and papers (Schweber, Bethe, Hoffmann, 1957; Georgi, 1981; etc.). We will consider briefly the results of this theory, as classical picture.

In QED the physical vacuum represents conditionally a number of enclosed in each other specific vacuums, formed by the various sorts of the virtual particles. For the dielectric permeability of such "dielectric" we can *conditionally* write:

$$d_V = d_V(\breve{\gamma}, \breve{\lambda}, \breve{\mu}, \breve{\beta}), \tag{9.4.1}$$

where ε_V is a full dielectric constant of the physical vacuum as a mixture of the vacuums $\breve{\gamma}, \breve{\lambda}, \breve{\mu}, \breve{\beta}$ of the virtual photons, leptons, mesons and barions accordingly.

The Coulomb potential energy of systems of two charges q' and q'' in a dielectric is equal to:

$$W(r) = \frac{q' q''}{d_d r}\ , \tag{9.4.2}$$

where d_d is a dielectric constant of a medium. In classical physics was accepted, that the vacuum cannot be polarized and consequently $d_V = d_d = 1$. In this case we have some theoretical value of the Coulomb potential energy:

$$W(r) = \frac{q_0' q_0''}{r}, \tag{9.4.3}$$

The electron characteristics, e.g. a charge q_0, considered without the influence of polarization, are referred in QED as "bare": $q_0 \equiv q_{bar}$.

On the other hand the electric charge, observable in the experiments, is not equal to theoretical value q_0, but some value $q_{\exp}$. So in the quantum field theory the Coulomb potential energy of two charges in physical vacuum is equal to

$$W(r) = \frac{q_0' q_0''}{d_V r} = \frac{q_{\exp}' q_{\exp}''}{r}, \tag{9.4.4}$$

Then

$$q_{\exp}' = \frac{q_0'}{\sqrt{d_V}},\quad q_{\exp}'' = \frac{q_0''}{\sqrt{d_V}}, \tag{9.4.5}$$

An electron polarizes the physical vacuum and creates around itself the screening layer from dipoles of the electron-positron pairs. The polarization charge of physical vacuum are referred in QED as polarized or screening: $q_{pol} \equiv q_{scr}$.

The theoretical calculation in QED leads to the conclusion that both the “bare” charge and the screening charge are equal to infinity:

$$q_{bare}^{QED} = \infty_1,\ \ q_{scr}^{QED} = \infty_2, \tag{9.4.6}$$

The measured charge can be considered as a difference among the “bare” charge and the screening charge:

$$q_{\exp} = q_{bare}^{QED} - q_{scr}^{QED} = \infty_1 - \infty_2 = const = q_{\exp} \equiv e, \tag{9.4.7}$$

The calculation procedure, which leads to the obtaining of experimental values, is named in the QED the renormalization procedure.

Thus, if it could be possible to measure an electron charge in very small distances from electron, it would be found that in the process of penetration behind the screening layer, this charge increases. The known measured constant is observed on atomic distances about 10^{-8} cm.

What we have now for the electromagnetic (fine structure) constant? Since $\alpha_{\exp} = \frac{e}{\hbar c}$, we can suppose that $\alpha_{bar} = \frac{q_{bar}}{\hbar c}$, $\alpha_{scr} = \frac{q_{scr}}{\hbar c}$, so as in the QED $\alpha_{bar} = \infty$, $\alpha_{scr} = \infty$ and $\alpha_{bare} > \alpha_{\exp}$, we have that $\alpha_{\exp} = \alpha_{bare} - \alpha_{scr} = \frac{e}{\hbar c}$ is finite value. The direct consequence is, that the electromagnetic constant is not actually a constant, but it grows with reduction of distance between the interacting particles.

4.2. Peculiarities of renormalization procedure in NTEP

The question arises: what is the basis of the subtraction of infinities in the renormalization procedure?

As this is noted above, the screening charge is a function of distance:

$$q_{bare}^{QED} \equiv q_{bar}^{QED}(r) \underset{r\to 0}{\to} \infty_1 \quad q_{scr}^{QED} \equiv q_{scr}^{QED}(r) \underset{r\to 0}{\to} \infty_1, \tag{9.4.6'}$$

Thus

$$q_{\exp} = q_{bar}^{QED}(r) - q_{scr}^{QED}(r) \underset{r\to 0}{=} e, \tag{9.4.6''}$$

In NTEP a "bare" charge has a final size. Obviously, the screening charge also has a final size:

$$q_{bar}^{NTEP} \underset{r\to 0}{=} q_1, \quad q_{scr}^{NTEP} \underset{r\to 0}{=} q_2, \tag{9.4.8}$$

Obviously $q_{bare} > q_{scr}$ or $q_1 > q_2$. Then the experimental charge in NTEP should be defined by their difference:

$$q_{\exp}^{NTEP} = q_1 - q_2 = e, \tag{9.4.9}$$

Thus, NTEP does not need the renormalization procedure, which has place in QED, as transition from infinite value to finite value. In NTEP the renormalization takes only into account the polarization of the physical vacuum, when transition from the "bare" characteristics to the experimental characteristics occurs.

Hence it follows that renormalization in the QED and generally in the quantum field theory is needed to take into account the sizes of particles and polarization of physical vacuum.

With respect to electromagnetic constant we have following. According to (9.4.5) in general case we have $e = q_{\exp} = \frac{q_{bar}}{\sqrt{d_V}}$ so that

$$q_{bare} = e\sqrt{d_V}, \tag{9.4.10}$$

Then

$$\alpha_{bar} = \frac{q_{bar}^2}{\hbar c} = \frac{e^2}{\hbar c} d_V = \alpha_{\exp} d_V, \tag{9.4.11}$$

Using (9.4.11) we receive for the fine structure constant:

$$\alpha_{\exp} \equiv \alpha = \frac{\alpha_{bare}}{d_V}, \tag{9.4.12}$$

4.2.2. The size of electron in NTEP taking into account the physical vacuum polarization

According to the representation (9.3.5) of the Dirac electron equation in NTEP, the bare (theoretical) size of electron in NTEP is equal to the Compton wavelength. How will change this size in the physical vacuum?

Taking into account (9.4.10) and (9.4.12), we can obtain:

$$r_0 = \frac{e^2}{mc^2} = \frac{q_{bar}^2}{d_V m_e c^2}\frac{\hbar c}{\hbar c} = \frac{\alpha_{bar}}{d_V} r_C = \alpha_{\exp} \cdot r_C \equiv \alpha \cdot r_C , \qquad (9.4.13)$$

where $r_0 = \frac{e^2}{mc^2}$ is the known characteristic of the electron energy distribution: the so-called classical electron radius $r_0 = \frac{e^2}{mc^2}$. In other words we obtained the result, known from the quantum mechanics: at large distance from electron the relation occurs:

$$r_0 = \alpha\, r_C \quad \text{or} \quad \frac{r_0}{r_C} = \alpha \approx \frac{1}{137} , \qquad (9.4.14)$$

Taking into account that $r_C \equiv r_{bar}$ and $r_0 \equiv r_{\exp}$, we obtain from (9.4.13):

$$r_0 \equiv r_{\exp} = \frac{\alpha_{bar} r_{bar}}{d_V} , \qquad (9.4.15)$$

The relation (9.4.13-15) can be interpreted in the following way: the classical radius of electron corresponds to the "bare" (theoretical) electron into polarized physical vacuum.

5.0. The consideration of the vacuum effects in electromagnetic view

After the appearance of the relativistic Dirac electron equation (Dirac, 1928a, b), calculations were carried out on several effects of electromagnetic interactions of electrons: the scattering of light by light, light by an electron, the annihilation of an electron-positron pair, the scattering of electrons by electrons, etc.

Practical calculations on real effects were carried out primarily by means of the perturbation theory developed by Dirac (Dirac, 1926, 1927b) for time-dependent perturbations. That theory corresponds to the method of the variation of constants in the theory of linear differential equations.

In all these cases, the results found in lowest-order perturbation theory turned out to agree well with experimental data, thereby confirming that this new theory was sound. However, attempts to refine the predictions through calculations of higher-order approximations led to integrals which diverge at large momenta: *ultraviolet divergences.*

Until now we examined the nonlinearities, which correspond to the free equation of Dirac. At the same time (Collection of article transl., 1959) there are nonlinearities, induced or vacuum, obliged to interaction fields with each other, or, clearly speaking, that appear because of the mutual transformation of particles' fields. The simplest examples are the nonlinearities in Maxwell's equations, induced by the mutual transformations of electron-positrons and photons, examined for the first time by Euler and Kockel and by Heisenberg, and also by Weisskopf and by Schwinger. It was shown that it is possible to select nonlinear Lagrangian of electrodynamics as the function of two invariants of EM field in the form of the series:

$$L = \alpha\left(E^2 - H^2\right)^2 + \beta(EH)^2 + \gamma\left(E^2 - H^2\right)^4 + \ldots \qquad (9.5.1)$$

so to obtain the scattering of light by light. Later Heisenberg and Schwinger obtained closed expressions for nonlinear Lagrangian of electrodynamics, without the expansion in series, for the

case of arbitrarily strong, but slowly changed fields (the details see (Akhiezer and Berestetskii, 1965).

They found the effective Lagrangian to order α^3 induced by static, homogeneous, external fields when no real electron-positron pairs could be produced. In their papers they also pointed out that the theory contained divergent vacuum self energy contributions that had to be subtracted. They also noted that in QED the fourth order contribution to Compton scattering diverged, as did the sixth order contribution to the scattering of light by light.

Below we present briefly the bases and results of the theory of scattering of light by light. They show that the quantum-mechanical tasks can be completely solved within the framework of nonlinear electromagnetic theory. As we noted previously this is possible because these two theories are two equivalent descriptions of reality.

5.1. On the scattering of light by light according to the Dirac electron theory in electromagnetic form (according to Heisenberg, Euler and Kockel)

Below (Heisenberg, 1934; Euler and Kockel, 1935; Euler, 1936; Heisenberg and Euler, 1936) *m is* mass of electron, *e* is charge of electrons, *c is* speed of light, $h = 2\pi\hbar$ is Planck's constant; p_1, p_2 are light quanta before the collision, $-p_3, -p_4$ are light quanta after the collision, $\left(p_1 p_2 |0| - p_3 - p_4\right)$ is matrix element of the operator 0, H^4 is 4th order matrix element of the Dirac theory; F_{ik} is field-strength of EM field, τ is volume of the area.

Halpern (Halpern, 1934) and Debye (in a discussion with W. Heisenber) have pointed out that according to the Dirac theory, there must be scattering of light by light. In this process one must differentiate two cases:

1) If the energies cp_1 and cp_2 of the two light quanta and the angle between its momentums $\vec{p}_1, \vec{p}_2$ are so large that energy and law of momentum permit the production of a real pair $\left(p_1 p_2 - \left(\vec{p}_1 \cdot \vec{p}_2\right)\right) > 2(mc)^2$. Then we will receive the probability of the dispersion of the light quanta by summing up the probabilities of the pair creation. This was accomplished by (Breit and Wheeler, 1934) .

2) Or if energy and momentum of two light quanta are not sufficient for the production of a real pair:

$$p_1 p_2 - \left(\vec{p}_1 \cdot \vec{p}_2\right) < 2(mc)^2, \qquad (9.5.2)$$

(i.e. in suitable reference system: $p_1 < mc$, $p_2 < mc$), nevertheless the light quanta $\vec{p}_1, \vec{p}_2$ can, due to the virtual pair creation, in two other light quanta be converted, and this must a dispersion of light by light give.

The probability of the transition of two light quanta $\vec{p}_1, \vec{p}_2$ in two different $-\vec{p}_3$, $-\vec{p}_4$ is given by the square of the matrix element H^4 of the Dirac theory.

The direct calculation of this matrix H^4 of the Dirac theory would be very difficult. It can be attributed however to the simpler problem of the calculation of two matrix elements by the following general views.

If two light waves disperse themselves one to other, this means a deviation from the superposition principle. The optical superposition principle is expressed by the linearity of the vacuum Maxwell equations. Thus the dispersion of light by light could be described by a nonlinear additive to the Maxwell vacuum equations, if a direct description is possible. This direct description, possibility of which we later proves, is suggested by the following analogy, which exists in the Dirac theory between light quanta and electrons.

Two electrons can produce the light quanta and enter into mutual interaction, which can be expressed in the dispersion of the electrons or in the Coulomb law. Likewise two light quanta can produce a virtual quantity of electron-positron pairs so that between them an interaction develops,

which leads to the dispersion of light by light. Also for this interaction of the light quanta with one another one should expect a simple, similar to the Coulomb law, direct expression.

It is the analogue of the Coulomb interaction of electrons:

$$\iint \frac{\rho_1 \cdot \rho_2}{\mathrm{r}_{12}} d\tau_1 d\tau_2 , \tag{9.5.3}$$

where ρ_1, ρ_2 is charge densities, r_{12} is a distance between two charges.

The Coulomb interaction in a matter field, which is described by a density operator $\psi^* \psi$, is

$$U = \frac{e^2}{2} \iint \frac{\psi^*(\xi)\psi(\xi)\psi^*(\xi')\psi(\xi')}{(\xi - \xi')} d\tau \, d\tau , \tag{9.5.4}$$

The effective cross-section for the dispersion of an electron with an electron is received from the square of the matrix element (9.5.4) for a transition in the matter field, which means the dispersion of two electrons with each other.

In order to find an interaction of the light quanta, similar to (9.5.4), one must search for a function U_1 of the degrees of freedom of the radiation field, and also for the field strengths F_{ik}. The matrix element F_{ik} for a transition in the radiation field, which means the dispersion of two light quanta with each other, is equal to this, which was discussed above. Then matrix element H^4_{in} of the Dirac theory for this process can be computed. About this interaction U_1 of the light quanta as function of the field strengths we can the following say.

Since it must lead to processes, during which two light quanta disappear and originate two other, the field strengths or their derivatives in the 4^{th} power then must contain:

$$U_1 = const \int \left[FFFF + const' \frac{\partial F}{\partial x} \frac{\partial F}{\partial x} FF + \ldots \right] d\tau , \tag{9.5.5}$$

(indices of tensors and vectors are here and in the following omitted).

Since the interaction U_1, which have dimension of an energy, but (as term of 4^{th} order of the Dirac theory) the electron charge in the 4^{th} power must contain (and due to the fact that from 4 universal units e, m, c, h only one dimensionless number, which can form the Sommerfeld fine structure constant $e^2/\hbar c \sim 1/137$), is the constant up to a numeric factor determined: $const = \frac{\hbar c}{e^2} \frac{1}{E_0^2}$, so that $E_0 = \frac{e}{\left(e^2/mc^2\right)^2}$ is the "field strength at the edge of the electron".

For the same reason the terms with the derivatives of the field strengths must contain still another length independent of the electron charge, such as the Compton wave length h/mc as additional factor.

First of all the fact is surprising that in the vacuum electrodynamics the electron mass is to occur, while is nevertheless presupposed that only light quanta and no electrons are present.

Although the terms, regarded here, have only validity as long as no real pairs are produced, they come off only by the virtual possibility of the pair creation and are disclosed itself in the occurrence of the electron mass. It can expect also (close to the Maxwell energy of the individual light quanta) a mutual interaction energy of the light quanta of the form:

$$U_1 = \frac{\hbar c}{e^2} \frac{1}{E_0^2} \int \left[FFFF + \left(\frac{\hbar}{mc} \frac{\partial F}{\partial x} \right) \left(\frac{\hbar}{mc} \frac{\partial F}{\partial x} \right) FF + \ldots \right] d\tau , \tag{9.5.6}$$

It will be shown later that the matrix element H^4 discussed above, which follows from the Dirac theory, can be also really transformed into the matrix element such as an expression (9.5.6).

Since we want by soft light $\left(|\vec{p}| < mc\right)$ and also by slowly variable fields $\left(\left|\frac{\hbar}{mc}\frac{\partial F}{\partial x}\right| < F\right)$ according to (9.5.2) to be limited, we can into (9.5.6) the terms with the derivatives of the field strengths let go away.

We take also now, as a subject to the later proof that the dispersion of soft light by light by an additional energy density in the radiation field of the form

$$u_1 = \frac{\hbar c}{e^2}\frac{1}{E_0^2} FFFF\,, \tag{9.5.7}$$

can be described as:

$$\mathrm{H}^4 = \left(p_1 p_2 \left|\int u_1 d\tau\right| - p_3 - p_4\right), \tag{9.5.8}$$

For this, by the ordinary perturbation of the Dirac theory, the matrix element of 4th order for this process is calculated and designed for light quantum energies cp (or $\frac{h\nu}{mc^2}$)

$$\mathrm{H}^4 = \frac{-1}{12\pi^2}\left(\frac{e^2}{\hbar c}\right)^2 \cdot \frac{1}{\hbar c} \cdot \lim_{\mathrm{r}\to 0}\int d\xi\left(A(\xi), \frac{\mathrm{r}}{r}\right)^4, \tag{9.5.9}$$

($\vec{A}$ is a potential of the radiation field).

The terms of 1st , 2nd and 3rd order disappeared, and the element of 4th order by $\frac{h\nu}{mc^2}$ could be represented formally as a matrix element of a function of the radiation field, so that for the considered processes, the ordinary Hamiltonian, which contains the energies of light and matter (see (Heisenberg and Pauli, 1930)), can be replaced by the following, which depends only on the radiation field:

$$\int u d\tau = \frac{1}{8\pi}\int\left(\vec{B}^2 + \vec{D}^2\right)d\tau - \frac{1}{360\pi^2}\frac{\hbar c}{e^2}\frac{1}{E_0^2}\int\left[\left(\vec{B}^2 - \vec{D}^2\right)^2 + 7\left(\vec{B}\cdot\vec{D}\right)^2\right]d\tau\,, \tag{9.5.10}$$

Here $\vec{D}$ is the electric displacement, $\vec{B}$ is the magnetic induction, $E_0 = \frac{e}{\left(e^2/mc^2\right)^2}$ is the value of "field strength at the edge of the electron" This stands $\vec{B} = rot\vec{A}$, $\vec{D}$ is canonically conjugate to $\vec{A}$, i.e.:

$$D_i(\xi)A_k(\xi') - A_k(\xi')D_i(\xi) = 2\hbar ci\cdot\delta(\xi-\xi')\delta_{ik}\,, \tag{9.5.11}$$

or

$$D_i(\xi)B_k(\xi') - B_k(\xi')D_i(\xi) = 2\hbar ci\frac{\partial}{\partial\xi'}\cdot\delta(\xi-\xi')\,, \tag{9.5.12}$$

(with cyclic i,k,l)

If one introduce by the usual way the quantities $\vec{E}$ and $\vec{H}$ by the equations:

$$\begin{cases} -\frac{1}{c}\dot{\vec{A}} = \vec{E}, \;\text{ i.e.: } \frac{1}{c}\dot{\vec{B}} + rot\vec{E} = 0 \\ -\frac{1}{c}\dot{\vec{D}} + rot\vec{H} = 0 \end{cases}, \tag{9.5.13}$$

then follows:

$$\frac{1}{4\pi}\vec{E}=\frac{\partial L}{\partial \vec{D}}=\frac{1}{4\pi}\vec{D}-\frac{1}{360\pi^2}\frac{\hbar c}{e^2}\frac{1}{E_0^2}\int\left[-4\left(\vec{B}^2-\vec{D}^2\right)\vec{D}+14\left(\vec{B}\cdot\vec{D}\right)\vec{B}\right]d\tau$$
$$\frac{1}{4\pi}\vec{H}=\frac{\partial L}{\partial \vec{B}}=\frac{1}{4\pi}\vec{B}-\frac{1}{360\pi^2}\frac{\hbar c}{e^2}\frac{1}{E_0^2}\int\left[-4\left(\vec{B}^2-\vec{D}^2\right)\vec{B}+14\left(\vec{B}\cdot\vec{D}\right)\vec{D}\right]d\tau \quad , \tag{9.5.14}$$

The relationship between the variables $\vec{B}$ and $\vec{D}$ on the one hand, $\vec{E}$ and $\vec{H}$ on the other hand, in this theory is not linear, because the scattering of light by light means a deviation from the principle of superposition.

The addition to Maxwell's energy in (9.5.10):

$$-\frac{1}{360\pi^2}\frac{\hbar c}{e^2}\frac{1}{E_0^2}\int\left[\left(\vec{B}^2-\vec{D}^2\right)^2+7\left(\vec{B}\cdot\vec{D}\right)^2\right]d\tau \quad , \tag{9.5.15}$$

can vividly as the interaction energy of the light-quantum to be interpreted. It is analog to the Coulomb interaction of electrons (9.5.3). The fact that in (9.5.15), unlike (9.5.3) only a simple integral is, means that two photons can interact in the same place only.

The non-linear correction of Maxwell's equations of the vacuum will be essential, when the field strengths are close to "the edge of the electron"; the derived here formulas are valid only as long as they are not became too large ($\left|\vec{E}\right|,\left|\vec{B}\right|,\left|\vec{D}\right|,\left|\vec{H}\right| << E_0$)..

It is interesting, that this additive, which is added to the Maxwell energy, can be compared with the Born (Born and Infeld, 1934) considerations proposed in the classical theory, where first expansion term is:

$$\frac{-(1{,}2361)^4}{32\pi}\frac{\hbar c}{e^2}\frac{1}{E_0^2}\int\left[\left(\vec{B}^2-\vec{D}^2\right)^2+4\left(\vec{B}\cdot\vec{D}\right)^2\right]d\tau \, , \tag{9.5.16}$$

Apart from the fact that the ratio of the coefficients of the two additional terms by Born is 1: 4, and by us is 1: 7, two expressions differ by a factor:

$$\frac{4}{45\pi\cdot(1{,}2361)^4}\cdot\frac{\hbar c}{e^2} \, , \tag{9.5.17}$$

Due to the actual value of the Sommerfeld fine structure constant the numeric value of this factor is ~ 1.7, and it is remarkable that the quantum-theoretic modification of Maxwell's equations in any case has the magnitude that would be expected from classical notion due to the self energy.

The equations (9.5.10), (9.5.13), (9.5.14), which follows from the Dirac theory, have a place only in case of the condition that the wavelengths of light are large than the Compton wavelength. Otherwise, in contrast to Born's theory, a developing higher terms on the light quantum energy levels will arise, and also other additives to the interaction of the 4th order in the derivatives of the field strengths (multiplied with $\frac{\hbar}{mc}$).

The experimental test of the deviations from the Maxwell's theory is difficult because the alleged effects are extremely small. The cross section for scattering of light by light of average wavelength λ is in the Dirac theory by (9.5.10) of the order: $Q\sim\left(e^2/mc^2\right)^4(\hbar/mc)^4\left(1/\lambda^6\right)$

So, it is about $10^{-30}\,cm^2$ for gamma-rays and $10^{-70}\,cm^2$ for visible light.

6.0. Modern calculation of particle interactions (S-matrix, Feynman diagrams, etc)

6.1. Calculations in quantum field theory

The quantum-mechanical calculations in contemporary QFT are produced with the aid of the mathematics, which includes the operators of creation and annihilation of particles, and also with

the aid of the apparatus of Feynman's diagrams, connected with them. In the following paragraphs we will consider the representation of calculated apparatus both in QFT and in NTEP.

As it was described above, in the quantum field theory, the functions ψ, ψ^+, A_μ are substituted by the operators $\hat{\psi}$, $\hat{\psi}^+, \hat{A}_\mu$, which satisfy the same commutation relations (for one and the same moment of time), as for the free fields.

We have for the operator of the energy density of interaction (Hamiltonian):

$$\hat{\mathrm{H}}_{\mathrm{int}} = -L_{\mathrm{int}} = -\frac{1}{c}\hat{j}_\mu \hat{A}_\mu = -e\left(\psi^+ \hat{\alpha}_\mu \hat{A}_\mu \hat{\psi}\right), \tag{9.6.1}$$

where $\hat{j}_\mu = -ec\left(\psi^+ \alpha^+{}_\mu \hat{\psi}\right)$ is current density.

Introducing designation $\hat{A} = \hat{\alpha}_\mu \hat{A}_\mu$, we rewrite formula (9.6.1) as follows:

$$\hat{\mathrm{H}}_{\mathrm{int}} = -e\left(\hat{\psi}^+ \hat{A} \hat{\psi}\right), \tag{9.6.2}$$

Solutions of the free-field equations are proportional to the creation and annihilation operators of stationary states of particles. In order to incorporate cases in which certain particles affect the motion of others or convert into others, we need to make the equations of motion nonlinear. In other words, we need to introduce in the Lagrangian terms L_{int} of higher powers in addition to the quadratic terms.

From the standpoint of the above theory, such interaction Lagrangians L_{int} might be any functions of the fields and their first derivatives, provided only that they satisfy some simple conditions of invariance.

$$i\hbar \frac{\partial \psi(t)}{\partial t} = \hat{\mathrm{H}}(t)\psi(t) \quad , \tag{9.6.3}$$

On the other hand, one can, as in ordinary quantum mechanics, transform by means of a unitary transformation $\psi(t) = e^{i\hat{\mathrm{H}}t}$ from the Heisenberg representation with constant state amplitudes to the Schrödinger representation, in which the state amplitude evolves in time in accordance with a Schrödinger equation,

$$i\hbar \frac{\partial \psi(t)}{\partial t} = \hat{\mathrm{H}}(t)\psi(t)\,, \tag{9.6.3}$$

and the field operators are constant.

In quantum field theory, a third representation proved to be most convenient. This representation is usually called the *interaction representation* $\Theta(t)$.

$$i\hbar \frac{\partial \Theta(t)}{\partial t} = \hat{\mathrm{H}}(t)\Theta(t)\,, \tag{9.6.4}$$

where

$$\hat{\mathrm{H}}(t) = e^{i\hat{\mathrm{H}}_0 t}\hat{\mathrm{H}}e^{-i\hat{\mathrm{H}}_0 t} \quad , \tag{9.6.5}$$

is the Schrödinger Hamiltonian of the interaction representation, which, as can be seen from (9.6.5), depends on time.

The general solution of (9.6.4) can be written in the form $\Theta(t)S(t,t_0)\Theta(t_0)$, where the evolution operator *S(t, t_0)* satisfies the same equation (9.6.4) in terms of *t* and can be written as a chronological exponential function:

$$S(t,t_0) = \hat{T}\left\{\exp\left(-i\int_{t_0}^{t}\hat{\mathrm{H}}(t')dt'\right)\right\}, \tag{9.6.7}$$

where $\hat{T}$ is the operator of normal product, who makes it possible to pass in the interaction representation to the products for different time moments.

For a comparison with experiment, the most interesting problem is that of scattering, for which we need an evolution operator over an infinite time interval, which transforms a stationary state $\Theta_{-\infty}$, in which the system is before the scattering, at $t \to -\infty$, into a stationary state $\Theta_{+\infty}$, which the system reaches after the scattering, at $t \to +\infty$:

$$\Theta_{+\infty} = S\,\Theta_{-\infty}, \tag{9.6.8}$$

where *S* is the scattering matrix (Heisenberg, 1943). Taking the limit $t \to +\infty$, $t \to -\infty$ in (9.6.7), and expressing the Hamiltonian $\hat{\mathrm{H}}(t)$ in terms of a spatial integral of the interaction Lagrangian $\hat{\bar{\mathrm{H}}}(t) = -\int_t d^3x\, L_{\mathrm{int}}(x)$; where it is to be understood that the interaction Lagrangian is written not in terms of Heisenberg fields, but in the form of the same function of the fields (15) in the Dirac representation, we find a compact expression for the scattering matrix:

$$S = \hat{T}\left\{\exp\left(-i\int_{-\infty}^{+\infty} dt\hat{\mathrm{H}}(t)\right)\right\} = \hat{T}\left\{\exp\left(-i\int_{-\infty}^{+\infty} d^3x L_{\mathrm{int}}(x)\right)\right\}, \tag{9.6.9}$$

This expression is explicitly relativistically invariant. The scattering matrix can be used to find probabilities for physical processes without plunging into the details of the time evolution, described by the amplitude $\Theta(t)$.

We need to underline, however, that expression (9.6.9), despite its simple form, is not a ready solution for further use; it is only a compact symbolic equation.

For this reason is necessary to resort to the assumption that the interaction is weak and to assume that the interaction Lagrangian L_{int} is proportional to a small interaction constant *g*. It then becomes possible to expand the chronological exponential function (9.6.9) in a power series according to perturbation theory: $S = 1 + \sum_{n\geq 1} g^n S_n$.

In this case the matrix elements for each order of the perturbation theory are expressed by terms of the matrix elements of chronological products of the corresponding number of interaction Lagrangians: $\int \left\langle \Theta^* \hat{T}\{L(x_1)L(x_2)...L(x_n)\}\Theta \right\rangle dx_1 dx_2 ... dx_n$.

Individual terms of *S*-matrix are integrals of the mixed products of operators $\hat{\psi}$, $\hat{\bar{\psi}}$, $\hat{A}$. Using the second Wick theorem, we can reduce arbitrary products of operators to sums of products of pairs of these operators.

A practical calculation of the matrix elements and integrals over $x_1,...,x_n$ of these elements is carried out by a technique proposed by R. Feynman in 1949. This technique includes the known Feynman diagrams (graphs) and correspondent rules.

Each of normal product it is possible to compare with the Feynman diagram (Akhiezer and Berestetskii, 1965).

Graphs with *n* vertexes correspond to the term *n* of *S*-matrix. Because in the electrodynamics an n^{th} term is proportional to the factor $(e/\hbar c)^n = (1/137)^n$, the matrix elements, obtained from it, will be also proportional to $(e/\hbar c)^n$. The graphs, which depict such processes, are called graphs of n-order. One and the same graph, which corresponds to a certain normal product of field operators, can describe a number of different processes of scattering. The terms, the Feynman graphs of which are characterized only by the transposition of the indices of vertexes, are called equivalent. The equivalent products describe one and the same totality of processes and they are equal to each other.

To the first term of S-matrix one graph of Feynman of the first order of the type $\left(\hat{\bar{\psi}}\hat{A}\hat{\psi}\right)$ corresponds. To the second term S-matrix 6 different graphs of Feynman of the second order of the type $\left(\hat{\bar{\psi}}\hat{A}\hat{\psi}\right)\left(\hat{\bar{\psi}}\hat{A}\hat{\psi}\right)\left(\hat{\bar{\psi}}\hat{A}\hat{\psi}\right)$ correspond. The third term S-matrix describes all effects of the third order and count 15 graphs, which have a type $\left(\hat{\bar{\psi}}\hat{A}\hat{\psi}\right)\left(\hat{\bar{\psi}}\hat{A}\hat{\psi}\right)\left(\hat{\bar{\psi}}\hat{A}\hat{\psi}\right)$. And so forth.

6.2. Calculations in framework of NTEP

The Hamiltonian and Lagrangian of the NTEP, as the non-linear theory, must contain all possible invariants of non-linear electromagnetic field theory. Thus we can suppose that a Lagrangian must be some function of the field invariants:

$$L = f_L(I_1, I_2), \tag{9.6.10}$$

where $I_1 = \left(\vec{E}^2 - \vec{H}^2\right), I_2 = \left(\vec{E}\cdot\vec{H}\right)$.

Hamiltonian is fully defined through the Lagrangian. Thus, if the function (9.6.10) is known, it is easy, using the formulas (12.1.13), to calculate the Hamiltonian, which will be now the function of the various powers of electromagnetic field vectors:

$$\hat{\mathrm{H}} = f_{\mathrm{H}}(\vec{E}, \vec{H}), \tag{9.6.11}$$

Apparently, for each problem the functions f_L and f_{H} will have their special form, which is unknown before the solution of problem. As it is known the approximate form of the function f_{H} can be found on the basis of the Schrodinger or Dirac wave equation, using the so-called perturbation method.

We suppose here that an expansion of the function f_{H} in Taylor–MacLaurent power series with unknown expansion coefficient exists. Then the problem is to calculate these coefficients.

The solution for each term of expansion is searched separately, starting from first. Usually this is the problem for a free particle, whose solution is already known. Then using the equation with the two first terms, we find the coefficient of the second term. Further using the equation for the three first terms, we find the coefficient for the third term of expansion, etc. In many cases by this method it is possible to obtain the solution with any desirable accuracy.

In case of function of two variables $\xi = f(x, y)$ the Taylor – MacLaurent power series nearly to a point (x_0, y_0) is:

$$f(x,y) = f(x_0,y_0) + \sum_{k=1}^{n}\frac{1}{k!}\left((x-x_0)\frac{\partial}{\partial x} + (y-y_0)\frac{\partial}{\partial y}\right)^k f(x_0,y_0) + O\left(\rho^n\right), \tag{9.6.12}$$

where $\rho = \sqrt{(x-x_0)^2 + (y-y_0)^2}$,

$$\left((x-x_0)\frac{\partial}{\partial x} + (y-y_0)\frac{\partial}{\partial y}\right) f(x_0,y_0) \equiv (x-x_0)\frac{\partial f(x_0,y_0)}{\partial x} + (y-y_0)\frac{\partial f(x_0,y_0)}{\partial y}, \tag{9.6.13}$$

$$\left((x-x_0)\frac{\partial}{\partial x} + (y-y_0)\frac{\partial}{\partial y}\right)^2 f(x_0,y_0) \equiv$$

$$\equiv (x-x_0)^2\frac{\partial f^2(x_0,y_0)}{\partial x^2} + 2(x-x_0)(y-y_0)\frac{\partial^2 f(x_0,y_0)}{\partial x \partial y} + (y-y_0)^2\frac{\partial f^2(x_0,y_0)}{\partial y^2}, \tag{9.6.14}$$

Etc. (In case when $x_0 = 0,\ y_0 = 0$ we obtain the MacLaurent series).

Obviously, for the most types of the functions the expansion contains approximately the same set of the terms, which distinguish only by the constant coefficients, any of which can be equal to zero (as examples, see the expansions of the quantum electrodynamics Lagrangian for particle at the presence of physical vacuum (Akhiezer and Berestetskii,. 1965; Schwinger, 1951; Weisskopf, 1936). Generally the expansion will look like:

$$L_M = \frac{1}{8\pi}\left(\vec{E}^2 - \vec{B}^2\right) + L', \tag{9.6.15}$$

where

$$L' = \alpha\left(\vec{E}^2 - \vec{B}^2\right)^2 + \beta\left(\vec{E}\cdot\vec{B}\right)^2 + \gamma\left(\vec{E}^2 - \vec{B}^2\right)\left(\vec{E}\cdot\vec{B}\right) + \\ + \xi\left(\vec{E}^2 - \vec{B}^2\right)^3 + \zeta\left(\vec{E}^2 - \vec{B}^2\right)\left(\vec{E}\cdot\vec{B}\right)^2 + \ldots \tag{9.6.16}$$

is the part, which is responsible for the non-linear interaction (here $\alpha, \beta, \gamma, \xi, \zeta, \ldots$ are some constants).

The corresponding Hamiltonian will be defined as follows:

$$\hat{\mathrm{H}} = \sum_i E_i \frac{\partial L}{\partial E_i} - L = \frac{1}{8\pi}\left(\vec{E}^2 + \vec{B}^2\right) + \overline{H}', \tag{9.6.17}$$

where the Hamiltonian part responsible for non-linear interaction is:

$$\hat{\mathrm{H}}' = \alpha\left(\vec{E}^2 - \vec{B}^2\right)\left(3\vec{E}^2 - \vec{B}^2\right) + \beta\left(\vec{E}\cdot\vec{B}\right)^2 + \\ \xi\left(\vec{E}^2 - \vec{B}^2\right)\left(5\vec{E}^2 + \vec{B}^2\right) + \zeta\left(3\vec{E}^2 - \vec{B}^2\right)\left(\vec{E}\cdot\vec{B}\right)^2 + \ldots \tag{9.6.18}$$

It is not difficult to obtain the quantum representation of Hamiltonian (9.6.18) of non-linear theory. Replacing the electromagnetic wave field vectors by quantum wave function, we will obtain a series of type:

$$\hat{\mathrm{H}} = \left(\psi^+\hat{\alpha}_0\psi\right) + \sum c_{1i}\left(\psi^+\hat{\alpha}_i\psi\right)\left(\psi^+\hat{\alpha}_j\psi\right) + \\ + \sum c_{2i}\left(\psi^+\hat{\alpha}_i\psi\right)\left(\psi^+\hat{\alpha}_j\psi\right)\left(\psi^+\hat{\alpha}_k\psi\right) + \ldots \tag{9.6.19}$$

where $\hat{\alpha}_i, \hat{\alpha}_j, \hat{\alpha}_k$ are Dirac matrixes, c_i are the coefficients of expansion.

7.0. Comparison of representations QED and NTEP

7.1. On the physical sense of the terms of Feynman graphs

As it was said above, the terms, Feynman's graphs of which are characterized only by the transposition of the indices of vertexes, are called equivalent. The equivalent products describe one and the same totality of processes and are equal to each other. The question arises: how can explain this equivalence (i.e. the presence of the certain number of terms of identical degree)?

The electromagnetic Hamiltonian of interaction in the quantum form, given above, in the first approximation, contains the sum of terms of the type of the diagrams of the second order of type $\left(\hat{\bar{\psi}}\hat{A}\hat{\psi}\right)\left(\hat{\bar{\psi}}\hat{A}\hat{\psi}\right)$. Continuing expansion and calculating the terms of the expansion of the third order, we will , obviously, obtain the sum of termss of the type of the diagrams of the third order of type $\left(\hat{\bar{\psi}}\hat{A}\hat{\psi}\right)\left(\hat{\bar{\psi}}\hat{A}\hat{\psi}\right)\left(\hat{\bar{\psi}}\hat{A}\hat{\psi}\right)$, and so forth. Thus, it is not difficult to see that expression (9.6.18) is the electromagnetic representation of expansion of S-the matrix of the quantum field theory. This proves that all special features of the quantum-field calculations of interaction energy of elementary particles can be represented by electrodynamics forms NTEP.

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Note that using the analysis, which is outlined above, can explain the electrodynamicc sense of the special features of S-matrix theory within the framework of NTEP.

7.2. On the physical sense of the terms of the expansion of the Hamiltonian

As we see, the terms of the series of Lagrangian and Hamiltonian expansions contain the limited number of uniform elements, such as $\left(\vec{E}^2+\vec{B}^2\right)$, $\left(\vec{E}\cdot\vec{B}\right)^2$, $\left(\vec{E}^2-\vec{B}^2\right)$, and some other. It is possible to assume that each term of the expansion of the Hamiltonian of nonlinear theory and each element of term of series have a certain constant physical sense.

For example, it is obvious that the first term of the expansion of Hamiltonian $\left(\vec{E}^2+\vec{B}^2\right)$ (which corresponds to the term of Lagrangian $\left(\vec{E}^2-\vec{B}^2\right)$), is charged with the interaction of the currents of electrons and photons. The second term of expansion of Hamiltonian $\left(\vec{E}\cdot\vec{B}\right)^2$ corresponds to the term $\left(\vec{E}\cdot\vec{B}\right)$ of Lagrangian, which, according to studies, is charged with the currents of the helicity of electromagnetic field (according to NTEP the currents of helicity there are own currents of neutral elementary particles).

In this case it is not difficult to see the analogy with expansion of fields on the electromagnetic moments, and also with decomposition of a S-matrix on the elements (Akhiezer and Berestetskii,, 1965), each of which corresponds to the particularities of interaction of separate particles. But the detailed analysis of the physical sense of the terms of the expansion of Lagrangian and Hamiltonian of nonlinear theory requires a separate study and lies beyond the limits of the purpose of this monograph.

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