Three fundamental masses derived by dimensional analysis

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Abstract

Three new mass dimension quantities m_i have been derived by dimensional analysis, in addition to the Planck mass $m_P \sim \sqrt{\frac{\hbar c}{G}} \sim 2.17\times 10^{-8}$ *kg*. Four fundamental constants – the speed of light in vacuum (*c*), gravitational constant (*G*), reduced Plank constant ($\hbar$) and Hubble constant (*H*) have been involved in the dimensional analysis. The first derived mass dimension quantity $m_1 \sim \frac{c^3}{GH} \sim 10^{53}$ *kg* practically coincides with the Fred Hoyle formula for the mass of the universe obtained by totally different approach. The exceptionally small mass dimension quantity $m_2 \sim \frac{\hbar H}{c^2} \sim 10^{-33}$ *eV* has been identified with the mass of the hypothetical graviton. The third derived mass $m_3 \sim \sqrt[5]{\frac{H\hbar^3}{G^2}} \sim 10^7$ *GeV* hits in the mass interval of hypothetic very heavy particles like sterile neutrino but it could not be identified unambiguously at the present time. The identification of two found masses reinforces the trust in the suggested approach. Besides, the order of magnitude of the total density of the universe has been estimated by this approach.

Key words: dimensional analysis, Planck mass, mass of the universe, graviton mass

1. Introduction

The famous Planck mass $m_P \sim \sqrt{\frac{\hbar c}{G}}$ has been introduced in [1] by means of three fundamental constants – the speed of light in vacuum (*c*), gravitational constant (*G*) and the reduced Plank constant ($\hbar$). Since the constants *c*, *G* and $\hbar$ represent three very basic aspects of the universe (i.e. the relativistic, gravitational and quantum phenomena), the Plank mass appears to a certain degree a unification of these phenomena. The Plank mass have many important aspects in modern physics. One of them is that the energy equivalent of Planck mass $E_P = m_P c^2 \sim \sqrt{\frac{\hbar c^5}{G}} \sim 10^{19}$ *GeV* appears unification energy of the fundamental interactions [2]. As a result, at Planck temperature $T_P = \frac{m_P c^2}{k} \sim 10^{32} K$, all symmetries broken since the early Big Bang would be restored, and the four fundamental forces of contemporary physical theory would become one force. On the other hand, Planck particle is a hypothetical subatomic particle, defined as a tiny black hole whose Compton wavelength is the same as its Schwarzschild radius.

The Plank mass formula has been derived by dimensional analysis using fundamental constants *c*, *G* and $\hbar$. The dimensional analysis is a conceptual tool often applied in physics to understand physical situations involving certain physical quantities [3-5]. It is routinely used to check the plausibility of derived equations and computations. When it is known, the certain quantity with which other determinative quantities would be connected, but the form of this connection is unknown, the dimensional equation was composed for its finding. In the left side of the equation, the unit of this quantity q_0 with its dimensional exponent has placed. In the right side of the equation, the product of units of the determinative quantities q_i rise to the unknown exponents n_i has placed $[q_0] \sim \prod_{i=1}^{n} [q_i]^{n_i}$, where *n* is positive integer and the exponents n_i are rational numbers. The problem of determination of the unknown equation transforms to a problem of finding of exponents. Most often the dimensional analysis has applied in the mechanics and other topics of the modern physics where there are many problems having a few determinative quantities. Also, the Planck mass can be approximately derived by setting it as the mass whose Compton wavelength and Schwartzchild radius are equal [6].

The discovery of the linear relationship between recessional velocity of distant galaxies, and distance $v = Hr$ [7] introduces new fundamental constant in physics and cosmology – the famous Hubble constant (*H*). Even seven years before, Friedman [8] derived his equations from the Einstein field equations [9], showing that the universe might expand at a rate calculable by the equations. Hubble constant determines age of the universe H^{-1}, Hubble distance cH^{-1}, critical density of the universe $\rho_c = \frac{3H^2}{8\pi G}$ [10], and other large-scale properties of the universe.

Because of the importance of the Hubble constant, in the present paper we include *H* in dimensional analysis together with *c*, *G* and $\hbar$ aiming to find the new mass dimension quantities $m_i \sim \prod_{j=1}^{3} q_j^{n_j}$, where every triad q_1, q_2, q_3 consists of three constants *c*, *G*, $\hbar$ and *H*. Thus, the Hubble constant will represent cosmological phenomena in derived new fundamental masses.

2. Three fundamental masses derived by dimensional analysis

2a. Fundamental mass derived by means of *c, G* and *H*

Below, we obtain a mass dimension quantity m_1 constructed from the fundamental constants – the speed of light (*c*), gravitational constant (*G*) and Hubble constant (*H*) using dimensional analysis. A quantity m_1 having dimension of mass could be constructed by means of the fundamental constants *c*, *G* and *H*:

$$m_1 = kc^{n_1} G^{n_2} H^{n_3} \quad (1),$$

where n_1, n_2 and n_3 are unknown exponents to be determined by matching the dimensions of both sides of the equation and *k* is dimensionless parameter of the order of magnitude of unit. Using the symbol *L* for length, *T* for time, *M* for mass, and writing "[*x*]" for the dimensions of some physical quantity *x*, we have the following:

$$\begin{aligned}&[c]=LT^{-1}\\&[G]=M^{-1}L^{3}T^{-2}\\&[H]=T^{-1}\end{aligned} \qquad (2)$$

The dimensions of the left and right sides of the equation (1) must be equal. Therefore:

$$[m_1]=[c]^{n_1}[G]^{n_2}[H]^{n_3} \qquad (3)$$

Taking into account the dimensions of quantities in formula (3) we obtain:

$$L^0T^0M^1=(LT^{-1})^{n_1}(L^3T^{-2}M^{-1})^{n_2}(T^{-1})^{n_3}=L^{n_1+3n_2}T^{-n_1-2n_2-n_3}M^{-n_2} \qquad (4)$$

From (4) we find the system of linear equations:

$$\begin{aligned}&n_1+3n_2=0\\&-n_1-2n_2-n_3=0\\&-n_2=1\end{aligned} \qquad (5)$$

The determinant Δ of the system is:

$$\Delta=\begin{vmatrix}1&3&0\\-1&-2&-1\\0&-1&0\end{vmatrix}=-1 \qquad (6)$$

The determinant $\Delta\neq 0$. Therefore, the system has a unique solution. We find this solution by Kramer's formulae (7):

$$\begin{aligned}&n_1=\frac{\Delta_1}{\Delta}\\&n_2=\frac{\Delta_2}{\Delta}\\&n_3=\frac{\Delta_3}{\Delta}\end{aligned} \qquad (7),$$

where $\Delta_1=\begin{vmatrix}0&3&0\\0&-2&-1\\1&-1&0\end{vmatrix}$ = - 3, $\Delta_2=\begin{vmatrix}1&0&0\\-1&0&-1\\0&1&0\end{vmatrix}=1$ and $\Delta_3=\begin{vmatrix}1&3&0\\-1&-2&0\\0&-1&1\end{vmatrix}=1$.

Therefore, the exponents $n_1=3, n_2=-1, n_3=-1$. Replacing obtained values of exponents in equation (1) we find formula (8) for the mass m_1:

$$m_1\sim\frac{c^3}{GH} \qquad (8)$$

First of all, the formula (8) has been derived by dimensional analysis in [11]. This formula practically coincides with Fred Hoyle formula for the mass of the universe $M = \frac{c^3}{2GH}$ [12] obtained by totally different approach.

The recent experimental values of *c*, *G* and *H* are used - $c = 299\ 792\ 458\ m\ s^{-1}$, $G = 6.673 \times 10^{-11}\ m^3\ kg^{-1}\ s^{-2}$ [13] and $H \approx 70\ km\ s^{-1}\ Mps^{-1}$ [14]. Replacing this values in (8) we obtain $m_1 \sim 1.76 \times 10^{53}$ *kg*. Therefore, the enormous mass m_1 would be identified with mass of the observable universe.

2b. Fundamental mass derived by means of c, $\hbar$ and *H*

Analogously, by means of the fundamental constants *c*, $\hbar$ and *H*, a quantity m_2 having dimension of mass could be constructed:

$$m_2 = kc^{n_1} \hbar^{n_2} H^{n_3} \tag{9}$$

We determine unknown exponents n_1, n_2, n_3 by dimensional analysis again. Matching the dimensions of both sides of equation (9) we find:

$$L^0 T^0 M^1 = (LT^{-1})^{n_1} (ML^2 T^{-1})^{n_2} (T^{-1})^{n_3} = L^{n_1 + 2n_2} T^{-n_1 - n_2 - n_3} M^{n_2} \tag{10}$$

From (10) we find the system of linear equations:

$$\begin{aligned} &n_1 + 2n_2 = 0 \\ &-n_1 - n_2 - n_3 = 0 \\ &n_2 = 1 \end{aligned} \tag{11}$$

The determinant of the system is $\Delta = \begin{vmatrix} 1 & 2 & 0 \\ -1 & -1 & -1 \\ 0 & 1 & 0 \end{vmatrix} = 1 \neq 0$. Therefore, the system has a unique solution, which we obtain by the Kramer's formulae again:

$$\begin{aligned} n_1 &= \frac{\Delta_1}{\Delta} = -2 \\ n_2 &= \frac{\Delta_2}{\Delta} = 1 \\ n_3 &= \frac{\Delta_3}{\Delta} = 1 \end{aligned} \tag{12}$$

Replacing obtained values of the exponents in equation (9) we find formula (13) for the mass m_2:

$$m_2 \sim \frac{\hbar H}{c^2} \tag{13}$$

Replacing the recent values of the constants c, $\hbar$ and H in (13) we obtain $m_2 \sim 2.70\times10^{-69}$ kg $=1.52\times10^{-33}$ eV. This exceptionally small mass coincides with so called "Hubble mass" [15, 16], which is close to the mass of the hypothetical graviton m_g estimated in [17, 18]. From equation (13) we find that the reduced Compton wavelength $\bar{\lambda}_2$ of this mass is equal to the Hubble distance cH^{-1}:

$$\bar{\lambda}_2 = \frac{\hbar}{m_2 c} = cH^{-1} \sim 1.3\times10^{26}\, m \tag{14}$$

2c. Fundamental mass derived by means of *G*, $\hbar$ and *H*

The third quantity m_3 having mass dimension could be constructed by means of the fundamental constants G, $\hbar$ and H:

$$m_3 = kG^{n_1}\hbar^{n_2}H^{n_3} \tag{15}$$

Matching the dimensions of both sides of equation (15) we find the system of linear equations:

$$\begin{aligned} &3n_1 + 2n_2 = 0 \\ &-2n_1 - n_2 - n_3 = 0 \\ &-n_1 + n_2 = 1 \end{aligned} \tag{16}$$

The determinant of the system is $\Delta = \begin{vmatrix} 3 & 2 & 0 \\ -2 & -1 & -1 \\ -1 & 1 & 0 \end{vmatrix} = 5 \neq 0$. Therefore, the system has a unique solution, which we obtain by Kramer's formulae:

$$\begin{aligned} n_1 &= \frac{\Delta_1}{\Delta} = -\frac{2}{5} \\ n_2 &= \frac{\Delta_2}{\Delta} = \frac{3}{5} \\ n_3 &= \frac{\Delta_3}{\Delta} = \frac{1}{5} \end{aligned} \tag{17}$$

Replacing the obtained values of the exponents in equation (15) we find formula (18) for the mass m_3:

$$m_3 \sim \sqrt[5]{\frac{H\hbar^3}{G^2}} \tag{18}$$

Replacing the recent values of the constants G, $\hbar$ and H, the mass m_3 takes value m_3 ~ 1.43×10^{-20} $kg \approx 8.0\times 10^{6}$ GeV. This mass is a dozen orders of magnitude lighter than the Planck mass and several orders of magnitude heavier than the heaviest known particles like the top quark $m_t \approx 174.3$ GeV [19]. According to seasaw models, a small neutrino mass is induced by mixing between an active neutrino and a heavy Majorana sterile neutrino, whose mass M_N ranges from TeV scale to grand unification scale ~ 10^{15} GeV. Thus, a possible candidate having mass m_3 appears the hypothetical sterile neutrino. At the present time, the mass m_3 could not yet be unambiguously identified.

3. Discussions and conclusions

Three new mass dimension quantities have been found by dimensional analysis, in addition to the Planck mass $m_P \sim \sqrt{\frac{\hbar c}{G}} \approx 2.17\times 10^{-8}$ kg. These masses have been derived by means of fundamental constants c, G, $\hbar$ and H. The mass dimension quantity $m_1 \sim \frac{c^3}{GH} \sim 10^{53}$ kg has been identified with the mass of the observable universe. The mass dimension quantity $m_2 \sim \frac{\hbar H}{c^2} \sim 10^{-33}$ eV has been identified with the mass of the hypothetical graviton. The third derived mass $m_3 \sim \sqrt[5]{\frac{H\hbar^3}{G^2}} \sim 10^{7}$ GeV hits in the mass interval of hypothetic very heavy particles like sterile neutrino but it could not yet be unambiguously identified at the present time.

Below, we demonstrate the heuristic value of the suggested approach deriving the total density of the universe by dimensional analysis. Actually, a quantity ρ having dimension of density could be constructed by means of the fundamental constants c, G and H:

$$\rho = kc^{n_1}G^{n_2}H^{n_3} \tag{19}$$,

where k is a dimensionless parameter of the order of magnitude of unit.

By dimensional analysis, we have found the exponents $n_1 = 0, n_2 = -1, n_3 = 2$. Therefore:

$$\rho \sim \frac{H^2}{G} \approx 7.93\times 10^{-26}\, kg\, m^{-3} \tag{20}$$

The recent Cosmic Microwave Background (CMB) observations show that the total density of the universe $\overline{\rho}$ is [20-22]:

$$\overline{\rho} = \Omega\rho_c \approx \rho_c = \frac{3H^2}{8\pi G} \sim 10^{-26}\, kg\, m^{-3} \tag{21}$$

Evidently, the density dimension quantity ρ, found by dimensional analysis, differs from the total density of the universe with the dimensionless parameter $k = \frac{3}{8\pi} \approx 0.12$ of the order of magnitude of unit. Besides, the formula (20) could be derived by means of other triad of fundamental constants, namely G, $\hbar$ and H.

According to the Big Bang cosmology, the Hubble constant decreases with the age of the universe. Therefore, in the framework of the Big Bang cosmology, the mass of the universe $m_1 \sim \frac{c^3}{GH}$ slowly increases, whereas the graviton mass $m_2 \sim \frac{\hbar H}{c^2}$ and mass $m_3 \sim \sqrt[5]{\frac{H\hbar^3}{G^2}}$ decrease with time. It is well known that according to the Steady State theory [23] the mass of the universe slowly increases with time. On the other hand, according to the Tired Light model [24], the Hubble constant H is truly a constant, not only in all directions, but all the time. Therefore, the derived fundamental masses are truly constants in the framework of the Tired Light model.

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