TWO GRAVITATIONAL SINGULARITIES

Abstract: This article presents a virtual gravitational potential, which could explain some recent astronomical singularities: the secular increase of the eccentricity of the orbit of the Moon and the increase of the Astronomical Unit. Anyway, it is a theoretical potential without any proof of its physical reality.

Javier Bootello.
Astrophysics student email : jbootello@ciccp.es

0.- Introduction

We consider singularities or anomalies, those observational results which does not fit with the theoretical assumptions obtained as result of the application of General Relativity. A collection of these unexplained phenomena within our Solar system, is summarize by Lämmerzahl [1]. The most recent singularity was presented in 2010 by J.D. Anderson [2] related with the increase of the eccentricity of the orbit of the Moon.

1 .- Perturbing gravitational potential *S(ϕ)*

Potential *S (ϕ)* is an update of the Tisserand mathematical potential developed in 1872, added with a gravitational background which could explain its real application. It is only a theoretical and virtual potential with undeveloped physical meaning.

Potential *S (ϕ)* is defined as a slight perturbation to the newtonian gravitational potential, linked with the radial velocity of the object. The potential *S (ϕ)* should produce in every object with mass, a "transit action" because of the larger/shorter time of action of the field over the target. That time depends on the ratio between the radial velocity of the target and that of the potential, assumed these to be equal to the speed of light. The potential, as a field with energy dimension, is linked with the square of that ratio.

We define the disturbing potential as:

$$S(\phi) = -\frac{3GM}{r}\left(\frac{Vr}{c}\right)^2$$

where Vr is the radial velocity of the object.

S (ϕ) is a perturbation added to the newtonian potential, and produces exactly the same orbit perihelion advance as General Relativity, when $Vr \ll c$. [3]

2 .- Increase of the eccentricity of the orbit of the Moon.

We will analyse the effects of any small perturbing acceleration on the eccentricity of an elliptical orbit. According to Gauss formulation, the eccentricity parameter adjustment, is linked with the perturbing acceleration, whatever could be its physical origin :

$$\frac{de}{dt} = \frac{\sqrt{1-e^2}}{n\,a}\; A_r\, sen(\phi) \qquad (1)$$

where Ar is the radial disturbing acceleration and ϕ the true anomaly.

The disturbing acceleration is the derivative of the potential *S (ϕ)* related to *r*, the same as we do to obtain the newtonian acceleration from the classic gravitational potential.

Point out also that Vr is instantaneously independent of *r* as it is the essential definition of the disturbing potential *S (ϕ)*. In fact, Vr could have any value not related with *r* .

Disturbing acceleration will have the following expression:

$$Ar = \frac{dS(\phi)}{d\,r} = \frac{3\,GM}{r^2}\;\frac{Vr^2}{c^2}$$

If we develop equation (1) and change derivates related to time (*t*) with that related to ϕ

$$\frac{de}{dt} = \frac{de}{d\phi}\;\frac{d\phi}{dt} = \frac{de}{d\phi}\cdot\omega = \frac{de}{d\phi}\;\frac{h}{r^2} \qquad (2)$$

where h is the angular momentum by unit of mass and *p* is the semi- latus.

For a keplerian ellipse, we have also

Intelectual Property Register of Spain : RTA - 68-11

$$Vr = \dot{r} = \frac{e\,h\,sin\,\phi}{p} \quad ; \quad \text{and} \quad \frac{\sqrt{1-e^2}}{n\,a} = \frac{p}{h}$$

inserting these in (2)

$$\frac{de}{d\phi}\ \frac{h}{r^2} = \frac{3GM}{r^2}\ \frac{e^2 h^2 \sin^2\phi}{c^2 p^2}\ \frac{p}{h} \cdot sin(\phi)$$

$$\frac{de}{d\phi} = \frac{3GM}{c^2 p}\ e^2\ sin^3(\phi)$$

This expression indicates the increase of eccentricity related with the true anomaly. The integration will give the result of its increase along one orbit of the Moon around the Earth. This potential always produces a positive and symmetrical effect about the axis of the ellipse. [3]

$$e_{orbit} = \frac{3GM}{c^2\ p}\ e^2\ 2\int_0^\pi sin^3(\phi)\ d\phi$$

the definite integral is

$$e_{orbit} = \frac{6GM}{c^2\ p} \cdot e^2 \cdot \frac{4}{3} = \frac{8GM}{c^2\ p}\ e^2$$

For the Earth – Moon parameters, we have :

$GM = 3{,}986 \times 10^{14}$ m s $^{-2}$
$e = 0{,}0647$
$p = 3{,}796 \times 10^8$ m

so the result of the increase of the eccentricity in each orbit is :

$e_{orbit} = 0{,}395 \times 10^{-12}$

and referred to a year :

$$e_{year} = \frac{365}{27{,}3} \cdot 0{,}395 \cdot 10^{-12} = 5{,}28 \cdot 10^{-12}$$

The increase of the eccentricity produced by the potential *S (ϕ)* in the orbit of the Moon around the Earth, is near consistent with the value obtained by astronomical detection through the Lunar Laser Ranging along the last 39 years [4] that is $(9 \pm 3) \times 10^{-12}$.

3 .- Increase of the Astronomical Unit.

The perturbing potential *S (ϕ)* produces an increase in the semi-major axis of the ellipse that according to Gauss, will have the following expression:

$$\frac{da}{dt} = \frac{2}{n\sqrt{1-e^2}}\ e \cdot A_r \cdot sin(\phi)$$

Using similar formulations as in paragraph before, we will have :

$$\frac{da}{d\phi} = \frac{6GM}{c^2\,p}\ \frac{a}{1-e^2}\ e^3\ sin^3(\phi)$$

For one orbit of the Earth around the Sun, we have :

$$a_{orbit} = \frac{6GM}{c^2}\ \frac{a}{p(1-e^2)} \cdot e^3\ 2\int_0^\pi sin^3(\phi)\ d\phi$$

$$a_{year} = \frac{16GM}{c^2}\ \frac{a}{p(1-e^2)} \cdot e^3$$

The Earth – Sun parameters :

$GM = 13{,}27 \times 10^{19}$ m s $^{-2}$
$e = 0{,}0167$
$p = 148{,}96 \times 10^9$ m

and the increase of Astronomical Unit in one year is :

$\text{A.U.}_{year} = 11{,}06$ *cm /year*

The increase of the Astronomical Unit produced by the potential *S (ϕ)* in the orbit of the Earth around the Sun, is consistent with the value obtained by astronomical detection through the analysis of radiometric measurements of distances between the Earth and the major planets including observations from Martian orbiters from 1971 [5]that is of 15 ± 4 cm /year.

If we apply potential *S (ϕ)* to the orbit of Mercury and Mars, we should obtain an increase of the semi-major axis of :

$a_{Mercury/year}$ = 224,3 *m / year.*
$a_{Mars/year}$ = 19,4 *m /year.*

4 .- Conclusions.

The potential *S (ϕ)* applied to the Moon's orbit around the Earth, produces an increase of eccentricity that is consistent with the observed values.

Applied to the Earth's orbit around the Sun, produces an increase of the Astronomical Unit which is consistent with the observed values. Applied to the orbit of Mercury, predicts an increase of the semi-major axis that perhaps could be determined in the next insertion of aircraft Messenger.

In any case, it is a virtual and theoretical potential, with no evidence of its physical reality suitable with material objects inside our universe. It would be necessary to implement a detailed study about it.

References

[1] Lämmerzahl C, Preuss O, Dittus H. *Is the physics within the Solar system really understood?* arXiv:gr-qc/0604052 2006

[2] Anderson J D., Nieto M M. *Astrometric solar-system anomalies 2010,* Proceedings IAU Symposium No.261, Cambridge University Press

[3] Bootello J. *The General Relativity precession of Mercury* Registered (RTA- 65 –11). Submitted : Classical and Quantum Gravity. (not published yet)

[4] Iorio L. *On the anomalous secular increase of the eccentricity of the orbit of the Moon*. arXiv:1102.0212

[5] Krasinsky G A, Brumberg V A. *Secular increase of the Astronomical Unit from analysis of the major planets motions and its interpretation*. Celestial Mechanics and Dynamical Astronomy 2004