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The potential function and entropy function of a system that carries large number of charged particles

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Abstract

If a system is consisted of a large number of charged particles, any one of the system's particles would couple with its neighbors by dissimilar strengths. Therefore, the system's particles would produce dissimilar potentials, which satisfy the probability distribution. To make the potential induced by wave number k an exact differential, we introduced the function λ. In this way, we defined the potential function Φ and entropy function S of the system.

Key words: particles system, potential function, particle entropy.

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1 Introduction

In a system that is consisted of a large number of charged particles, any one of the system's particles would couple with its neighbors to produce dissimilar potential function Φ. Based on the dissimilar coupling strengths, those particles would possess different potentials, which satisfy the probability distribution function f. To make energy exerted on wave number k by the system's particles an exact differential,we introduced the function λ. After combining the two charged systems into a stable one, we defined the potential function Φ and entropy function S of the stable system. If the system's potential is moved from the curved surface C_i to C_j, the particles would take ordered motions under the potential Φ.

2 The system's potential function and entropy function when charged particles couple with one another

In ith particle, that carries charge q and couples with field $\mathbb{E}_{k_i}$ in a system of large amount particles, when this electric charge q is coupled with a system in instable state. ith particle would produce potential [1]

$$\varphi_i = \int_{a_i}^{b_i} \mathbb{E}_{k_i} \cdot dk, \tag{1}$$

in (1), k is the field's wave number of a carries charge q, at the coupling, and the ith particle would have energy

$$\epsilon_i = q\varphi_i. \tag{2}$$

It is assumed that the system has N particles number. Owing to that coupling between individual particles shows differences, those particles would have dissimilar energy. Under the assumption that the energy of those particles satisfies the probability distribution function f, we get the following for the phase space Ω and particles numbers in the system, based on (1) integral, the potential function φ_i of a particle exist the upper limit b_i and lower limit a_i.

When exist large numbers particles in a system, for coupling potential function $\varphi = \int_a^b \mathbb{E} \cdot dk$, the integral both the upper limit b and lower limit a satisfy same probability distribution function f, hence the upper limit become $\langle b \rangle$ and lower limit is $\langle a \rangle$[2]. Every one of particles there is expected potential function $\langle \varphi \rangle = \int_{\langle a \rangle}^{\langle b \rangle} \mathbb{E} \cdot dk$ and expected energy $\langle \epsilon \rangle = q\langle \varphi \rangle$. For (2), when N particles exist in the system, we has energy

$$\mathcal{E} = \int_{\langle a \rangle}^{\langle b \rangle} Nq \, \mathbb{E}_k \cdot dk, \tag{3}$$

if the system's electric charge $Q = Nq$ and potential $\Phi(r, k)$, the $\Phi(r, k)$ is a potential function of spatial position r and wave number k.

We make electric field in instable state, $\mathbb{E}_k = \partial\Phi/\partial k$ and force $\mathbb{F}_k = Q\partial\Phi/\partial k = Q\mathbb{E}_k$, then the energy of the particle becomes

$$\delta\mathcal{E}' = Q\partial\Phi/\partial k \cdot dk. \tag{4}$$

For a potential function Φ in the stable state, there is field $\mathbb{E}_r = \partial\Phi/\partial r$, the electric charge Q has energy [3]

$$\delta \mathcal{E}'' = Q\partial\Phi/\partial r \cdot dr, \tag{5}$$

and force $\mathbb{F}_r = Q\partial\Phi/\partial r = Q\mathbb{E}_r$, which is associated with the spatial position of Q. It can be observed from (5) that energy of the charge Q. Hence (4) and (5) the potential energy of Q against would be

$$\delta \mathcal{E} = Q\partial\Phi/\partial r \cdot dr + Q\partial\Phi/\partial k \cdot dk = Q\mathbb{E}_r \cdot dr + Q\mathbb{E}_k \cdot dk, \tag{6}$$

by calculating the closed curve integral calculus of (6), we have

$$\mathcal{E} = \oint \delta\mathcal{E} = \oint Q\mathbb{E}_r \cdot dr + \oint Q\mathbb{E}_k \cdot dk, \tag{7}$$

for conservation system because $\oint Q\mathbb{E}_r \cdot dr = 0$ [4], we have

$$\mathcal{E}' = \oint Q\mathbb{E}_k \cdot dk, \tag{8}$$

the equation that is related with the integral wave number k. Owing to (8) is not equal to zero. For the integral path, do not make exact differential.

Now we apply the Method of Caratheodory to a system of charged particles [5, 6], to showed that the concepts of potential and entropy

function. By rewriting (4) into its component form, we get

$$\delta\mathcal{E} = \delta\mathcal{E}' = \sum_{j}^{n} Q\mathbb{E}_{k_j} \cdot dk_j. \tag{9}$$

Equation (9) must be multiplied by a function $1/\lambda$ to make it an exact differential, let (8) becomes a exact differential. Hence (9) we obtain

$$\frac{1}{\lambda}\delta\mathcal{E} = \frac{1}{\lambda}\sum_{j}^{n} Q\mathbb{E}_{k_j} \cdot dk_j, \tag{10}$$

in (10), we get following solution

$$F = A \quad (A \text{ is constant}), \tag{11}$$

make $dF = 0$, from (10) and (11) then become

$$dF = \frac{1}{\lambda}\delta\mathcal{E} = \frac{1}{\lambda}\sum_{j=1}^{n} Q\mathbb{E}_{k_j} \cdot dk_j. \tag{12}$$

Take $n = 3$ for the spatial component, made $dF = 0$, and (10) and (12) becomes

$$dF = \frac{1}{\lambda}(Q\mathbb{E}_{k_1} \cdot dk_1 + Q\mathbb{E}_{k_2} \cdot dk_2 + Q\mathbb{E}_{k_3} \cdot dk_3) = 0, \tag{13}$$

$$dF = \partial F/\partial k_1 \cdot dk_1 + \partial F/\partial k_2 \cdot dk_2 + \partial F/\partial k_3 \cdot dk_3 = 0, \tag{14}$$

the dF is a exact differential, from (13) and (14), we have

$$(\partial F/\partial k_1)/Q\mathbb{E}_{k_1} = (\partial F/\partial k_2)/Q\mathbb{E}_{k_2} = (\partial F/\partial k_3)/Q\mathbb{E}_{k_3} = 1/\lambda, \tag{15}$$

the (13) is an exact differential. In the case of 2 systems respectively of system 1 and system 2, we get the following from (13)

$$dF_1 = \frac{1}{\lambda_1}\delta\mathcal{E}_1, \quad dF_2 = \frac{1}{\lambda_2}\delta\mathcal{E}_2. \tag{16}$$

Let the two systems merge into one, their potentials should be equal when the two systems reach the equilibrium point. In the new system formed by the

afore-listed two we have

$$dF = \frac{1}{\lambda}\delta\mathcal{E}. \tag{17}$$

let τ be the potential of systems 1 and 2 in the equilibrium process, we get the following from (12), (16) and (17) from energy conservation:

$$dF = \frac{\delta\mathcal{E}}{\lambda} = \frac{\delta\mathcal{E}_1 + \delta\mathcal{E}_2}{\lambda} = \frac{\lambda_1}{\lambda}dF_1 + \frac{\lambda_2}{\lambda}dF_2, \tag{18}$$

$$dF = \partial F/\partial F_1 dF_1 + \partial F/\partial F_2 dF_2 + \partial F/\partial\tau d\tau, \tag{19}$$

comparing (18) with (19), we get

$$\partial F/\partial F_1 = \lambda_1/\lambda, \quad \partial F/\partial F_2 = \lambda_2/\lambda \text{ and } \partial F/\partial\tau = 0. \tag{20}$$

It would not be difficult to see from (20) that F is irrelevant with τ potential, by taking $\partial F/\partial\tau = 0$ and $\partial F_l/\partial\tau = 0 \quad (l = 1, 2)$, namely,

$$\partial(\lambda_1/\lambda)/\partial\tau = \partial(\lambda_2/\lambda)/\partial\tau = 0, \tag{21}$$

from (21) we have

$$\partial(\ln\lambda_1)/\partial\tau = \partial(\ln\lambda_2)/\partial\tau = \partial(\ln\lambda)/\partial\tau = L(\tau), \tag{22}$$

$L(\tau)$ is a common function that is irrelevant with the system. consider (22) , we draw into a function $M(F_l)$ of the variable F_l, from (20), by taking $\partial F/\partial\tau = 0$ and $\partial F_l/\partial\tau = 0 \quad (l = 1, 2)$ becomes

$$\ln\lambda_l = \int L(\tau)d\tau + \ln M(F_l), \tag{23}$$

or

$$\lambda_l = M(F_l)\exp(\int L(\tau)d\tau) \quad (l = 1, 2). \tag{24}$$

Now we may define the electric system's potential function $\Phi(\tau)$

$$\Phi(\tau) = C\exp(\int L(\tau)d\tau) \quad \text{(C is constant)}, \tag{25}$$

and entropy function S[7]

$$S_l = \frac{1}{C}\int M(F_l)dF_l \quad (l = 1, 2), \tag{26}$$

from the energy relation $\delta\mathcal{E} = \delta\mathcal{E}_1 + \delta\mathcal{E}_2 = \lambda_1 dF_1 + \lambda_2 dF_2$, formulas (16), (24), (25) and (26), we have

$$\delta\mathcal{E} = \Big[\sum_{l=1}^{2}\frac{1}{C}\, M(F_l)dF_l\Big]\Big[C\exp(\int L(\tau)d\tau)\Big] = \sum_{l=1}^{2}\Phi(\tau)dS_l. \tag{27}$$

When the system's energy shifts from the curved surface $F = C_1$ to $F = C_2$, factor $1/\Phi$ is introduced into (27) to make the entropy differential, $dS = \delta\mathcal{E}/\Phi$ are an exact differential. By now, we may illustrate the order motion of the charged particles by using potential Φ , or by using the system's entropy S. In statistical mechanics, the thermodynamical motion of particles should satisfy the Boltzmann equation, the disorder motions of particles could be represented by using entropy $S' = k\ln W$ [8], where k is Boltzmann constant, W is probability. If temperature T and potential Φ simultaneously exist in the same system, these particles would move in both states respectively of disorder thermodynamical motion and order potential motion.

References

[1] X.Z. Zhong. *WHEN COUPLED WITH ANOTHER PARTICLE, A CHARGED PARTICLE PRODUCES THE COULOMB POTENTIAL, WEAK COUPLING POTENTIAL AND YUKAWA POTENYIAL.* preprint., [83] vixra: 1101.0059.

[2] A.N. Shiryaev. *PROBABILITY.* 2nd ed. Springer-Verlag, New York, Inc., 1996.

[3] J.D. Jackson. *CLASSICAL ELECTRODYNAMICS*, 3rd ed. John-Wiley & Sons, Inc., 1999.

[4] W.R. Smythe. *STATIC AND DYNAMIC ELECTRICITY.* McGraw-Hill, Inc., 1968.

[5] T.Y. Wu. *THERMODYNAMICS GAS KINEMATICS AND STATISTICAL MECHANICS.* LianJing, TaiWan, Inc., 1977.

[6] L.D. Landau and E.M. Lifshts. *STATISTICAL PHYSICS.* Part 1, 3rd ed. Reed Education and Professional Ltd. Inc., 1980.

[7] Edward F. Obert. *CONCEPTS OF THERMODYNAMICS.* McGraw-Hill, Book Co. Inc., 1960.

[8] K. Huang. *STATISTICAL MECHANICS.* John Wile & Sons. Inc., 1963.