Fermat Last Theorem
and
Riemann Hypothesis (6)

Automorphic Functions
And
Fermat's Last Theorem（6）

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Abstract

In 1637 Fermat wrote: "*It is impossible to separate a cube into two cubes, or a biquadrate into two biquadrates, or in general any power higher than the second into powers of like degree: I have discovered a truly marvelous proof, which this margin is too small to contain.*"

This means: $x^n + y^n = z^n (n > 2)$ has no integer solutions, all different from 0(i.e., it has only the trivial solution, where one of the integers is equal to 0). It has been called Fermat's last theorem (FLT). It suffices to prove FLT for exponent 4 and every prime exponent P. Fermat proved FLT for exponent 4. Euler proved FLT for exponent 3[8].

In this paper using automorphic functions we prove FLT for exponents $12P$ and $4P$, where P is an odd prime. The proof of FLT must be direct. But indirect proof of FLT is disbelieving.

In 1974 Jiang found out Euler formula of the cyclotomic real numbers in the cyclotomic fields

$$\exp\left(\sum_{i=1}^{4m-1} t_i J^i\right) = \sum_{i=1}^{4m} S_i J^{i-1}, \tag{1}$$

where J denotes a $4m$ th root of negative unity, $J^{4m}=-1$, m=1, 2, 3,…, t_i are the real numbers.

S_i is called the automorphic functions (complex trigonometric functions) of order $4m$ with $(4m-1)$ variables [5,7].

$$S_i=\frac{1}{2m}\left[(-1)^{i-1}\sum_{j=0}^{m-1} e^{B_j}\cos\left(\theta_j+\frac{(i-1)(2j+1)\pi}{4m}\right)\right.$$

$$\left.+\sum_{j=0}^{m-1} e^{D_j}\cos\left(\phi_j-\frac{(i-1)(2j+1)\pi}{4m}\right)\right], \tag{2}$$

where $i=1,\ldots,4m$;

$$B_j=\sum_{\alpha=1}^{4m-1} t_\alpha(-1)^\alpha\cos\frac{(2j+1)\alpha\pi}{4m},\quad \theta_j=\sum_{\alpha=1}^{4m-1} t_\alpha(-1)^{1+\alpha}\sin\frac{(2j+1)\alpha\pi}{4m},$$

$$D_j=\sum_{\alpha=1}^{4m-1} t_\alpha\cos\frac{(2j+1)\alpha\pi}{4m},\quad \phi_j=\sum_{\alpha=1}^{4m-1} t_\alpha\sin\frac{(2j+1)\alpha\pi}{4m},$$

$$2\sum_{j=0}^{m-1}(B_j+D_j)=0. \tag{3}$$

From (2) we have its inverse transformation[5,7]

$$e^{B_j}\cos\theta_j=S_1+\sum_{i=1}^{4m-1} S_{1+i}(-1)^i\cos\frac{(2j+1)i\pi}{4m},$$

$$e^{B_j}\sin\theta_j=\sum_{i=1}^{4m-1} S_{1+i}(-1)^{1+i}\sin\frac{(2j+1)i\pi}{4m},$$

$$e^{D_j}\cos\phi_j=S_1+\sum_{i=1}^{4m-1} S_{1+i}\cos\frac{(2j+1)i\pi}{4m},$$

$$e^{D_j}\sin\phi_j=\sum_{i=1}^{4m-1} S_{1+i}\sin\frac{(2j+1)i\pi}{4m}. \tag{4}$$

(3) and (4) have the same form.

From (3) we have

$$\exp\left[2\sum_{j=0}^{m-1}(B_j+D_j)\right]=1. \tag{5}$$

From (4) we have

$$\exp\left[2\sum_{j=0}^{m-1}(B_j+D_j)\right]=\begin{vmatrix} S_1 & -S_{4m} & \cdots & -S_2 \\ S_2 & S_1 & \cdots & -S_3 \\ \cdots & \cdots & \cdots & \cdots \\ S_{4m} & S_{4m-1} & \cdots & S_1 \end{vmatrix}=\begin{vmatrix} S_1 & (S_1)_1 & \cdots & (S_1)_{4m-1} \\ S_2 & (S_2)_1 & \cdots & (S_2)_{4m-1} \\ \cdots & \cdots & \cdots & \cdots \\ S_{4m} & (S_{4m})_1 & \cdots & (S_{4m})_{4m-1} \end{vmatrix} \quad (6)$$

where

$$(S_i)_j=\frac{\partial S_i}{\partial t_j}[7]$$

From (5) and (6) we have circulant determinant

$$\exp\left[2\sum_{j=0}^{m-1}(B_j+D_j)\right]=\begin{vmatrix} S_1 & -S_{4m} & \cdots & -S_2 \\ S_2 & S_1 & \cdots & -S_3 \\ \cdots & \cdots & \cdots & \cdots \\ S_{4m} & S_{4m-1} & \cdots & S_1 \end{vmatrix}=1 \quad (7)$$

If $S_i \neq 0$, where $i=1,2,...,4m$, then (7) has infinitely many rational solutions.

Assume $S_1 \neq 0, S_2 \neq 0,$ and $S_i=0$, where $i=3,...,4m$. $S_i=0$ are $(4m-2)$ indeterminate equations with $(4m-1)$ variables. From (4) we have

$$e^{2B_j}=S_1^2+S_2^2-2S_1S_2\cos\frac{(2j+1)\pi}{4m},\ e^{2D_j}=S_1^2+S_2^2+2S_1S_2\cos\frac{(2j+1)\pi}{4m}, \quad (8)$$

Example. Let $m=15$. From (3) and (8) we have Fermat's equations

$$\exp[2\sum_{j=0}^{14}(B_j+D_j)]=S_1^{60}+S_2^{60}=(S_1^{20})^3+(S_2^{20})^3=1. \quad (9)$$

From (3) we have

$$\exp[2\sum_{j=0}^{4}(B_{3j+1}+D_{3j+1})]=[\exp(-t_{20}+t_{40})]^{20}. \quad (10)$$

From (8) we have

$$\exp[2\sum_{j=0}^{4}(B_{3j+1}+D_{3j+1})]=S_1^{20}+S_2^{20} \quad (11)$$

From (10) and (11) we have Fermat's equation

$$\exp[2\sum_{j=0}^{4}(B_{3j+1}+D_{3j+1})]=S_1^{20}+S_2^{20}=[\exp(-t_{20}+t_{40})]^{20} \quad (12)$$

Euler prove that (9) has no rational solutions for exponent 3[8]. Therefore we prove that (12) has no rational solutions for exponent 20.

Theorem. Let $m=3P$, where P is an odd prime. From (3) and (8) we have Fermat's equation.

$$\exp[2\sum_{j=0}^{3P-1}(B_j+D_j)]=S_1^{12P}+S_2^{12P}=(S_1^{4P})^3+(S_2^{4P})^3=1 \quad (13)$$

From (3) we have

$$\exp[2\sum_{j=0}^{3P-1}(B_{3j+1}+D_{3j+1})]=[\exp(-t_{4P}+t_{8P})]^{4P}. \quad (14)$$

From (8) we have

$$\exp[2\sum_{j=0}^{P-1}(B_{3j+1}+D_{3j+1})]=S_1^{4P}+S_2^{4P}. \quad (15)$$

From (14) and (15) we have Fermat's equation

$$\exp[2\sum_{j=0}^{P-1}(B_{3j+1}+D_{3j+1})]=S_1^{4P}+S_2^{4P}=[\exp(-t_{4P}+t_{8P})]^{4P} \quad (16)$$

Euler prove that (13) has no rational solutions for exponent 3 [8]. Therefore we prove that (16) has no rational solutions for exponent $4P$ [5,7].

Remark. It suffices to prove FLT for exponent 4. Let $n=4P$, where P is an odd prime. We have the Fermat's equation for exponent $4P$ and the Fermat's equation for exponent P [5,7]. This is the proof that Fermat thought to have had. In complex hyperbolic functions let exponent n be $n=\Pi P$, $n=2\Pi P$ and $n=4\Pi P$. Every factor of exponent n has Fermat's equation [1-7]. In complex trigonometric functions let exponent n be $n=\Pi P$, $n=2\Pi P$ and $n=4\Pi P$. Every factor of exponent n has Fermat's equation [1-7]. Using modular elliptic curves Wiles and Taylor prove FLT [9,10]. This is not the proof that Fermat thought to have had.

The classical theory of automorphic functions, created by Klein and Poincarè was concerned with the study of analytic functions in the unit circle that are invariant under a discrete group of transformation. Automorphic funciton are the generalization of trigonometric, hyperbolic, elliptic, and certain other functions of elementary analysis. The automorphic functions (complex trigonometric functions and complex hyperbolic functions) have a wide application in mathematics and physics.

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DISPROOFS OF RIEMANN'S HYPOTHESIS

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Abstract

As it is well known, the Riemann hypothesis on the zeros of the $\zeta(s)$ function has been assumed to be true in various basic developments of the 20-th century mathematics, although it has never been proved to be correct. The need for a resolution of this open historical problem has been voiced by several distinguished mathematicians. By using preceding works, in this paper we present comprehensive disproofs of the Riemann hypothesis. Moreover, in 1994 the author discovered the arithmetic function $J_n(\omega)$ that can replace Riemann's $\zeta(s)$ function in view of its proved features: if $J_n(\omega) \neq 0$, then the function has infinitely many prime solutions; and if $J_n(\omega) = 0$, then the function has finitely many prime solutions. By using the Jiang $J_2(\omega)$ function we prove the twin prime theorem, Goldbach's theorem and the prime theorem of the form $x^2 + 1$. Due to the importance of resolving the historical open nature of the Riemann hypothesis, comments by interested colleagues are here solicited.

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1. Introduction

In 1859 Riemann defined the zeta function[1]

$$\zeta(s) = \prod_p (1 - p^{-s})^{-1} = \sum_{n=1}^{\infty} \frac{1}{n^s}, \tag{1}$$

where $s = \sigma + ti, i = \sqrt{-1}, \sigma$ and t are real, p ranges over all primes. $\zeta(s)$ satisfies the functional equation [2]

$$\pi^{-\frac{s}{2}}\Gamma(\frac{s}{2})\zeta(s) = \pi^{-\frac{(1-s)}{2}}\Gamma(\frac{1-s}{2})\zeta(1-s). \tag{2}$$

From (2) we have

$$\zeta(ti) \neq 0. \tag{3}$$

Riemann conjectured that $\zeta(s)$ has infinitely many zeros in $0 \leq \sigma \leq 1$, called the critical strip. Riemann further made the remarkable conjecture that the zeros of $\zeta(s)$ in the critical strip all lie on the central line $\sigma = 1/2$, a conjecture called the famous **Riemann hypothesis** (RH).

It was stated by Hardy in 1914 that infinitely many zeros lie on the line; A. Selberg stated in 1942 that a positive proportion at least of all the zeros lie on the line; Levinson stated in 1974 that more than one third of the zeros lie on the line; Conrey stated in 1989 that at least two fifths of the zeros lie on the line.

The use of the RH then lead to many mathematical problems, such as the generalized Riemann conjecture, Artin's conjecture, Weil's conjecture, Langlands' program, quantum chaos, the hypothetical Riemann flow [3, 4], the zeta functions and L-functions of an algebraic variety and other studies. Similarly, it is possible to prove many theorems by using the RH.

However, the RH remains a basically unproved conjecture to this day. In fact, Hilbert properly stated in 199 that the problem of proving or disproving the RH is one of the most important problems confronting 20th century mathematicians. In 2000 Griffiths and Graham pointed out that the RH is the first challenging problem for the 21st century. The proof of the RH then become the millennium prize problem.

In 1997 we studied the tables of the Riemann zeta function [5] and reached preliminary results indicating that the RH is false [6, 7, 8]. In this paper we present a comprehensive disproof of the RH and show that the computation of all zeros of the $\zeta(1/2+ti)$ function done during the past 100 years is in error, as preliminarily indicated in Refs. [9, 10, 11]. Since the RH is false, all theorems and conjectures based to the same are also false.

2. Disproofs of Riemann's Hypothesis

Theorem 1. $\zeta(s)$ has no zeros in the critical strip, that is $\zeta(s) \neq 0$, where $0 \leq \sigma \leq 1$.

Proof 1. From (1) we have

$$\frac{1}{\zeta(s)} = \prod_p (1 - \frac{1}{p^s}) = Re^{\theta i}, \tag{4}$$

where

$$R = \prod_p R_p, \qquad R_p = \sqrt{1 - \frac{2\cos(t \log p)}{p^\sigma} + \frac{1}{p^{2\sigma}}}, \tag{5}$$

$$\theta = \sum_p \theta_p, \qquad \theta_p = \tan^{-1} \frac{\sin(t \log p)}{p^\sigma - \cos(t \log p)}. \tag{6}$$

If $\sigma = 0$, from (5) we have $R_p = \sqrt{2}\sqrt{1-\cos(t\log p)}$. If $\cos(t\log p) = 1$, we have $R_p = 0$ then $R = 0$. If $\sigma > 0$ from (5) we have $R_p \neq 0$. $\zeta(s) = 0$ if and only if $\operatorname{Re}\zeta(s) = 0$ and $\operatorname{Im}\zeta(s) = 0$, that is $R = \infty$. From (5) we have that if $\cos(t\log p) \leq 0$ then $R_p > 1$ and if $\cos(t\log p) > 0$ then $R_p < 1$. $\cos(t\log p)$ is independent of the real part σ, but may well depend on primes p and imaginary part t. We write $m_+(t)$ for the number of primes p satisfying $\cos(t\log p) > 0$, $m_-(t)$ for the number of primes p satisfying $\cos(t\log p) \leq 0$.

For $\cos(t\log p) > 0$, we have

$$1 > R_p(1+ti) > R_p(0.5+ti). \tag{7}$$

If $m_+(t_1)$ is much greater than $m_-(t_1)$ such that $R(0.5+t_1 i) = \min$. From (5), (6) and (7) we have for given t_1

$$\begin{aligned} \min R(\sigma_1 + t_1 i) &> \min R(1+t_1 i) > \min R(0.5+t_1 i) \\ &> \min R(\sigma_2 + t_1 i) \to 0, \end{aligned}$$

$$ \tag{8} $$

$$\theta(\sigma_1 + t_1 i) = \theta(1 + t_1 i) = \theta(0.5 + t_1 i) = \theta(\sigma_2 + t_1 i) = \text{const} \tag{9}$$

where $\sigma_1 > 1$ and $0 \leq \sigma_2 < 0.5$.
Since $\mid \zeta(s) \mid = \frac{1}{R}$ from (8) we have

$$\max \mid \zeta(\sigma_1 + t_1 i) \mid < \max \mid \zeta(1 + t_1 i) \mid$$

$$< \max \mid \zeta(0.5 + t_1 i) \mid < \max \mid \zeta(\sigma_2 + t_1 i) \mid \to \infty. \tag{10}$$

For $\cos(t \log p) < 0$ we have

$$1 < R_p(0.5 + ti) < R_p(0.4 + ti) < R_p(0.3 + ti). \tag{11}$$

If $m_-(t_1)$ is much greater than $m_+(t_1)$ such that $R(0.5 + t_1 i) = \max$. From (5), (6) and (11) we have for given t_1

$$\max R(\sigma_1 + t_1 i) < \max R(0.5 + t_1 i) < \max R(0.4 + t_1 i),$$

$$< \max R(0.3 + t_1 i) < \max R(\sigma_2 + t_1 i) \neq \infty, \tag{12}$$

$$\begin{aligned} \theta(\sigma_1 + t_1 i) &= \theta(0.5 + t_1 i) = \theta(0.4 + t_1 i) \\ &= \theta(0.3 + t_1 i) = \theta(\sigma_2 + t_1 i) = \text{const}, \end{aligned}$$

$$ \tag{13} $$

where $\sigma_1 > 0.5$ and $0 \leq \sigma_2 < 0.3$.
Since $\mid \zeta(s) \mid = \frac{1}{R}$ from (12) we have

$$\min \mid \zeta(\sigma_1 + t_1 i) \mid > \min \mid \zeta(0.5 + t_1 i) \mid > \min \mid \zeta(0.4 + t_1 i) \mid >$$

$$\min \mid \zeta(0.3 + t_1 i) \mid > \min \mid \zeta(\sigma_2 + t_1 i) \mid \neq 0. \tag{14}$$

Proof 2. We define the beta function

$$\beta(s) = \prod_p (1 + p^{-s})^{-1} = \sum_{n=1}^{\infty} \frac{\lambda(n)}{n^s}, \tag{15}$$

where $\lambda(1) = 1, \lambda(n) = (-1)^{a_1 + \cdots + a_k}$ if $n = p_1^{a_1} \cdots p_k^{a_k}, t \neq 0$.

From (15) we have

$$\frac{1}{\beta(s)} = \prod_p (1 + \frac{1}{p^s}) = \bar{R} e^{\bar{\theta} i}, \tag{16}$$

where

$$\bar{R} = \prod_p \bar{R}_p, \bar{R}_p = \sqrt{1 + \frac{2\cos(t \log p)}{p^\sigma} + \frac{1}{p^{2\sigma}}}, \tag{17}$$

$$\bar{\theta} = \sum_p \bar{\theta}_p, \bar{\theta}_p = \tan^{-1} \frac{-\sin(t \log p)}{p^\sigma + \cos(t \log p)}. \tag{18}$$

For $\cos(t \log p) < 0$, we have

$$1 > \bar{R}_p(1 + ti) > \bar{R}_p(0.5 + ti). \tag{19}$$

If $m_-(t_1)$ is much greater than $m_+(t_1)$ such that $\bar{R}(0.5 + t_1 i) = \min$. From (17), (18) and (19) we have for given t_1

$$\min \bar{R}(\sigma_1 + t_1 i) > \min \bar{R}(1 + t_1 i) > \min \bar{R}(0.5 + t_1 i)$$

$$> \min \bar{R}(\sigma_2 + t_1 i) \to 0, \tag{20}$$

$$\bar{\theta}(\sigma_1 + t_1 i) = \bar{\theta}(1 + t_1 i) = \bar{\theta}(0.5 + t_1 i) = \bar{\theta}(\sigma_2 + t_1 i) = \text{const}, \tag{21}$$

where $\sigma_1 >$ and $0 \le \sigma_2 < 0.5$.
Since $\mid \beta(s) \mid = \frac{1}{R}$ from (20) we have

$$\max \mid \beta(\sigma_1 + t_1 i) \mid < \max \mid \beta(1 + t_1 i) \mid$$

$$< \max \mid \beta(0.5 + t_1 i) \mid < \max \mid \beta(\sigma_2 + t_1 i) \mid \to \infty. \tag{22}$$

For $\cos(t \log p) > 0$ we have

$$1 < \bar{R}_p(0.5 + ti) < \bar{R}_p(0.4 + ti) < \bar{R}_p(0.3 + ti). \tag{23}$$

If $m_+(t_1)$ is much greater than $m_-(t_1)$ such that that $\bar{R}(0.5 + t_1 i) = \max$. From (17), (18) and (23) we have for given t_1

$$\max \bar{R}(\sigma_1 + t_1 i) < \max \bar{R}(0.5 + t_1 i) < \max \bar{R}(0.4 + t_1 i)$$

$$< \max \bar{R}(0.3 + t_1 i) < \max \bar{R}(\sigma_2 + t_1 i) \ne \infty, \tag{24}$$

$$\begin{aligned}\bar{\theta}(\sigma_1 + t_1 i) &= \bar{\theta}(0.5 + t_1 i) = \bar{\theta}(0.4 + t_1 i) \\ &= \bar{\theta}(0.3 + t_1 i) = \bar{\theta}(\sigma_2 + t_1 i) = \text{const},\end{aligned} \tag{25}$$

where $\sigma_1 > 0.5$ and $0 \leq \sigma_2 < 0.3$.
Since $\mid \beta(s) \mid = \frac{1}{R}$ from (24) we have

$$\min \mid \beta(\sigma_1 + t_1 i) \mid > \min \mid \beta(0.5 + t_1 i) \mid$$

$$> \min \mid \beta(0.4 + t_1 i) \mid >$$

$$\min \mid \beta(0.3 + t_1 i) \mid > \min \mid \beta(\sigma_2 + t_1 i) \mid \neq 0. \tag{26}$$

From (1) and (15) we have

$$\zeta(2s) = \zeta(s)\beta(s). \tag{27}$$

In 1896 J. Hadamard and de la Vallee Poussin proved independently $\mid \zeta(1 + ti) \mid \neq 0$. From (27) we have

$$\mid \zeta(1 + 2ti) \mid = \mid \zeta(\frac{1}{2} + ti) \mid \mid \beta(\frac{1}{2} + ti) \mid \neq 0. \tag{28}$$

From (28) we have

$$\mid \zeta(\frac{1}{2} + ti) \mid \neq 0 \quad \text{and} \quad \mid \beta(\frac{1}{2} + ti) \mid \neq 0. \tag{29}$$

$\zeta(s)$ and $\beta(s)$ are the dual functions. From (22) we have

$$\mid \beta(\frac{1}{2} + ti) \mid \neq \infty. \tag{30}$$

Therefore we have

$$\mid \zeta(\frac{1}{2} + ti) \mid \neq 0. \tag{31}$$

In the same way we have

$$\mid \zeta(\frac{1}{2} + 2ti) \mid = \mid \zeta(\frac{1}{4} + ti) \mid \mid \beta(\frac{1}{4} + ti) \mid \neq 0. \tag{32}$$

From (32) we have

$$| \zeta(\frac{1}{4} + ti) | \neq 0 \quad \text{and} \quad | \beta(\frac{1}{4} + ti) | \neq 0. \tag{33}$$

In the same way we have

$$| \zeta(\frac{1}{2^n} + ti) | \neq 0. \tag{34}$$

As $n \to \infty$ we have

$$| \zeta(ti) | \neq 0. \tag{35}$$

Proof 3. For $\sigma > 1$ we have

$$\log \zeta(s) = \sum_p \sum_{m=1}^{\infty} m^{-1} p^{-m\sigma} \exp(-itm \log p). \tag{36}$$

If $\zeta(s)$ had a zero at $\frac{1}{2} + ti$, then $\log | \zeta(\sigma + ti) |$ would tend to $-\infty$ as σ tends to $\frac{1}{2}$ from the right. From (36) we have

$$\log | \zeta(s) | = \sum_p \sum_{m=1}^{\infty} m^{-1} p^{-m\sigma} \cos(tm \log p), \tag{37}$$

with t replaced by $0, t, 2t, \cdots, Ht$, it gives

$$\sum_{j=0}^{H-1} \begin{pmatrix} 2H \\ j \end{pmatrix} \log | \zeta(\sigma + (H-j)ti) | + \frac{1}{2} \begin{pmatrix} 2H \\ H \end{pmatrix}$$

$$\log \zeta(\sigma) = \sum_p \sum_{m=1}^{\infty} m^{-1} p^{-m\sigma} A \geq 0, \tag{38}$$

where

$$\begin{aligned} A &= \sum_{j=0}^{H-1} \begin{pmatrix} 2H \\ j \end{pmatrix} \cos\left((H-j)tm \log p\right) + \tfrac{1}{2} \begin{pmatrix} 2H \\ H \end{pmatrix} \\ &= 2^{H-1} \left[1 + \cos(tm \log p)\right]^H \geq 0, \end{aligned} \tag{39}$$

H is an even number.

From (38) we have

$$(\zeta(\sigma))^{\frac{1}{2}\binom{2H}{H}} \prod_{j=0}^{H-1} |\zeta(\sigma+(H-j)ti)|^{\binom{2H}{j}} \geq 1. \quad (40)$$

Since $|\ \zeta(\frac{1}{2}+eti)\ | \neq \infty$[5], where $e = 1, 2, \cdots, H$, from (40) we have $|\ \zeta(\frac{1}{2}+eti)\ | \neq 0$ for sufficiently large even number H.

Min $|\ \zeta(\frac{1}{2}+ti) \approx 0$ but $\neq 0$. The computation of all zeros of $\zeta(\frac{1}{2}+ti)$ is error, which satisfies the the error RH.

From (39) we have

$$\cos 2\theta + 4\cos\theta + 3 = 2(1+\cos\theta)^2,$$

$$\cos 4\theta + 8\cos 3\theta + 28\cos 2\theta + 56\cos\theta + 35 = 8(1+\cos\theta)^4,$$

$$\cos 5\theta + 12\cos 5\theta + 66\cos 4\theta + 220\cos 3\theta$$

$$+495\cos 2\theta + 792\cos\theta + 462 = 32(1+\cos\theta)^6.$$

3. The Arithmetic Function $J_n(\omega)$ Replacing Riemann's Hypothesis

In view of the preceding results, the RH has no value for the study of prime distributions. In 1994 the author discover the arithmetic function $J_n(\omega)$ [12, 13] that is able to take the place of Riemann's zeta-functions and L-functions because of the following properties: $J_n(\omega) \neq 0$, then the function has infinitely many prime solutions; and if $J_n(\omega) = 0$, then the function has finitely many prime solutions.

By using Jiang's $J_n(\omega)$ function we have proved numerous theorems including the twin prime theorem, Goldbach's theorem, the prime k-tuples theorem, Santilli's theory for a prime table, the theorem of finite Fermat primes, the theorem of finite Mersenne primes, the theorem of finite repunit primes, there are infinitely many triples of n, n+1 and n+2 that each is the product of k distinct primes, there are infinitely many Carmichael numbers which are product of exactly five primes, there there are finitely many Carmichael numbers which are product of exactly six primes $\cdots$ in the prime distributions [14]. We give some theorems below

Theorem 2. Twin prime theorem: $p_1 = p + 2$.

We have the arithmetic function

$$J_2(\omega) = \prod_{3\le p\le p_i}(p-2) \ne 0.$$

Since $J_2(\omega) \ne 0$, there are infinitely many primes p such that p_1 is a prime.

Theorem 3. Goldbach theorem: $N = p_1 + p_2$.
We have the arithmetic function

$$J_2(\omega) = \prod_{3\le p\le p_i}(p-2)\prod_{p|N}\frac{p-1}{p-2} \ne 0.$$

Since $J_2(\omega) \ne 0$, every even number N greater than 4 is the sum of two odd primes.

Theorem 4. $p_1 = (p+1)^2 + 1$.
We have the arithmetic function

$$J_2(\omega) = \prod_{3\le p\le p_i}(p-2-(-1)^{\frac{p-1}{2}}) \ne 0.$$

Since $J_2(\omega) \ne 0$, there are infinitely many primes p such that p_1 is a prime.

Theorem 5. $p_1 = p^2 - 2$.
We have

$$J_2(\omega) = \prod_{3\le p\le p_i}(p-2-(\frac{2}{p})) \ne 0.$$

Since $J_2(\omega) \ne 0$, there are infinitely many primes p such that p_1 is a prime.

Theorem 6. $p_1 = p+4 \quad$ and $p_2 = 4p+1$.
We have

$$J_2(\omega) = 3\prod_{7\le p\le p_i}(p-3) \ne 0.$$

Since $J_2(\omega) \ne 0$, there are infinitely many primes p such that p_1 and p_2 are primes.

Theorem 7. $p_1=(p+1)^2+1 \quad$ and $\quad p_2=(p+1)^2+3$.
We have

$$J_2(\omega)=\prod_{5\le p\le p_i}(p-3-(-1)^{\frac{p-1}{2}}-(\frac{-3}{p}))\neq 0.$$

Since $J_2(\omega)\neq 0$, there are infinitely many primes p such that p_1 and p_2 are primes.

Theorem 8. The prime 13-tuples theorem: $p+b$: $b=0,4,6,10,16,18,24,28,34,40,46,48,90$.
Since $J_2(13)=0$, there are no prime 13-tuples if $p\neq 13$.

Theorem 9. The prime 14-tuples theorem: $p+b$: $b=0,2,6,8,12,18,20,26,30,32,36,42,48,50$.
We have

$$J_2(\omega)=300\prod_{29\le p\le p_i}(p-14)\neq 0.$$

Since $J_2(\omega)\neq 0$, there are infinitely many prime 14-tuples.

Theorem 10. $p_1=6m+1, p_2=12m+1, p_3=18m+1, p_4=36m+1, p_5=72m+1$.
We have

$$J_2(\omega)=12\prod_{13\le p\le p_i}(p-6)\neq 0.$$

Since $J_2(\omega)\neq 0$, there are infinitely many integers m such that p_1,p_2,p_3,p_4 and p_5 are primes. $n=p_1p_2p_3p_4p_5$ is the Carmichael numbers.

Theorem 11. $p_3=p_1+p_2+p_1p_2$.
We have

$$J_3(\omega)=\prod_{3\le p\le p_i}(p^2-3p+3)\neq 0.$$

Since $J_3(\omega)\neq 0$, there are infinitely many primes p_1 and p_2 such that p_3 is a prime.

Theorem 12. $p_3=(p_1+1)^5+p_2$.
We have

$$J_3(\omega)=\prod_{3\le p\le p_i}(p^2-3p+3)\neq 0.$$

Since $J_3(\omega) \neq 0$, there are infinitely many primes p_1 and p_2 such that p_3 is a prime.

Theorem 13. $p_4 = p_1(p_2 + p_3) + p_2 p_3$.

We have

$$J_4(\omega) = \prod_{3 \le p \le p_i} \left(\frac{(p-1)^4 - 1}{p} + 1 \right) \neq 0.$$

Since $J_4(\omega) \neq 0$, there are infinitely many primes p_1, p_2 and p_3 such that p_4 is a prime.

Theorem 14. Each of $n, n+1$ and $n+2$ is the product of k distinct primes.

Suppose that each of $m_1, m_2 = m_1 + 1$ and $m_3 = m_1 + 2$ is the product of $k-1$ distinct primes. We define

$$p_1 = 2m_2m_3x + 1, \quad p_2 = 2m_1m_3x + 1, \quad p_3 = 2m_1m_2x + 1. \quad (41)$$

We have the arithmetic function

$$J_2(\omega) = \prod_{3 \le p \le p_i} (p - 4 - \chi(p)) \neq 0, \quad (42)$$

where $\chi(p) = -2$ if $p \mid m_1m_2m_3$; $\chi(p) = 0$ otherwise.

Since $J_2(\omega) \neq 0$, there exist infinitely many integers x such that p_1, p_2 and p_3 are primes.

¿From (41) we have $n = m_1p_1 = 2m_1m_2m_3x + m_1, n+1 = m_1p_1 + 1 = 2m_1m_2m_3x + m_1 + 1 = m_2(2m_1m_3x + 1) = m_2p_2, n+2 = m_1p_1 + 2 = 2m_1m_2m_3x + m_1 + 2 = m_3(2m_1m_2x + 1) = m_3p_3$. If p_1, p_2 and p_3 are primes, then each of $n, n+1$ and $n+2$ is the product of k distinct primes. For example, $n = 1727913 = 3 \times 11 \times 52361, n+1 = 1727914 = 2 \times 17 \times 50821, n+2 = 1727915 = 5 \times 7 \times 49369$.

$J_n(\omega)$ is a generalization of Euler's proof for the existence of infinitely many primes. It has a wide application in various fields.

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