Reduced Total Energy Requirements For The Original Alcubierre and Natario Warp Drive Spacetimes-The Role Of Warp Factors.

Fernando Loup *
Residencia de Estudantes Universitas Lisboa Portugal

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Abstract

Warp Drives are solutions of the Einstein Field Equations that allows Superluminal Travel within the framework of General Relativity. There are at the present moment two known solutions: The Alcubierre Warp Drive discovered in 1994 and the Natario Warp Drive discovered in 2001. However as stated by both Alcubierre and Natario themselves the Warp Drive violates all the known energy conditions because the stress energy momentum tensor(the right side of the Einstein Field Equations) for the Einstein tensor G_{00} is negative implying in a negative energy density. While from a classical point of view the negative energy is forbidden the quantum theory allows the existence of very small amounts of it being the Casimir effect a good example as stated by Alcubierre himself. But the stress energy momentum tensor of both Alcubierre and Natario Warp Drives have the speed of the ship raised to the square inside its mathematical structure which means to say that as fast the ship goes by then more and more amounts of negative energy are needed in order to maintain the Warp Drive. Since the total energy requirements to maintain the Warp Drive are enormous and since quantum theory only allows small amounts of it,many authors regarded the Warp Drive as unphysical and impossible to be achieved. We compute the negative energy density requirements for a Warp Bubble with a radius of 100 meters(large enough to contain a ship) moving with a speed of 200 times light speed(fast enough to reach stars at 20 light-years away in months not in years)and we verify that the negative energy density requirements are of about 10^{28} times the positive energy density of Earth!!!(We multiply the mass of Earth by c^2 and divide by Earth volume for a radius of $6300km$). However both Alcubierre and Natario Warp Drives as members of the same family of the Einstein Field Equations requires the so-called Shape Functions in order to be mathematically defined. We present in this work two new Shape Functions one for the Alcubierre and another for the Natario Warp Drive Spacetimes that allows arbitrary Superluminal speeds while keeping the negative energy density at "low" and "affordable" levels.We do not violate any known law of quantum physics and we maintain the original geometries of both Alcubierre and Natario Warp Drive Spacetimes.

*spacetimeshortcut@yahoo.com

1 The Problem of the Negative Energy in both Alcubierre and Natario Warp Drive Spacetimes-The Unphysical Nature of Warp Drive

The Einstein Field Equation of General Relativity without the Cosmological term is given by the following expression

$$G_{pq} = \frac{8\pi G}{c^4} T_{pq} \tag{1}$$

For the negative energy we need to compute only the following component:

$$G_{00} = \frac{8\pi G}{c^4} T_{00} \tag{2}$$

In the equation above G_{pq} is the Einstein tensor related to the curvature of the Spacetime G is the Gravitational Constant $(6,67 \times 10^{-11} \frac{Nm^2}{kg^2}$ c is the light speed $(3 \times 10^8 m/s)$ and T_{pq} is the stress energy momentum tensor associated to the Spacetime curvature.In order to generate a Spacetime curvature we need to generate first the matter-energy density associated to such a curvature.Remember that Mass curves the Spacetime.

Using Einstein words "Matter tells Spacetime how to curve and Spacetime tells Matter how to behave".

Some authors often writes the equation in Geometrized Units $c = G = 1$ and the equation in this case becomes:

$$G_{pq} = 8\pi T_{pq} \tag{3}$$

$$G_{00} = 8\pi T_{00} \tag{4}$$

Writing the Einstein Field Equation to compute the stress energy momentum tensor associated to a given Spacetime curvature we have:

$$T_{pq} = \frac{c^4}{8\pi G} G_{pq} \tag{5}$$

$$T_{00} = \frac{c^4}{8\pi G} G_{00} \tag{6}$$

Note that $\frac{c^4}{G}$ is a huge number of about $10^{32}/10^{-11} = 10^{43}$ also known as the Planck number. This means to say that a large concentration of mass-energy only generates small amounts of Spacetime curvature.

Writing now the stress energy momentum tensor for the energy density in both Alcubierre and Natario Warp Drive Spacetimes;

- 1)-Negative Energy Density in the Alcubierre Warp Drive(pg 4 in [2])(pg 8 in [1]):

$$\rho = -\frac{1}{32\pi} vs^2 \left[f'(rs)\right]^2 [\frac{y^2 + z^2}{rs^2}] \tag{7}$$

$$\rho = -\frac{1}{32\pi} vs^2 \left[\frac{df(rs)}{drs}\right]^2 [\frac{y^2 + z^2}{rs^2}] \tag{8}$$

These are the original Alcubierre expressions of 1994 written in Geometrized Units.Converting to normal units we would have[1]:

$$\rho = -\frac{c^2}{G}\frac{1}{32\pi}vs^2\left[f'(rs)\right]^2\left[\frac{y^2+z^2}{rs^2}\right] \tag{9}$$

$$\rho = -\frac{c^2}{G}\frac{1}{32\pi}vs^2\left[\frac{df(rs)}{drs}\right]^2\left[\frac{y^2+z^2}{rs^2}\right] \tag{10}$$

In the expression above vs is the speed of a hypothetical Warp Drive starship,rs is the distance travelled by an Eulerian observer from the center of the Warp Drive Bubble to the Warp Drive Bubble Walls and $f(rs)$ is the Alcubierre Shape Function.For more details about the geometrical features of the Alcubierre Warp Drive see pg 4 in [1].

$$rs = \sqrt{(x-xs)^2+y^2+z^2} \tag{11}$$

$$f(rs) = \frac{tanh[@(rs+R)] - tanh[@(rs-R)]}{2tanh(@R)} \tag{12}$$

xs is the center of the Warp Drive Bubble where the ship resides.

R is the radius of the Warp Bubble and @ is the Alcubierre parameter related to the thickness.According to Alcubierre these can have arbitrary values. This is very important as we will see later in this work

The Shape Function $f(rs)$ have a value of 1 inside the Warp Bubble and zero outside the Warp Bubble while being $0 < f(rs) < 1$ in the Warp Bubble Walls.Since $f(rs)$ is always 1 inside the Warp Bubble and zero outside the Warp Bubble we need to take only the derivatives where $f(rs)$ varies from 1 to zero which means to say that we take the derivatives of $f(rs)$ in the region where $0 < f(rs) < 1$.This region is known as the Alcubierre Warped region.

Note that for a Superluminal Warp Drive ship the speed vs appears in the expression of the negative energy density raised to a power of 2. Imagine that we have a Warp Ship moving at a Superluminal speed $vs = 200$ which means to say 200 times light speed in order to make a round trip from Earth to a nearby star at 20 light-years away in a reasonable amount of time(in months not in years). So in the expression of the negative energy we have the factor $c^2 = (3 \times 10^8)^2 = 9 \times 10^{16}$ being divided by $6,67 \times 10^{-11}$ giving $1,35 \times 10^{27}$ and this is multiplied by $(6 \times 10^{10})^2 = 36 \times 10^{20}$ coming from the term $vs = 200$ giving $1,35 \times 10^{27} \times 36 \times 10^{20} = 1,35 \times 10^{27} \times 3,6 \times 10^{21} = 4,86 \times 10^{48}$!!!

A number with 48 zeros[2]!!!!

Note also that we must integrate this factor for the negative energy density over the volume of the Warp Bubble and a Bubble large enough to contains a starship inside must at least have a radius of 100 meters or more and an Alcubierre parameter of about 20 meters giving a thickness of 2 meters[3].

[1]Note that we uses c^2 and not c^4.See the Appendix on Dimensional Reduction

[2]This is a factor.We are interested in the huge numerical magnitude only

[3]This will be discussed later in our Microsoft Excel simulations

So we are facing this scenario:

- 1)-$\frac{c^2}{G}$ in normal units raises the negative energy density requirements by the enormous factor of 10^{27} while quantum theory only allows the existence of very small amounts of it
- 2)-as fast the Warp Drive starship moves the negative energy density requirements becomes even more bigger due to the term vs^2
- 3)-a Warp Bubble must be large enough to contains a starship inside,Then we are integrating at least a negative energy density with a factor of 10^{48} by a sphere about 100 meters of radius

Our Earth have a mass of about $6 \times 10^{24} kg$ and multiplying this by c^2 in order to get the total positive energy "stored" in the Earth according to the Einstein equation $E = mc^2$ we would find the value of $54 \times 10^{40} = 5,4 \times 10^{41} Joules$.

Earth have a positive energy of $10^{41} Joules$ and dividing this by the volume of the Earth(radius $R_{Earth} =$ 6300 km approximately) we would find the positive energy density of the Earth.Taking the radius of the Earth $(6300000m)^3 = 2,5 \times 10^{20}$ and dividing $5,4 \times 10^{41}$ by $(4/3)\pi R_E arth^3$ we would find the value of $4,77 \times 10^{20} \frac{Joules}{m^3}$. So Earth have a positive energy density of $4,77 \times 10^{20} \frac{Joules}{m^3}$ and we are talking about negative energy densities with a factor of 10^{48} for the Warp Drive while the quantum theory allows only microscopical amounts of negative energy density.

So we would need to generate in order to maintain a Warp Drive with 200 times light speed the negative energy density equivalent to the positive energy density of 10^{28} Earths!!!!

A number with 28 zeros!!!.Unfortunately we must agree with the major part of the scientific community that says:"Warp Drive is impossible!!"

Et Voila The Unphysical Nature Of The Warp Drive

Our scenario look too bad for the Warp Drive

But look again to the expression of the negative energy density:

$$\rho = -\frac{c^2}{G}\frac{1}{32\pi}vs^2\left[f'(rs)\right]^2[\frac{y^2+z^2}{rs^2}] \tag{13}$$

$$\rho = -\frac{c^2}{G}\frac{1}{32\pi}vs^2\left[\frac{df(rs)}{drs}\right]^2[\frac{y^2+z^2}{rs^2}] \tag{14}$$

In order to reduce the total energy density requirements for the Alcubierre Warp Drive we need to work with very low derivatives of the Shape Function since we cannot overcome the other terms (c,G and vs)

For the Alcubierre Shape Function

$$f(rs) = \frac{tanh[@(rs+R)] - tanh[@(rs-R)]}{2tanh(@R)} \tag{15}$$

the derivative is:

$$f'(rs) = \frac{1}{2tanh(@R)}[\frac{@}{cosh^2[@(rs+R)]} - \frac{@}{cosh^2[@(rs-R)]}] \tag{16}$$

For a Warp Bubble of radius $R = 100$ meters and an Alcubierre parameter $@ = 20$ meters giving a thickness of 2 meters, the factors of $\varepsilon^{[@(rs+R)]}$ are very large numbers because we are raising $2,718^{20\times100} = 2,718^{2000}$ even when $rs = 0$ the center of the Warp Bubble.This number is enormous.On the other hand the factors of $\varepsilon^{[-@(rs+R)]} = \frac{1}{\varepsilon^{[@(rs+R)]}} = \frac{1}{2,718^{2000}}$ and this factor reduces to zero.Hence the term in $cosh^2[@(rs+R)]$ reduces to $\varepsilon^{[@(rs+R)]}$ and dividing an Alcubierre parameter $@ = 20$ by $2,718^{2000}$ will reduces this term to zero so the term in $cosh^{-2}[@(rs+R)]$ can be neglected.Then the derivative really accounts for:

$$f'(rs) = \frac{1}{2tanh(@R)}[-\frac{@}{cosh^2[@(rs-R)]}] \tag{17}$$

its square would then be:

$$f'(rs)^2 = \frac{1}{4tanh^2(@R)}[\frac{@^2}{cosh^4[@(rs-R)]}] \tag{18}$$

We mentioned before that we need to take the derivatives of the Alcubierre Shape Function in the Alcubierre Warped Region which means to say the region where $0 < f(rs) < 1$.A single plot in Microsoft Excel for a Warp Bubble radius $R = 100$ and an Alcubierre parameter $@ = 20$ shows that $f(rs)$ is always 1 from $rs = 0$ the center of the Bubble to $rs = 99$ meters.Between $rs = 99$ and $rs = 100$ meters the $f(rs)$ falls from 1 to $0,5$ with $f(rs) = 0,5$ when $rs = 100$ meters and reaches the value of 0 at $rs = 101$ meters maintaining the value of 0 as rs moves farther away from R in agreement with the Alcubierre dynamics that says $f(rs) = 1$ inside the Warp Bubble,$f(rs) = 0$ outside the Warp Bubble and $0 < f(rs) < 1$ in the Warp Bubble Walls.The region between 99 and 101 meters is the thickness of the Warp Bubble and have a value of 2 meters.When $rs = R$ the square of the derivative of the Alcubierre Shape Function becomes

$$f'(rs)^2 = \frac{1}{4tanh^2(@R)}@^2 \tag{19}$$

it is easy to see that for a Warp Bubble of large radius and Alcubierre parameter @ ,the term $tanh^2(@R) = 1$ and when $rs = R$ we are left with:

$$f'(rs)^2 = \frac{1}{4}@^2 \tag{20}$$

Note that this derivative square have a value of 100 when $rs = R$ because $20^2 = 400$ for an Alcubierre parameter of $@ = 20$ meters.

Multiplying 10^{48} by 100 will give an incredible amount of negative energy density requirements of 10^{50} which means to say a negative energy density of 10^{30} times the energy density of the Earth rendering the Alcubierre Warp Drive impossible and Unphysical.

Unless we can find a different Shape Function to ameliorate these huge negative energy densities.

- 2)-Negative Energy Density in the Natario Warp Drive-(pg 5 in [2])

$$\rho = -\frac{vs^2}{8\pi}\left[3(n'(rs))^2\cos^2\theta + \left(n'(rs) + \frac{rs}{2}n''(rs)\right)^2\sin^2\theta\right]. \tag{21}$$

$$\rho = -\frac{vs^2}{8\pi}\left[3(\frac{dn(rs)}{dr})^2\cos^2\theta + \left(\frac{dn(rs)}{dr} + \frac{rs}{2}\frac{d^2n(rs)}{dr^2}\right)^2\sin^2\theta\right]. \tag{22}$$

Above are the original Natario expressions of 2001 written in Geometrized Units.Converting to normal units we would have[4]:

$$\rho = -\frac{c^2}{G}\frac{vs^2}{8\pi}\left[3(n'(rs))^2\cos^2\theta + \left(n'(rs) + \frac{rs}{2}n''(rs)\right)^2\sin^2\theta\right]. \tag{23}$$

$$\rho = -\frac{c^2}{G}\frac{vs^2}{8\pi}\left[3(\frac{dn(rs)}{dr})^2\cos^2\theta + \left(\frac{dn(rs)}{dr} + \frac{rs}{2}\frac{d^2n(rs)}{dr^2}\right)^2\sin^2\theta\right]. \tag{24}$$

Note that the terms in $\frac{c^2}{G}vs^2$ rendering the factor of 10^{48} and in consequence Natario Warp Drive unphysical also appears here and this is a natural and expected consequence of both Natario and Alcubierre Warp Drive Spacetimes being members of the same family of solutions of the Einstein Field Equations.

The term $n(rs)$ is the Natario Shape Function defined by:(eqs 74 and 75 pg 13 in [4])(eqs 74 and 75 pg 13 in [3])

$$n(rs) = \frac{1}{2}[1 - f(rs)] \tag{25}$$

$$n(rs) = \frac{1}{2}[1 - \frac{tanh[@(rs+R)] - tanh[@(rs-R)]}{2tanh(@R)}] \tag{26}$$

The Natario Shape Function is defined as being $n(rs) = 0$ inside the Warp Bubble and $n(rs) = \frac{1}{2}$ outside the Warp Bubble while being $0 < n(rs) < \frac{1}{2}$ in the Natario Warped Region.(see pg 5 in [2])(see also Section 3 in [3] and [4] for an explanation about how the Natario Warp Drive geometry at Superluminal speeds works).

Note that we used the Alcubierre Shape Function to define the Natario Shape Function and this is a proof that both Warp Drives belongs to the same family of solutions of the Einstein Field Equations of General Relativity.

It is easy to figure out when $f(rs) = 1$(interior of the Alcubierre Bubble) then $n(rs) = 0$(interior of the Natario Bubble) and when $f(rs) = 0$(exterior of the Alcubierre Bubble)then $n(rs) = \frac{1}{2}$(exterior of the Natario Bubble)

Then we can see that the Natario Warp Drive Bubble can be defined in a way almost similar to the Alcubierre Warp Drive Bubble[5].

The Natario Warp Drive must also have a radius R, an Alcubierre parameter @ and an Eulerian observer travelling from the center of the Natario Bubble(which lies also at xs) will travel a distance rs to approach the Warp Bubble Walls in a way similar to the one of the Alcubierre Warp Drive.

It is easy to see that the derivatives of the Natario Shape Function are given by:

[4]Note that we uses c^2 and not c^4.See the Appendix on Dimensional Reduction

[5]However the Natario Warp Bubble is different than the Alcubierre one.See Sections 2 and 3 in [3] and [4]

$$n'(rs) = -\frac{1}{2}f'(rs) \quad (27)$$

$$n''(rs) = -\frac{1}{2}f''(rs) \quad (28)$$

The square of the derivatives are

$$n'(rs)^2 = \frac{1}{4}f'(rs)^2 \quad (29)$$

$$n''(rs)^2 = \frac{1}{4}f''(rs)^2 \quad (30)$$

Plotting a simulation in Microsoft Excel for a Natario Warp Bubble with a radius $R = 100$ meters and an Alcubierre parameter of $@ = 20$ meters when $rs = 0$ then $n(rs) = 0$ and $n(rs) = 0$ from $rs = 0$ meters to $rs = 99$ meters.At $rs = 99$ meters $n(rs)$ starts to grow reaching the value of $n(rs) = 0,25$ at $rs = 100$ meters and when $rs = 101$ meters $n(rs) = \frac{1}{2}$ maintaining the same value of $\frac{1}{2}$ as rs moves farther away from R.This satisfies the Natario requirements for a $n(rs) = 0$ inside the Bubble,$n(rs) = \frac{1}{2}$ outside the Bubble while being $0 < n(rs) < \frac{1}{2}$ in the Bubble Walls(Natario Warped Region). The thickness of the Warp Bubble is 2 meters.

Note that if $f(rs)$ is the same of the Alcubierre Warp Drive then all our results concerning $cosh[@(rs+R)]$ and $cosh[@(rs-R)]$ remains valid and we need to take the derivatives of $n(rs)$ only when $n(rs)$ varies which means to say the Natario Warped Region.

If all our results for the Alcubierre $f(rs)$ holds then when $rs = R$ as previously seen before we have $cosh[@(rs-R)] = 1$ and we have our eq 20 divided by $\frac{1}{4}$ giving the value of 25.

Neglecting for a while the term in $sin(\theta)$ in the Natario Energy Density we have the factor of 10^{48} multiplied by $75 \times cos(\theta)^2$.Since the term in $sin(\theta)^2$ is in an addiction then independently of the value of the derivative of second order of the Natario Shape Function $n(rs)$ we are left with an unphysical term of $10^{49} \times cos(\theta)^2$ which means at least the energy density of 10^{29} Earths!!!

Just like in the Alcubierre case the Natario Warp Drive is unphysical and impossible to be achieved.Unless we can find different Shape Functions to ameliorate these Negative Energy Density requirements and with very low derivatives since the terms c G and vs cannot be overruled.

2 Reduced Total Energy Requirements For The Original Alcubierre and Natario Warp Drive Spacetimes-The Role Of Warp Factors

- 1)-The Negative Energy Density in the Natario Warp Drive is given by(pg 5 in [2]):

$$\rho = -\frac{c^2}{G}\frac{vs^2}{8\pi}\left[3(n'(rs))^2\cos^2\theta + \left(n'(rs) + \frac{rs}{2}n''(rs)\right)^2\sin^2\theta\right]. \tag{31}$$

$$\rho = -\frac{c^2}{G}\frac{vs^2}{8\pi}\left[3(\frac{dn(rs)}{dr})^2\cos^2\theta + \left(\frac{dn(rs)}{dr} + \frac{rs}{2}\frac{d^2n(rs)}{dr^2}\right)^2\sin^2\theta\right]. \tag{32}$$

Being the Natario Shape Function defined as(eqs 74 and 75 pg 13 in [4])(eqs 74 and 75 pg 13 in [3]):

$$n(rs) = \frac{1}{2}[1 - f(rs)] \tag{33}$$

$$n(rs) = \frac{1}{2}[1 - \frac{tanh[@(rs+R)] - tanh[@(rs-R)]}{2tanh(@R)}] \tag{34}$$

The Natario Shape Function is defined as being $n(rs) = 0$ inside the Warp Bubble and $n(rs) = \frac{1}{2}$ outside the Warp Bubble while being $0 < n(rs) < \frac{1}{2}$ in the Natario Warped Region.(see pg 5 in [2]).

Then in the Natario Warped Region $n(rs)$ is a fractionary number and for a Warp Bubble of radius $R = 100$ meters and an Alcubierre parameter $@ = 20$ meters when $rs = 100$ meters,$n(rs)$ possesses the value of $n(rs) = 0,25$ and raising $0,25$ to a power of 2 we will get the value of $0,06$,raising $0,25$ to a power of 3 we will get the value of $0,02$ and raising $0,25$ to a power of 4 we will get the value of $3,91 \times 10^{-3}$.Note that these values are smaller than the original $0,25$ so raising the Shape Function to an arbitrary power we will get a number smaller than the original value of the Shape Function.Arbitrary powers of the Shape Function will generate arbitrary small numbers able to cope with the factor 10^{48} because we are raising to powers a fractionary number.

- 2)-The Negative Energy Density in the Alcubierre Warp Drive is given by(pg 4 in [2])(pg 8 in [1]):

$$\rho = -\frac{c^2}{G}\frac{1}{32\pi}vs^2\left[f'(rs)\right]^2[\frac{y^2+z^2}{rs^2}] \tag{35}$$

$$\rho = -\frac{c^2}{G}\frac{1}{32\pi}vs^2\left[\frac{df(rs)}{drs}\right]^2[\frac{y^2+z^2}{rs^2}] \tag{36}$$

Being the Alcubierre Shape Function defined as(pg 4 in [1]):

$$f(rs) = \frac{tanh[@(rs+R)] - tanh[@(rs-R)]}{2tanh(@R)} \tag{37}$$

The Shape Function $f(rs)$ have a value of 1 inside the Warp Bubble and zero outside while being $0 < f(rs) < 1$ in the Warp Bubble Walls.(pg 4 in [1])

Note that inside the Alcubierre Warped Region $f(rs)$ is also a fractionary number and what was said for the Shape Function in the Natario Warped Region is also valid for the Alcubierre Warped Region

We already know that the term $-\frac{c^2}{G}vs^2$ implies in a negative energy density requirements with a factor possessing a magnitude of 10^{48} or at least 10^{28} times the positive energy density of Earth and this terms cannot be overcomed.We know also that quantum physics allows only the existence of small microscopical amounts of negative energy density while we need enormous quantities of it.We cannot overrule the principles of quantum mechanics and we do not want to modify the original geometry of the Alcubierre and Natario Warp Drive Spacetimes.

How can this problem be solved??

Starting with the Natario Warp Drive(we will examine the Alcubierre case later):

We can low the derivatives of the Natario Shape Function to levels so close to zero that these levels will absorb the term 10^{48} resulting in a Negative Energy Density very low and compatible with the one of the quantum physics allowing ourselves to restore the physical feasibility of the Natario Warp Drive.

Without violating any known laws of quantum physics we will introduce here the Scale of Warp Factors

But first we must remind ourselves that Alcubierre defined the radius R and the Alcubierre parameters @ as arbitrary parameters(see pg 4 in [1]) and Natario defined the Natario Shape Function as being 0 in the ship and $\frac{1}{2}$ far from it (see pg 5 in [2])but note that the original Natario work of 2001 did not presented an algebraic expression for the Natario Shape Function because the 2001 work was conceived as a generic work to show that any function that gives 0 in the ship and $\frac{1}{2}$ far from it can performs as a Natario Shape Function. Natario included generic expressions of Extrinsic Curvatures and Negative Energy Density.

We must keep in mind these important points of view before we can proceed:

- 1)-Natario wrote his work in a generic form where any function that gives 0 in the ship and $\frac{1}{2}$ far from it can perform as a Natario Shape Function independently of the form of the function
- 2)-Alcubierre defined the radius R and the thickness @ as arbitrary parameters

We are now ready to introduce the new Natario Shape Function using a scale of Warp Factors

In our Microsoft Excel plot of both Alcubierre and Natario Shape Functions in the same worksheet we introduced another arbitrary numerical parameter like Alcubierre did in 1994 for R and @:

We introduced a Warp Factor WF as a dimensionless parameter and the Natario Shape Function is raised to a power of this Warp Factor.

Note that when we redefine the Natario Shape Function raised to a power of a Warp Factor WF as follows :

$$n(rs) = [\frac{1}{2}][1 - f(rs)]^{WF} \tag{38}$$

the new Natario Shape Function gives the following results:

- 1)-When $f(rs) = 1$(inside the Alcubierre Bubble) then $[1 - f(rs)]^{WF} = 0^{WF} = 0$ and $n(rs) = [\frac{1}{2}][1 - f(rs)]^{WF} = 0$ (inside the Natario Bubble)
- 2)-When $f(rs) = 0$(outside the Alcubierre Bubble) then $[1 - f(rs)]^{WF} = 1^{WF} = 1$ and $n(rs) = [\frac{1}{2}][1 - f(rs)]^{WF} = \frac{1}{2}$ (outside the Natario Bubble)

Note that this function is valid as a Natario Shape Function
Its derivative becomes:

$$n'(rs) = -[\frac{1}{2}]WF[1 - f(rs)]^{WF-1}f'(rs) \tag{39}$$

its square becomes

$$n'(rs)^2 = [\frac{1}{4}]WF^2[1 - f(rs)]^{2(WF-1)}f'(rs)^2 \tag{40}$$

Note that inside the Alcubierre or Natario Warped Region[6]the term $1 - f(rs)$ is a fractionary number which means to say that $0 < 1 - f(rs) < 1$ and $[1 - f(rs)]^{WF-1} << 1$ or $[1 - f(rs)]^{WF-1} \cong 0$ and this allows ourselves to get inside the Warped Region a derivative of the Natario Shape Function $n'(rs) \cong 0$ able to cope with the factor 10^{48} if WF is arbitrarily large.[7]

In our Microsoft Excel simulation we used a Warp Factor $WF = 200$[8]a radius $R = 100$ meters and an Alcubierre parameter $@ = 20$ meters. From $rs = 0$ meters to $rs = 99$ meters the original Natario Shape Function was $n(rs) = 0$.From $rs = 99$ meters to $rs = 100$ meters $n(rs)$ grew from 0 to 0,25 becoming $n(rs) = 0,25$ at $rs = 100$ meters and from $rs = 100$ to $rs = 101$ $n(rs)$ grew again from 0,25 reaching the value of $\frac{1}{2}$ when $rs = 101$ and for greater values of rs,$n(rs)$ kept the value of $\frac{1}{2}$ and the new Natario Shape Function gave also the same values as expected from a Natario Shape Function except in the Natario Warped Region.The thickness here is 2 meters starting at 99 meters and ending at 101 meters.

Note that when $rs = R$ as seen before the Alcubierre Shape Function is $f(rs) = 0,5$.This is the center of the Warp Bubble thickness that starts at 99 meters and ends at 101 meters.This is valid in both Alcubierre and Natario Warp Drive.

The original Natario Shape Function when $rs = R$ becomes:

$$n(rs) = \frac{1}{2}[1 - f(rs)] = \frac{1}{2}[1 - 0,5] = \frac{1}{2}[0,5] = 0,25 \tag{41}$$

But looking to the new Natario Shape Function when $rs = R$ we have the new result

$$n(rs) = [\frac{1}{2}][1 - 0,5]^{200} = [\frac{1}{2}][1 - 0,5]^{200} = 3,111 \times 10^{-61} \tag{42}$$

[6]Although the Natario Warp Bubble and Shape Function is different than its Alcubierre counterparts the Natario Warped Region is equal to its Alcubierre counterpart because we defined the Natario Shape Function as a function of the Alcubierre Shape Function

[7]independently of the expression of $f(rs)$ if the Shape Function gives fractionary numbers in both Alcubierre and Natario Warped Regions then raising these numbers to powers we will get even smaller numbers

[8]Although the Warp Factor is a dimensionless parameter we took inspiration from the speed of the Warp Ship.In this case $vs = 200$ 200 times light speed

Notice that both Natario Shape Functions gives 0 in the ship and $\frac{1}{2}$ far from it but in the Natario Warped Region $0 < n(rs) < \frac{1}{2}$ one Shape Function is very different from the other and the reason is the Warp Factor WF because we are raising $0,5$ to a power of 200 which means to say $[0,5]^{200} = \frac{1}{2^{200}}$ and this number is very small and enough to cope with the factor of 10^{48} because 2^{200} is enormous.

And now the derivatives in the point where $rs = R$ the center of the Warp Bubble thickness keeping in mind the fact that $f(rs)$ is the Alcubierre Shape Function and its derivative in this point is equal to $\frac{@}{2}$:

$$n'(rs) = -[\frac{1}{2}]200[0,5]^{199} \times 10 \tag{43}$$

its square becomes

$$n'(rs)^2 = [\frac{1}{4}]4 \times 10^4[0,5]^{398} \times 100 \tag{44}$$

Note that $[0,5]^{398} = 1,55 \times 10^{-120}$ is a number so small that will overcome the factor of 10^{48}.Note also that in the center of the Warp Bubble thickness the square of the derivative have a high value $f'(rs)^2 = 100$. So the Warp Factor allows ourselves to low the negative energy density requirements by lowering the values of the derivatives of the Natario Shape Function without violations of quantum physics at least in the center of the Warp Bubble thickness because it is better to multiply $1,55 \times 10^{-120}$ by 10^{48} than to multiply $f'(rs)^2$ by 10^{48} and $f'(rs)^2 = 100$.

Then for the center of the Warp Bubble thickness we have the following scenario:

$$\rho = -\frac{c^2}{G}\frac{vs^2}{8\pi}\left[3(n'(rs))^2\cos^2\theta + \left(n'(rs) + \frac{rs}{2}n''(rs)\right)^2\sin^2\theta\right]. \tag{45}$$

$$n'(rs)^2 = [\frac{1}{4}]WF^2[1 - f(rs)]^{2(WF-1)}f'(rs)^2 \tag{46}$$

Inside the Alcubierre or Natario Warped Region:

$0 < 1 - f(rs) < 1$ and $[1 - f(rs)]^{2(WF-1)} << 1$ or $[1 - f(rs)]^{2(WF-1)} \cong 0$ giving $n'(rs)^2 \cong 0$ able to cope with the factor 10^{48} coming from $\frac{c^2}{G}vs^2$ if WF is arbitrarily large.

Since the derivative square $f'(rs)^2$ in the center of the Warp Bubble thickness when analyzed by Excel simulations do not help too much we rewrite the power expression $pe(rs)$ of the derivative square $n'(rs)^2$ separated from $f'(rs)^2 = 100$ as follows:

$$pe(rs) = [\frac{1}{4}]WF^2[1 - f(rs)]^{2(WF-1)} \tag{47}$$

For our case of $rs = R$ we have :

$$pe(rs) = [\frac{1}{4}]4 \times 10^4[0,5]^{398} = 10^4 \times 1,55 \times 10^{-120} = 1,55 \times 10^{-116} \tag{48}$$

In agreement with the Excel simulations.

but we also have

$$f'(rs)^2 = 100 \tag{49}$$

Note that we must multiply the power expression $pe(rs) = 1,55 \times 10^{-116}$ by $f'(rs)^2 = 100$ the derivative square to get the real result of $n'(rs)^2$ that is $n'(rs)^2 = 1,55 \times 10^{-114}$.

then the expression of the square of the derivative of the Natario Shape Function

$$n'(rs)^2 = [\frac{1}{4}]WF^2[1 - f(rs)]^{2(WF-1)}f'(rs)^2 \tag{50}$$

can be written as:

$$n'(rs)^2 = pe(rs)f'(rs)^2 \tag{51}$$

Note that in the center of the Warp Bubble thickness when $rs = R$, $pe(rs) = 1,55 \times 10^{-116}$ and $f'(rs)^2 = 100$, so the term that generates a low $n'(rs)^2$ being $n'(rs)^2 = 1,55 \times 10^{-114}$,is the power expression $pe(rs)$ and the derivative square $f'(rs)^2$ do not accounts too much.

But what happens when rs approaches the end of the Warp Bubble thickness??

What happens when rs approaches the value of 101 meters ???

At 101 meters our Excel simulation shows that the power expression $pe(rs) = 10000$ because:

$$pe(rs) = [\frac{1}{4}]WF^2[1 - f(rs)]^{2(WF-1)} = [\frac{1}{4}]4 \times 10^4[1]^{398} = 10^4 \tag{52}$$

This can easily be seen because as far as rs moves away from 100 meters approaching 101 meters in a continuous growth,the Alcubierre Shape Function decreases from $f(rs) = 0,5$ at 100 meters to 0 at 101 meters also at a continuous decrease rate so the term $1-f(rs)$ approaches 1 making the Warp Factor useless.

Like we pointed out before the Warp Factor ameliorates the Negative Energy Density requirements in the inner zones of the Warp Bubble thickness but not in its limits.[9]

But the square of the derivative $f'(rs)^2$ have a different behavior:

When $rs = R$ in the center of the Warp Bubble thickness then $f'(rs)^2 = 100$ but when $rs = 101$ then $f'(rs)^2 = 2,89 \times 10^{-32}$ and multiplying a power expression $pe(rs) = 10 \times 10^4$ by a square derivative $f'(rs)^2 = 2,89 \times 10^{-32}$ we get the value for $n'(rs)^2$ as being $n'(rs)^2 = 2,89 \times 10^{-28}$

This is very important:

- 1)-In the center of the Warp Bubble thickness the power expression $pe(rs)$ have low values and the derivative square $f'(rs)^2$ have high values.
- 2)-In the Outer Limit of the Warp Bubble thickness, $pe(rs)$ have high values and $f'(rs)^2$ have low values which means to say that the rules are reverted

Back to the square of the derivative of the Natario Shape Function in the Outer Limit of the Warp Bubble thickness $n'(rs)^2 = 2,89 \times 10^{-28}$ we must multiply this by 10^{48} to get a real estimative of the Negative Energy Density[10] that is of the order of $-2,89 \times 10^{20} \frac{Joules}{meters^3}$!!![11] Dividing by c^2 we get the Negative

[9]in the Natario case we are concerned about the Outer Limit when $rs = 101$ meters.In the Inner Limit $rs = 99$ and $pe(rs) = 0$ while $f'(rs) = 2,89 \times 10^{-32}$.Then we have $n'(rs)^2 = pe(rs)f'(rs)^2 = 0$

[10]See Appendix on Dimensional Reduction

[11]equal in magnitude to the positive energy density of the Earth

Mass Density of $-2,89 \times 10^{4} \frac{kilograms}{meters^{3}}$.These values are "acceptable" or "affordable"

In the center of the Warp Bubble thickness we have: $n'(rs)^2 = 1,55 \times 10^{-114}$ and multiplying this by 10^{48} we have an estimative of the Negative Energy Density of $n'(rs)^2 = -1,55 \times 10^{-66} \frac{Joules}{meters^3}$ and a Negative Mass Density of $n'(rs)^2 = -1,55 \times 10^{-82} \frac{kilograms}{meters^3}$ and these values are so low that we will not continue the analysis in the center of the Warp Bubble Thickness.

We could perform a numerical integration over the entire Warp Bubble thickness to determine the total Negative Mass that must be generated to sustain our Warp Bubble but we will terminate this with a rapid estimative.

From $rs = 100$ to $rs = 101$ the Negative Density of Mass and Energy grows from a very low value to an acceptable value.Assunimg that the acceptable value is uniform along the Warp Bubble thickness[12] and this region encompasses the region between $99 <= rs <= 101$ we can obtain the volume of this Warp Shell as follows:

$$V_{shell} = \frac{4}{3} \times \pi \times (101 - 99)^3 = 3,349 \times 10^1 \tag{53}$$

And multiplying the Negative Mass Density of $-2,89 \times 10^{4} \frac{kilograms}{meters^{3}}$ by $3,349 \times 10^1$ we get a real Negative Mass of $-9,67 \times 10^5 kilograms$. Multiplying the Negative Energy Density by the same value we get the Total Negative Energy Requirement of $9,67 \times 10^{21} Joules$.

These quantities are perhaps "affordable" for a future Quantum Gravity Theory.At least we are not talking about ten times the mass of the Universe needed to generate the Warp Drive or 10^{28} times the positive energy density of the Earth!!!.

We are talking about $96Tons$!!! of Negative Mass.

Note that the same arguments we used for the Natario Warp Drive can be applied for an Alcubierre Warp Drive with an Alcubierre Shape Function defined as:

$$a(rs) = f(rs)^{WF} \tag{54}$$

When $f(rs) = 1$ $a(rs) = 1$ and when $f(rs) = 0$ $a(rs) = 0$ but in the Alcubierre Warped Region $0 < a(rs) < 1$ have very small values and hence its derivatives .

$$a'(rs) = (WF)f(rs)^{WF-1}f'(rs) \tag{55}$$

If in the distant future an advanced civilization with capabilities in spacetime metric engineering can generate a Natario Shape Function like this one

$$n(rs) = \frac{1}{2}[1 - f(rs)] \tag{56}$$

Perhaps generating this Natario Shape Function would be more recommendable

$$n(rs) = [\frac{1}{2}][1 - f(rs)]^{WF} \tag{57}$$

[12] the real values of Mass and Energy Densities are lower than the ones we are about to obtain

Or even this Alcubierre Shape Function

$$a(rs) = f(rs)^{WF} \tag{58}$$

3 Conclusion-The Role of the Warp Factor in the Alcubierre and Natario Warp Drive Spacetimes

Alcubierre and Natario Warp Drive were regarded as unphysical by the scientific community due to the large negative energy density requirements demanded to create these Spacetimes.The argument of the scientific community is valid due to the factor $\frac{c^4}{G}$ from the Einstein Field Equations multiplied by the square of the Warp Drive speed vs when attaining Superluminal velocities that demands large outputs of negative energy to maintain the Warp Drive while quantum theory only allow microscopical amounts of it and we know that we cannot violate quantum physics.

In this work we demonstrated that $\frac{c^4}{G}vs^2$ demands in negative energy density at least 10^{28} times the positive energy density of the Earth for a Warp Drive ship with a velocity of 200 times light speed using the known Shape Functions for Alcubierre and Natario Warp Drive Spacetimes

But without violating quantum physics we introduced two new Shape Functions one for the Alcubierre and another for the Natario Warp Drive Spacetimes that allowed ourselves to low the negative energy requirements to "low" and "affordable" levels perhaps allowing ourselves to restore the physical feasibility of these Spacetimes.

We introduced here a Warp Factor that must be taken into account in further studies of Alcubierre or Natario Warp Drive Spacetimes

How long will we need to wait in order to have an affordable Alcubierre or Natario Warp Drive?
We cannot answer this question by now but if the scientific community regains interest again in the Warp Drive as a Dynamical Spacetime perhaps we will not have to wait too much longer

4 Appendix:Dimensional Reduction from $\frac{c^4}{G}$ to $\frac{c^2}{G}$

The Alcubierre expressions for the Negative Energy Density in Geometrized Units $c = G = 1$ are given by[13]:

$$\rho = -\frac{1}{32\pi} vs^2 \left[f'(rs)\right]^2 [\frac{y^2 + z^2}{rs^2}] \tag{59}$$

$$\rho = -\frac{1}{32\pi} vs^2 \left[\frac{df(rs)}{drs}\right]^2 [\frac{y^2 + z^2}{rs^2}] \tag{60}$$

In this system all physical quantities are identified with geometrical entities such as lengths,areas or dimensionless factors.Even time is interpreted as the distance travelled by a pulse of light during that time interval,so even time is given in lengths.Energy,Momentum and Mass also have the dimensions of lengths.We can multiply a mass in kilograms by the conversion factor $\frac{G}{c^2}$ to obtain the mass equivalent in meters.On the other hand we can multiply meters by $\frac{c^2}{G}$ to obtain kilograms.The Energy Density($\frac{Joules}{meters^3}$)in Geometrized Units have a dimension of $\frac{1}{length^2}$ and the conversion factor for Energy Density is $\frac{G}{c^4}$.Again on the other hand by multiplying $\frac{1}{length^2}$ by $\frac{c^4}{G}$ we retrieve again ($\frac{Joules}{meters^3}$). [14].

This is the reason why in Geometrized Units the Einstein Tensor have the same dimension of the Stress Energy Momentum Tensor(in this case the Negative Energy Density)and since the Einstein Tensor is associated to the Curvature of Spacetime both have the dimension of $\frac{1}{length^2}$.

$$G_{00} = 8\pi T_{00} \tag{61}$$

Passing to normal units and computing the Negative Energy Density we multiply the Einstein Tensor (dimension $\frac{1}{length^2}$) by the conversion factor $\frac{c^4}{G}$ in order to retrieve the normal unit for the Negative Energy Density ($\frac{Joules}{meters^3}$).

$$T_{00} = \frac{c^4}{8\pi G} G_{00} \tag{62}$$

Examine now the Alcubierre equations:

$vs = \frac{dxs}{dt}$ is dimensionless since time is also in lengths.$\frac{y^2+z^2}{rs^2}$ is dimensionless since both are given also in lengths. $f(rs)$ is dimensionless but its derivative $\frac{df(rs)}{drs}$ is not because rs is in meters. So the dimensional factor in Geometrized Units for the Alcubierre Energy Density comes from the square of the derivative and is also $\frac{1}{length^2}$.Remember that the speed of the Warp Bubble vs is dimensionless in Geometrized Units and when we multiply directly $\frac{1}{length^2}$ from the Negative Energy Density in Geometrized Units by $\frac{c^4}{G}$ to obtain the Negative Energy Density in normal units $\frac{Joules}{meters^3}$ the first attempt would be to make the following:

$$\rho = -\frac{c^4}{G}\frac{1}{32\pi} vs^2 \left[f'(rs)\right]^2 [\frac{y^2 + z^2}{rs^2}] \tag{63}$$

$$\rho = -\frac{c^4}{G}\frac{1}{32\pi} vs^2 \left[\frac{df(rs)}{drs}\right]^2 [\frac{y^2 + z^2}{rs^2}] \tag{64}$$

[13]See Geometrized Units in Wikipedia

[14]See Conversion Factors for Geometrized Units in Wikipedia

But note that in normal units vs is not dimensionless and the equations above do not lead to the correct dimensionality of the Negative Energy Density because the equations above in normal units are being affected by the dimensionality of vs.

In order to make vs dimensionless again,the Negative Energy Density is written as follows:

$$\rho = -\frac{c^4}{G}\frac{1}{32\pi}(\frac{vs}{c})^2 \left[f'(rs)\right]^2 [\frac{y^2+z^2}{rs^2}] \tag{65}$$

$$\rho = -\frac{c^4}{G}\frac{1}{32\pi}(\frac{vs}{c})^2 \left[\frac{df(rs)}{drs}\right]^2 [\frac{y^2+z^2}{rs^2}] \tag{66}$$

Giving:

$$\rho = -\frac{c^2}{G}\frac{1}{32\pi}vs^2 \left[f'(rs)\right]^2 [\frac{y^2+z^2}{rs^2}] \tag{67}$$

$$\rho = -\frac{c^2}{G}\frac{1}{32\pi}vs^2 \left[\frac{df(rs)}{drs}\right]^2 [\frac{y^2+z^2}{rs^2}] \tag{68}$$

As already seen.The same results are valid for the Natario Energy Density

Note that from

$$\rho = -\frac{c^4}{G}\frac{1}{32\pi}(\frac{vs}{c})^2 \left[f'(rs)\right]^2 [\frac{y^2+z^2}{rs^2}] \tag{69}$$

$$\rho = -\frac{c^4}{G}\frac{1}{32\pi}(\frac{vs}{c})^2 \left[\frac{df(rs)}{drs}\right]^2 [\frac{y^2+z^2}{rs^2}] \tag{70}$$

Making $c = G = 1$ we retrieve again

$$\rho = -\frac{1}{32\pi}vs^2 \left[f'(rs)\right]^2 [\frac{y^2+z^2}{rs^2}] \tag{71}$$

$$\rho = -\frac{1}{32\pi}vs^2 \left[\frac{df(rs)}{drs}\right]^2 [\frac{y^2+z^2}{rs^2}] \tag{72}$$

5 Epilogue

- "The only way of discovering the limits of the possible is to venture a little way past them into the impossible."-Arthur C.Clarke[15]
- "The supreme task of the physicist is to arrive at those universal elementary laws from which the cosmos can be built up by pure deduction. There is no logical path to these laws; only intuition, resting on sympathetic understanding of experience, can reach them"-Albert Einstein[16][17]

[15] special thanks to Maria Matreno from Residencia de Estudantes Universitas Lisboa Portugal for providing the Second Law Of Arthur C.Clarke

[16] "Ideas And Opinions" Einstein compilation, ISBN $0-517-88440-2$, on page 226."Principles of Research" ([Ideas and Opinions],pp.224-227), described as "Address delivered in celebration of Max Planck's sixtieth birthday (1918) before the Physical Society in Berlin"

[17] appears also in the Eric Baird book Relativity in Curved Spacetime ISBN $978-0-9557068-0-6$

6 Remarks

We performed all the numerical calculus of our simulations for the Alcubierre and Natario Warp Drive using Microsoft Excel[18].We can provide our Excel files to those interested[19] and although Excel is a licensed program there exists another program that can read Excel files available in the Internet as a freeware for those that perhaps may want to examine our files:the OpenOffice[20] at http://www.openoffice.org

[18]Copyright(R) by Microsoft Corporation
[19]perhaps referees for future conventional Journals
[20]Copyright(R) by Oracle Corporation

7 Legacy

This work is dedicated to the 10*th* anniversary of the Natario Warp Drive Spacetime.The first version appeared in the arXiv.org as gr-qc/0110086 in 19 October 2001.
It is also dedicated to the Alcubierre Warp Drive Spacetime.The first version appeared in the arXiv.org as gr-qc/0009013 in 5 September 2000.
But above everything else this work is dedicated to the Memory of the American science fiction novelist Eugene Wesley Roddenberry(1921-1991) the creator of Star Trek. When everything seemed to be lost for both Alcubierre and Natario Warp Drive Spacetimes and the unphysical nature seemed to be an unsurmountable obstacle the old Gene appeared to "save the day"

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