The k-Number Sieve and k-Inclusion-exclusion Formula, Principle and Harvest

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Abstract: 1-Number Sieve: It is the Eratosthenes'-Number Sieve and the da Silva-Sylvester formula. 2-Number Sieve: We can obtain result of Goldbach' conjecture and the number of solutions of Goldbach problem. 3-Number Sieve: We can obtain result p_3 in N= $p_3+p_i P_1$ and 3-Inclusion-exclusion formula. ($p_i < \sqrt{N}$, $P_1 > \sqrt{N}$.)

1 The principle.

N: Even number. This paper to discuss the $N \geqslant 50$. ($r \geqslant 4$.)

p_i, p_r, p_{r+1}: Prime, $2 \leqslant p_i \leqslant p_r < \sqrt{N} < p_{r+1} < N$. i=1,2,…,r. r= π ($\sqrt{N}$).

[p]、[p_1, p_2]、[p_3] 、[p_4]: A prime lying in the interval [p_r+1, $N-p_r-1$],

"1+1": N= p_1+p_2。 (Goldbach' conjecture.)

"1+2": N= $p_i+p_i P_1$= $p_3+p_i P_1$= $p_4+p_i P_2$. (prime $P_1 > p_r$, prime $P_2 < p_r$.) This paper to discuss the N= $p_3+p_i P_1$.

If the N= $N_i + n_N p_i$, $N_i \neq 0$, We have p=$N_i + np_i$.

If the N= $N_{ij} + n_N p_i p_j$, $N_{ij} \neq 0$, We have p=$N_{ij} + np_i p_j$.

Diagram 1 is the principle of k-Number Sieve. (When N=152.)

Natural number 1 ~ 152

1-Number Sieve:—— ↓ — ↓ ——→ **Harvest:** ① p=13 ~ 139.②1-Inclusion-exclusion formula.

Out: np_i.($n \geqslant 2$.)**2-Number Sieve:** —— ↓ — ↓ ——→**Harvest:**①p_1,p_2.②2-Inclusion-exclusion formula.

↓ ↓

Out: $N_i + np_i$ =prime

3-Number Sieve:—— ↓ — ↓ ——→**Harvest:**①p_3。 ②3-Inclusion-exclusion formula.

↓ ↓

Out: $N_{ij} + np_i p_j$=prime

↓ ↓ ↓

(Analyse)

Diagram 1 the principle of k-Number Sieve

2 1-Number Sieve.

Object:

Natural number 1 ~ 152.

Out:

n p_i. $1 \leqslant i \leqslant r$, $n \geqslant 2$.

Harvest:

①Primes 2, 5, 7, 11, [13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 71, 73, 79, 83, 89, 97, 101, 101, 103, 107, 109, 113, 127, 131, 137, 139,]149, 151.

②1-Inclusion-exclusion formula . (da Silva-Sylvester formula.)

Literature:

① Textbook.

②Goldbach' Conjecture (3): Goldbach' Primes and Eratosthenes' sieve method.

(http://vixra.org/abs/1007.0045)

③Goldbach' Conjecture (4): The expression of the number of Goldbach' Primes.

(http://vixra.org/abs/1007.0046)

3 **2-Number Sieve.**

Object:

Primes: [13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 71, 73, 79, 83, 89, 97, 101, 101, 103, 107, 109, 113, 127, 131, 137, 139.]

Out:

$N_i + np_i$ =prime. $1 \leqslant i \leqslant r$。

They are:

2+3n = [17, 23, 29, 41, 47, 53, 59, 71, 83, 89, 101, 107, 113, 131, 137.]

2+5n = [17, 37, 47, 67, 97, 107, 127, 137.]

5+7n = [19, 47, 61, 89, 103, 131.]

9+11n = [31, 53, 97.]

Positioning for size: [17, 19, 23, 29, 31, 37, 41, 47, 53, 59, 61, 71, 83, 89, 97, 101, 101, 103, 107, 113, 127, 131, 137.]

Harvest:

①N= p_1+p_2。 They are: 13, 43, 73, 79, 109, 139.

And: 152=13+139=43+109=73+79=79+73=109+43=139+13。

②2-Inclusion-exclusion formula .

Literature:

①Tong Xin Ping, Even Number Formulae When Calculating Goldbach Problems, Journal of Youjiang Teachers' College for Nationalities Vol.10-No.3 Sep.1997 10-12.

②Goldbach' Conjecture (3): Goldbach' Primes and Eratosthenes' sieve method.

(http://vixra.org/abs/1007.0045)

③Goldbach' Conjecture (4): The expression of the number of Goldbach' Primes.

(http://vixra.org/abs/1007.0046)

④Goldbach' Conjecture (6): The Chinese Remainder Theorem and Goldbach' Primes.

(http://vixra.org/abs/1007.0049)

⑤Goldbach' Conjecture (8): Upper Bound Estimation of Number of Goldbach' Primes.

(http://vixra.org/abs/1008.0064)

⑥Goldbach' Conjecture (9): Proved Hardy-Littlewood Conjecture (A)

(http://vixra.org/abs/1008.0088)

⑦Goldbach' Conjecture (10): The Six Details in the Hardy-Littlewood Conjecture (A).

(http://vixra.org/abs/1012.0004)

4 **3-Number Sieve.**

Object:

$N_i + np_i$ =prime, $1 \leqslant i \leqslant r$.

They are: [17, 19, 23, 29, 31, 37, 41, 47, 53, 59, 61, 71, 83, 89, 97, 101, 103, 107, 113, 127, 131, 137.]

Out:

$N_{ij} + np_i\, p_j$=prime, $1 \leqslant i < j \leqslant r$.

They are:

8+3 × 3n = [17, 53, 71, 89, 107.]

2+3 × 5n = [17, 47, 107, 137.]

5+3 × 7n = [47, 89, 131.]

$2+5\times 5n$ = [127.]
$20+3\times 11n$ = [53.]
$12+5\times 7n$ = [47.]
$54+7\times 7n$ = [103.]
$42+5\times 11n$ = [97.]
$75+7\times 11n$ = [0]
$31+11\times 11n$ = [0]
Positioning for size: [17, 47, 53, 71, 89, 97, 103, 107, 131, 137.]

Harvest:

①N= $p_3+p_i P_1$。 p_3 are 19, 23, 29, 31, 37, 41, 59, 61, 67, 83, 101, 113.

They are: 152 = 19+7×19 = 23+3×43 = 29+3×41 = 31+11×11 = 37+5×23 = 41+3×37 = 59+3×31 = 61+7×23 = 67+5×17 = 83+3×23 = 101+3×17 = 113+3×13。($P_1 > p_r$.)

②3-Inclusion-exclusion formula .

Literature:

① When N > 10, the N as the sum of a prime and the product of two primes…
(http://prep.istic.ac.cn/eprint/operat/showfile.jsp?d=FILEINFO&org=1134633007022&r=1134633007022)

②The number of solution for $N=p_a+p_bp_c$.
(http://prep.istic.ac.cn/eprint/Upload//2010/1268703639030.doc)
(http://www.mathchina.com/cgi-bin/topic.cgi?forum=12&topic=1430&show=550)

5 The primes in $N_{ij} + np_i p_j$.

The primes in $N_{ij} + np_i p_j$: [17, 47, 53, 71, 89, 97, 103, 107, 131, 137.]
17: N=17+3 × 3 × 3 × 5= “1+4”。
47: N=47+3 × 5 × 7= “1+3”。
53: N=53+3 × 3 × 11= “1+3”。
71: N=71+3 × 3 × 3 × 3= “1+4”。
89: N=89+3 × 3 × 7= “1+3”。
97: N=97+5 × 11= “1+2” ($=p_4+p_i P_2$)。
103: N=103+7 × 7= “1+2” ($=p_4+p_i P_2$)。
107: N=107+3 × 3 × 5= “1+3”。
131: N=131+3 × 7= “1+2” ($=p_4+p_i P_2$)。
137: N=137+3 × 5= “1+2” ($=p_4+p_i P_2$)。
They are: four “1+2” ($=p_4+p_i P_2$, $P_2 < p_r$.), four “1+3” , two “1+4” .

6 Conclusion.

When $1 \leqslant i \leqslant r$, if $(p, p_i)=1$, then p is a prime. The 1-Inclusion-exclusion formula is da Silva-Sylvester formula.

When $1 \leqslant i \leqslant r$, if p_1, $p_2 \neq N_i + np_i$, then N= p_1+p_2= “1+1” , and we can obtain 2-Inclusion-exclusion formula.

When $1 \leqslant i < j \leqslant r$, if $p_3=N_i + np_i \neq N_{ij} + np_i p_j$, then N= $p_3+p_i P_1$= “1+2” , and we can obtain 3-Inclusion-exclusion formula. ($P_1 > p_r$.)

When $1 \leqslant i < j \leqslant r$, if $p_4=N_i + np_i=N_{ij} + np_i p_j$, then N= $p_4+p_i P_2$= “1+2” , but we can not obtain 4-Inclusion-exclusion formula. ($P_2 < p_r$.)

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