





The ‘non-metric’ component,  $\delta\phi\hat{\rho} = d\phi\hat{\rho} \sin\phi\hat{\psi}_P$ , projected into  $\phi\hat{c}\phi\hat{t}$  along the vertical by interval  $d\phi\hat{\rho}$  about point P of curved  $\phi\hat{\rho}$  in Fig. 1 is now projected into the proper intrinsic metric space  $\phi\rho^{0'}$  along the vertical in Fig. 3 and the ‘non-metric’ component  $\delta\phi\hat{\rho}^0 = d\phi\hat{\rho}^0 \sin\phi\hat{\psi}_{P^0}$ , projected into  $\phi\hat{c}\phi\hat{t}^0$  along the horizontal by interval  $d\phi\hat{\rho}^0$  about point  $P^0$  along the curved  $\phi\hat{\rho}^0$  in Fig. 2 is now projected into the proper intrinsic metric space  $\phi\rho'$  along the horizontal in Fig. 3.

Although the ‘non-metric’ components  $\delta\phi\hat{\rho}$  and  $\delta\phi\hat{\rho}^0$  actually exist as shown in Fig. 3, they cannot appear in the intrinsic metric coordinate interval projection relations (5a) and (5b). This is so because any interval  $\delta\phi\hat{\rho}$  of the ‘non-metric’ absolute intrinsic space is equivalent to zero interval of the relative (i.e without hat label) proper intrinsic metric space  $\phi\rho'$  into which  $\delta\phi\hat{\rho}$  is projected in Fig. 3.

If we temporarily take into account the projective ‘non-metric’ components in the intrinsic coordinate projection relations that can be derived from Fig. 3, then we have the following

$$(d\phi\rho') = (d\phi\hat{\rho}) \cos\phi\hat{\psi}_P; \quad \delta\phi\hat{\rho}^0 = d\phi\hat{\rho}^0 \sin\phi\hat{\psi}_{P^0};$$

(w.r.t. 3 – observers in  $E'^3$ ) (6a)

$$d\phi\rho^{0'} = d\phi\hat{\rho}^0 \cos^2\phi\hat{\psi}_{P^0}; \quad \delta\phi\hat{\rho} = d\phi\hat{\rho} \sin\phi\hat{\psi}_P;$$

; (w.r.t. 3 – observers in  $E^{0'3}$ ) (6b)

Now there is equality of square of intrinsic coordinate interval  $d\phi\hat{\rho}^2$  along the curved absolute intrinsic space  $\phi\hat{\rho}$  and the sum of squares of the intrinsic coordinate intervals  $d\phi\rho'$  and  $\delta\phi\hat{\rho}^0$  along the straight line proper intrinsic space  $\phi\rho'$  projected along the horizontal by  $d\phi\hat{\rho}$  and  $d\phi\hat{\rho}^0$  respectively in Fig. 3, expressed as follows

$$(d\phi\hat{\rho})^2 = (d\phi\rho')^2 + (\delta\phi\hat{\rho}^0)^2$$

This can be seen as invariance of partial intrinsic ‘line element’ between the curved  $\phi\hat{\rho}$  and its projective straight line  $\phi\rho'$  along the horizontal with respect to 3-observers in  $E'^3$  in Fig. 3. Hence

$$(d\phi\rho')^2 = (d\phi\hat{\rho})^2 - (\delta\phi\hat{\rho}^0)^2,$$

which upon using system (6a) gives,

$$(d\phi\rho')^2 = (d\phi\rho^{0'})^2 \sec^2\phi\hat{\psi}_P - (d\phi\hat{\rho}^0)^2 \sin^2\phi\hat{\psi}_{P^0}$$

This simplifies further as follows by virtue of Eq. (5b):

$$(d\phi\rho')^2 = (d\phi\rho^{0'})^2 \sec^2\phi\hat{\psi}_P - (d\phi\rho^{0'})^2 \tan^2\phi\hat{\psi}_{P^0};$$

(w.r.t. 3 – observers in  $E'^3$ ) (7a)

There is likewise invariance of partial intrinsic line element between the the curved absolute intrinsic space  $\phi\hat{\rho}^0$

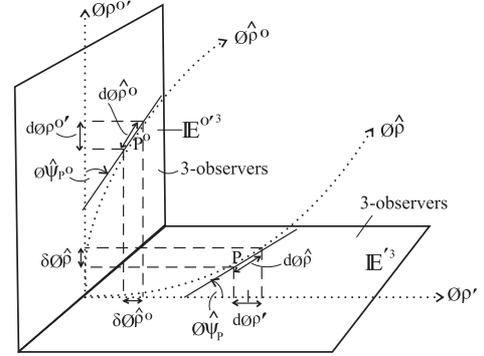


Fig. 3: Curved ‘two-dimensional’ absolute intrinsic metric space  $(\phi\hat{\rho}, \phi\hat{\rho}^0)$  and its projective flat two-dimensional proper intrinsic metric space  $(\phi\rho', \phi\rho^{0'})$  underlying flat six-dimensional proper physical space  $(E'^3, E^{0'3})$  with respect to 3-observers in  $E'^3$  in our universe and 3-observers in  $E^{0'3}$  in the positive time-universe, obtained by uniting Fig. 1 and Fig. 2.

and its projective straight line proper intrinsic space  $\phi\rho^{0'}$  along the vertical in Fig. 3, expressed as follows

$$(d\phi\hat{\rho}^0)^2 = (d\phi\rho^{0'})^2 + (\delta\phi\hat{\rho})^2$$

or

$$(d\phi\rho^{0'})^2 = (d\phi\hat{\rho}^0)^2 - (\delta\phi\hat{\rho})^2,$$

which upon using system (6b) gives,

$$(d\phi\rho^{0'})^2 = (d\phi\rho^{0'})^2 \sec^2\phi\hat{\psi}_{P^0} - (d\phi\hat{\rho})^2 \sin^2\phi\hat{\psi}_P$$

This simplifies further as follows by virtue of Eq. (5a):

$$(d\phi\rho^{0'})^2 = (d\phi\rho^{0'})^2 \sec^2\phi\hat{\psi}_{P^0} - (d\phi\rho')^2 \tan^2\phi\hat{\psi}_P;$$

(w.r.t. 3 – observers in  $E^{0'3}$ ) (7b)

Since the point P along the curved  $\phi\hat{\rho}$  and the point  $P^0$  along the curved  $\phi\hat{\rho}^0$  are symmetry-partner points, the absolute intrinsic angles  $\phi\hat{\psi}_P$  and  $\phi\hat{\psi}_{P^0}$  are equal; we can let  $\phi\hat{\psi}_P = \phi\hat{\psi}_{P^0} \equiv \phi\hat{\psi}$ . By using this fact and adding Eqs. (7a) and (7b) we have

$$(d\phi\rho^{0'})^2 + (d\phi\rho')^2 = (d\phi\rho^{0'})^2 (\sec^2\phi\hat{\psi} - \tan^2\phi\hat{\psi}) + (d\phi\rho')^2 (\sec^2\phi\hat{\psi} - \tan^2\phi\hat{\psi})$$

(8)

Eq. (8) expresses intrinsic local Euclidean invariance ( $\phi$ LEI) in terms of proper intrinsic coordinate intervals partially with respect to 3-observers in  $E'^3$  and partially with respect to 3-observers in  $E^{0'3}$ , by virtue of relation,  $\sec^2\phi\hat{\psi} - \tan^2\phi\hat{\psi} = 1$ . The full invariance of intrinsic line element (8) between the curved ‘two-dimensional’ absolute intrinsic space  $(\phi\hat{\rho}^0, \phi\hat{\rho})$  and its projective flat two-dimensional proper intrinsic space  $(\phi\rho^{0'}, \phi\rho')$  with respect to 3-observers in  $E'^3$  and 3-observers in  $E^{0'3}$  has been written partially as invariance of intrinsic

line element (7a) between the curved  $\phi\hat{\rho}$  and its projective straight line  $\phi\rho'$  with respect to 3-observers in  $E'^3$  and partially as invariance of intrinsic line element (7b) between the curved  $\phi\hat{\rho}^0$  and its projective straight line  $\phi\rho^{0'}$  with respect to 3-observers in  $E^{0'3}$  in Fig. 3 earlier.

Now the invariance of intrinsic line element between the curved 'two-dimensional' absolute intrinsic metric space  $(\phi\hat{\rho}^0, \phi\hat{\rho})$  and its projective flat two-dimensional proper intrinsic metric space  $(\phi\rho^{0'}, \phi\rho')$  in Fig. 3 can be expressed as follows

$$(d\phi\hat{s})^2 = (d\phi s')^2$$

or

$$(d\phi\hat{\rho}^0)^2 + (d\phi\hat{\rho})^2 = (d\phi\rho^{0'})^2 + (d\phi\rho')^2 \quad (9)$$

It then follows that the proper intrinsic space intervals  $d\phi\rho^{0'}$  and  $d\phi\rho'$  can be replaced by the absolute intrinsic space intervals  $d\phi\hat{\rho}^0$  and  $d\phi\hat{\rho}$  respectively in Eq. (8) to have

$$\begin{aligned} (d\phi\hat{\rho}^0)^2 + (d\phi\hat{\rho})^2 &= (d\phi\hat{\rho}^0)^2(\sec^2\phi\hat{\psi} - \tan^2\phi\hat{\psi}) \\ &\quad + (d\phi\hat{\rho})^2(\sec^2\phi\hat{\psi} - \tan^2\phi\hat{\psi}) \end{aligned} \quad (10)$$

Eq. (10) expresses intrinsic local Euclidean invariance ( $\phi$ LEI) on the curved 'two-dimensional' absolute intrinsic space  $(\phi\hat{\rho}, \phi\hat{\rho}^0)$  in terms of absolute intrinsic coordinate intervals partially with respect to 3-observers in  $E'^3$  and partially with respect to 3-observers in  $E^{0'3}$ , by virtue of relation,  $\sec^2\phi\hat{\psi} - \tan^2\phi\hat{\psi} = 1$ . Let us replace  $(d\phi\hat{\rho}^0)^2 + (d\phi\hat{\rho})^2$  by the square of absolute intrinsic Euclidean line element  $(d\phi\hat{s})^2$  at the left-hand side of (10) to have

$$\begin{aligned} (d\phi\hat{s})^2 &= (d\phi\hat{\rho}^0)^2(\sec^2\phi\hat{\psi} - \tan^2\phi\hat{\psi}) \\ &\quad + (d\phi\hat{\rho})^2(\sec^2\phi\hat{\psi} - \tan^2\phi\hat{\psi}) \end{aligned} \quad (11)$$

or

$$(d\phi\hat{s})^2 = (d\phi\hat{\rho}^0)^2 + (d\phi\hat{\rho})^2 \quad (12)$$

The absolute intrinsic Euclidean line element (11) or (12) obtains at every point along the curved  $\phi\hat{\rho}$  and at the symmetry-partner point along the curved  $\phi\hat{\rho}^0$  partially with respect to 3-observers in  $E'^3$  and partially with respect to 3-observers in  $E^{0'}$  in Fig. 3, in so far as both the metric and 'non-metric' intrinsic coordinate interval projections are taken into account in deriving intrinsic coordinate interval projection relations from Fig. 3, as done in systems (6a) and (6b) and Eqs. (7a) and (7b). This, then, is validation of intrinsic local Euclidean invariance on the curved '2-dimensional' absolute intrinsic space  $(\phi\hat{\rho}^0, \phi\hat{\rho})$  with respect to 3-observers in  $E'^3$  and 3-observers in  $E^{0'3}$  in Fig. 3.

Now let us as done on 'two-dimensional' and 'three-dimensional' absolute intrinsic metric spaces  $\phi\hat{M}^2$  and  $\phi\hat{M}^3$  in sub-section 1.1 of [1], separate the absolute intrinsic Euclidean line element  $(d\phi\hat{s})^2$  of Eq. (11) into the metric component  $(d\phi\hat{s}_m)^2$  and the 'non-metric' component  $(d\phi\hat{s}_{nm})^2$  as

follows

$$\begin{aligned} (d\phi\hat{s})^2 &= (d\phi\hat{s}_m)^2 + (d\phi\hat{s}_{nm})^2 \\ &= \sum_{i,j=0}^1 \phi\hat{g}_{ij}d\phi\hat{x}^i d\phi\hat{x}^j - \sum_{i,j=0}^1 \phi\hat{R}_{ij}d\phi\hat{x}^i d\phi\hat{x}^j \end{aligned} \quad (13)$$

$$\begin{aligned} &= \left( \sec^2\phi\hat{\psi}(d\phi\hat{\rho}^0)^2 + \sec^2\phi\hat{\psi}(d\phi\hat{\rho})^2 \right) \\ &\quad - \left( \tan^2\phi\hat{\psi}(d\phi\hat{\rho}^0)^2 + \tan^2\phi\hat{\psi}(d\phi\hat{\rho})^2 \right) \end{aligned} \quad (14)$$

The absolute intrinsic metric line element on the curved 'two-dimensional' absolute intrinsic metric space  $(\phi\hat{\rho}, \phi\hat{\rho}^0)$ , which is valid partially with respect to 3-observers in  $E'^3$  and partially with respect to 3-observers in  $E^{0'3}$  in Fig. 3 that follows from Eqs. (13) and (14) is the following

$$\begin{aligned} (d\phi\hat{s}_m)^2 &= \sum_{i,j=0}^1 \phi\hat{g}_{ij}d\phi\hat{x}^i d\phi\hat{x}^j \\ &= \phi\hat{g}_{00}(d\phi\hat{\rho}^0)^2 + \phi\hat{g}_{11}(d\phi\hat{\rho})^2 \end{aligned} \quad (15)$$

$$= \sec^2\phi\hat{\psi}(d\phi\hat{\rho}^0)^2 + \sec^2\phi\hat{\psi}(d\phi\hat{\rho})^2 \quad (16)$$

$$= \frac{(d\phi\hat{\rho}^0)^2}{1 - \phi\hat{k}^2} + \frac{(d\phi\hat{\rho})^2}{1 - \phi\hat{k}^2} \quad (17)$$

The implied absolute intrinsic metric tensor is

$$\phi\hat{g}_{ij} = \begin{pmatrix} \sec^2\phi\hat{\psi} & 0 \\ 0 & \sec^2\phi\hat{\psi} \end{pmatrix} = \begin{pmatrix} \frac{1}{1 - \phi\hat{k}^2} & 0 \\ 0 & \frac{1}{1 - \phi\hat{k}^2} \end{pmatrix} \quad (18)$$

The derived circular absolute intrinsic metric line element (16) or (17) is the absolute intrinsic line element on the curved 'two-dimensional' absolute intrinsic metric space  $(\phi\hat{\rho}^0, \phi\hat{\rho})$  in Fig. 3. It is effectively the union of the partial absolute intrinsic line element (1) derived with respect to 3-observers in the proper physical Euclidean 3-space  $E'^3$  from Fig. 1 and partial absolute intrinsic line element (3) derived with respect to 3-observers in the proper physical Euclidean 3-space  $E^{0'3}$  from Fig. 2, just as Fig. 3 from which (16) or (17) has been derived is union of Figs. 1 and 2.

It must be noted, as explicitly stated by Eqs. (1) and (3) that the term,  $\phi\hat{g}_{00}(d\phi\hat{\rho}^0)^2 = (d\phi\hat{\rho}^0)^2/(1 - \phi\hat{k}^2)$ , of the absolute intrinsic line element (17) has been derived by and is hence valid with respect to 3-observers in the proper physical Euclidean 3-space  $E^{0'3}$  of the positive time-universe, while the term,  $\phi\hat{g}_{11}(d\phi\hat{\rho})^2 = (d\phi\hat{\rho})^2/(1 - \phi\hat{k}^2)$ , has been derived by and is hence valid with respect to 3-observers in the proper physical Euclidean 3-space  $E'^3$  of our (or positive) universe in Fig. 3. Thus the components  $\phi\hat{g}_{00}$  and  $\phi\hat{g}_{11}$  of the derived circular absolute intrinsic metric tensor  $\phi\hat{g}_{ij}$  of Eq. (18) are valid with respect to 3-observers in  $E^{0'3}$  and  $E'^3$  respectively.

In essence, the curved 'two-dimensional' absolute intrinsic metric space  $(\phi\hat{\rho}, \phi\hat{\rho}^0)$  is an absolute intrinsic Riemannian



$\phi c\phi t'$  and  $d\phi\hat{\rho}^0 \rightarrow \phi\hat{c}d\phi\hat{t}$  in relation (5b), while retaining relation (5a).

Again the ‘non-metric’ component  $\phi\hat{c}d\phi\hat{t}$  projected into  $\phi\rho'$  along the horizontal by interval  $\phi\hat{c}d\phi\hat{t}$  about point  $P^0$  along the curved  $\phi\hat{c}\phi\hat{t}$  and the ‘non-metric’ component  $\delta\phi\hat{\rho}$  projected into  $\phi c\phi t'$  along the vertical by interval  $d\phi\hat{\rho}$  at the symmetry-partner point P along the curved  $\phi\hat{\rho}$  in Fig. 4 have not been taken into consideration in the intrinsic metric coordinate interval projection relations (19a) and (19b), since our interest is in deriving the absolute intrinsic metric line element and the implied absolute intrinsic metric tensor or to construct absolute intrinsic Riemann geometry on the curved ‘two-dimensional’ absolute intrinsic metric spacetime ( $\phi\hat{\rho}$ ,  $\phi\hat{c}\phi\hat{t}$ ) in Fig. 4, partially with respect to 3-observers in  $E'^3$  and partially with respect to 1-observers in  $ct'$  in that figure. The projective ‘non-metric’ absolute intrinsic coordinate intervals  $\delta\phi\hat{\rho}$  and  $\phi\hat{c}d\phi\hat{t}$  cannot appear in an absolute intrinsic metric line element.

However let us temporarily take into account the projective ‘non-metric’ components in the intrinsic coordinate projection relations that can be derived from Fig. 4 to have as follows

$$\phi c d\phi t' = \phi\hat{c}d\phi\hat{t} \cos \phi\hat{\psi}_{P^0}; \quad \delta\phi\hat{\rho} = d\phi\hat{\rho} \sin \phi\hat{\psi}_P; \quad (\text{w.r.t. } 1 - \text{observers in } ct') \quad (20a)$$

$$d\phi\rho' = d\phi\hat{\rho} \cos \phi\hat{\psi}_P \quad \phi\hat{c}d\phi\hat{t} = \phi\hat{c}d\phi\hat{t} \sin \phi\hat{\psi}_{P^0}; \quad (\text{w.r.t. } 3 - \text{observers in } E'^3) \quad (20b)$$

There is invariance of partial intrinsic line element between the curved absolute intrinsic space  $\phi\hat{\rho}$  and its projective straight line proper intrinsic space  $\phi\rho'$  along the horizontal with respect to 3-observers in  $E'^3$  in Fig. 4, expressed as follows

$$(d\phi\hat{\rho})^2 = (d\phi\rho')^2 + \phi\hat{c}^2(\delta\phi\hat{t})^2$$

or

$$(d\phi\rho')^2 = (d\phi\hat{\rho})^2 - \phi\hat{c}^2(\delta\phi\hat{t})^2,$$

which upon using system (20b) gives

$$(d\phi\rho')^2 = (d\phi\rho')^2 \sec^2 \phi\hat{\psi}_P - \phi\hat{c}^2(d\phi\hat{t})^2 \sin^2 \phi\hat{\psi}_{P^0}$$

This simplifies further as follows by virtue of Eq. (19a)

$$(d\phi\rho')^2 = (d\phi\rho')^2 \sec^2 \phi\hat{\psi}_P - \phi\hat{c}^2(d\phi\hat{t})^2 \tan^2 \phi\hat{\psi}_{P^0}; \quad (\text{w.r.t. } 3 - \text{observers in } E'^3) \quad (21a)$$

There is likewise invariance of partial intrinsic line element between the curved absolute intrinsic time ‘dimension’  $\phi\hat{c}\phi\hat{t}$  and its projective straight line proper intrinsic time dimension  $\phi c\phi t'$  along the vertical with respect to 1-observers in  $ct'$  in Fig. 4, expressed as follows

$$\phi\hat{c}^2(d\phi\hat{t})^2 = \phi\hat{c}^2(d\phi\hat{t})^2 + (\delta\phi\hat{\rho})^2$$

or

$$\phi\hat{c}^2(d\phi\hat{t})^2 = (\phi\hat{c}^2(d\phi\hat{t})^2 - (\delta\phi\hat{\rho})^2),$$

which upon using system (20a) gives

$$\phi\hat{c}^2(d\phi\hat{t})^2 = (\phi\hat{c}^2(d\phi\hat{t})^2 \sec^2 \phi\hat{\psi}_{P^0} - (d\phi\hat{\rho})^2 \sin^2 \phi\hat{\psi}_P)$$

This simplifies further as follows by virtue of Eq. (19b)

$$\phi\hat{c}^2(d\phi\hat{t})^2 = (\phi\hat{c}^2(d\phi\hat{t})^2 \sec^2 \phi\hat{\psi}_{P^0} - (d\phi\hat{\rho})^2 \tan^2 \phi\hat{\psi}_P); \quad (\text{w.r.t. } 1 - \text{observers in } ct') \quad (21b)$$

Now the point P along the curved  $\phi\hat{\rho}$  and the point  $P^0$  along the curved  $\phi\hat{c}\phi\hat{t}$  in Fig. 4 are symmetry-partner points. Consequently the absolute intrinsic angles  $\phi\hat{\psi}_P$  and  $\phi\hat{\psi}_{P^0}$  are equal. Thus we shall let,  $\phi\hat{\psi}_P = \phi\hat{\psi}_{P^0} \equiv \phi\hat{\psi}$ . By using this fact and adding Eqs. (21a) and (21b) we have

$$\phi\hat{c}^2(d\phi\hat{t})^2 + (d\phi\rho')^2 = \phi\hat{c}^2(d\phi\hat{t})^2(\sec^2 \phi\hat{\psi} - \tan^2 \phi\hat{\psi}) + (d\phi\rho')^2(\sec^2 \phi\hat{\psi} - \tan^2 \phi\hat{\psi}) \quad (22)$$

Eq. (22) expresses intrinsic local Euclidean invariance ( $\phi$ LEI) in terms of proper intrinsic coordinate intervals partially with respect to 3-observers in  $E'^3$  and partially with respect to 1-observers in  $ct'$  in Fig. 4, by virtue of the relation,  $\sec^2 \phi\hat{\psi} - \tan^2 \phi\hat{\psi} = 1$ . The full invariance of intrinsic Euclidean line element (22) has been written partially as Eq. (21a) with respect to 3-observers in  $E'^3$  from the lower half of Fig. 4 and partially as Eq. (21b) with respect to 1-observers in  $ct'$  from the upper half of Fig. 4 above.

The invariance of intrinsic line element between the curved absolute intrinsic spacetime ( $\phi\hat{\rho}$ ,  $\phi\hat{c}\phi\hat{t}$ ) and its projective flat proper intrinsic spacetime ( $\phi\rho'$ ,  $\phi c\phi t'$ ) in Fig. 4 is expressed formally as follows

$$(d\phi\hat{s})^2 = (d\phi s')^2$$

or

$$\phi\hat{c}^2(d\phi\hat{t})^2 - (d\phi\hat{\rho})^2 = \phi\hat{c}^2(d\phi\hat{t})^2 - (d\phi\rho')^2 \quad (23)$$

It follows from Eq. (23) that the proper intrinsic space interval  $d\phi\rho'$  and the proper intrinsic time dimension interval  $\phi c d\phi t'$  can be replaced by the absolute intrinsic time ‘dimension’ interval  $\phi\hat{c}d\phi\hat{t}$  respectively in Eq. (22) to have

$$\phi\hat{c}^2(d\phi\hat{t})^2 + (d\phi\hat{\rho})^2 = \phi\hat{c}^2(d\phi\hat{t})^2(\sec^2 \phi\hat{\psi} - \tan^2 \phi\hat{\psi}) + (d\phi\hat{\rho})^2(\sec^2 \phi\hat{\psi} - \tan^2 \phi\hat{\psi}) \quad (24)$$

Eq. (24) expresses intrinsic local Euclidean invariance ( $\phi$ LEI) in terms of absolute intrinsic coordinate intervals partially with respect to 3-observers in  $E'^3$  and partially with

respect to 1-observers in  $ct'$  in Fig. 4. Let us replace the left-hand side of Eq. (24) by the square of absolute intrinsic line element to be denoted by  $(d\phi\hat{s}^*)^2$  to have

$$(d\phi\hat{s}^*)^2 = \phi\hat{c}^2(d\phi\hat{t})^2(\sec^2\phi\hat{\psi} - \tan^2\phi\hat{\psi}) + (d\phi\hat{\rho})^2(\sec^2\phi\hat{\psi} - \tan^2\phi\hat{\psi}) \quad (25)$$

or

$$(d\phi\hat{s}^*)^2 = \phi\hat{c}^2(d\phi\hat{t})^2 + (d\phi\hat{\rho})^2 \quad (26)$$

The reason for introducing the dummy star label on the intrinsic line element in Eq. (25) or (26) shall be given shortly in this paper.

The absolute intrinsic Euclidean line element (25) or (26) obtains at every point along the curved  $\phi\hat{\rho}$  and the symmetry-partner point along the curved  $\phi\hat{c}\phi\hat{t}$  with respect to 3-observers in  $E'^3$  and 1-observers in  $ct'$  in Fig. 4, in so far as both the metric and 'non-metric' intrinsic coordinate interval projections have been taken into account in deriving intrinsic coordinate projection relations from Fig. 4, as done in systems (20a) and (20b) and in Eqs. (21a) and (21b). This, then, is validation of intrinsic local Euclidean invariance ( $\phi$ LEI) on the curved 'two-dimensional' absolute intrinsic spacetime  $(\phi\hat{\rho}, \phi\hat{c}\phi\hat{t})$  partially with respect to 3-observers in  $E'^3$  and partially with respect to 1-observers in  $ct'$  in Fig. 4.

As done with  $(d\phi\hat{s})^2$  in Eq. (11) earlier, let us separate the absolute intrinsic Euclidean line element  $(d\phi\hat{s}^*)^2$  in Eq. (25) into the metric and 'non-metric' components  $(d\phi\hat{s}_m^*)^2$  and  $(d\phi\hat{s}_{nm}^*)^2$  as follows

$$\begin{aligned} (d\phi\hat{s}^*)^2 &= (d\phi\hat{s}_m^*)^2 + (d\phi\hat{s}_{nm}^*)^2 \\ &= \sum_{i,j=0}^1 \phi\hat{g}_{ij}^* d\phi\hat{x}^i d\phi\hat{x}^j - \sum_{i,j=0}^1 \phi\hat{R}_{ij}^* d\phi\hat{x}^i d\phi\hat{x}^j \\ &= \left( \sec^2\phi\hat{\psi}(d\phi\hat{\rho}^0)^2 + \sec^2\phi\hat{\psi}(d\phi\hat{\rho})^2 \right) \\ &\quad - \left( \tan^2\phi\hat{\psi}(d\phi\hat{\rho}^0)^2 + \tan^2\phi\hat{\psi}(d\phi\hat{\rho})^2 \right) \end{aligned} \quad (27)$$

$$(28)$$

The absolute intrinsic line element on the curved 'two-dimensional' absolute intrinsic spacetime  $(\phi\hat{\rho}, \phi\hat{c}\phi\hat{t})$ , which is valid partially with respect to 3-observers in  $E'^3$  and partially with respect to 1-observers in  $ct'$  in Fig. 4 that follows from Eqs. (27) and (28), is the following

$$\begin{aligned} (d\phi\hat{s}_m^*)^2 &= \sum_{i,j=0}^1 \phi\hat{g}_{ij}^* d\phi\hat{x}^i d\phi\hat{x}^j \\ &= \phi\hat{g}_{00}^* \phi\hat{c}^2 (d\phi\hat{t})^2 + \phi\hat{g}_{11}^* (d\phi\hat{\rho})^2 \quad (29) \\ &= \sec^2\phi\hat{\psi} \phi\hat{c}^2 (d\phi\hat{t})^2 + \sec^2\phi\hat{\psi} (d\phi\hat{\rho})^2 \quad (30) \\ &= \frac{\phi\hat{c}^2 (d\phi\hat{t})^2}{1 - \phi\hat{k}^2} + \frac{(d\phi\hat{\rho})^2}{1 - \phi\hat{k}^2} \quad (31) \end{aligned}$$

where the relation,  $\phi\hat{k} = \sin\phi\hat{\psi}$ , derived in sub-section 1.1 of [1] and presented as Eq. (13) of that paper has been used.

The absolute intrinsic metric tensor implied by the absolute intrinsic line element (30) or (31) is the following

$$\phi\hat{g}_{ij}^* = \begin{pmatrix} \sec^2\phi\hat{\psi} & 0 \\ 0 & \sec^2\phi\hat{\psi} \end{pmatrix} \quad (32)$$

or

$$\phi\hat{g}_{ij}^* = \begin{pmatrix} \frac{1}{1 - \phi\hat{k}^2} & 0 \\ 0 & \frac{1}{1 - \phi\hat{k}^2} \end{pmatrix} \quad (33)$$

Again, the component  $\phi\hat{g}_{00}^* \phi\hat{c}^2 (d\phi\hat{t})^2 = \phi\hat{c}^2 d\phi\hat{t}^2 / (1 - \phi\hat{k}^2)$  in the absolute intrinsic line element (29), (30) or (31) has been derived by and is hence valid with respect to 1-observers in the proper time dimension  $ct'$ , while the component  $\phi\hat{g}_{11}^* (d\phi\hat{\rho})^2 = \sec^2\phi\hat{\psi} (d\phi\hat{\rho})^2 = (d\phi\hat{\rho})^2 / (1 - \phi\hat{k}^2)$  has been derived by and is hence valid with respect to 3-observers in the proper physical Euclidean 3-space  $E'^3$  in Fig. 4.

Now the absolute intrinsic line element (16) or (17) on the curved 'two-dimensional' absolute intrinsic metric space  $(\phi\hat{\rho}, \phi\hat{\rho}^0)$  in Fig. 3, obtained by uniting Fig. 1, (which is valid with respect to 3-observers in  $E'^3$ ) and Fig. 2, (which is valid with respect to 3-observers in  $E^{0/3}$ ), possesses the circular structure like the absolute intrinsic line element on curved 'two-dimensional' and 'three-dimensional' absolute intrinsic metric spaces  $\phi\hat{M}^2$  or  $\phi\hat{M}^3$  encountered in part two of this paper [1]; compare the absolute intrinsic line element (16) or (17) on the curved  $(\phi\hat{\rho}, \phi\hat{\rho}^0)$  in Fig. 3 above with the absolute intrinsic line elements (2d) and (3) on  $\phi\hat{M}^2$  and  $\phi\hat{M}^3$  in [1].

The absolute intrinsic line element (30) or (31) on the curved 'two-dimensional' absolute intrinsic metric spacetime  $(\phi\hat{\rho}, \phi\hat{c}\phi\hat{t})$  in Fig. 4, which is valid partially with respect to 3-observers in  $E'^3$  and partially with respect to 1-observers in  $ct'$  in that figure, likewise possesses the circular structure like the absolute intrinsic line element (2d) on curved 'two-dimensional' absolute intrinsic metric space  $\phi\hat{M}^2$  in [1].

It follows from the foregoing two paragraphs that the pair of absolute intrinsic tensor equations derived for curved 'two-dimensional' and 'three-dimensional' absolute intrinsic metric spaces  $\phi\hat{M}^2$  and  $\phi\hat{M}^3$  in [1] and presented as Eqs. (32) and (44) of that paper, are equally valid for the curved 'two-dimensional' absolute intrinsic metric spacetime  $(\phi\hat{\rho}, \phi\hat{c}\phi\hat{t})$  in Fig. 4 above. Let us then write those absolute intrinsic tensor equations in terms of starred absolute intrinsic metric tensor and starred absolute intrinsic Ricci tensor on the curved 'two-dimensional' absolute intrinsic spacetime  $(\phi\hat{\rho}, \phi\hat{c}\phi\hat{t})$  in Fig. 4 as follows

$$\phi\hat{g}_{ij}^* - \phi\hat{R}_{ij}^* = \delta_{ij} \quad (\phi$$
LEI) (34)

And for the second absolute intrinsic tensor equation, let us start with the intermediate equation (42) of [1] in the

process of derivation of that equation in that paper,

$$\phi \hat{R}_{ij}^* - \frac{\text{Tr} \phi \hat{R}_i^{i*}}{n} \phi \hat{g}_{ij}^* = 0 \quad (35)$$

where  $n$  is the dimensionality of the absolute intrinsic Riemann space and of the matrix  $\phi \hat{R}_i^{i*}$ . For the  $(\phi \hat{\rho}, \phi \hat{c}\phi \hat{t})$  in Fig. 4 being considered here,  $n = 2$ , thereby simplifying (35) as follows

$$\phi \hat{R}_{ij}^* - \frac{1}{2} \phi \hat{R}^* \phi \hat{g}_{ij}^* = 0 \quad (36)$$

where the absolute intrinsic Riemann scalar  $\phi \hat{R}^*$  is the trace of the  $2 \times 2$  diagonal matrix  $\phi \hat{R}_i^{i*}$ .

Interestingly Eq. (36) in absolute intrinsic Riemann geometry on curved ‘two-dimensional’ absolute intrinsic metric spacetimes  $(\phi \hat{\rho}, \phi \hat{c}\phi \hat{t})$  in Fig. 4, takes on its form in the context of conventional Riemann geometry,

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = 0 \quad (37)$$

However while the factor  $\frac{1}{2}$  in the term  $-\frac{1}{2} \phi \hat{R}^* \phi \hat{g}_{ij}^*$  in (36) restricts absolute intrinsic Riemann spaces to curved ‘two-dimensional’ absolute intrinsic metric spaces of the type  $(\phi \hat{\rho}, \phi \hat{\rho}^0)$  in Fig. 3, which must be replaced by the curved ‘two-dimensional’ absolute intrinsic Riemannian metric spacetime  $(\phi \hat{\rho}, \phi \hat{c}\phi \hat{t})$  in Fig. 4, the factor  $\frac{1}{2}$  in the term  $-\frac{1}{2} R g_{\mu\nu}$  in (37) in conventional Riemann geometry is not known to restrict the dimensionality of a conventional Riemann space or conventional Riemann spacetime to 2. A conventional Riemann space  $M^p$  can be of any dimension  $p$  and a conventional Riemannian spacetime  $M^{p+q}$  can be of any dimension  $p + q$ ; for instance  $p = 3, q = 1$  in the case of curved four-dimensional spacetime of the general theory of relativity. Eq. (37) is known to apply to all conventional Riemann spaces and conventional Riemannian spacetimes without restriction on their dimensionality.

As derived in [1], Eq. (36) admits of further simplification as follows

$$\phi \hat{R}_{ij}^* - \phi \hat{k}^2 \phi \hat{g}_{ij}^* = 0 \quad (38)$$

where  $\phi \hat{k}$  is the equal absolute intrinsic curvature parameter of an arbitrary point along the curved  $\phi \hat{\rho}$  and its symmetry-partner point along the curved  $\phi \hat{\rho}^0$  in Fig. 3, which become an arbitrary point along the curved  $\phi \hat{\rho}$  and its symmetry-partner point along the curved  $\phi \hat{c}\phi \hat{t}$  in Fig. 4.

The perfect symmetry of state between our (or positive) universe and the positive time-universe makes absolute intrinsic curvature parameters  $\phi \hat{k}_P$  and  $\phi \hat{k}_{P_0}$  at every pair of symmetry-partner points along the curved  $\phi \hat{\rho}$  and  $\phi \hat{c}\phi \hat{t}$  respectively in Fig. 4 to be identical; that is,  $\phi \hat{k}_P = \phi \hat{k}_{P_0} \equiv \phi \hat{k}$ . It is the square of the identical absolute intrinsic curvature parameters  $\phi \hat{k}^2$  that appears as the diagonal entries of the  $2 \times 2$  diagonal matrix  $\phi \hat{R}_i^{i*}$  in Eq. (35), for  $n = 2$ . Hence  $\text{Tr} \phi \hat{R}_i^{i*} = \phi \hat{R}^* = 2\phi \hat{k}^2$  and  $\frac{1}{2} \phi \hat{R}^* = \phi \hat{k}^2$ , which makes Eq. (38) the same as Eq. (36).

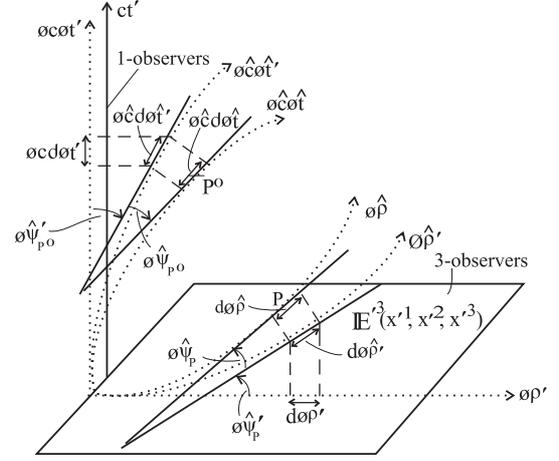


Fig. 5: A pair of co-existing ‘two-dimensional’ absolute intrinsic metric spacetimes and their underlying flat two-dimensional proper intrinsic metric spacetime and the outward manifestation of the latter namely, the flat four-dimensional proper metric spacetime; the lower half of this figure is valid with respect to 3-observers in the proper physical Euclidean 3-space and the upper half is valid with respect to 1-observers in the proper time dimension.

It is the pair of absolute intrinsic tensor equations (34) and (38), written as Eqs. (32) and (44) in [1], (and not (34) and (36) above), that shall be found directly applicable in absolute intrinsic Riemann geometry on curved ‘two-dimensional’ absolute intrinsic metric spacetime  $(\phi \hat{\rho}, \phi \hat{c}\phi \hat{t})$ , partially with respect to 3-observers in  $E^3$  and partially with respect to 1-observers in  $ct'$  in Fig. 4. For instance, the (algebraic) solution to Eqs. (34) and (38) are the starred absolute intrinsic metric tensor (33) and the following starred absolute intrinsic Ricci tensor,

$$\phi \hat{R}_{ij}^* = \begin{pmatrix} \frac{\phi \hat{k}^2}{1 - \phi \hat{k}^2} & 0 \\ 0 & \frac{\phi \hat{k}^2}{1 - \phi \hat{k}^2} \end{pmatrix} \quad (39)$$

Now let us consider the superposition of a pair of ‘two-dimensional’ absolute intrinsic metric spacetimes (a pair of ‘two-dimensional’ absolute intrinsic Riemannian metric spacetimes)  $(\phi \hat{\rho}, \phi \hat{c}\phi \hat{t})$  and  $(\phi \hat{\rho}', \phi \hat{c}\phi \hat{t}')$ , such that  $(\phi \hat{\rho}, \phi \hat{c}\phi \hat{t})$  lies over (or is curved relative to)  $(\phi \hat{\rho}', \phi \hat{c}\phi \hat{t}')$ , as illustrated in Fig. 5.

The pair of absolute intrinsic tensor equations (34) and (38) must be written in terms of resultant starred absolute intrinsic metric tensor and resultant starred absolute intrinsic Ricci tensor as follows

$$\phi \hat{g}_{ij}^* - \phi \hat{R}_{ij}^* = \delta_{ij} \quad (40)$$

$$\phi \hat{R}_{ij}^* - (\phi \hat{k})^2 \phi \hat{g}_{ij}^* = 0 \quad (41)$$

where the resultant absolute intrinsic curvature parameter  $\phi \hat{k}$

for the purpose of writing the resultant absolute intrinsic line element and resultant absolute intrinsic metric tensor at an arbitrary point of the upper curved absolute intrinsic space  $\phi\hat{\rho}$  relative to  $\phi\rho'$  along the horizontal or at the symmetry-partner point of the upper curved absolute intrinsic time 'dimension'  $\phi\hat{c}\phi\hat{t}$  relative to  $\phi c\phi t'$  along the vertical, is given in terms of the individual absolute intrinsic curvature parameters  $\phi\hat{k}'$  at point P' of the lower curved absolute intrinsic space  $\phi\rho'$  and  $\phi\hat{k}$  at point P of the upper curved absolute intrinsic space  $\phi\hat{\rho}$  prior to their superposition as follows, as derived in sub-section 1.6 of part one of this paper [1]:

$$(\phi\hat{k})^2 = (\phi\hat{k}')^2 + \phi\hat{k}^2 \quad (42)$$

The resultant starred absolute intrinsic tensors  $\phi\hat{g}_{ij}^*$  and  $\phi\hat{R}_{ij}^*$  that satisfy equations (40) and (41) are the following

$$\begin{aligned} \phi\hat{g}_{ij}^* &= \begin{pmatrix} \frac{1}{1 - (\phi\hat{k})^2} & 0 \\ 0 & \frac{1}{1 - (\phi\hat{k})^2} \end{pmatrix} \\ &= \begin{pmatrix} \frac{1}{1 - (\phi\hat{k}')^2 - \phi\hat{k}^2} & 0 \\ 0 & \frac{1}{1 - (\phi\hat{k}')^2 - \phi\hat{k}^2} \end{pmatrix} \end{aligned} \quad (43)$$

and

$$\begin{aligned} \phi\hat{R}_{ij}^* &= \begin{pmatrix} \frac{(\phi\hat{k})^2}{1 - (\phi\hat{k})^2} & 0 \\ 0 & \frac{(\phi\hat{k})^2}{1 - (\phi\hat{k})^2} \end{pmatrix} \\ &= \begin{pmatrix} \frac{(\phi\hat{k}')^2 + \phi\hat{k}^2}{1 - (\phi\hat{k}')^2 - \phi\hat{k}^2} & 0 \\ 0 & \frac{(\phi\hat{k}')^2 + \phi\hat{k}^2}{1 - (\phi\hat{k}')^2 - \phi\hat{k}^2} \end{pmatrix} \end{aligned} \quad (44)$$

The resultant absolute intrinsic line element on the curved 'two-dimensional' absolute intrinsic spacetime with curved absolute intrinsic 'dimensions'  $\phi\hat{\rho}$  and  $\phi\hat{c}\phi\hat{t}$  in Fig. 5 is then given partially with respect to 3-observers in  $E'^3$  and partially with respect to 1-observers in  $ct'$  (or with respect to (3+1)-observers in  $(E'^3, ct')$ ) as follows

$$\begin{aligned} (d\phi\hat{s}^*)^2 &= \phi\hat{g}_{00}^* \phi\hat{c}^2 (d\phi\hat{t})^2 + \phi\hat{g}_{11}^* (d\phi\hat{\rho})^2 \\ &= \frac{\phi\hat{c}^2 (d\phi\hat{t})^2}{1 - (\phi\hat{k}')^2 - \phi\hat{k}^2} + \frac{(d\phi\hat{\rho})^2}{1 - (\phi\hat{k}')^2 - \phi\hat{k}^2} \end{aligned} \quad (45)$$

On the other hand, the projection of the elementary coordinate interval  $d\phi\hat{\rho}$  about point P of the upper curved absolute intrinsic space  $\phi\hat{\rho}$  into proper intrinsic space  $\phi\rho'$  along the horizontal and of interval  $\phi\hat{c}d\phi\hat{t}$  about point P<sup>0</sup> of the upper curved absolute intrinsic time 'dimension'  $\phi\hat{c}\phi\hat{t}$  into the proper intrinsic time dimension  $\phi c\phi t'$  along the vertical, are given in terms of the resultant absolute intrinsic angle,  $\phi\hat{\psi}_{\text{res}} = \phi\hat{\psi} + \phi\hat{\psi}'$ , as follows, as derived in sub-sub-section 1.6.2 of part one of this paper [1]:

$$\begin{aligned} d\phi\rho' &= d\phi\hat{\rho} \cos \phi\hat{\psi}_{\text{res}} \\ &= d\phi\hat{\rho} \cos \phi\hat{\psi}' \cos \phi\hat{\psi} \\ &= d\phi\hat{\rho} (1 - \phi\hat{k}'^2)^{1/2} (1 - \phi\hat{k}^2)^{1/2}; \end{aligned} \quad (46)$$

(w. r. t. 3-observers in  $E'^3$ )

$$\begin{aligned} \phi c d\phi t' &= \phi\hat{c} d\phi\hat{t} \cos \phi\hat{\psi}_{\text{res}} \\ &= \phi\hat{c} d\phi\hat{t} \cos \phi\hat{\psi}' \cos \phi\hat{\psi} \\ &= \phi\hat{c} d\phi\hat{t} (1 - \phi\hat{k}'^2)^{1/2} (1 - \phi\hat{k}^2)^{1/2}; \end{aligned} \quad (47)$$

(w. r. t. 1-observers in  $ct'$ ). Extension of relations (42) through Eq. (47) to a situation of the superposition of three and larger number of curved 'two-dimensional' absolute intrinsic metric spacetimes is easy and straight forward.

It is at the first step of the modification of Fig. 3 to the form in which it is valid for absolute intrinsic Riemann geometry in our universe, when Fig. 3 is converted to Fig. 4, that the absolute intrinsic tensor equations (34) and (38) must be solved to obtain the starred absolute intrinsic metric tensor (33) and starred absolute intrinsic Ricci tensor (39). The starred absolute intrinsic metric tensor (33), the starred absolute intrinsic Ricci tensor (39) and Fig. 4 they are associated with, all of which are valid partially with respect to 3-observers in the proper physical Euclidean 3-space  $E'^3$  and partially with respect to 1-observers in the proper time dimension  $ct'$  in Fig. 4, shall now be modified to the forms in which they are valid with respect to 3-observers in the proper Euclidean 3-space  $E'^3$  solely, at the second (and final) step of converting Fig. 3 and the associated absolute intrinsic line element (16) or (17), the implied absolute intrinsic metric tensor (18) and the absolute intrinsic Ricci tensor (39) to the forms in which they are valid with respect to 3-observers in the physical proper Euclidean 3-space  $E'^3$  solely.

The modified form of Fig. 4 to be derived is the valid diagram and the associated modified absolute intrinsic line element, absolute intrinsic metric tensor and absolute intrinsic Ricci tensor are the valid forms in the context of absolute intrinsic Riemann geometry in our universe.

### 1.1 The form of spacetime/intrinsic spacetime diagram of absolute intrinsic Riemann geometry that is valid with respect to 3-observers in the proper physical Euclidean 3-space solely

Now the starred absolute intrinsic line element (30) or (31) and the implied starred absolute intrinsic metric tensor (32) or

(33), although have been derived on the curved ‘two-dimensional’ absolute intrinsic spacetime (or ‘two-dimensional’ absolute nospacetime)  $(\phi\hat{\rho}, \phi\hat{c}\phi\hat{t})$  in Fig. 4, do not possess the hyperbolic structure of the metric tensors on Riemannian metric spacetime manifolds of the type  $M^{p+q}$ ;  $p = 3, q = 1$ . Rather they have the circular/elliptical structure of the metric tensors of Riemannian metric spaces without time dimension, of the class  $M^p$ . The fact that the proper time dimension  $ct'$ , the proper intrinsic time dimension  $\phi c\phi t'$  and the curved absolute intrinsic time ‘dimension’  $\phi\hat{c}\phi\hat{t}$  appear in Fig. 4 (to replace  $E^{0/3}$ ,  $\phi\rho^{0'}$  and  $\phi\hat{\rho}^0$  respectively in Fig. 3), has not shown up in the structure of the absolute intrinsic line element (30) or (31) and the implied absolute intrinsic metric tensor (32) or (33) on the curved ‘two-dimensional’ absolute intrinsic metric spacetime  $(\phi\hat{\rho}, \phi\hat{c}\phi\hat{t})$  in that figure.

The circular structure of the absolute intrinsic line element (30) or (31) and of the absolute intrinsic metric tensor (32) or (33) arise because they have been derived partially from the upper half of Fig. 4 by or with respect to 1-observers in the proper time dimension  $ct'$  along the vertical and partially from the lower half of that figure by or with respect to 3-observers in the proper Euclidean 3-space  $E'^3$  along the horizontal. The existence of the curved absolute intrinsic time ‘dimension’  $\phi\hat{c}\phi\hat{t}$  does not appear in physics formulated by 3-observers in  $E'^3$ , just as the curved absolute intrinsic space  $\phi\hat{\rho}$  does not appear in physics formulated by 1-observers in  $ct'$  in Fig. 4. Consequently the absolute intrinsic line element (30) or (31) obtained by uniting the partial absolute intrinsic line element derived on the curved  $\phi\hat{c}\phi\hat{t}$  by 1-observers in  $ct'$  and the partial absolute intrinsic line element derived on the curved  $\phi\hat{\rho}$  by 3-observers in  $E'^3$  in Fig. 4 has not assumed the hyperbolic structure expected on a curved ‘two-dimensional’ absolute intrinsic spacetime.

The purpose of this sub-section is to derive the form of Fig. 4 that is valid with respect to 3-observers in the proper physical Euclidean 3-space  $E'^3$  solely in that figure and to derive the corresponding modified forms of the starred absolute intrinsic metric tensor (32) or (33) and the starred absolute intrinsic Ricci tensor (39) from the modified diagram. It is the modified diagram and the associated modified absolute intrinsic line element and modified absolute intrinsic metric tensor and modified absolute intrinsic Ricci tensor that are valid for absolute intrinsic Riemann geometry in our universe, as shall be justified.

Now let us present the reference geometry to the geometry of Fig. 4 as Fig. 6. Fig. 6 will exist in the absence of absolute intrinsic Riemann geometry, thereby making the curved absolute intrinsic metric space  $\phi\hat{\rho}$  and curved absolute intrinsic metric time ‘dimension’  $\phi\hat{c}\phi\hat{t}$  in Fig. 4 to become the extended straight line absolute intrinsic metric space  $\phi\hat{\rho}$  along the horizontal and the extended straight line absolute intrinsic metric time ‘dimension’  $\phi\hat{c}\phi\hat{t}$  along the vertical respectively in Fig. 6.

The reference geometry to absolute intrinsic Riemannian

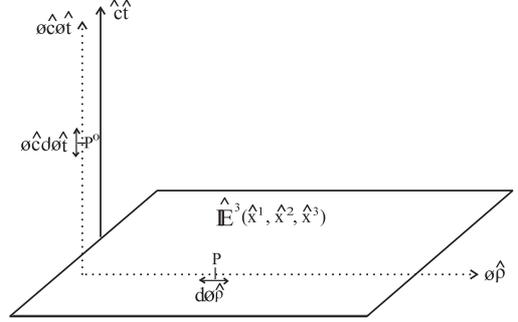


Fig. 6: Flat ‘four-dimensional’ absolute metric spacetime underlied by flat ‘two-dimensional’ absolute intrinsic metric spacetime, which exists in the absence of a long-range metric force field (or in the absence of absolute intrinsic Riemannian spacetime geometry).

spacetime geometry of Fig. 6 will persist in the absence of a long-range metric force field. However let us introduce the source of a long-range absolute metric force field at a point  $S$  on the flat absolute space  $\hat{E}^3$ . Then its underlying source of long-range absolute intrinsic metric force field in the underlying straight line absolute intrinsic space  $\phi\hat{\rho}$  will appear automatically directly underneath the source of absolute metric force field introduced at point  $S$  in  $\hat{E}^3$ . When we particularize to gravitational field, as shall be done fully elsewhere, this means that the absolute rest mass  $\hat{M}_0$  of a gravitational field source is introduced at point  $S$  in  $\hat{E}^3$  and the absolute intrinsic rest mass  $\phi\hat{M}_0$  of the gravitational field source automatically appears in  $\phi\hat{\rho}$  underneath  $\hat{M}_0$  in  $\hat{E}^3$ . As shall be explained to some extent in the last section of this paper and completely elsewhere with further development, this action will cause Fig. 6 to evolve into Fig. 4.

Now the absolute time ‘dimension’  $\hat{c}\hat{t}$  and the absolute intrinsic time ‘dimension’  $\phi\hat{c}\phi\hat{t}$  remain unchanged, that is, do not transform into proper time dimension  $ct'$  (usually denoted by  $c\tau$ ) and proper intrinsic time dimension  $\phi c\phi t'$  respectively in absolute physics/absolute intrinsic physics, such as associated with the presence of absolute metric force field in absolute spacetime and absolute intrinsic metric force field in absolute intrinsic spacetime, which causes Fig. 6 to transform into Fig. 4 discussed above. Graphically, this means that  $\hat{c}\hat{t}$  and  $\phi\hat{c}\phi\hat{t}$  along the vertical in Fig. 6 must remain along the vertical with respect to 3-observers in the proper Euclidean 3-space  $E'^3$ , as happens in Fig. 1, which is valid with respect to 3-observers in  $E'^3$  solely, in the context of absolute physics/absolute intrinsic physics. The absolute time ‘dimension’  $\hat{c}\hat{t}$  and the absolute intrinsic time ‘dimension’  $\phi\hat{c}\phi\hat{t}$  must likewise remain along the vertical in the modified form of Fig. 4 being sought, which is valid with respect to 3-observers in the Euclidean 3-space  $E'^3$  solely, in the context of absolute physics/absolute intrinsic physics.

Thus for the purpose of deriving absolute intrinsic line element and its implied absolute intrinsic metric tensor on

the curved ‘two-dimensional’ absolute intrinsic metric spacetime  $(\phi\hat{\rho}, \phi\hat{c}\phi\hat{t})$  and for formulating the non-detectable absolute metric theories of physics as 3-geometry theories in the proper Euclidean 3-space - absolute time parameter  $(E'^3; \hat{t})$  and absolute intrinsic metric theories of physics as intrinsic 1-geometry theories in the underlying proper intrinsic space - absolute intrinsic time parameter  $(\phi\rho'; \phi\hat{t})$ , the preceding paragraph makes it mandatory for us to modify Fig. 4 in such a way that the absolute time ‘dimension’  $\hat{c}\hat{t}$  and the absolute intrinsic time ‘dimension’  $\phi\hat{c}\phi\hat{t}$  remain along the vertical in the modified diagram. The resulting diagram will then contain flat  $(E'^3; \hat{c}\hat{t})$  underlied by flat  $(\phi\rho'; \phi\hat{c}\phi\hat{t})$ . It will hence be a 3-geometry/intrinsic 1-geometry diagram, which is valid with respect to 3-observers in the proper physical Euclidean 3-space  $E'^3$  solely, (unlike the 4-geometry/intrinsic 2-geometry diagram of Fig. 4 that is valid partially with respect to 3-observers in the proper Euclidean 3-space and partially with respect to 1-observers in the proper time dimension). The required modified form of Fig. 4, which is valid for absolute intrinsic Riemann geometry in our universe is derived hereunder.

Now the anti-clockwise sense of rotation by positive absolute intrinsic angle  $\phi\hat{\psi}_P$  of the absolute intrinsic coordinate interval  $d\phi\hat{\rho}$  relative to its projective proper intrinsic coordinate interval  $d\phi\rho'$  along the horizontal is valid with respect to 3-observers in the proper physical Euclidean 3-space  $E'^3$  in Fig. 4, since anti-clockwise rotation is positive with respect to these observers. Likewise the clockwise rotation by positive absolute intrinsic angle  $\phi\hat{\psi}_{P_0}$  of the absolute intrinsic time coordinate interval  $\phi\hat{c}d\phi\hat{t}$  relative to its projective proper intrinsic coordinate interval  $\phi cd\phi t'$  along the vertical is valid with respect to 1-observers in the time dimension  $ct'$  in Fig. 4, since clockwise rotation is positive with respect to these observers, (as explained in detail in section 4 of [3]). On the other hand, the clockwise rotation by positive absolute intrinsic angle  $\phi\hat{\psi}_{P_0}$  of the absolute intrinsic time coordinate interval  $\phi\hat{c}d\phi\hat{t}$  relative to  $\phi cd\phi t'$  along the vertical is invalid with respect to 3-observers in 3-space  $E'^3$ . Consequently the upper half of Fig. 4 is valid with respect to 1-observers in  $ct'$  and the lower half is valid with respect to 3-observers in  $E'^3$ .

In order to make Fig. 4 valid with respect to observers in the physical 3-space  $E'^3$  solely, we must change the positive sign of the absolute intrinsic angle  $\phi\hat{\psi}_{P_0}$  of inclination of  $\phi\hat{c}d\phi\hat{t}$  to  $\phi cd\phi t'$  without changing its clockwise sense. However we can do this only if we also interchange the interval  $\phi\hat{c}d\phi\hat{t}$  along the curved  $\phi\hat{c}\phi\hat{t}$  and its projection  $\phi cd\phi t'$  into  $\phi c\phi t'$  along the vertical. Doing this about every point along the curved  $\phi\hat{c}\phi\hat{t}$  implies interchanging the curved  $\phi\hat{c}\phi\hat{t}$  and the straight line proper intrinsic time dimension  $\phi c\phi t'$ . By implementing these in Fig. 4 we have Fig. 7, which is now valid with respect to 3-observers in the physical proper Euclidean 3-space  $E'^3$  solely.

It must be observed that since the absolute intrinsic time ‘dimension’  $\phi\hat{c}\phi\hat{t}$  is a straight line along the vertical, its out-

ward manifestation is the straight line absolute time ‘dimension’  $\hat{c}\hat{t}$  along the vertical in Fig. 7. Thus the proper time dimension  $ct'$  does not appear with respect to 3-observers in the proper physical 3-space  $E'^3$  in the absolute intrinsic Riemannian spacetime geometry of Fig. 7. Fig. 7 contains the curved absolute intrinsic metric space  $\phi\hat{\rho}$  and straight line absolute intrinsic metric time ‘dimension’  $\phi\hat{c}\phi\hat{t}$  on which to construct absolute intrinsic Riemann geometry with respect to 3-observers in the proper physical Euclidean 3-space  $E'^3$ , as shall be done below.

Fig. 7 also contains the flat proper Euclidean 3-space - absolute time ‘dimension’  $(E'^3, \hat{c}\hat{t})$  (the Galileo space of absolute physics), in which to formulate non-observable absolute physics, such as absolute gravity, absolute motion, absolute electromagnetism, etc, (as 3-geometry classical theories) and its underlying proper intrinsic space - absolute intrinsic time ‘dimension’  $(\phi\rho', \phi\hat{c}\phi\hat{t})$  (the intrinsic Galileo space of absolute intrinsic physics), in which to formulate absolute intrinsic physics, such as absolute intrinsic gravity, absolute intrinsic motion, absolute intrinsic electromagnetism, etc, (as intrinsic 1-geometry classical theories) by 3-observers in the proper physical Euclidean space  $E'^3$ , as shall be developed elsewhere with further development.

The intrinsic metric coordinate interval projection relations derivable from Fig. 7, from which absolute intrinsic metric line element can be derived on  $(\phi\hat{\rho}, \phi\hat{c}\phi\hat{t})$  with respect to 3-observers in  $E'^3$  solely in that figure are the following

$$\phi\hat{c}d\phi\hat{t} = \phi cd\phi t' \cos(-\phi\hat{\psi}_{P_0}) \quad d\phi\rho' = d\phi\hat{\rho} \cos \phi\hat{\psi}_P$$

or

$$\phi cd\phi t' = \phi\hat{c}d\phi\hat{t} \sec \phi\hat{\psi} \quad (48a)$$

$$d\phi\rho' = d\phi\hat{\rho} \cos \phi\hat{\psi} \quad (48b)$$

Equation (48a) derived by 3-observers in  $E'^3$  in Fig. 7 replaces Eq. (19a) derived by 1-observers in the proper time dimension  $ct'$  in Fig. 4. Equations (48a) and (48b) are intrinsic ‘time dilation’ and intrinsic ‘length contraction’ formulae with respect to 3-observers in the proper physical 3-space  $E'^3$  in the absolute intrinsic Riemannian spacetime geometry of Fig. 7.

The projective intrinsic metric coordinate intervals  $d\phi\rho'$  and  $\phi cd\phi t'$  in Fig. 7 have been put into consideration in relations (48a) and (48b), while the ‘non-metric’ intrinsic coordinate intervals  $\delta\phi\hat{\rho}$  and  $\phi\hat{c}d\phi\hat{t}$  have been disregarded. Indeed the absolute intrinsic line element on the curved absolute intrinsic spacetime  $(\phi\hat{\rho}, \phi\hat{c}\phi\hat{t})$ , which is valid with respect to 3-observers in the physical proper Euclidean 3-space  $E'^3$  solely in Fig. 4, must be synthesized from the intrinsic metric coordinate interval projection relations (48a) and (48b) derived from Fig. 7. However the appropriate structure (or signature) of that absolute intrinsic line element to adopt is yet unknown and cannot be determined from Eqs. (48a) and (48b).

In order to determine the structure (or signature) of the absolute intrinsic line element and consequently the absolute

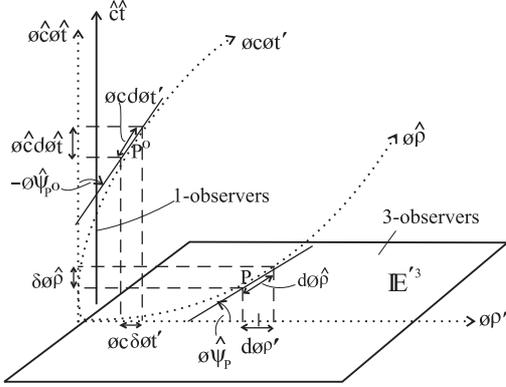


Fig. 7: The form of Fig. 4 that is valid with respect to 3-observers in the proper physical Euclidean 3-space solely; the correct diagram for absolute intrinsic Riemannian spacetime geometry in our universe.

intrinsic metric tensor on curved ‘two-dimensional’ absolute intrinsic spacetime  $(\phi\hat{\rho}, \phi\hat{c}\phi\hat{t})$  that is valid with respect to 3-observer in  $E'^3$  solely in Fig. 4, we must first determine which of intrinsic local Euclidean invariance ( $\phi$ LEI) and intrinsic local Lorentz invariance ( $\phi$ LLI) on  $(\phi\hat{\rho}, \phi\hat{c}\phi\hat{t})$  is valid with respect to 3-observers in  $E'^3$  solely in Fig. 7. Thus let us take into account the projective ‘non-metric’ components in the intrinsic coordinate projection relations that can be derived from Fig. 7 to have as follows

$$\phi\hat{c}d\phi\hat{t} = \phi\hat{c}d\phi\hat{t}' \cos(-\phi\hat{\psi}_{P0}); \quad \delta\phi\hat{\rho} = d\phi\hat{\rho} \sin \phi\hat{\psi}_P$$

or

$$\phi\hat{c}d\phi\hat{t}' = \phi\hat{c}d\phi\hat{t} \sec \phi\hat{\psi}_{P0}; \quad \delta\phi\hat{\rho} = d\phi\hat{\rho} \sin \phi\hat{\psi}_P \quad (49a)$$

and

$$d\phi\hat{\rho}' = d\phi\hat{\rho} \cos \phi\hat{\psi}_P; \quad \phi\hat{c}\delta\phi\hat{t}' = \phi\hat{c}d\phi\hat{t}' \sin(-\phi\hat{\psi}_{P0})$$

or

$$d\phi\hat{\rho}' = d\phi\hat{\rho} \cos \phi\hat{\psi}_P; \quad \phi\hat{c}\delta\phi\hat{t}' = -\phi\hat{c}d\phi\hat{t}' \sin \phi\hat{\psi}_{P0} \quad (49b)$$

or

$$d\phi\hat{\rho}' = d\phi\hat{\rho} \cos \phi\hat{\psi}_P; \quad \phi\hat{c}\delta\phi\hat{t}' = -\phi\hat{c}d\phi\hat{t}' \tan \phi\hat{\psi}_{P0} \quad (49c)$$

where Eq. (48b) has been used in the second equation between systems (49a) and (49b).

The intrinsic coordinate projection relations of systems (49a) and (49b) derived from Fig. 7 are valid with respect to 3-observers in the proper physical Euclidean 3-space  $E'^3$  solely in that figure. They correspond to systems (20a) and (20b) derived from Fig. 4, which are valid with respect to 1-observers in  $ct'$  and 3-observers in  $E'^3$  respectively.

Now the only absolute intrinsic space coordinate interval  $d\phi\hat{\rho}$  about point O along the absolute intrinsic space  $\phi\hat{\rho}$  when

$\phi\hat{\rho}$  was a straight line along the horizontal in Fig. 6, becomes replaced by components  $d\phi\hat{\rho}'$  and  $\phi\hat{c}\delta\phi\hat{t}'$  projected along the horizontal in Fig. 7, upon evolution of the geometry of Fig. 7 from Fig. 6. There is invariance of the squares of intrinsic coordinate intervals along the horizontal between Fig. 6 and Fig. 7, expressed as follows:

$$(d\phi\hat{\rho}')^2 + \phi\hat{c}^2(\delta\phi\hat{t}')^2 = (d\phi\hat{\rho})^2$$

or

$$\begin{aligned} (d\phi\hat{\rho}')^2 &= (d\phi\hat{\rho})^2 - \phi\hat{c}^2(\delta\phi\hat{t}')^2 \\ &= (d\phi\hat{\rho}')^2 \sec^2 \phi\hat{\psi}_P - \phi\hat{c}^2(\delta\phi\hat{t}')^2 \sin^2 \phi\hat{\psi}_{P0} \end{aligned} \quad (50)$$

where systems (49a) and (49b) have been used. In effect, this equation expresses invariance of partial intrinsic line element between the curved ‘one-dimensional’ absolute intrinsic space  $\phi\hat{\rho}$  and its projective straight line proper intrinsic space  $\phi\hat{\rho}'$  in Fig. 7.

The following relation likewise obtains between the intrinsic coordinates interval  $\phi\hat{c}d\phi\hat{t}'$  along the curved proper intrinsic time dimension  $\phi\hat{c}\phi\hat{t}'$  and the absolute intrinsic coordinate intervals  $\phi\hat{c}d\phi\hat{t}$  and  $\delta\phi\hat{\rho}$  projected into the straight line absolute intrinsic time ‘dimension’  $\phi\hat{c}\phi\hat{t}$  along the vertical in the upper half of Fig. 7

$$\begin{aligned} \phi\hat{c}^2(d\phi\hat{t}')^2 &= \phi\hat{c}^2(d\phi\hat{t})^2 + (\delta\phi\hat{\rho})^2 \\ &= \phi\hat{c}^2(d\phi\hat{t}')^2 \cos^2 \phi\hat{\psi}_{P0} + (d\phi\hat{\rho})^2 \sin^2 \phi\hat{\psi}_P \\ &= \phi\hat{c}^2(d\phi\hat{t}')^2 \cos^2 \phi\hat{\psi}_{P0} + (d\phi\hat{\rho}')^2 \tan^2 \phi\hat{\psi}_P \end{aligned} \quad (51)$$

where, again, systems (49a) and (49b) have been used and Eq. (48b) has been used between the last two lines of equations. Again, Eq. (51) expresses invariance of intrinsic line element between the curved one-dimensional proper intrinsic time dimension  $\phi\hat{c}\phi\hat{t}'$  and the straight line absolute intrinsic time ‘dimension’  $\phi\hat{c}\phi\hat{t}$  along the vertical in Fig. 7. Both relations (50) and (51) have been derived by 3-observers in the proper Euclidean 3-space  $E'^3$  in Fig. 7 solely, with respect to whom Fig. 7 is valid.

Now the addition of Eqs. (50) and (51) does not lead to intrinsic local Euclidean invariance ( $\phi$ LEI), as can be easily verified. It may be recalled that the addition of the corresponding Eqs. (21a) and (21b) derived from Fig. 4, leads to intrinsic local Euclidean invariance expressed by Eq. (22).

On the other hand, let us subtract Eq. (50) from Eq. (51) to have as follows

$$\begin{aligned} \phi\hat{c}^2(d\phi\hat{t}')^2 - (d\phi\hat{\rho}')^2 &= \phi\hat{c}^2(d\phi\hat{t}')^2 \cos^2 \phi\hat{\psi} \\ &\quad + (d\phi\hat{\rho}')^2 \tan^2 \phi\hat{\psi} \\ &\quad - (d\phi\hat{\rho}')^2 \sec^2 \phi\hat{\psi} \\ &\quad + \phi\hat{c}^2(d\phi\hat{t}')^2 \sin^2 \phi\hat{\psi} \end{aligned} \quad (52)$$

where the fact that  $\phi\hat{\psi}_P = \phi\hat{\psi}_{P0} \equiv \phi\hat{\psi}$  has been used. Eq. (52) is given as follows by associating like terms at the right-hand side

$$\begin{aligned} \phi c^2 (d\phi t')^2 - (d\phi\rho')^2 &= \phi c^2 (d\phi t')^2 (\cos^2 \phi\hat{\psi} + \sin^2 \phi\hat{\psi}) \\ &\quad - (d\phi\rho')^2 (\sec^2 \phi\hat{\psi} - \tan^2 \phi\hat{\psi}) \end{aligned} \quad (53)$$

Eq. (53) expresses intrinsic local lorentz invariance ( $\phi$ LLI) in terms of proper intrinsic coordinate intervals by virtue of relations  $\cos^2 \phi\hat{\psi} + \sin^2 \phi\hat{\psi} = 1$  and  $\sec^2 \phi\hat{\psi} - \tan^2 \phi\hat{\psi} = 1$ .

Now the invariance of intrinsic line element between the absolute intrinsic spacetime  $(\phi\hat{\rho}, \phi\hat{c}\phi\hat{t})$  and the proper intrinsic spacetime  $(\phi\rho', \phi c d\phi t')$  in Fig. 7 allows us to write the following

$$(d\phi\hat{s})^2 = (d\phi s')^2$$

or

$$\phi\hat{c}^2 (d\phi\hat{t})^2 - (d\phi\hat{\rho})^2 = \phi c^2 (d\phi t')^2 - (d\phi\rho')^2 \quad (54)$$

where  $d\phi\hat{s}$  pertains to the ‘two-dimensional’ absolute intrinsic spacetime  $(\phi\hat{\rho}, \phi\hat{c}\phi\hat{t})$  bounded by curved  $\phi\hat{\rho}$  and straight line  $\phi\hat{c}\phi\hat{t}$ , while  $d\phi s'$  pertains to  $(\phi\rho', \phi c d\phi t')$  in Fig. 7.

It follows from Eq. (54) that the proper intrinsic coordinate intervals  $d\phi\rho'$  and  $\phi c d\phi t'$  can be replaced by the absolute intrinsic coordinate intervals  $d\phi\hat{\rho}$  and  $\phi\hat{c} d\phi\hat{t}$  respectively in Eq. (53) to have

$$\begin{aligned} \phi\hat{c}^2 (d\phi\hat{t})^2 - (d\phi\hat{\rho})^2 &= \phi\hat{c}^2 (d\phi\hat{t})^2 (\cos^2 \phi\hat{\psi} + \sin^2 \phi\hat{\psi}) \\ &\quad - (d\phi\hat{\rho})^2 (\sec^2 \phi\hat{\psi} - \tan^2 \phi\hat{\psi}) \end{aligned} \quad (55)$$

Again Eq. (55) expresses intrinsic local Lorentz invariance in terms of absolute intrinsic coordinate intervals. Let us replace  $\phi\hat{c}^2 (d\phi\hat{t})^2 - (d\phi\hat{\rho})^2$  by  $(d\phi\hat{s})^2$  at the left-hand side of (55) to have

$$\begin{aligned} (d\phi\hat{s})^2 &= \phi\hat{c}^2 (d\phi\hat{t})^2 (\cos^2 \phi\hat{\psi} + \sin^2 \phi\hat{\psi}) \\ &\quad - (d\phi\hat{\rho})^2 (\sec^2 \phi\hat{\psi} - \tan^2 \phi\hat{\psi}) \end{aligned} \quad (56)$$

or

$$(d\phi\hat{s})^2 = \phi\hat{c}^2 (d\phi\hat{t})^2 - (d\phi\hat{\rho})^2 \quad (57)$$

The absolute intrinsic Lorentzian line element (56) or (57) obtains at every point along the curved  $\phi\hat{\rho}$  and the symmetry-partner point along the straight line absolute intrinsic time ‘dimension’  $\phi\hat{c}\phi\hat{t}$  with respect to 3-observers in  $E'^3$  in Fig. 7, in so far as both the metric and ‘non-metric’ intrinsic coordinate interval projections are taken into account in deriving intrinsic coordinate projection relations from Fig. 7, as done in systems (49a-c) and in Eqs. (50) and (51).

In brief, it is intrinsic local Lorentz invariance ( $\phi$ LLI) (and not intrinsic local Euclidean invariance ( $\phi$ LEI)) that obtains on the ‘two-dimensional’ absolute intrinsic spacetime

bounded by curved  $\phi\hat{\rho}$  and straight line  $\phi\hat{c}\phi\hat{t}$ , with respect to 3-observers in the physical proper Euclidean 3-space  $E'^3$  in Fig. 7, in so far as both the metric and ‘non-metric’ intrinsic coordinate interval projections are taken into account in deriving intrinsic coordinate projection relations from Fig. 7.

Now let us separate  $(d\phi\hat{s})^2$  in Eq. (56) into the metric and ‘non-metric’ components as follows

$$\begin{aligned} (d\phi\hat{s})^2 &= (d\phi\hat{s}_m)^2 + (d\phi\hat{s}_{nm})^2 \\ &= \sum_{i,j=0}^1 \phi\hat{g}_{ij} d\phi\hat{x}^i d\phi\hat{x}^j - \sum_{i,j=0}^1 \phi\hat{R}_{ij} d\phi\hat{x}^i d\phi\hat{x}^j \end{aligned} \quad (58)$$

$$\begin{aligned} &= \left( \cos^2 \phi\hat{\psi} \phi\hat{c}^2 (d\phi\hat{t})^2 - \sec^2 \phi\hat{\psi} (\phi\hat{\rho})^2 \right) \\ &\quad - \left( -\sin^2 \phi\hat{\psi} \phi\hat{c}^2 (d\phi\hat{t})^2 - \tan^2 \phi\hat{\psi} (d\phi\hat{\rho})^2 \right) \end{aligned} \quad (59)$$

The absolute intrinsic line element without star label on the curved ‘two-dimensional’ absolute intrinsic metric spacetime  $(\phi\hat{\rho}, \phi\hat{c}\phi\hat{t})$  that is valid with respect to 3-observers in  $E'^3$  solely in Fig. 4, (which is the same as the absolute intrinsic line element on  $(\phi\hat{\rho}, \phi\hat{c}\phi\hat{t})$  in Fig. 7), which follows from Eqs. (58) and (59) is the following

$$\begin{aligned} (d\phi\hat{s}_m)^2 &= \sum_{i,j=0}^1 \phi\hat{g}_{ij} d\phi\hat{x}^i d\phi\hat{x}^j \\ &= \phi\hat{g}_{00} \phi\hat{c}^2 (d\phi\hat{t})^2 + \phi\hat{g}_{11} (d\phi\hat{\rho})^2 \\ &= \cos^2 \phi\hat{\psi} \phi\hat{c}^2 (d\phi\hat{t})^2 - \sec^2 \phi\hat{\psi} (d\phi\hat{\rho})^2 \end{aligned} \quad (60)$$

or

$$(d\phi\hat{s}_m)^2 = (1 - \phi\hat{k}^2) \phi\hat{c}^2 (d\phi\hat{t})^2 - \frac{(d\phi\hat{\rho})^2}{1 - \phi\hat{k}^2} \quad (62)$$

The derived hyperbolic absolute intrinsic line element of Eq. (61) or (62) on the ‘two-dimensional’ absolute intrinsic spacetime  $(\phi\hat{\rho}, \phi\hat{c}\phi\hat{t})$  in Fig. 7, which is valid with respect to 3-observers in the proper physical Euclidean 3-space  $E'^3$  solely, implies the following hyperbolic absolute intrinsic metric tensor with respect to 3-observers in  $E'^3$  solely

$$\phi\hat{g}_{ij} = \begin{pmatrix} \cos^2 \phi\hat{\psi} & 0 \\ 0 & -\sec^2 \phi\hat{\psi} \end{pmatrix} \quad (63)$$

or

$$\phi\hat{g}_{ij} = \begin{pmatrix} 1 - \phi\hat{k}^2 & 0 \\ 0 & -\frac{1}{1 - \phi\hat{k}^2} \end{pmatrix} \quad (64)$$

The absolute intrinsic line element (61) or (62) and the absolute intrinsic metric tensor (63) or (64), on the curved absolute intrinsic space - straight line absolute intrinsic time ‘dimension’  $(\phi\hat{\rho}, \phi\hat{c}\phi\hat{t})$  in Fig. 7, which are valid with respect

to 3-observers in the proper physical Euclidean 3-space  $E'^3$  solely in that figure, are now hyperbolic as known for space-time metrics. It can also be said that Eqs. (61) - (64) are valid on the curved 'two-dimensional' absolute intrinsic spacetime  $(\phi\hat{\rho}, \phi\hat{c}\phi\hat{t})$  in Fig. 4, with respect to 3-observers in the proper physical Euclidean 3-space  $E'^3$  solely in that figure.

Eq. (61) or (62) and Eq. (63) or (64) give the final forms of the absolute intrinsic line element and absolute intrinsic metric tensor in the context of 'two-dimensional' absolute intrinsic Riemannian spacetime geometry (or absolute Riemannian nospacetime geometry) in our universe. The absolute intrinsic curvature parameter  $\phi\hat{k}$  that appears in them shall be related to the absolute intrinsic parameters of the metric force field that gives rise to absolute intrinsic Riemannian spacetime geometry within a region of the universal spacetime elsewhere with further development.

The absolute intrinsic metric tensor (without star label)  $\phi\hat{g}_{ij}$ , (on a manifold of the type  $M^{p+q}$ , which is  $\phi\hat{M}^{1+1}$  in the present case), is the modified form of the starred absolute intrinsic metric tensor  $\phi\hat{g}_{ij}^*$  of Eq. (32) or (33), (on a manifold of the type  $M^p$ , which is  $\phi\hat{M}^2$  in the present case). The components of  $\phi\hat{g}_{ij}^*$  and  $\phi\hat{g}_{ij}$  are related by comparing Eqs. (33) and (64) as follows

$$\phi\hat{g}_{00} = \frac{1}{\phi\hat{g}_{00}^*}; \phi\hat{g}_{11} = -\phi\hat{g}_{11}^*; \phi\hat{g}_{ij} = \phi\hat{g}_{ij}^* = 0; i \neq j \quad (65a)$$

The following relations also follow among the components of  $\phi\hat{g}_{ij}^*$  in Eq. (33) and among the components of  $\phi\hat{g}_{ij}$  in Eq. (64)

$$\phi\hat{g}_{11}^* = \phi\hat{g}_{00}^*; \phi\hat{g}_{11} = -\frac{1}{\phi\hat{g}_{00}} \quad (65b)$$

The validity of systems (65a) and (65b) in all situations is guaranteed by the fact that there is perfect symmetry of state between the positive time-universe and our universe and indeed among the four universes isolated in [2-5], as mentioned earlier. This fact guarantees that the curvature of the absolute intrinsic space  $\phi\hat{\rho}$  relative to the proper intrinsic space  $\phi\rho'$  at every point along  $\phi\hat{\rho}$  is identical to the curvature of the absolute intrinsic time 'dimension'  $\phi\hat{c}\phi\hat{t}$  relative to the proper intrinsic time dimension  $\phi c\phi t'$  at the symmetry-partner point along  $\phi\hat{c}\phi\hat{t}$  in Fig. 4. Hence the absolute intrinsic curvature parameter  $\phi\hat{k}_P$  at point P on curved  $\phi\hat{\rho}$  is identical to the absolute intrinsic curvature parameter  $\phi k_{P^0}$  at the symmetry-partner point  $P^0$  of the curved  $\phi\hat{c}\phi\hat{t}$ . That is,  $\phi\hat{\psi}_P = \phi\psi_{P^0} \equiv \phi\hat{\psi}$ . Hence  $\phi\hat{k}_P = \phi k_{P^0} \equiv \phi\hat{k}$  in Fig. 4, as mentioned earlier, and this is true in all situations and implies that systems (65a) and (65b) are true in all situations.

In obtaining the final absolute intrinsic metric tensor without star label  $\phi\hat{g}_{ij}$  of Eq. (63) or (64) tensorially, one must solve the pair of starred absolute intrinsic tensor equations (34) and (38) simultaneously to obtain the starred absolute intrinsic metric tensor  $\phi\hat{g}_{ij}^*$  of Eq. (33) and the starred ab-

solute intrinsic Ricci tensor  $\phi\hat{R}_{ij}^*$  of Eq. (39). One must then apply relations (65a) and (65b) to obtain the absolute intrinsic metric tensor without star label  $\phi\hat{g}_{ij}$  from the starred absolute intrinsic metric tensor  $\phi\hat{g}_{ij}^*$  so obtained.

In order to obtain the absolute intrinsic Ricci tensor without star label  $\phi\hat{R}_{ij}$ , which is compatible with the absolute intrinsic metric tensor without star label  $\phi\hat{g}_{ij}$  obtained from the programme in the foregoing paragraph, we shall make use of the validity of intrinsic local Lorentz invariance ( $\phi\text{LLI}$ ) on  $(\phi\hat{\rho}, \phi c\phi t')$  with respect to 3-observers in the proper physical Euclidean 3-space  $E'^3$  in Fig. 7 in so far as both the metric and 'non-metric' intrinsic coordinate interval projections are taken into account in deriving intrinsic coordinate projection relations from Fig. 7 demonstrated earlier. This implies that Eq. (34) must now be written in terms of absolute intrinsic tensors without star label  $\phi\hat{g}_{ij}$  and  $\phi\hat{R}_{ij}$  and with the Euclidean metric tensor  $\delta_{ij}$  in that equation replaced by the Lorentzian metric tensor  $\eta_{ij}$ . In other words, the following equation, written as Eq. (58) earlier, must be satisfied

$$\phi\hat{g}_{ij} - \phi\hat{R}_{ij} = \eta_{ij} \quad (\phi\text{LLI}) \quad (66)$$

With  $\phi\hat{g}_{ij}$  given by Eq. (63) or (64), the absolute intrinsic Ricci tensor without star label  $\phi\hat{R}_{ij}$  that satisfies Eq. (66) is the following

$$\phi\hat{R}_{ij} = \begin{pmatrix} -\sin^2 \phi\hat{\psi} & 0 \\ 0 & -\tan^2 \phi\hat{\psi} \end{pmatrix} \quad (67)$$

$$= \begin{pmatrix} -\phi\hat{k}^2 & 0 \\ 0 & -\frac{\phi\hat{k}^2}{1 - \phi\hat{k}^2} \end{pmatrix} \quad (68)$$

Now let us consider a situation where a pair of 'two-dimensional' absolute intrinsic metric spacetimes co-exist. One will naturally be curved relative to the other as illustrated in Fig. 5. The lower half of Fig. 5 is valid with respect to 3-observers in  $E'^3$ , while the upper half is valid with respect to 1-observers in  $ct'$ . In order to make Fig. 5 valid with respect to 3-observers in  $E'^3$  solely, it must be modified as Fig. 8.

The resultant intrinsic metric coordinate interval projection relations, or the resultant intrinsic length contraction and resultant intrinsic time dilation formulae, which are valid with respect to 3-observers in  $E'^3$  solely in Fig. 8 are the following

$$\begin{aligned} d\phi\rho' &= d\phi\hat{\rho} \cos \phi\hat{\psi}_{\text{res}} = d\phi\hat{\rho} \cos \phi\hat{\psi}' \cos \phi\hat{\psi} \\ &= d\phi\hat{\rho} (1 - \phi\hat{k}'^2)^{1/2} (1 - \phi\hat{k}^2)^{1/2} \end{aligned} \quad (69)$$

and

$$\begin{aligned} \phi c d\phi t' &= \phi\hat{c} d\phi\hat{t} \sec \phi\hat{\psi}_{\text{res}} = \phi\hat{c} d\phi\hat{t} \sec \phi\hat{\psi}' \sec \phi\hat{\psi} \\ &= \phi\hat{c} d\phi\hat{t} (1 - \phi\hat{k}'^2)^{-1/2} (1 - \phi\hat{k}^2)^{-1/2} \end{aligned} \quad (70)$$

The resultant absolute intrinsic metric tensor without star label  $\phi\hat{g}_{ij}$  and the resultant absolute intrinsic Ricci tensor

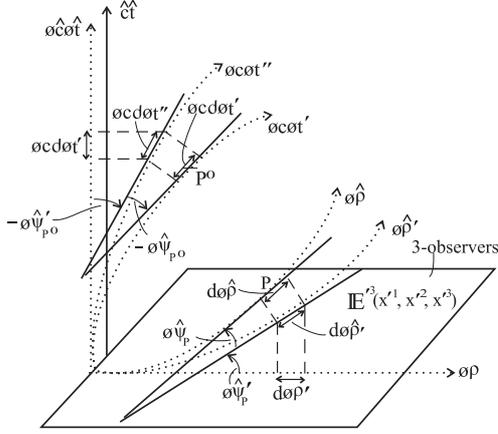


Fig. 8: Deriving resultant intrinsic coordinate projection relations with respect to 3-observers in the underlying proper physical Euclidean 3-space solely, when two curved absolute intrinsic metric spacetimes (or absolute intrinsic Riemannian metric spacetimes) co-exist.

without star label  $\hat{\phi}\hat{R}_{ij}$ , which are valid with respect to 3-observers in  $E'^3$  in Fig. 8, are given by writing Eqs. (63) and (64) in terms of the resultant absolute intrinsic angle  $\hat{\phi}\hat{\psi}$  and resultant absolute intrinsic curvature parameter  $\hat{\phi}\hat{k}$  as follows

$$\hat{\phi}\hat{g}_{ij} = \begin{pmatrix} 1 - \sin^2 \hat{\phi}\hat{\psi} & 0 \\ 0 & -\frac{1}{1 - \sin^2 \hat{\phi}\hat{\psi}} \end{pmatrix} \quad (71)$$

where  $\sin^2 \hat{\phi}\hat{\psi} = \sin^2 \hat{\phi}\hat{\psi}' + \sin^2 \hat{\phi}\hat{\psi}$ , as follows from the derived relation (90) of [1]. Eq. (71) corresponds to the following in terms of resultant absolute intrinsic curvature parameter

$$\hat{\phi}\hat{g}_{ij} = \begin{pmatrix} 1 - (\hat{\phi}\hat{k})^2 & 0 \\ 0 & -\frac{1}{1 - (\hat{\phi}\hat{k})^2} \end{pmatrix} \quad (72)$$

where  $(\hat{\phi}\hat{k})^2 = (\hat{\phi}\hat{k}')^2 + \hat{\phi}\hat{k}^2$ , as derived in [1] and presented as Eq. (91) of that paper.

And by writing equations (67) and (68) in terms of the resultant absolute intrinsic angle  $\hat{\phi}\hat{\psi}$  and resultant absolute intrinsic curvature parameter  $\hat{\phi}\hat{k}$  we have as follows

$$\hat{\phi}\hat{R}_{ij} = \begin{pmatrix} -\sin^2 \hat{\phi}\hat{\psi} & 0 \\ 0 & -\frac{\sin^2 \hat{\phi}\hat{\psi}}{1 - \sin^2 \hat{\phi}\hat{\psi}} \end{pmatrix} \quad (73)$$

or

$$\hat{\phi}\hat{R}_{ij} = \begin{pmatrix} -\hat{\phi}\hat{k}^2 & 0 \\ 0 & -\frac{\hat{\phi}\hat{k}^2}{1 - \hat{\phi}\hat{k}^2} \end{pmatrix}$$

$$= \begin{pmatrix} -(\hat{\phi}\hat{k}')^2 - \hat{\phi}\hat{k}^2 & 0 \\ 0 & -\frac{(\hat{\phi}\hat{k}')^2 - \hat{\phi}\hat{k}^2}{1 - (\hat{\phi}\hat{k}')^2 - \hat{\phi}\hat{k}^2} \end{pmatrix} \quad (74)$$

Thus the resultant absolute intrinsic line element on the upper curved 'two-dimensional' absolute intrinsic metric spacetime  $(\hat{\phi}\hat{\rho}, \hat{\phi}\hat{c}\hat{t})$  in Fig. 5, which is valid with respect to 3-observers in  $E'^3$  solely in that figure, derived via Fig. 8 is the following

$$\begin{aligned} (d\hat{\phi}\hat{s})^2 &= \hat{\phi}\hat{g}_{00}\hat{\phi}\hat{c}^2 d\hat{t}^2 - \hat{\phi}\hat{g}_{11} d\hat{\rho}^2 \\ &= (1 - \sin^2 \hat{\phi}\hat{\psi}' - \sin^2 \hat{\phi}\hat{\psi})\hat{\phi}\hat{c}^2 d\hat{t}^2 - \\ &\quad - \frac{d\hat{\rho}^2}{1 - \sin^2 \hat{\phi}\hat{\psi}' - \sin^2 \hat{\phi}\hat{\psi}} \end{aligned} \quad (75)$$

or

$$\begin{aligned} (d\hat{\phi}\hat{s})^2 &= (1 - \hat{\phi}\hat{k}'^2 - \hat{\phi}\hat{k}^2)\hat{\phi}\hat{c}^2 d\hat{t}^2 - \\ &\quad - \frac{d\hat{\rho}^2}{1 - \hat{\phi}\hat{k}'^2 - \hat{\phi}\hat{k}^2} \end{aligned} \quad (76)$$

The extension of relations (69) through (76) to the situation where three and a larger number of curved 'two-dimensional' absolute intrinsic metric spacetimes (or absolute intrinsic Riemannian metric spacetimes) co-exist is straight forward.

## 2 Isolating non-uniform absolute intrinsic static speeds along the curved absolute intrinsic space and curved absolute intrinsic time 'dimension'

Figs. 9a and 9b are valid with respect to 1-observers in the proper time dimension  $ct'$  of our universe and 1-observers in the proper time dimension  $ct'^0$  of the positive time-universe respectively, as indicated. The elementary interval  $\hat{\phi}\hat{c}d\hat{t}$  of the curved absolute intrinsic time 'dimension'  $\hat{\phi}\hat{c}\hat{t}$  at point  $P^0$  along  $\hat{\phi}\hat{c}\hat{t}$  spans interval  $\hat{\phi}cd\hat{t}'$  of  $\hat{\phi}c\hat{t}'$  along the vertical and interval  $d\hat{\rho}$  of  $\hat{\phi}\hat{\rho}$  along the horizontal in Fig. 9a. The trigonometric sine ratio of the absolute intrinsic angle  $\hat{\phi}\hat{\psi}_{P^0}$  of inclination of the curved  $\hat{\phi}\hat{c}\hat{t}$  to  $\hat{\phi}c\hat{t}'$  along the vertical at point  $P^0$  along  $\hat{\phi}\hat{c}\hat{t}$  is given as

$$\sin \hat{\phi}\hat{\psi}_{P^0} = \frac{d\hat{\rho}}{\hat{\phi}\hat{c}d\hat{t}} = \frac{\hat{\phi}\hat{V}_{s,P^0}}{\hat{\phi}\hat{c}} \quad (77)$$

where,  $d\hat{\rho}/d\hat{t} = \hat{\phi}\hat{V}_{s,P^0}$ , shall be referred to as absolute intrinsic static speed of the curved absolute intrinsic time 'dimension'  $\hat{\phi}\hat{c}\hat{t}$  at point  $P^0$  along  $\hat{\phi}\hat{c}\hat{t}$ , with respect to all 1-observers in the proper time dimension  $ct'$  of our universe along the vertical in Fig. 9a.

The trigonometric sine ratio of the absolute intrinsic angle  $\hat{\phi}\hat{\psi}_P$  of inclination of the curved  $\hat{\phi}\hat{c}\hat{t}^0$  relative to  $\hat{\phi}c\hat{t}^0$  along the horizontal at point P along  $\hat{\phi}\hat{c}\hat{t}^0$  in Fig. 9b is likewise given as

$$\sin \hat{\phi}\hat{\psi}_P = \frac{d\hat{\rho}^0}{\hat{\phi}\hat{c}d\hat{t}^0} = \frac{\hat{\phi}\hat{V}_{s,P}}{\hat{\phi}\hat{c}} \quad (78)$$

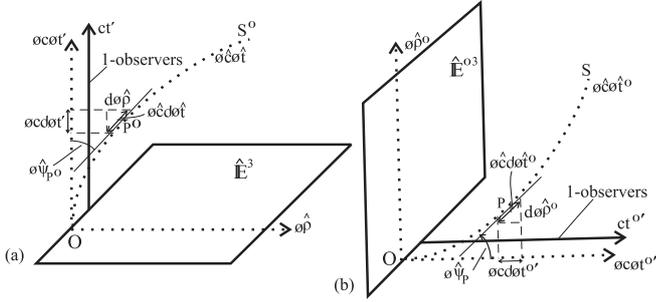


Fig. 9: Deriving absolute intrinsic static speeds along curved absolute intrinsic metric time ‘dimensions’ with respect to 1-observers in the proper metric time dimensions in our universe and positive time-universe.

where, again,  $d\phi^0/d\phi^0 = \phi_{\hat{V}_{s,P}}$ , is the absolute intrinsic static speed of the curved absolute intrinsic time ‘dimension’  $\phi^0$  at point P along  $\phi^0$ , with respect to 1-observers in the proper time dimension  $ct^{0'}$  of the positive time-universe along the horizontal in Fig. 9b.

Although the point  $P^0$  of the curved absolute intrinsic time ‘dimension’  $\phi^0$  possesses absolute intrinsic static speed  $\phi_{\hat{V}_{s,P^0}}$  relative to 1-observers in  $ct'$  in Fig. 9a and the point P of the curved  $\phi^0$  possesses absolute intrinsic static speed  $\phi_{\hat{V}_{s,P}}$  relative to 1-observers in  $ct^{0'}$  in Fig. 9b, the points  $P^0$  of  $\phi^0$  and P of  $\phi^0$  are not in absolute intrinsic motion (or absolute intrinsic flow), hence the reference to  $\phi_{\hat{V}_{s,P^0}}$  and  $\phi_{\hat{V}_{s,P}}$  as static (and not dynamical) absolute intrinsic speeds.

The pair of points  $P^0$  along the curved  $\phi^0$  in Fig. 9a and P along the curved  $\phi^0$  in Fig. 9b are symmetry-partner points. Another pair of symmetry-partner points  $Q^0$  along the curved  $\phi^0$  in Fig. 9a and Q along the curved  $\phi^0$  in Fig. 9b likewise possesses absolute intrinsic speeds  $\phi_{\hat{V}_{s,Q^0}}$  relative to 1-observers in  $ct'$  in Fig. 9a and  $\phi_{\hat{V}_{s,Q}}$  relative to 1-observers in  $ct^{0'}$  in Fig. 9b respectively. The absolute intrinsic static speeds  $\phi_{\hat{V}_{s,P^0}}$  and  $\phi_{\hat{V}_{s,Q^0}}$  along the curved  $\phi^0$  are illustrated in Fig. 10a and the corresponding absolute intrinsic static speeds  $\phi_{\hat{V}_{s,P}}$  and  $\phi_{\hat{V}_{s,Q}}$  along the curved  $\phi^0$  are illustrated in Fig. 10b.

The half-geometry of Fig. 10a with respect to 1-observers in the proper time dimension  $ct'$  of our universe and the half-geometry of Fig. 10b with respect to 1-observers in the proper time dimension  $ct^{0'}$  of the positive time-universe co-exist and must be united into a single full diagram. In doing this and making the resulting full diagram to contain the spacetime and intrinsic spacetime dimensions of our (or positive) universe solely, we must, as derived in [3], let  $ct^0 \rightarrow E'^3$ ;  $\phi c\phi t^{0'} \rightarrow \phi^0$  and  $\phi^0 \rightarrow \phi^0$  in Fig. 10b and unite the lower half of the resulting diagram with the upper half of Fig. 10a to have Fig. 11.

We have again recovered the 4-geometry/intrinsic 2-geometry diagram of Fig. 4, in which the ‘one-dimensional’ ab-

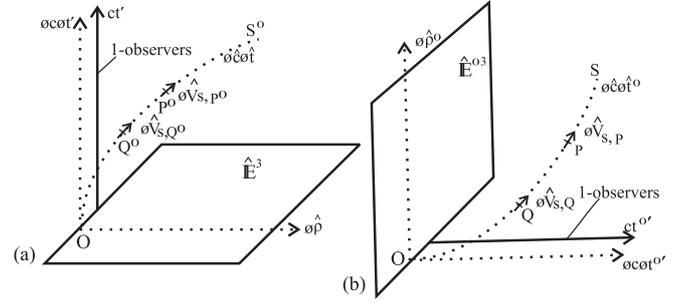


Fig. 10: Non-uniform absolute intrinsic static speeds along curved absolute intrinsic metric time ‘dimensions’ with respect to 1-observers in the proper time dimensions of our universe and positive time-universe, which are established by the sources of symmetry-partner absolute intrinsic metric force fields located at symmetry-partner positions S and  $S^0$  on the curved absolute intrinsic time dimensions.

solute intrinsic metric space  $\phi^0$  is curved relative to its projective straight line proper intrinsic metric space  $\phi^0$  along the horizontal and the absolute intrinsic metric time ‘dimension’  $\phi^0$  is curved relative to its projective straight line proper intrinsic metric time dimension  $\phi c\phi t'$  along the vertical. The new addition to Fig. 4 in Fig. 11 are the non-uniform absolute intrinsic static speeds at every point along the curved absolute intrinsic metric space  $\phi^0$  and along the curved absolute intrinsic metric time ‘dimension’  $\phi^0$ , where absolute intrinsic static speeds at only two points P and Q along  $\phi^0$  and at the symmetry-partner points  $P^0$  and  $Q^0$  along  $\phi^0$  are shown in Fig. 11. The lower half of Fig. 11 is valid with respect to 3-observers in  $E'^3$ , while the upper half is valid with respect to 1-observers in  $ct'$ .

As illustrated in Fig. 11, the absolute intrinsic static speeds  $\phi_{\hat{V}_{s,Q}}$  and  $\phi_{\hat{V}_{s,P}}$  along the curved absolute intrinsic metric space  $\phi^0$  are projected invariantly as absolute intrinsic static speeds  $\phi_{\hat{V}_{s,Q}}$  and  $\phi_{\hat{V}_{s,P}}$  into the straight line proper intrinsic metric space  $\phi^0$  along the horizontal with respect to 3-observers in  $E'^3$ . The absolute intrinsic static speed  $\phi_{\hat{V}_{s,Q^0}}$  and  $\phi_{\hat{V}_{s,P^0}}$  along the curved absolute intrinsic time ‘dimension’  $\phi^0$  are likewise projected invariantly as absolute intrinsic static speeds  $\phi_{\hat{V}_{s,Q^0}}$  and  $\phi_{\hat{V}_{s,P^0}}$  into the proper intrinsic metric time dimension  $\phi c\phi t'$  along the vertical with respect to all 1-observers in  $ct'$ .

The projective absolute intrinsic static speeds  $\phi_{\hat{V}_{s,Q}}$  and  $\phi_{\hat{V}_{s,P}}$  along  $\phi^0$  are then made manifest in absolute static speeds  $\hat{V}_{s,Q}$  and  $\hat{V}_{s,P}$  in the proper Euclidean 3-space  $E'^3$ , just as the projective absolute intrinsic static speeds  $\phi_{\hat{V}_{s,Q^0}}$  and  $\phi_{\hat{V}_{s,P^0}}$  along  $\phi c\phi t'$  are made manifest in absolute static speeds  $\hat{V}_{s,Q^0}$  and  $\hat{V}_{s,P^0}$  along the proper time dimension  $ct'$ , as shown in Fig. 11.

One would expect the absolute intrinsic static speeds  $\phi_{\hat{V}_{s,Q}}$  and  $\phi_{\hat{V}_{s,P}}$  along the curved  $\phi^0$  to project proper in-

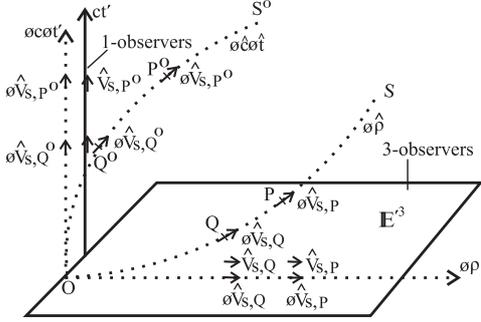


Fig. 11: Non-uniform absolute intrinsic static speeds along curved absolute intrinsic metric space  $\phi\hat{\rho}$  and curved absolute intrinsic metric time 'dimension'  $\phi\hat{c}\phi\hat{t}$ , established by the sources of a long-range absolute intrinsic metric force field at positions S on  $\phi\hat{\rho}$  and  $S^0$  on  $\phi\hat{c}\phi\hat{t}$ , are invariantly projected as non-uniform absolute intrinsic static speeds along the projective straight line isotropic proper intrinsic metric space  $\phi\rho'$  along the horizontal and the projective straight line proper intrinsic metric time dimension  $\phi c\phi t'$  along the vertical, which are made manifest in non-uniform absolute static speeds in the proper Euclidean 3-space  $E'^3$  and along the proper metric time dimension  $ct'$  in our universe.

intrinsic static speeds  $\phi V'_{s,Q}$  and  $\phi V'_{s,P}$  into the proper intrinsic space  $\phi\rho'$  along the horizontal, even as the curved absolute intrinsic space  $\phi\hat{\rho}$  is projected along the horizontal as proper intrinsic space  $\phi\rho'$ , which should then be made manifest in proper static speeds  $V'_{s,Q}$  and  $V'_{s,P}$  in the proper Euclidean 3-space  $E'^3$  with respect to 3-observers in  $E'^3$ . One would likewise expect the absolute intrinsic static speeds  $\phi\hat{V}_{s,Q^0}$  and  $\phi\hat{V}_{s,P^0}$  along the curved  $\phi\hat{c}\phi\hat{t}$  to project proper intrinsic static speeds  $\phi V'_{s,Q^0}$  and  $\phi V'_{s,P^0}$  into the proper intrinsic time dimension  $\phi c\phi t'$  along the vertical, even as the curved absolute intrinsic time 'dimension'  $\phi\hat{c}\phi\hat{t}$  is projected along the vertical as proper intrinsic time dimension  $\phi c\phi t'$ , which should then be made manifest in proper static speeds  $V'_{s,Q^0}$  and  $V'_{s,P^0}$  along the proper time dimension  $ct'$  with respect to 1-observers in  $ct'$ .

The proper intrinsic static speeds  $\phi V'_{s,Q}$ ,  $\phi V'_{s,P}$ ,  $\phi V'_{s,Q^0}$  and  $\phi V'_{s,P^0}$  that are expected to be projected along  $\phi\rho'$  and  $\phi c\phi t'$  in Fig. 11, as discussed in the foregoing paragraph, being without hat label, are relative intrinsic static speeds, just as the proper intrinsic space  $\phi\rho'$  and proper intrinsic time dimension  $\phi c\phi t'$  are relative intrinsic space and relative intrinsic dimension. The proper static speeds  $V'_{s,Q}$ ,  $V'_{s,P}$ ,  $V'_{s,Q^0}$  and  $V'_{s,P^0}$  expected to appear in  $E'^3$  and  $ct'$  in Fig. 11, being without hat label, are relative static speeds, just as the proper Euclidean 3-space  $E'^3$  and the proper time dimension  $ct'$  are relative space and relative dimension.

The concept of relative intrinsic static speed and relative static speed, (which should convey no meaning at this point, since we have grown accustomed to relative kinematical speeds only), shall be adequately appropriated into the

present theory with further development.

The fact that absolute intrinsic static speeds along the curved absolute intrinsic space  $\phi\hat{\rho}$  and curved absolute intrinsic time 'dimension'  $\phi\hat{c}\phi\hat{t}$  are projected invariantly into proper intrinsic space  $\phi\rho'$  and proper intrinsic time dimension  $\phi c\phi t'$  as absolute intrinsic static speeds  $\phi\hat{V}_{s,Q}$ ,  $\phi\hat{V}_{s,P}$ ,  $\phi\hat{V}_{s,Q^0}$  and  $\phi\hat{V}_{s,P^0}$  in Fig. 11, instead of proper intrinsic static speeds  $\phi V'_{s,Q}$ ,  $\phi V'_{s,P}$ ,  $\phi V'_{s,Q^0}$  and  $\phi V'_{s,P^0}$ , is a graphical illustration of the invariance of intrinsic static speeds in the context of absolute intrinsic Riemann geometry (or in the context of the absolute intrinsic metric phenomena that give rise to absolute intrinsic Riemann geometry). This invariance is stated as follows

$$\phi V'_s = \phi\hat{V}_s \quad (79a)$$

Hence

$$V'_s = \hat{V}_s \quad (79b)$$

where (79a) has been written at an arbitrary point along the curved  $\phi\hat{\rho}$  and its symmetry-partner point along the curved  $\phi\hat{c}\phi\hat{t}$  and (79b) has been written at the corresponding point in  $E'^3$  and its symmetry-partner point along  $ct'$ .

Let us re-write Eqs. (77) and (78), while letting  $\phi\hat{\psi}_{P^0} = \phi\hat{\psi}_P \equiv \phi\hat{\psi}$  and  $\phi\hat{V}_{s,P^0} = \phi\hat{V}_{s,P} \equiv \phi\hat{V}_s$  in those equations as the following singular equation, which is valid along both  $\phi\hat{\rho}$  and  $\phi\hat{c}\phi\hat{t}$ :

$$\sin \phi\hat{\psi} = \phi\hat{V}_s / \phi\hat{c} \quad (80a)$$

But the relation for the identical absolute intrinsic curvature parameters,  $\phi\hat{k}_{P^0} = \phi\hat{k}_P \equiv \phi\hat{k}$ , at any given point P along the curved absolute intrinsic space  $\phi\hat{\rho}$  with respect to 3-observers in  $E'^3$  and at the symmetry-partner point  $P^0$  along the curved absolute intrinsic metric time 'dimension'  $\phi\hat{c}\phi\hat{t}$  with respect to 1-observers in  $ct'$ , has been related to the absolute intrinsic angle,  $\phi\hat{\psi}_{P^0} = \phi\hat{\psi}_P \equiv \phi\hat{\psi}$ , in sub-section 1.1 of [1] as

$$\sin \phi\hat{\psi} = \phi\hat{k} \quad (80b)$$

The absolute intrinsic curvature parameter at an arbitrary point along the curved  $\phi\hat{\rho}$  and at the symmetry-partner point along the curved  $\phi\hat{c}\phi\hat{t}$  is therefore related to the absolute intrinsic static speed at the same point from Eqs. (80a) and (80b) as

$$\phi\hat{k} = \phi\hat{V}_s / \phi\hat{c} \quad (80c)$$

The absolute intrinsic metric tensor and absolute intrinsic Ricci tensor without star label, given in terms of absolute intrinsic curvature parameter as Eqs. (64) and (68) in the case of one absolute intrinsic Riemann space, that is, in the case of a singular curved absolute intrinsic metric spacetime, can then be written in terms of absolute intrinsic static speed respectively as follows

$$\phi\hat{g}_{ij} = \begin{pmatrix} 1 - \phi\hat{V}_s^2 / \phi\hat{c}^2 & 0 \\ 0 & -\frac{1}{1 - \phi\hat{V}_s^2 / \phi\hat{c}^2} \end{pmatrix} \quad (81)$$

and

$$\phi \hat{R}_{ij} = \begin{pmatrix} -\phi \hat{V}_s'^2 / \phi \hat{c}^2 & 0 \\ 0 & -\frac{\phi \hat{V}_s'^2 / \phi \hat{c}^2}{1 - \phi \hat{V}_s'^2 / \phi \hat{c}^2} \end{pmatrix} \quad (82)$$

The absolute intrinsic line element (62) likewise becomes the following in terms of absolute intrinsic static speed,

$$d\phi \hat{s}^2 = (1 - \phi \hat{V}_s'^2 / \phi \hat{c}^2) \phi \hat{c}^2 d\phi \hat{t}^2 - \frac{d\phi \hat{\rho}^2}{1 - \phi \hat{V}_s'^2 / \phi \hat{c}^2} \quad (83)$$

The resultant absolute intrinsic metric tensor, resultant absolute intrinsic Ricci tensor and resultant absolute intrinsic line element (72), (74) and (76) in a situation where two absolute intrinsic Riemannian metric spacetimes co-exist become the following in terms of absolute intrinsic static speed:

$$\phi \hat{g}_{ij} = \begin{pmatrix} 1 - \frac{\phi \hat{V}_s'^2}{\phi \hat{c}^2} - \frac{\phi \hat{V}_s'^2}{\phi \hat{c}^2} & 0 \\ 0 & -\frac{1}{1 - \frac{\phi \hat{V}_s'^2}{\phi \hat{c}^2} - \frac{\phi \hat{V}_s'^2}{\phi \hat{c}^2}} \end{pmatrix}, \quad (84)$$

$$\phi \hat{R}_{ij} = \begin{pmatrix} -\frac{\phi \hat{V}_s'^2}{\phi \hat{c}^2} + \frac{\phi \hat{V}_s'^2}{\phi \hat{c}^2} & 0 \\ 0 & -\frac{\frac{\phi \hat{V}_s'^2}{\phi \hat{c}^2} + \frac{\phi \hat{V}_s'^2}{\phi \hat{c}^2}}{1 - \frac{\phi \hat{V}_s'^2}{\phi \hat{c}^2} - \frac{\phi \hat{V}_s'^2}{\phi \hat{c}^2}} \end{pmatrix} \quad (85)$$

and

$$(d\phi \hat{s})^2 = \left(1 - \frac{\phi \hat{V}_s'^2}{\phi \hat{c}^2} - \frac{\phi \hat{V}_s'^2}{\phi \hat{c}^2}\right) \phi \hat{c}^2 d\phi \hat{t}^2 - \frac{d\phi \hat{\rho}^2}{1 - \frac{\phi \hat{V}_s'^2}{\phi \hat{c}^2} - \frac{\phi \hat{V}_s'^2}{\phi \hat{c}^2}} \quad (86)$$

Extension of Eqs. (84) - (86) to situations where three or a larger number of absolute intrinsic Riemannian metric spacetimes (or curved 'two-dimensional' absolute intrinsic metric spacetimes) co-exist (or are superposed) is straight forward.

The absolute intrinsic curvature parameter  $\phi \hat{k}$  is a geometrical parameter, as follows from its derivation in sub-section 1.1 of part two of this paper [1]. The non-uniform absolute intrinsic static speeds  $\phi \hat{V}_s$  along the curved absolute intrinsic space  $\phi \hat{\rho}$  and curved absolute intrinsic time 'dimension'  $\phi \hat{c} \phi \hat{t}$  in Fig. 11, which are related to the non-uniform absolute intrinsic curvature parameters  $\phi \hat{k}$  of the curved  $\phi \hat{\rho}$  and curved  $\phi \hat{c} \phi \hat{t}$  by Eq. (80c), is likewise an absolute intrinsic geometrical parameter. This is so because the definition,  $\phi \hat{V}_s = d\phi \hat{\rho} / d\phi \hat{t}$ , follows from the geometry of Figs. 9a and 9b, without relation to the absolute intrinsic parameters of the

absolute intrinsic metric force field that establishes absolute intrinsic Riemann geometry. The absolute intrinsic geometrical parameter  $\phi \hat{V}_s$  (or  $\phi \hat{k}$ ) that appears in the absolute intrinsic metric tensor, absolute intrinsic Ricci tensor and absolute intrinsic line element in absolute intrinsic Riemann geometry, shall be related to the absolute intrinsic parameters of the absolute intrinsic metric force field that gives rise to curved 'two-dimensional' absolute intrinsic metric spacetime ( $\phi \hat{\rho}, \phi \hat{c} \phi \hat{t}$ ) elsewhere with further development.

The explanation of the evolution of the curved absolute intrinsic space  $\phi \hat{\rho}$  and curved absolute intrinsic time 'dimension'  $\phi \hat{c} \phi \hat{t}$  in Fig. 11 or Fig. 4 from the reference geometry of Fig. 6, which follows from the validity of Eqs. (77) and (78), re-written as Eq. (80a), at every point along the curved  $\phi \hat{\rho}$  and  $\phi \hat{c} \phi \hat{t}$  in Fig. 11, is that non-uniform absolute intrinsic static speeds are identically established along the straight line absolute intrinsic space  $\phi \hat{\rho}$  and straight line absolute intrinsic time 'dimension'  $\phi \hat{c} \phi \hat{t}$  from a point  $(S, S^0)$  on the flat 'four-dimensional' absolute spacetime  $(\hat{E}^3, \hat{c} \hat{t})$  in Fig. 6. Then the geometry of Fig. 11 evolves as a consequence, since (80a) must be satisfied at every point along  $\phi \hat{\rho}$  and  $\phi \hat{c} \phi \hat{t}$ . The mechanism by which this is achieved requires explanation to be given elsewhere.

The geometry of Fig. 11 will evolve from Fig. 6, for instance, if the source of a long-range absolute metric force field (such as the source of an absolute gravitational field) located at a point S in the absolute space  $\hat{E}^3$  of our universe in Fig. 6, establishes non-uniform absolute static speeds  $\hat{V}_s$  along every radial direction from its centre in all its finite neighbourhood in  $\hat{E}^3$  and the source of absolute intrinsic metric force field in the absolute intrinsic space  $\phi \hat{\rho}$  underlying the source of absolute metric force field in  $\hat{E}^3$ , establishes non-uniform absolute intrinsic static speeds  $\phi \hat{V}_s$  along the straight line absolute intrinsic space  $\phi \hat{\rho}$  in all its finite neighbourhood in Fig. 6. This will give rise to the curved  $\phi \hat{\rho}$  and its projective straight line proper intrinsic space  $\phi \rho'$  along the horizontal in our universe as in Fig. 11.

The identical symmetry-partner source of long-range absolute metric force field in flat absolute space  $\hat{E}^{03}$  and identical source of long-range absolute intrinsic metric force field in straight line absolute intrinsic space  $\phi \hat{\rho}^0$  in the geometry in the positive universe that corresponds to that of Fig. 6 in our universe, will give rise to curved absolute intrinsic metric space  $\phi \hat{\rho}^0$  that projects straight line proper intrinsic metric space  $\phi \rho^{0'}$  along the vertical (as in Fig. 2) in the positive time-universe. This then corresponds to curved absolute intrinsic time 'dimension'  $\phi \hat{c} \phi \hat{t}$  and its projective proper intrinsic time dimension  $\phi c \phi t'$  of our universe along the vertical in Fig. 4 or Fig. 11.

Non-uniform absolute intrinsic static speeds  $\phi \hat{V}_s$  established along the straight line absolute intrinsic time 'dimension'  $\phi \hat{c} \phi \hat{t}$  from a point on the flat absolute spacetime  $(\hat{E}^3, \hat{c} \hat{t})$  in Fig. 6, being absolute intrinsic parameters, can cause curvature of the absolute intrinsic space  $\phi \hat{\rho}$  and absolute in-

trinsic time ‘dimension’  $\phi\hat{c}\phi\hat{t}$  from that point, thereby transforming Fig. 6 to Fig. 11. On the other hand, the non-uniform absolute intrinsic static speeds  $\phi\hat{V}_s$  projected along the straight line proper intrinsic space  $\phi\rho'$  and straight line proper intrinsic time dimension  $\phi c\phi t'$  in Fig. 11, cannot cause curvature of  $\phi\rho'$  and  $\phi c\phi t'$ . This is so because absolute intrinsic static speed  $\phi\hat{V}_s$  can produce no effect whatever on the relative proper intrinsic space  $\phi\rho'$  and relative proper intrinsic time dimension  $\phi c\phi t'$ . The non-uniform absolute static speeds  $\hat{V}_s$  in the relative proper Euclidean 3-space  $E'^3$  in Fig. 11, can likewise produce no detectable effect in  $E'^3$  with respect to 3-observers in  $E'^3$ .

The geometry of Fig. 11 will endure for as long as no other parameters/intrinsic parameters are introduced into it. This means that evolution of spacetime/intrinsic spacetime within a long-range metric force field will terminate at the first stage, where first stage is evolution from the reference geometry of Fig. 6 to the geometry of Fig. 4 or Fig. 11. However there is an inevitable second stage of evolution of spacetime/intrinsic spacetime in a long-range metric force field, in which the flat four-dimensional proper metric spacetime  $(E'^3, ct')$  and its underlying flat two-dimensional proper intrinsic metric spacetime  $(\phi\rho', \phi c\phi t')$  in Fig. 4 or Fig. 11 evolve into flat four-dimensional relativistic metric spacetime  $(E^3, ct)$  and its underlying flat two-dimensional relativistic intrinsic metric spacetime  $(\phi\rho, \phi c\phi t)$ . The four-dimensional relativistic spacetime is proposed to be curved in a gravitational field in the general theory of relativity (GR), but this fundamental assumption of GR shall be invalidated in the context of the present evolving theory in the fourth part of this paper.

Although the 3-geometry/intrinsic 1-geometry diagram of Fig. 7, which is valid with respect to 3-observers in  $E'^3$  solely, is the valid diagram for absolute intrinsic Riemann geometry in our universe, the 4-geometry/intrinsic 2-geometry diagram of Fig. 4 of Fig. 11, which is valid partially with respect to 3-observers in  $E'^3$  and partially with respect to 1-observers in  $ct'$ , is the geometry that evolves naturally from the reference geometry of Fig. 6. Fig. 7 is a manipulation of Fig. 4 or Fig. 11, done in order to obtain an equivalent diagram that is valid with respect to 3-observers in the proper physical Euclidean 3-space  $E'^3$  solely.

Fig. 4 or Fig. 11 at the first stage of evolution of spacetime/intrinsic spacetime within a long-range metric force field has important theoretical significance in physics, although it is not an observed geometry, since it endures for no moment before transforming into the enduring geometry at the second stage of evolution of spacetime/intrinsic spacetime in a long-range metric force field, to be discussed further shortly and developed fully in the fourth part of this paper. For instance, the flat four-dimensional proper metric spacetime  $(E'^3, ct')$  and its underlying flat two-dimensional proper intrinsic metric spacetime  $(\phi\rho', \phi c\phi t')$  in that figure are the spacetime of classical (or Newtonian) mechanics and intrinsic classical

mechanics in the assumed absence of (relative) gravity in our universe. It must be noted that only absolute static speeds (or absolute metric force field) and its underlying absolute intrinsic static speeds (or absolute intrinsic metric force field) are present in those figures. The concept of relative gravity shall be properly defined elsewhere with further development. It, in brief, means the presence of proper static speeds (or proper metric force field) and its underlying proper intrinsic static speeds (or proper intrinsic metric force field), as shall be found.

It is also on the flat four-dimensional proper metric spacetime  $(E'^3, ct')$  and its underlying flat two-dimensional proper intrinsic metric spacetime  $(\phi\rho', \phi c\phi t')$  that the special theory of relativity (SR) and intrinsic special theory of relativity ( $\phi$ SR) operate in our universe in the assumed absence of (relative) gravity, as developed in [2-5]. As noted at the end of [3], SR/ $\phi$ SR involve affine spacetime coordinates/affine intrinsic spacetime coordinates of particle’s frame and observer’s frame (or involve affine spacetime/intrinsic spacetime geometry). Consequently SR/ $\phi$ SR cannot alter the flat four-dimensional proper metric spacetime  $(E'^3, ct')$  and its underlying flat two-dimensional proper intrinsic metric spacetime  $(\phi\rho', \phi c\phi t')$  on which they operate in the assumed absence of (relative) gravity.

It must be recalled that the curved ‘two-dimensional’ absolute intrinsic spacetime  $(\phi\hat{\rho}, \phi\hat{c}\phi\hat{t})$  in Fig. 4 or Fig. 11, (which is being incorporated into physics newly in this third part of this paper and the second part [1]), did not appear in [2-5]. Only the flat four-dimensional proper metric spacetime  $(E'^3, ct')$  known in physics and the new flat two-dimensional proper intrinsic metric spacetime  $(\phi\rho', \phi c\phi t')$  underlying  $(E'^3, ct')$ , which was first introduced as *ansatz* in [2] and isolated formally in [5], are known in SR/ $\phi$ SR in [2-5].

In brief, the flat four-dimensional proper metric spacetime  $(E'^3, ct')$  and its underlying flat two-dimensional proper intrinsic metric spacetime  $(\phi\rho', \phi c\phi t')$  in Fig. 4 or Fig. 11, are the reference metric spacetime/intrinsic metric spacetime for the 4-geometry/intrinsic 2-geometry theories of relativity/intrinsic relativity. One such theories of relativity/intrinsic relativity is the special theory of relativity/intrinsic special theory of relativity (SR/ $\phi$ SR) – the theories of relative motion/relative intrinsic motions of material particles and objects – which operate on extended  $(E'^3, ct')$  and its underlying extended  $(\phi\rho', \phi c\phi t')$  in the absence of (relative) gravity and leave them unchanged, as mentioned above.

There are also the theory of relativity and theory of intrinsic relativity, which are associated with the presence of a long-range relative metric force field on four-dimensional metric spacetime and its underlying long-range relative intrinsic metric force field on two-dimensional intrinsic metric spacetime. These will convert the extended flat four-dimensional proper metric spacetime  $(E'^3, ct')$  and its underlying flat two-dimensional proper intrinsic metric spacetime  $(\phi\rho', \phi c\phi t')$  into extended flat four-dimensional relativistic

metric spacetime  $(E^3, ct)$  and its underlying extended flat two-dimensional relativistic intrinsic metric spacetime  $(\phi\rho, \phi c\phi t)$  within the long-range relative metric force field at the second stage of evolution of spacetime/intrinsic spacetime in a long-range metric force field, as shall be developed in the fourth part of this paper.

Submitted on Month Day, Year / Accepted on Month Day, Year

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