A CASE STUDY ON THE CLAY MILLENNIUM PRIZE PROBLEMS

JORMA JORMAKKA

Abstract. This article is a case study investigating the following issues: how well general purpose problem solving methods work on hard mathematical problems, how much time is required in order to learn enough from the fields in order to attack such hard problems, are there easy formulations and treatments of the Clay millennium prize problems, and what is the sociological response from the mathemacical community and media to proposed solutions to hard mathematical problems.

1. Introduction

A case study is an action study where a situation or scenario is performed and observations are made based on the experiences. Case studies are seldom made in mathematics, maybe because the field is abstract. However, this type of research is quite possible also in mathematics. This article describes a case study that investigates issues that have importance to development of problem solving methods, teaching of mathematics, multidisciplinary research, sociological research, and to studies on how media gives publicity on different topics.

The case study was quite straightforward, though it required a fair amount of tedious work. First a solution, or a solution attempt, was made to each of the seven Clay millennium prize problems. Next the solutions were placed on arxiv and efforts were made to publish the solutions in journals. Responses that these caused were collected. The solutions were also sent to a small number of selected experts and their responses were observed. Finally the experiences were written into this short report. Publishing this report is motivated by a reader asking me to post some text on the research methods used in creating [1]-[7].

The main results are as follows:

1) It is possible to grasp the problem and to work out a rather good solution attempt to any of the Clay problems by putting two to six months of intensive work to each problem. Some mathematical background is probably needed. A dissertation in mathematics from one of the more difficult fields is an advantage but much more important is a long experience in solving hard problems. Hard problems have a different nature than normal problems, and they cannot be approached in the same way. Especially, they are not so likely to be solved just by knowing the results of the particular field, by adapting existing methods from the field, or by combining several exisiting methods to a longer chain of logic. However, the case study indicates that these problems can be approached by a problem solving method that starts from studying concrete example cases and the problems that appear there.

Key words and phrases. Case study; problem solving; mathematics education; Clay problems.

2) The response from the mathematical community confirmed the largely negative expectations based on earlier experiences and observations of what kind of responses have arisen from similar attempts. For the most time the mathematical community and media have tried to ignore the results, but they have also tried to ridicule the author, and tried to claim that the results are wrong, however without presenting sound mathematical arguments to support such a judgement. They also tried to question the motives of the author, such as implying that 3-4 years of efforts on the Clay problems were made only because of self-promotion, while a much easier and clearly the favored way of self-promotion is writing a web blog. All of these methods are well known in opinion control and commonly used to supress inconvenient information. Why should they appear in mathematics is a mystery. No interest on whether the proposed solutions are correct or not could be detected in the mathematical community.

The rest of this paper is structured as follows. Section 2 discusses how much mathematical background is required for someone trying to solve hard mathematical problems. Section 3 describes general purpose problem solving methods. Only the last method was actually used and it proved effective in all Clay problems. Section 4 describes briefly the experiences of the solution attempts and of the responses. References point out to the actual efforts to solve the Clay problems. References to similar case studies or other relevant research in the scientific literature have so far not been found.

2. Can the Clay problems be solved by amateurs?

A common opinion among mathematicians is that only an expert of a particular area can meaningsfully work on open hard problems of the area. Should this be true, then there is no possibility of producing anything but gibberish in an effort to solve all of the seven Clay problems as clearly nobody is a world-class expert on the seven areas these problems address. However, there are certain reasons why this opinion may not be correct.

In the last 20 years a relatively large number of mathematicians and physicists moved from their fields to information technology and telecommunications, and apparently were able to contribute to research on those areas despite of not having formal education on the new fields. The only mathematical methods that could be used where mainly from basic mathematics. The specialists on those fields were originally just as sceptical as mathematicians still are towards outsiders, and expected that their fields require several years of formal studies. Yet, the transition was made in a short time by many people. Today it is not considered strange in Information Technology(IT) that people with different backgrounds work on a problem area that is unknown to them. It has been noticed that the result are good and the traditional resistance has given way to a positive attitude. After all, the goal is to get the job done, not to guard your own intellectual territory.

This being the case some educators of theoretical topics have claimed that the reason their students have done well on IT is that theoretical studies of mathematics, theoretical computer science or theoretical physics give a good basis on various fields. It is more reasonable to argue that the students succeeded despite of the theoretical studies because they were intelligent, hard working and willing to learn new things. While training logical thinking in theoretical areas itself is not harmful and usually just the opposite, there are some harmful thought patterns that theoretical studies create, which have to be unlearned. One of them is that it is necessary to learn a new topic fully before being able to work on the topic. That means students believing this myth are unable to tolerate uncertainty. It is much more pleasant to work in an area where you have time to learn everything than in a fast changing area where you all the time must learn new things and largely work in the state of complete uncertainty. Mathematics is a rather slowly developing field. It is really surpricing to notice how little has changed e.g. in the area of the Clay problems since I last time studied mathematics some 25 years ago. Most of these problems were known already at that time and I naturally had studied books on all of these fields before making my Ph.D. I still had the books on my bookshelves and very strangely they turned out to be still current and provided a sufficient base for starting with the Clay problems. Obviously, this is not the situation in Information Technology. Inability to work under uncertainty and unpreparedness to learn new things fast can be a fatal disadvantage in Information Technology.

Occasionally one also hears an argument that amateurs might even have some edge over experts in solving open problems. It is the myth, or folklore, of the fresh look claiming that people who have not been involved in some problem might easier find a solution or a new approach than people who have worked on the problem for a long time. This myth assumes that most fields contain problems that are not easily solved by the methods that have been developed in the field, and that the specialists on these methods would be too focused on trying the methods that they know. Indeed, should they use other methods than those developed on the field, they would also lose the advantage they have from being specialists on the field. Concerning this myth, I think there is no special advantage. All advantage comes from having more experience in working with different difficult problems, i.e., it is not an advantage of the lack of knowledge and experience but an advantage of having more knowledge and experience, which simply is not from the particular field.

The problem of how much time it should take before a person, amateur or not, learns enough of a new field in order to attack one of the Clay problems is easiest solved by looking at the example of students. A talented Ph.D. student can come to a new field and in half-a-year or one year write the first original research paper on a new topic. A professional person should need less time since the basic concepts of what is scientific work are already learned. We may conclude that any mathematician could work on any problem in mathematics, sciences and technics, if they wanted and were motivated enough to do the effort. It is not too much to claim that any mathematician could make a good effort on any of the well-known hard problems in mathematics, say e.g. the Clay Millennium Prize problems. However, in order to do so in a reasonable time without first becoming an expert on the new topic, he would have to change his style of work from the traditional mathematical ideal of knowing everything to the alternative way based on problem solving and fast learning.

3. Methods of problem solving

There are a number of known problem solving methods. The following list is not claimed to be exhaustive. As problem solving is an art, one should only suggest methods that one has tried in practise and found effective. The ones that I consider the best are analogy, negation, combination, gap finding, need identification, construction, innovation, and starting by studying a concrete example. There are other methods that consciously I use, but they are more difficult to explain in short report. Additionally there are problem solving methods that are not consciously used but the solutions just appear from somewhere. Many problems also do not require any problem solving methods, it is enough to structure the problem and use existing methods to solve it in parts. Finally, there are problems of medium-level difficulty that can be attacked in a natural way by the tools of the existing theory. In the case of hard mathematical problems, such as the Clay problems, solutions simply do not appear from somewhere and there is no clear way how to structure the problem and turn it into ordinary work. The tools of the existing theory have already been tried, and they have not worked. All of the mentioned general problem solving methods are briefly described but only the last was actually used in the Clay problems. Indeed, I consider it as the only working method in the case of hard mathematical problems. The other described methods work in other types of problems, and have proven to be quite effective in several areas. The descriptions show the limitations of these methods but these limitations do not change the fact that all of these methods work well if one knows how to use them.

Analogy

Analogy means starting from some similar mechanism and modifying the mechanims to a new area. It is commonly used in IT in service creation where e.g. a new network service can be invented by adding the prefix e- or mobile- to any existing activity in the society. Analogy is also the most common research method taught to the students of mathematics in the form, extend an existing method to a different problem. Analogy is a fast way to create a new result but the result is never really original. Often the result is also not very useful because analogy modifies a good and useful method but there is no guarantee that the modified method is good and useful. It may be artificial and unnecessary. Analogy is also not a promising method for attacking hard problems because as analogy is the easiest, it has almost certainly been tried. It is good on a new field where lots of concepts and mechanisms can simply be lifted from other fields by analogy.

Negation

Negation starts from the assumption that while there are existing solutions, they are not good. Usually the starting point is that the existing solutions are very good indeed and it is difficult to see how and why they should be discarded. The reason is that they should be discarded not because there is anything wrong with them, but because the task is to create something that has not yet been invented. Therefore we decide to discard the existing solutions by starting from the statement that the existing solution is totally wrong, we simply have to notice what is wrong with it. As the existing solution in reality is good, we usually have to change the assumptions. Therefore, the problem is posed in the way: the existing solutions are fine but they answer the wrong question. Thus, what is the correct question? It is often true that if there is a good existing solution, then inverting the assumptions, turning something to inverse, or in some other way negating the solution, also gives good solution to some different problem. This is why it may be said that true statements remain true when they are inverted. Naturally this statement is not to be taken literally, but when there is a good solution based on a strong paradigm, the paradigm gets a status of the accepted correct way of thinking of the problem. Therefore, negating the paradigm gets added interest from being in contradiction with what people have been accustomed to think. Therefore it is new, and if it is new and correct, it feels like a fine solution. It is not difficult to make a negation which is also correct, since every paradigm must limit the reality. As an example, in telecommunications properties such as statelessness and scalability have been raised to accepted paradigms. It does not take much effort to notice that state-keeping protocols also work if designed correctly, and they have many excellent properties. Also scalability can be negated, e.g. network management is by nature limited to managing your own network. Thus, we can easily make a solution that clearly has better properties than the current solutions and is in contradiction with current accepted paradigms. By carefully choosing the wording in a research paper, one can make the result sufficiently interesting.

Combination

Combination is one of the typical suggestions that are offered for finding important results. If means combining either several theories, or methods from some theory to problems in another area. A simple form of combination is a suggestion, do what they do in Crakow but with the methods they use in Warsaw. Very often a multidisciplinary group is also suggested. The basic assumption in this method is that between the fields there are fertile areas that nobody has investigated. This idea can be compared to a situation in arid terrain: there are a few fertile oasis, and a desert between them. The suggestion is to go digging water in the desert since nobody seems to be doing it. There often is a good reason why there are spaces between different branches of science: the areas between were simply not so fertile. Combination has also the problem that it tends to create clumsy inefficient solutions that nobody can understand, and which are prone to errors. Combination is commonly used in software development in the form of taking large software systems and glueuing them together. This is done in order to save time, since potentially reusing existing work is better than redoing anything. What happened with these glued pieces of software is that it turns out that the pieces do not actually fit together and there are very often misunderstandings of how the pieces should behave in the interfaces. While there is a theoretically sound solution of specifying clearly defined interfaces, they are still often misunderstood because an interface is an illusion, a facade that tries to make a complicated system appear simple. The illusion works well until one meets the cases that were not thought through carefully. In mathematics there is a facade: a proof has assumptions and a statement. Problems arise if the assumptions do not actually describe correctly what is assumed, or the reader misunderstands what is stated. These cases do appear in mathematics, and modern mathematics is very much based on building proofs by using other proved statements. The concept of elementary proof has a special position. A proof is elementary if it does not use results that cannot be easily checked by the reader. In many cases it means that an elementary proof is self-contained, but it does not necessarily need to be so. If may be based on simple concepts, such as simplices. Preferably, one should not combine simplices to complexes in an elementary proof. Combination is an intentional violation of the principle of elementary proofs, and its weakness derives from that property.

Gap finding

Gap finding is a reseach method that can be compared to zoology. You classify what exists and locate from the classification what has not been studied yet. Classification is based on literature, which makes this method very easy - though somewhat tiresome and dull - to do. Lots of graphs and figures should be produced from different view points in order to find gaps, because we want gaps that show some useful new research ideas. In general, looking at the graphs one needs a philosophical attitude comparing the solutions to properties, needs, ideas, and structures, in order to get anything interesting out of them. What commonly is the result when a gap finding exercise is made with students is that they do the graphs fine but cannot locate interesting gaps. They only produce obvious gaps. This is because they do not consider enough view points. One must try to obtain as much information as possible from those graphs. Typical view points include time development: prediction of the future trends, estimation of the times to reach some point, consideration of what making some breakthrough or evolution would cause. Additionally one should also consider the business model aspects, especially moves that invalidate certain business model, and protection moves to prevent that. These may be keys for predicting technology. How well gap finding suits to mathematics is unclear, but certainly one may identify a missing theory and develop it. If it is motivated as a natural, but missing, part of the whole, the reception of the new theory may be more positive. In order to make a mathematical theory one needs new problems that the theory addresses, new concepts, and a small number of key theorems. These key theorems inspire other theorems, and the problems give a natural area for applications. Too often a new theory is based only on a different mathematical approach without finding new problem setting. Such a theory will be competing with an existing one and will very possibly remain a curiosity.

Need identification

Need identification is one of the all time dullest research methods, but it is widely used in technical areas. You ask the customers what they want, and they try to be helpful and answer things they have heard are coming, or things they have heard are missing - meaning things that have been very difficult to make, like good natural language translation. They often also add some small gimmics from their own experience, which they turn out not to be so popular anyway. In mathematics there are no needs. Nobody seems to want any paper to be written, in fact mathematicians are either completely uninterested or hostile if a new paper is presented to them. Thus, need identification hardly is a suitable research method here. One may of course produce the need oneself finding an idea for some method that could be useful in other research.

Construction

Constructive results should not be strange to mathematicians but they are unfamiliar to many researchers who work in more mathematical or scientifical parts of technical sciences. They like to consider as results only something that has mathematical manipulations - like solving equations, measurements, or simulations. Constructive results mean proposing a construction. In technology, these constructions are usually not optimal in any sense and it is difficult to motivate why this construction should be accepted rather than some other. This is why one should find principles, design guide lines, paradigms, properties etc. By these it is possible to make a logical structure to the proposed construction such that it - under the selected properties - appears as a good solution filling these properties, rather than an ad hoc choice. Figures are essential in this method, and some formalism is needed. Preferably the formalism should be an existing one because this adds credibility to the result. The general problem in results obtained by construction is that the readers cannot be persuaded to think that the construction is a good one, as it is essentially a new concept. Therefore the readers tend to think that this is not from their field and not interesting. Using an existing formalism gives the work the character of an application of a formalism that they know, and for that reason it is easier for them to see the result as interesting.

Innovation

Innovation differs from invention by having clear practical usefulness. Innovation is usually not technically or scientifically as demanding as an invention. Typical innovations are new usages of existing solutions, new business ideas, and useful practical applications of a very small new invention. This research method is possible, as one of the few, also to a person who does not have technical or scientific background because the method can fully utilize practical knowledge of the real problems and usage situations. As the inventions in the innovation method are typically quite small, one may proceed with the methods that are best suited for finding small inventions fast, i.e., analogy and negation.

Start by studying a concrete example

This method actually means starting from the scratch. I have earlier characterized this method by functional descriptions such as: First try all possible methods, then all impossible methods, and then again try the most promising possible methods. Or: Take a sledge hammer and hit the problem with the hammer. If it does not crack, take a twice as heavy hammer and hit twice as hard. After each failed effort do these doublings and finally the problem will crack. Or: First make several hundred pages of calculations to get some feeling of the problem. The problem starts to become clearer after filling 3-4 200 page A4 notebooks with calculations. After explaining these functional aspects, I noticed that many people have no clue of what they should calculate, i.e., how to get started, and this is why I renamed this method to indicate how to start. A concrete example is the best way to start approaching a problem and it gives easily possibilities to calculations. The important thing here is that you should not start by studying the literature. This is because you will learn the existing ways of approaching the problem and these ways have been tried too many times. Later you may, indeed should, look at the literature - unless you solved the problem in a so simple and certain way that there is no need for checking if it is in contradiction with known results. The amount of work this method requires is of course very large but it would be incorrect to call it trial and error. The goal is to understand the structure of the problem, often by visualization, symmetry, or other regularity that becomes evident after many trials. Indeed, it is extremely rare to stumble to a solution by accident. All calculations are made in order to understand the structure. There are some good approaches. If the problem is a nonlinear partial differential equation of several variables and people seem to think that such a problem cannot be directly solved, then try solving it directly. The guidelines are that if the solution should be smooth everywhere, the highest orders must cancel. Find the highest orders and cancel them, then introduce small modifications to get the lower orders match. Then look for a series form solution to give a more general result. If it is a topological, geometric, or algebraic problem, or a nasty mixture called algebraic geometry, ignore algebraic methods when looking for a solution and visualize it. Find symmetries or other regularities and try to make the visualized intuition algebraic. A proof must be based on clear arguments, which is best with algebraic means, while the way the proof idea was found can be intuitive and visual.

4. Experiences on solving the problems and trying to get them reviewed

Only one solution has been checked and published [1]. It is a proof that there exist solutions that cannot be continued to the whole space-time for the space-periodic Navier-Stokes problem as formulated by Clay. The Clay formulation poses the problem in $\mathbb{R}^3$ and not on a three-torus, and does not require the pressure to be space-periodic. The problem statement does not exclude feedback forces as external forces. Under these conditions the problem can be solved. There exists a solution where pressure is not space-periodic and where the external force is a feedback force having the value zero everywhere. The solution develops a singularity in a finite time. A similar solution can be made for the non-periodic case. It was apparently not intentional that the solutions were not unique. The version of the official problem statement stated still in the Spring 2008 that the solutions are known to be unique, and referees of two journals stated the same. Thus, it was not known. The official problem statement seems to have been revised sometime in 2009, as the statement is no longer there. As for the external force being a feedback force, since it is not forbidden, it is allowed. Indeed, it is difficult to see how feedback forces could be forbidden without changing the intent of the problem statement: in the Navier-Stokes equations the external force is a general force that is often a feedback force. For instance, every steering and propulsion system creates a force that takes into account the movement of the fluid, and is thus a feedback force.

Two solutions [2] and [3] are believed to be correct, but on these fields it is practically impossible to get a solution attempt correctly reviewed. [3] was actually written over 20 years ago and several attempts were made to get it reviewed. No serious error was found, but finally in 2001 I gave up the efforts to get it checked as it was hopeless. [2] seems to have the same fate. One eminent expert states having erected a wall of silence such that no efforts for solving the problem will be read by experts. This kind of a wall seems indeed to exist, in violation of the basic principles of science.

Not all problems were solved. Notably, [4], [5] and [7] are only approaches. All of these approaches originally seemed to be tentative proofs or counterexample, but errors were found. I did not consider any of them as strong candidates because [5] and [7] were created in only two months, which is generally not sufficient for a hard problem, which [4] is a notoriously hard problem. However, feedback from possible solutions should be asked from experts when one had created a solution and cannot find an error in it. The possibility of errors is always high when solving hard mathematical problems. It follows simply from the difficulty of the problem. Nobody makes an error when solving $x = 2 + 2$. With a mathematical problem where an expert knows how to solve it while a non-expert does not, the expert will not make an error, while a non-expert may make an error but may also solve it. In the case of open hard problems, such as the Clay problems, neither the expert nor the non-expert know how to solve them. Errors are more than probable, it is likely that several attempts contain errors. Yet, there is a chance that they are correct and should therefore be checked, and even failed efforts may lead to better attacks. For these reasons, if you try to solve one of the well-known problems, do not entitle your paper as solution to this problem and post it to arxiv. It may very well turn out that it is not a solution, and you cannot change the title in arxiv.

The papers [6] seems to be a solution to the Clay problem, and again, as in the Navier-Stokes problem, the Clay problem may not correctly describe the actual problem. [6] shows that there are classical solutions to the Yang-Mills field equations that allow arbitrarily small energy. This is a new result and not in doubt. The question is whether it follows that a nontrivial quantum Yang-Mills field theory does not have a mass gap. The argument presented in [6] is that based on what is written in the official problem statement we are not expected to change the classical Yang-Mills equations e.g. by changing the coupling constant to a dimensioned property. It is a very reasonable opinion that there should be changes to the Yang-Mills equations - a quantum Yang-Mills theory can very well be quite different from the classical theory. It should be a unique theory and reflect the real physical world. In the Clay problem it seems that the goal is not to find the real physical theory. The task seems to be simply to start from the classical Yang-Mills equations and to quantizise them in some commonly accepted way. A commonly accepted quantization should in some way be compatible with the path integral quantization. As there are the classical solutions from [6], a question arises how they could be excluded in order to show that there is a mass gap. If there is no natural reason to exclude them, and we still do so, it certainly seems that we solve the mass gap problem in a trivial way that can be always made: There are these solutions that have arbitrarily small energy but we excude them for no other reason than that they have small energy. Consequently, we must get a theory where all nonvacuum solutions have energy that is above some minimum. The most simple is to create a theory containing only the vacuum and one other eigenstate field. Obviously, this must be what the Clay problem statement calls a trivial theory with a mass gap and the question was whether there is a non-trivial theory with a mass gap. A nontrivial theory cannot simply exclude state fields deriving from solutions to the classical Yang-Mills.

It is rather interesting to notice that experts responded to the atttempts [4], [5] and [7], which all had errors. They also kindly explained where I had made an error. After a few emails the case was settled and I made a correction to my solution. In the case of [1], [2], [3] and [6] the experts did not explain where they see the error. Either they did not answer at all, or they stated that there is an error in some place but did not respond when I explained how I thought it was. Thus, these cases were not resolved, and I could not make any correction to my solution since I could not verify correctness of the expert's comment. In the case [1] I actually verified that the experts claiming an error were wrong.

The papers [1]-[7] were all written in a particular style, which is rather uncommon in mathematics and science today. Notably, the proofs do not use existing theorems, there are very few references, and no related work is described. These are characteristics that partly follow from the problem solving approach, and partially are conscious choices.

To describe why they follow from the problem solving approach, let us remind ourselves that the main tools a mathematician uses are powerful theorems in his field. Thus, he creates new proofs to new theorems by using these tools. His expertice in the field is largely knowing these tools and being able to use them. The tools are very area specific and thus the specialist's expertise is limited to his area.

The problem solving methods in Section 3 are clearly no tools of this type. They are approaches. Consequently, following these approaches the proofs are not referring to problem solving theorems, as such theorems do not even exist. As these problem solving methods are generic, they are not limited to any area, but they also do not give any toolset to be used. We can compare this difference by an analogy to computer programming. A system programmer uses system calls and creates programs and scripts that in an efficient way solve certain type of problems. Knowing the system calls requires special expertise, but the code is not portable. When writing portable code, you do not use system calls unless completely necessary for accessing protected resources. Often you rewrite or reinvent something that actually would be ready in some library in order not to need to include the library, because including a library means that the whole library needs to be taken. Everything except for accessing protected resources can be written without system calls as a general purpose programming language can do everything you can do in a computer. Which style is better? It is generally agreed that usually it is better to write portable code, and if the system calls do not offer a clear way for solving the task, it is always better to write portable code because it is easier to understand and more flexible.

Thus, following the problem solving methods the proofs are not based on existing theorems but mainly self-contained, and there is no real need for including references. However, the scientific style usually requires references and related work for showing competence. In the case of [1]-[7] it was a conscious decision to leave these parts out, while in a paper intended for a Ph.D. thesis these parts are naturally required. References and related work is today very easy to write because there are databases. Earlier these parts did indeed indicate some familiarity with the topic, but not any more. I took a calculated risk that the papers might be discarded because they do not have the related work and references, as some people might use these parts for evaluation of the competence of the author. A nonexpert should not try to appear as an expert but stick to the common language of simple mathematics. Trying to write a strange language, such as mathematics of some field, is doomed to fail. Similarly as a native Finnish speaker can spot a foreigner trying to speak Finnish in about two seconds, it does not take long for an expert to notice that there are some unprecisely used terms in any text written by a nonexpert. Leaving out these parts has the purpose of disabling the referee's possibility of focusing on these parts rather than on the main argument. As referees do not want to check proofs of the Clay problems, they prefer to criticize other parts of the paper in their report, and if a nonexpert writes parts that describe the expert's field, he is giving too many chances for a referee to comment. Therefore, the paper should not have any other parts.

The goals were to write everything in a very simple way in order to make checking of the papers easy. This is also because the most common argument against proposed solutions of known problems is that they are written in an unclear way by using unknown and difficult notations, and this would be the reason that they are not checked. Certainly, the papers [1]-[7] are all very simple. Every competent mathematician should know enough of each of the areas in order to be able to check the papers [1]-[7], unless the level of mathematics education has catastrophically sunken in the last 25 years. As the mathematical community has not wanted to check them either, we know that the argument that the papers are difficult is not the real reason why they are not checked.

References to results in the literature were kept to minimum. This is because there are errors in published works and a referee would have to check also results taken from literature.

Concering the efforts to publish the papers, the following comment suffices: It is never easy to publish in a journal and often one is not satisfied with the comments of the referees. However, in the case that the submitted manuscript claims an exceptional result, like a solution to a Clay problem, the difficulties in publishing are much worse than normally.

Different ways of bypassing these difficulties have been suggested, and it is not uncommon that some friendly person suggests one of these ways without ever having tried them. As I have more experience on this issue than most, I will comment on the possibilities.

1) Arranging a seminar and inviting there the leading world experts without telling them what the seminar is about is an absurd idea. Nobody will come. You must have powerful friends to do this.

2) Posting solutions to arxiv and expecting that the mathematical community would enthustically read them is another absurd idea. Nobody will read them and only very few amateurs will comment them on some discussion boards. You must have powerful friends to do this.

3) Sending the paper to leading experts will not help. Usually the leading experts will not even answer to letters. Only if they find an error they may respond that there is an error. If it is not an error and you explain again, the expert will not respond. You should send to your powerful friends.

4) The following protocol is often suggested: Divide the paper into several papers such that the first $n-1$ make no references to the problem while the last paper simply collects the results. The last paper cannot be objected to as it directly follows from the other papers, and will be easily published. This protocol has two serious faults. Firstly, that the last paper directly follows from the others does not in any way guarantee that it be accepted. The referee simply denies that it follows, and nothing can be done. Secondly, if this is done, many other people may also notice that the result follows and will get the result published before you manage to pass the review process.

5) A modified version of 4) is writing a paper in such a way that it actually contains the whole result but does not clearly state it. After the paper is published you have to announce that the problem is solved since if you do not, the result is ignored. This modified protocol has a similar flaw as the first one in 4). There is no reason that the mathematical community should accept that the problem is solved. One eminent person is enough to deny it, and there is no way to prove yor proof. In general, if someone has decided not to understand something, there is nothing that can force him to understand.

6) There is a method of smuggling the results through the editor and the referees in order to get another review. This method depends on finding an honest editor, while the referee can be anything. One does not explain the main result of the paper in the title, abstract, introduction, or conclusions. Small lemmas are put to the first ten pages, while the result is the only theorem and comes after some 30 pages. A very likely outcome is that the paper is not noticed to be a solution to

a Clay problem, and the referee responds only to the lemmas. Then it is possible to ask for another review since the referee obviously did not notice the main result in the paper. Indeed, with any luck, the referee comments that all results on this paper are already known and they you can respond that the referee claims that a solution to the Clay problem is known. The editor may well grant another review.

More methods should be developed, unless there will be a change in the review process. Indeed, if a competition is set, the organizers of the competition should also arrage a realistic way for getting the proposed solutions checked.

There are three difficult issues in solving the Clay problems. The first one is solving the mathematical problem. As these are hard problems, it is very difficult and requires a major effort. This stage takes months. The second stage is even more difficult: getting the paper published. Journals are not giving these type of papers a fair review. The editor usually does not allow answering to the referee (there is always only one referee for these papers). This stage will take years. The last stage is getting the published result accepted by the mathematical community. This stage may be impossible to pass, as acceptance of any information is largely dependent on media. It is naive to think that if the solution is correct and published in an American peer-review journal of good reputation, the mathematical community would either show an error, or accept it. There is a third alternative: they can simply ignore it. As it is, there appears to be a promising topic for a sociological study of how scientific results are accepted, or ignored, and given, or denied, media coverage.

References

[1] J. Jormakka: Solutions to three-dimensional Navier-Stokes equations for incompressible fluids, Electronic Journal of Differential Equations, 2010, No 93, pp. 1-14.
[2] J. Jormakka: On the existence of polynomial-time algorithms to the subset sum problem, arXiv:0809.4935, 2008.
[3] J. Jormakka: On the existence of exotic homotopy 3-spheres, arxiv1012.0391, 2010.
[4] J. Jormakka: On the zeroes of the Riemann zeta function, arXiv:0806.2361, 2008.
[5] J. Jormakka: On the rank of elliptic curves, arXiv:0809.4091, 2008.
[6] J. Jormakka: Solutions to Yang-Mills equations, axviv:0146067, 2010.
[7] J. Jormakka: A note on the Hodge conjecture, arxiv:1101.0921, 2011.

Jorma Jormakka
Karhekuja 4, 01660 Vantaa, Finland
E-mail address: jorma.o.jormakka@kolumbus.fi