A Note on the Action at a Distance

February 14, 2012.

José Francisco García Juliá

jfgj1@hotmail.es

We consider that the action at a distance is carried out by virtual carriers of the force, which are created and annihilated in the vacuum by the field in a time less than that of the Heisenberg's uncertainty.

Key words: action at a distance, virtual carrier.

In this short and very simple note, we consider that the action at a distance is carried out by virtual carriers of the force [1], which are created and annihilated in the vacuum by the field in a time less than that of the Heisenberg's uncertainty

$$\Delta t \leq \frac{\hbar}{2\Delta E} \tag{1}$$

where t is the time, $\hbar = h/2\pi$ and h is the Planck's constant, and E the energy. There is no violation of the law of the energy conservation.

The source field creates and annihilates virtual carriers in the vacuum (ket $|0\rangle$) with the operators of creation (a^+) and annihilation (a, its adjoint) of the Dirac's second quantization [2]:

$$aa^+|0\rangle = |0\rangle \tag{2}$$

where

$$a^+|0\rangle = |\psi\rangle = |1\rangle_\psi,\ a|1\rangle_\psi = |0\rangle \tag{3}$$

and ψ is the wave function. The virtual quantum fluctuation, (2), is reproduced by all the space, although it is attenuated with the distance to the source: $d = ct = c\sum \Delta t_n$, where c is the light speed in the vacuum and n a positive integer, and $0 < \Delta t_1 \leq \Delta t_2 \leq \cdots \leq \Delta t_n$ and $\Delta E_1 \geq \Delta E_2 \geq \cdots \geq \Delta E_n > 0$. Because of the Heisenberg's uncertainty principle

$$\Delta E \Delta t \geq \frac{\hbar}{2} \tag{4}$$

a real particle, that is, a source field, creates, $E - \Delta E_1/2$, and reabsorbs, $E + \Delta E_1/2$, one or more virtual particles in the surrounding vacuum Δs_1 (where s is the space-time interval) in a time $\Delta t_1 \le \hbar/2\Delta E_1$ given by (1), E being the energy of the particle and $E + \Delta E_1/2 - (E - \Delta E_1/2) = \Delta E_1 = (<(E - <E>)^2>)^{1/2} = (<E^2> - 2<E><E> + <E>^2)^{1/2} = (<E^2> - <E>^2)^{1/2}$ its uncertainty (and in this case also a potential energy), where $<E> = <\psi|E|\psi> = \int_V \psi^* E \psi dV$ is the expected (or mean) value of E, $<\psi|$ the bra vector (the dual of the ket vector $|\psi>$), ψ^* the complex conjugate of ψ and V the volume. This first group of (one or more) virtual particles produces a second vibration in the contiguous vacuum Δs_2 that creates the second group of (one or more) virtual particles, which before being annihilated (being absorbed now by the vacuum) produces a third vibration and so on. Note that because of the principle of minimum energy, it is not $\Delta E_1 < \Delta E_2 < \ldots < \Delta E_n$, then $\Delta E_1 \ge \Delta E_2 \ge \ldots \ge \Delta E_n$, and from (1), $\Delta t_1 \le \Delta t_2 \le \ldots \le \Delta t_n$. The virtual particles, while they exist, follow the hypothetical force lines created by the source field. Note that from (1), $\Delta E_n \le \hbar c/2\Delta x_n$, where x is a distance and $\Delta x_n = c\Delta t_n$, which implies that these virtual particles (virtual carriers of the force) move in the vacuum with the speed c and therefore they have no rest mass; and $F_n = \Delta E_n/\Delta x_n \le \hbar c/2\Delta x_n^{\,2}$, where F is the force. F_1 besides producing attraction or repulsion inside of Δs_1 produces the vibration in Δs_2 which in turn produces the force F_2 and so on. For a classical field, it would be $U \propto 1/r$, where U is the potential energy and r the radial distance, and $F = dU/dr \propto 1/r^2$. Note that the typical coupling constants for the electric and gravitational interactions are, respectively, $k_e = e^2/4\pi\varepsilon_0\hbar c$ and $k_g = Gm^2/\hbar c$, where $-e$ and m are the electric charge and the mass of the electron, respectively, ε_0 the electric permittivity of the vacuum and G the Newton's gravitational constant; and then, $e^2/4\pi\varepsilon_0 k_e = \hbar c = Gm^2/k_g$. Hence, for the classical electric and gravitational fields, it would be, respectively, $U = -e^2/4\pi\varepsilon_0 r$ and $F = dU/dr = e^2/4\pi\varepsilon_0 r^2$, and $U = -Gm^2/r$ and $F = dU/dr = Gm^2/r^2$ (see the appendix).

For the electromagnetic field, the virtual carrier is a virtual photon that would follow the force lines, which are outward for a positive electric charge or a north magnetic pole and inward for a negative electric charge or a south magnetic pole. Therefore, virtual photons in opposite directions would produce repulsion and in the same direction would produce attraction; that is, electric charges of the same sign (or same magnetic poles) repel each other, and conversely, electric charges of opposite sign (or opposite magnetic poles) attract each other. The square of the incremental space-time interval is:

$$\Delta s_n^{\,2} = c^2 \Delta t_n^{\,2} - \Delta x_n^{\,2} - \Delta y_n^{\,2} - \Delta z_n^{\,2} \tag{5}$$

where x, y and z are the rectangular coordinates.

For the gravitational field, the virtual carrier would be the virtual graviton. However, the virtual graviton would not drag the bodies in the same form as the virtual photon, because so whether we consider the gravitational force lines outgoing or incoming, the force would always be repulsive instead of attractive. We assume that the virtual graviton produces the gravitational attraction bending the space and that this curvature decreases with the distance to the source field. The square of the incremental space-time interval would be [3]:

$$\Delta s_n{}^2 = \left(1 - \frac{r_g}{r_n}\right) c^2 \Delta t_n{}^2 - r_n{}^2 \left(\sin^2\theta_n \Delta\phi_n{}^2 + \Delta\theta_n{}^2\right) - \frac{\Delta r_n{}^2}{1 - \frac{r_g}{r_n}} \tag{6}$$

where $r_g = 2GM/c^2$ is the Schwarzschild's gravitational radius of the body of mass M that produces the gravitational source field, G the Newton's gravitational constant and r, θ and ϕ the spherical coordinates.

We deduce that the vacuum vibrates because of the presence of the fields, acting as a quantum oscillator. Hence, in absence of fields, the vacuum does not contain any energy, including the vacuum zero point energy, $\hbar\omega/2$, where $\omega = 2\pi\nu$ is the angular frequency and ν the frequency. With the electromagnetic field, as the positive and negative electric charges exist, the vacuum vibrates like an electric dipole quantum oscillator, which implies that the photon has a spin of $2\hbar/2 = 1\hbar$. With the gravitational field, as the negative mass does not exist, the vacuum vibrates like a mass quadrupole quantum oscillator, which implies that the graviton would have a spin of $4\hbar/2 = 2\hbar$.

For last, as (5) and (6) refer to the vacuum space-time, they have to be quantized. From (5), with $\Delta s_n = 0$ and

$$\Delta r_n{}^2 = \Delta x_n{}^2 + \Delta y_n{}^2 + \Delta z_n{}^2 \tag{7}$$

$$\Delta r_n = j_n \lambda_n \tag{8}$$

$$\Delta t_n = j_n / \nu_n \tag{9}$$

where r is the rectangular distance (equal to the spherical coordinate r, the radial distance), j a positive integer and λ the wavelength; we obtain that: $c = \Delta r_n / \Delta t_n = \lambda_n \nu_n$. From (6), with $\Delta s_n = 0$ and

$$\Delta \ell_n{}^2 = r_n{}^2 \left(\sin^2\theta_n \Delta\phi_n{}^2 + \Delta\theta_n{}^2\right) + \frac{\Delta r_n{}^2}{1 - \frac{r_g}{r_n}} \tag{10}$$

$$\Delta \tau_n{}^2 = \left(1 - \frac{r_g}{r_n}\right) \Delta t_n{}^2 \tag{11}$$

$$\Delta \ell_n = k_n \lambda_n \tag{12}$$

$$\Delta \tau_n = k_n / \nu_n \tag{13}$$

where ℓ and τ are the distance and the time in a curved space-time, respectively, and k a positive integer; we obtain again that: $c = \Delta \ell_n / \Delta \tau_n = \lambda_n \nu_n$. And (8) and (9), and (12) and (13), would be the quantization rules for the vacuum space-time, for the electromagnetic and gravitational fields, respectively.

We have showed, in a simple manner, that the action at a distance is carried out by virtual carriers of the force, travelling to the speed of the light in the vacuum (it is not instantaneous), and that these virtual carriers are produced by the fields through the quantum vibrations of the vacuum space-time.

Appendix

From a real scalar field ϕ (using the Fourier transform)

$$\phi(\vec{x}) = \int \frac{d^3k}{(2\pi)^3} e^{i\vec{k}\cdot\vec{x}} \widetilde{\phi}(\vec{k}) \tag{1}$$

where x is the distance and k the wave number; and for

$$-\nabla^2 \phi(\vec{x}) = \delta^{(3)}(\vec{x}) \tag{2}$$

where δ is the Dirac delta function; we would have

$$-\nabla^2 \int \frac{d^3k}{(2\pi)^3} e^{i\vec{k}\cdot\vec{x}} \widetilde{\phi}(\vec{k}) = \delta^{(3)}(\vec{x})$$

$$-\int \frac{d^3k}{(2\pi)^3} \nabla^2 \left(e^{i\vec{k}\cdot\vec{x}}\right) \widetilde{\phi}(\vec{k}) = \delta^{(3)}(\vec{x})$$

$$-\int \frac{d^3k}{(2\pi)^3} i^2 \vec{k}^2 e^{i\vec{k}\cdot\vec{x}} \widetilde{\phi}(\vec{k}) = \delta^{(3)}(\vec{x})$$

$$\int \frac{d^3k}{(2\pi)^3} \vec{k}^2 e^{i\vec{k}\cdot\vec{x}} \widetilde{\phi}(\vec{k}) = \delta^{(3)}(\vec{x})$$

As

$$\delta^{(3)}(\vec{x}) = \int \frac{d^3k}{(2\pi)^3} e^{i\vec{k}\cdot\vec{x}} \tag{3}$$

then $\vec{k}^2 \widetilde{\phi}(\vec{k}) = 1$ and $\widetilde{\phi}(\vec{k}) = 1/\vec{k}^2$; and, from (1),

$$\phi(\vec{x}) = \int \frac{d^3k}{(2\pi)^3} \frac{e^{i\vec{k}\cdot\vec{x}}}{\vec{k}^2} \tag{4}$$

And substituting $\vec{k}$ by $\vec{p} = \hbar\vec{k}$ ($\vec{p} = \vec{k}$, for $\hbar = 1$), where $\vec{p}$ is the momentum,

$$\phi(\vec{x}) = \int \frac{d^3p}{(2\pi)^3} \frac{e^{i\vec{p}\cdot\vec{x}}}{\vec{p}^2} \tag{5}$$

From which

$$U(\vec{r}) = -\frac{e^2}{\varepsilon_0} \int \frac{d^3p}{(2\pi)^3} \frac{e^{i\vec{p}\cdot\vec{r}}}{\vec{p}^2} \tag{6}$$

where r is the radial distance; and from (2) and (6), $\nabla^2 U(\vec{r}) = (e^2/\varepsilon_0)\delta^{(3)}(\vec{r})$, and

$$U(\vec{r}) = -\frac{e^2}{4\pi\varepsilon_0 r} \tag{7}$$

which is the attractive Coulomb potential energy between an electron (with an electric charge $-e$) and a positron (with an electric charge $+e$); since

$$\nabla^2 \frac{1}{r} = -4\pi\delta^{(3)}(\vec{r}) \tag{8}$$

and $\nabla^2 U(\vec{r}) = \nabla^2(-e^2/4\pi\varepsilon_0 r) = (-e^2/4\pi\,\varepsilon_0)\nabla^2(1/r) = (e^2/\varepsilon_0)\delta^{(3)}(\vec{r})$. And the force would be $F = dU/dr = e^2/4\pi\varepsilon_0 r^2$.

[1] Donald H. Perkins, Introduction to High Energy Physics, p. 188, Addison-Wesley, Reading, Massachusetts, 1972.

[2] P. A. M. Dirac, Principios de Mecánica Cuántica, pp. 149 and 244, in Spanish, Ariel, Barcelona, 1967. Original edition, The Principles of Quantum Mechanics, Oxford University Press, 1958.

[3] L. D. Landau and E. M. Lifshitz, Teoría Clásica de los Campos, p. 398, in Spanish, Reverté, Barcelona, 1973. Original edition by Nauka, Moscow, 1967.