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Category: Classical physics.

Electromagnetic field equation and field wave equation

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Abstract

Starting from the Gauss field equation, the author of this paper sets up a group of electromagnetic field functions and a continuity equation which depicts the electric and magnetic fields. This group of equations is in perfect conformity with the Maxwell equation. By using these function groups we derived another group of wave equation of the free electromagnetic field, in which the wave amplitude is the function of frequency ω and wave number k.

Key words: field equation, wave equation.

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1 Introduction

It is well known that Gauss field function is consisted of a group of functions that depict the electric and magnetic fields in the static, stable state. When the electric and magnetic fields show time- and space-dependent changes, the author deems that those fields in the instable state would still satisfy the Gauss equations. Based on this consideration, it would be possible to derive a group of electromagnetic field equations and a continuous equation of the electric and magnetic fields, all of which are in perfect conformity with the Maxwell equation. Starting from those equations, the author has established a group of wave equations of the free electromagnetic field, which somewhat show differences compared with the Maxwell wave equation, because wave amplitude is the function of frequency ω and wave number k.

2 Equations of the Electromagnetic Field

In accordance with the Coulomb's law, we get the strength of the electric field $\mathbb{E} = Q/r^2 \cdot \mathbf{e}_r$, the $\mathbf{e}_r$ is unit vector of the space. When the electric charge Q is encircled by a closed areas, we have the Gauss theorem [1]

$$\int_v \nabla \cdot \mathbb{E} d^3x = \int_v 4\pi\rho d^3x. \qquad (1)$$

When the charge density ρ changes, the electric field $\mathbb{E}$ also shows changes. In equation (1), we calculate the total derivative of time $d/dt = \partial/\partial t + \mathbb{V} \cdot \nabla$[2] and taken constant $\mathbb{C}$ is velocity of electric field. For unstable state, the following equation is obtained

$$\int_v \nabla \cdot (\partial \mathbb{E}/\partial t + \mathbb{C} \cdot \nabla \mathbb{E}) d^3x = \int_v 4\pi(\partial\rho/\partial t + \mathbb{V} \cdot \nabla\rho) d^3x. \qquad (2)$$

Therefore, electric current density[3] $\mathbb{J}=\rho\mathbb{V}$ equation (2) can be written as

$$\int_v [\frac{\partial}{\partial t}(\nabla\cdot\mathbb{E}-4\pi\rho)+\nabla\cdot(\mathbb{C}\cdot\nabla\mathbb{E}-4\pi\mathbb{J})]d^3x=0. \tag{3}$$

When $Q=0$, we have $\rho=0$ to the electric field equation. (2) becomes

$$\int_v \nabla\cdot(\partial\mathbb{E}/\partial t+\mathbb{C}\cdot\nabla\mathbb{E})d^3x=\oint_s(\partial\mathbb{E}/\partial t+\mathbb{C}\cdot\nabla\mathbb{E})d^2x=0.$$

Hence

$$\partial\mathbb{E}/\partial t+\mathbb{C}\cdot\nabla\mathbb{E}=0. \tag{4}$$

After tensor calculus and taken metric tensor in 4-dimensions of space-time $g_{\mu\nu}=1,-1,-1,-1,\ (\mu=\nu)$[4], the $\mathbb{C}$ is constant, (3) can be changed into

$$\partial\mathbb{E}/\partial t-\nabla\cdot\mathbb{C}\mathbb{E}=0. \tag{5}$$

From equation (4) into (3), we have

$$\frac{\partial}{\partial t}(-\nabla\cdot\mathbb{E}+4\pi\rho)+\nabla\cdot(\partial\mathbb{E}/\partial t+4\pi\mathbb{J})=0. \tag{6}$$

In the continuity equation (6), let

$$-\nabla\cdot\mathbb{E}+4\pi\rho=\nabla\cdot\mathbb{E}^{'},\quad \partial\mathbb{E}/\partial t+4\pi\mathbb{J}=4\pi\mathbb{J}^{'}. \tag{7}$$

From equation (7) into (6), it becomes

$$\nabla\cdot(\partial\mathbb{E}'/\partial t+4\pi\mathbb{J}^{'})=0. \tag{8}$$

For $\nabla\cdot(c\nabla\times\mathbb{L})=0$($c$ is constant), (8) becomes

$$\partial\mathbb{E}^{'}/\partial t+4\pi\mathbb{J}^{'}=c\nabla\times\mathbb{L}. \tag{9}$$

If the free magnetic pole does not exist *M=0*, and we have $d/dt=\partial/\partial t+\mathbb{C}\cdot\nabla$, the constant $\mathbb{C}$ is velocity of magnetic field. With Gauss theorem $\int_v\nabla\cdot\mathbb{B}\ d^3x$=0 (or $\nabla\cdot\mathbb{B}=0$) [5], we obtained

$$\int_v \nabla\cdot(\partial\mathbb{B}/\partial t+\mathbb{C}\cdot\nabla\mathbb{B})d^3x=\oint_s(\partial\mathbb{B}/\partial t+\mathbb{C}\cdot\nabla\mathbb{B})d^2x=0. \tag{10}$$

Hence

$$\partial\mathbb{B}/\partial t + \mathbb{C}\cdot\nabla\mathbb{B} = 0. \tag{11}$$

Similarly, from the tensor calculus, equation (11) can be changed into

$$\partial\mathbb{B}/\partial t - \nabla\cdot\mathbb{C}\mathbb{B} = 0, \tag{12}$$

and identical equation

$$\frac{\partial}{\partial t}(\nabla\cdot\mathbb{B}) + \nabla\cdot(-\partial\mathbb{B}/\partial t) = 0 \tag{13}$$

From $\nabla\cdot\mathbb{B} = 0$, we have $\partial(\nabla\cdot\mathbb{B})/\partial t = 0$, and $\nabla\cdot(-\partial\mathbb{B}/\partial t) = 0$, this result as

$$-\partial\mathbb{B}/\partial t = c\nabla\times\mathbb{N}. \tag{14}$$

can be obtained. Electric field $\mathbb{E}$ equation (5) and magnetic field $\mathbb{B}$ equation (12) are completely symmetrical in the form. Considering the Maxwell equation based on equations (1), (9) and (14), we get the electromagnetic field equation. Let $\mathbb{L} = \mathbb{B}$ and $\mathbb{N} = \mathbb{E}$, we have Maxwell equation[6]

$$\begin{aligned} \nabla\cdot\mathbb{E} &= 4\pi\rho, \\ \nabla\cdot\mathbb{B} &= 0, \\ \nabla\times\mathbb{B} &= \frac{1}{c}\frac{\partial\mathbb{E}}{\partial t} + \frac{4\pi}{c}\mathbb{J}, \\ \nabla\times\mathbb{E} &= -\frac{1}{c}\frac{\partial\mathbb{B}}{\partial t}, \end{aligned} \tag{15}$$

and field equations (5) and (12):

$$\begin{aligned} \partial\mathbb{E}/\partial t - \nabla\cdot\mathbb{C}\mathbb{E} &= 0, \\ \partial\mathbb{B}/\partial t - \nabla\cdot\mathbb{C}\mathbb{B} &= 0. \end{aligned} \tag{16}$$

3 Wave Equation of the Free Electromagnetic filed

For a free electromagnetic field $\mathbb{J} = \rho = 0$, from the 3rd and 4th equations of (15), we have

$$\nabla \times \mathbb{B} = \frac{1}{c}\frac{\partial \mathbb{E}}{\partial t}, \quad \nabla \times \mathbb{E} = -\frac{1}{c}\frac{\partial \mathbb{B}}{\partial t}. \tag{17}$$

In Cartesian coordinates, It is known that $\nabla \times \nabla \times \mathbb{E} = \nabla\nabla \cdot \mathbb{E} - \nabla^2\mathbb{E}$. Here let $\nabla \cdot \mathbb{E} = \delta_{ij}\nabla\mathbb{E}$, where $\delta_{ij} = 1, 1, 1, (i = j)$. Given the equation (4) $\partial\mathbb{E}/\partial t + \mathbb{C} \cdot \nabla\mathbb{E} = 0$(where the $\mathbb{C}$ is a constant), now we can discuss the wave equation of unstable state electric field. Using $\mathbb{C} \cdot \nabla$ to multiply (4), we obtained $\mathbb{C} \cdot \nabla\partial\mathbb{E}/\partial t + \mathbb{C} \cdot \nabla\mathbb{C}\nabla \cdot \mathbb{E} = 0$. I thought metric tensor in 4-dimensions of space-time taking $g_{\mu\nu} = 1, -1, -1, -1(\mu = \nu)$ and $\mathbb{C} \cdot \mathbb{C} = c^2$, so[7]

$$\nabla\nabla \cdot \mathbb{E} = \frac{1}{c^2}\mathbb{C} \cdot \nabla\partial\mathbb{E}/\partial t.$$

We have the wave equation

$$\frac{1}{c^2}\frac{\partial^2\mathbb{E}}{\partial^2 t} + \frac{\mathbb{C}}{c^2} \cdot \frac{\nabla\partial\mathbb{E}}{\partial t} - \nabla^2\mathbb{E} = 0. \tag{18}$$

For plane monochrome waves, and λ is constant, equation (18) may have the following solution

$$\mathbb{E} = \mathbb{E}_0 \exp[-\lambda k \cdot r(1 + i)] \exp[\lambda\omega t(1 + i)]. \tag{19}$$

Similarly and based on equations (17) and (11), the following equation is applicable to the magnetic field

$$\frac{1}{c^2}\frac{\partial^2\mathbb{B}}{\partial^2 t} + \frac{\mathbb{C}}{c^2} \cdot \frac{\nabla\partial\mathbb{B}}{\partial t} - \nabla^2\mathbb{B} = 0. \tag{20}$$

and its solution is

$$\mathbb{B} = \mathbb{B}_0 \exp[-\lambda k \cdot r(1+i)] \exp[\lambda \omega t(1+i)]. \tag{21}$$

In equations (19) and (21), the free electromagnetic field has wave amplitude $\mathbb{F}_0 \exp(-\lambda k \cdot r + \lambda \omega t)$. Therefore, equations (18) and (20) are ones which contain the wave number k and frequency ω in the wave amplitudes. Now let's calculate the mean values $\langle \mathbb{E}(r) \rangle_t$ and $\langle \mathbb{B}(r) \rangle_t$ over longer times:

$$\langle \mathbb{E}(r) \rangle_t = \int_0^\infty \mathbb{E}(r,t)/t \cdot dt = \frac{1}{2}\pi \mathbb{E}_0 \exp[-\lambda k \cdot r(1+i)], \tag{22}$$

$$\langle \mathbb{B}(r) \rangle_t = \int_0^\infty \mathbb{B}(r,t)/t \cdot dt = \frac{1}{2}\pi \mathbb{B}_0 \exp[-\lambda k \cdot r(1+i)]. \tag{23}$$

It would not be difficult to see from unstable state, that electromagnetic field equations yet satisfy the Gauss equations. From that we obtain Maxwell equations, electric field $\mathbb{E}$ magnetic field $\mathbb{B}$ equations and free electromagnetic field wave equations. These wave equations are different from formula of Maxwell wave equation, the solution of the equation contain wave number k and frequency ω in the wave amplitudes.

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