INTUITIONISTIC FUZZY $\Gamma-$IDEALS OF $\Gamma-$LA-SEMIGROUPS.

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Abstract. We consider the intuitionistic fuzzification of the concept of several $\Gamma-$ideals in $\Gamma-$LA-semigroup S, and investigate some related properties of such $\Gamma-$ideals. We also prove in this paper the set of all intuitionistic fuzzy left(right) $\Gamma-$ideal of S is become LA-semigroup. We prove In $\Gamma-$LA band intuitionistic fuzzy right and left $\Gamma-$ideals are coincide..

1. Indroduction

The notion of fuzzy set in a set theory was indroduced by L.A.Zadeh, (see [5]) and since then this concept has been applied to various algebraic structures. The idea of "Intuitionistic fuzzy set" was first introduced by K.T.Atanassov (see [1, 2]) as generalization of the notion of fuzzy set. The concept of LA-semigroup was first indroduced by Kazim and Naseerudin (see [4]). Let S be non empty set then $(S, \circ)$ is called LA-semigroup, if S is closed and satisfies the identity $(x \circ y) \circ z = (z \circ y) \circ x$ for all $x, y, z \in S$. Later, Q.Mushtaq and others have ivestigated the structure further an added many useful result to theory of LA-semigroups.T.Shah and Inayatur Rehman have introduced the concept of $\Gamma-$LA-semigroup (see [6]). Let S and Γ be any nonempty sets. If there exist a mapping $S \times \Gamma \times S \longrightarrow S$ written as (a, γ, b) by $a\gamma b$, S is called $\Gamma-$LA-semgroup if S satisfies the identity $(a\beta b)\gamma c = (c\beta b)\gamma a$ for all $a, b, c \in S$ and $\beta, \gamma \in \Gamma$. Whereas the $\Gamma-$LA-semigroups are a generalization of LA-semigroup.Tariq Shah and Inayatur Rehman introduce the notion of $\Gamma-$ideals in $\Gamma-$LA-semigroups. Whereas the $\Gamma-$ideals in $\Gamma-$LA-semigroups are infact a generalization of ideals in LA-semigroups.

In this paper, we indroduce the notion of an intuitionistic fuzzy left (right) of $\Gamma-$LA-semigroup S, and also introduce notion of intuitionistic fuzzy $\Gamma-$ideals of $\Gamma-$LA-semigroup S, then some related properties are investigated. Characterizations of intutionistic fuzzy left (right) $\Gamma-$ideals are given. A mapping f from a $\Gamma-$LA-semigroup S to a $\Gamma-$LA-semigroup T is called a homomorphism if $f(x\gamma y) = f(x)h(\gamma)f(y)$ for all $x, y \in S$ and $\gamma \in \Gamma$, Also for homomorphism f from a $\Gamma-$LA-semigroup S to a $\Gamma-$LA-semigroup T, if $B = (\mu_B, \gamma_B)$ is an intuitionistic fuzzy $\Gamma-$ideals of $\Gamma-$LA-semigroup T, then the preimage $f^{-1}(B) = (f^{-1}(\mu_B), f^{-1}(\gamma_B))$ of B under f is an intuitionistic fuzzy $\Gamma-$ideals of $\Gamma-$LA-semigroup S.

2. Preliminaries

Definition 1. *[6] Let $S = \{x, y, z, ...\}$ and $\Gamma = \{\alpha, \beta, \gamma, ...\}$ be two non-empty sets. Then S is called a $\Gamma-$LA-semigroup if it satisfies*

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1) $x\gamma y \in S$
2) $(x\beta y)\gamma z = (z\beta y)\gamma x$
for all $x, y, z \in S$ and $\beta, \gamma \in \Gamma$.

Definition 2. *[6] A non-empty set U of a $\Gamma-$LA-semigroup S is said to be a $\Gamma-$sub LA-semigroup S if $U\Gamma U \subseteq U$.*

Definition 3. *[6]A left (right) $\Gamma-$ideal of a $\Gamma-$LA-semigroup S is non-empty subset U of S such that $S\Gamma U \subseteq U$ $(U\Gamma S \subseteq U)$ if U is both a left and a right $\Gamma-$ideal of a $\Gamma-$LA-semigroup S, then we say that U is $\Gamma-$ideal of S.*

Definition 4. *[1, 2] Let X be a nonempty fixed set. An intuitionistic fuzzy set (briefly, IFS) A is object having the form*

$$A = \{(x, \mu_A(x), \gamma_A(x) : x \in X\}$$

where the functions $\mu_A : X \longrightarrow [0, 1]$ and $\gamma_A : X \longrightarrow [1, 0]$ denote the degre of membership (namely $\mu_A(x)$) and the degree of nonmembership (namely $\gamma_A(x)$) of each element $x \in X$ to the set A, respectively, and $0 \leq \mu_A(x) + \gamma_A(x) \leq 1$ for all $x \in S$ for the sake of simplicity, we use the symbol $A = (\mu_A, \gamma_A)$ for the IFS $A = \{(x, \mu_A(x), \gamma_A(x) : x \in X\}$.

Definition 5. *[9] A fuzzy set μ in a $\Gamma-$LA-semigroup S is called fuzzy $\Gamma-$subLA-semigroup of S, if $\mu_A(x\gamma y) \geq \mu_A(x) \wedge \mu_A(y)$ for all $x, y \in S$ and $\gamma \in \Gamma$.*

Definition 6. *[9] A fuzzy set μ in a $\Gamma-$LA-semigroup S is called fuzzy left(resp, right) $\Gamma-$ideal of S, if $\mu_A(x\gamma y) \geq \mu_A(y)$ (resp, $\mu_A(x\gamma y) \geq \mu_A(x)$) for all $x, y \in S$ and $\gamma \in S$. A fuzzy set μ in a $\Gamma-$LA-semigroup S is called fuzzy $\Gamma-$ideal of S, if fuzzy set μ is both fuzzy left $\Gamma-$ideal and fuzzy right $\Gamma-$ideal of $\Gamma-$LA-semigroup S.*

3. Intuitionistic fuzzy $\Gamma-$ideals.

In what follows, S denote as $\Gamma-$LA-semigroup, unless otherwisespecified.

Definition 7. *An IFS $A = (\mu_A, \gamma_A)$ in S is called an intuitionistic fuzzy $\Gamma-$subLA-semigroup of S if satisfies.*

(IF1) $\mu_A(x\gamma y) \geq \mu_A(x) \wedge \mu_A(y)$,
(IF2) $\gamma_A(x\gamma y) \leq \gamma_A(x) \vee \gamma_A(y)$,
for all $x, y \in S$.

Definition 8. *An IFS $A = (\mu_A, \gamma_A)$ in S is called an intuitionistic fuzzy right $\Gamma-$idealof S if satisfies.*

(IF3) $\mu_A(x\gamma y) \geq \mu_A(x)$,
(IF4) $\gamma_A(x\gamma y) \leq \gamma_A(x)$,
for all $x, y \in S$.

Definition 9. *An IFS $A = (\mu_A, \gamma_A)$ in S is called an intuitionistic fuzzy left $\Gamma-$ideal of S if satisfies.*

(IF5) $\mu_A(x\gamma y) \geq \mu_A(y)$,
(IF6) $\gamma_A(x\gamma y) \leq \gamma_A(y)$,
for all $x, y \in S$.

Example 1. *Let $S = \{-i, 0, i\}$ and $\Gamma = S$. Then by defining $S \times \Gamma \times S \to S$ as $a\gamma b = a.\gamma.b$ for all $a, b \in S$ and $\gamma \in \Gamma$. It can be easily verified that S is a $\Gamma-$LA-semigroup under complex number multiplication while S is not an LA-semigroup. Let $A = \langle \mu_A, \gamma_A \rangle$ be IFS on S. $\mu_A : S \longrightarrow [1, 0]$ by $\mu_A(0) = 0.7, \mu_A(i) = \mu_A(-i) = 0.5$ and $\gamma_A(0) = 0.2, \gamma_A(i) = \gamma_A(-i) = 0.4$, Then by routine calculation $A = \langle \mu_A, \gamma_A \rangle$ is an intuitionistic fuzzy $\Gamma-$ideal of S.*

Theorem 1. *Let S be $\Gamma-$LA-semigroup with left identity. Then every intuitionistic fuzzy right $\Gamma-$ ideal of S is an intuitionistic fuzzy left $\Gamma-$ideal,*

Proof. Let $A = \langle \mu_A, \gamma_A \rangle$ be an intuitionistic fuzzy right $\Gamma-$ ideal of S and let $x, y \in S$ and $\alpha, \beta \in \Gamma$. Then

$$\begin{aligned} \mu_A(x\alpha y) &= \mu_A((e\beta x)\alpha y) = \mu_A((y\beta x)\alpha e) \\ &\geq \mu_A(y\beta x) \geq \mu_A(y) \\ \mu_A(x\alpha y) &\geq \mu_A(y) \end{aligned}$$

and

$$\begin{aligned} \gamma_A(x\alpha y) &= \gamma_A((e\beta x)\alpha y) = \gamma_A((y\beta x)\alpha e) \\ &\leq \gamma_A(y\beta x) \leq \gamma_A(y) \\ \gamma_A(x\alpha y) &\leq \gamma_A(y) \end{aligned}$$

Hence $A = \langle \mu_A, \gamma_A \rangle$ is an intuitionistic fuzzy left $\Gamma-$ ideal of S. □

Corollary 1. *In $\Gamma-$LA-semigroup S with left identity, every intuitionistic fuzzy right $\Gamma-$ ideal of S is intuitionistic fuzzy $\Gamma-$ ideal of S.*

Theorem 2. *Let $\{A_i\}_{i\in\Lambda}$ be family of intuitionistic fuzzy $\Gamma-$ideals of $\Gamma-$LA-semigroup S. Then $\cap A_i$ is also an intuitionistic fuzzy $\Gamma-$ideals of S, where*

$$\begin{aligned} \cap A_i &= \langle \wedge\mu_{A_i}, \vee\gamma_{A_i} \rangle \text{ and} \\ \wedge\mu_{A_i}(x) &= \inf\{\mu_{A_i}(x) \ / \ i \in \Lambda, \ x \in S\} \\ \vee\gamma_{A_i}(x) &= \sup\{\gamma_{A_i}(x) \ / \ i \in \Lambda, \ x \in S\} \end{aligned}$$

Proof. Let $\{A_i\}_{i\in\Lambda}$intuitionistic fuzzy $\Gamma-$ideals of $\Gamma-$LA-semigroup S and let for any $x, y \in S$ and $\gamma \in \Gamma$. Then

$$\begin{aligned} \wedge\mu_{A_i}(x\gamma y) &\geq \wedge\mu_{A_i}(x) \\ \vee\gamma_{A_i}(x\gamma y) &\leq \vee\gamma_{A_i}(x) \end{aligned}$$

and

$$\begin{aligned} \wedge\mu_{A_i}(x\gamma y) &\geq \wedge\mu_{A_i}(y) \\ \vee\gamma_{A_i}(x\gamma y) &\leq \vee\gamma_{A_i}(y) \end{aligned}$$

Hence $\cap A_i = \langle \wedge\mu_{A_i}, \vee\gamma_{A_i} \rangle$ is an intuitionistic fuzzy $\Gamma-$ideals of $\Gamma-$LA-semigroup S, □

Theorem 3. *Let $A = \langle \mu_A, \gamma_A \rangle$ be an intuitionistic fuzzy left(resp, right) $\Gamma-$ideal of $\Gamma-$LA-semigroup S. Then $\Box A = \langle \mu_A, \bar{\mu_A} \rangle$ is an intuitionistic fuzzy left(resp,right) $\Gamma-$ ideal of S, where $\bar{\mu_A} = 1 - \mu_A$.*

Proof. Let $A = \langle \mu_A, \gamma_A \rangle$ be an intuitionistic fuzzy left $\Gamma-$ ideal of $\Gamma-$LA-semigroup S and let for any $x, y \in S$ and $\gamma \in \Gamma$. Then

$$\begin{aligned} \mu_A(x\gamma y) &\geq \mu_A(y) \\ -\mu_A(x\gamma y) &\leq -\mu_A(y) \\ 1 - \mu_A(x\gamma y) &\leq 1 - \mu_A(y) \\ \bar{\mu_A}(x\gamma y) &\leq \bar{\mu_A}(y) \end{aligned}$$

Hence $\Box A = \langle \mu_A, \bar{\mu_A} \rangle$ is an intuitionistic fuzzy left $\Gamma-$ ideal of $\Gamma-$LA-semigroup S $\Box$

Definition 10. *Let $A = \langle \mu_A, \gamma_A \rangle$ be an IFS in S and $\alpha \in [0, 1]$. Then sets*

$$\mu^{\geq}_{A,\alpha} := \{x \in S \ / \ \mu_A(x) \geq \alpha\}, \gamma^{\leq}_{A,\alpha} := \{x \in S \ / \ \gamma_A(x) \leq \alpha\}$$

are called a $\mu-$level $\alpha-$cut and $\gamma-$level $\alpha-$cut of A respectively.

Theorem 4. *Let $A = \langle \mu_A, \gamma_A \rangle$ be an IFS in $\Gamma-$LA-semigroup S. Then $A = \langle \mu_A, \gamma_A \rangle$ is an intuitionistic fuzzy left(resp,right) $\Gamma-$ ideal of $\Gamma-$LA-semigroup S if and only if $\mu-$level $\alpha-$cut and $\gamma-$level $\alpha-$cut of A are left(resp,right) $\Gamma-$ ideal of S.*

Proof. Let $\alpha \in [0, 1]$. Suppose $\mu^{\geq}_{A,\alpha}(= \Phi)$, and $\gamma^{\leq}_{A,\alpha}(= \Phi)$, are left $\Gamma-$ ideal of $\Gamma-$LA-semigroup S.We must show that $A = \langle \mu_A, \gamma_A \rangle$ an intuitionistic fuzzy left $\Gamma-$ ideal of S. Suppose $A = \langle \mu_A, \gamma_A \rangle$ is not an intuitionistic fuzzy left $\Gamma-$ ideal of S, then there exit $x_\circ, y_\circ$ in S and $\gamma \in \Gamma$ such that

$$\mu_A(x_\circ \gamma y_\circ) < \mu_A(y_\circ).$$

Taking

$$\alpha_\circ = \frac{1}{2}\{\mu_A(x_\circ \gamma y_\circ) + \mu_A(y_\circ)\}$$

we have $\mu_A(x_\circ \gamma y_\circ) < \alpha_\circ < \mu_A(y_\circ)$. It follows that $y_\circ \in \mu^{\geq}_{A,\alpha}$ and $x_\circ \in S$ and $\gamma \in \Gamma$ but $x_\circ \gamma y_\circ \notin \mu^{\geq}_{A,\alpha}$, which is a contradication. Thus

$$\mu_A(x\gamma y) \geq \mu_A(y)$$

for all $x, y \in S$ and $\gamma \in \Gamma$, and now

$$\gamma_A(x_\circ \gamma y_\circ) > \gamma_A(y_\circ).$$

Taking

$$\alpha_\circ = \frac{1}{2}\{\gamma_A(x_\circ \gamma y_\circ) + \gamma_A(y_\circ)\}$$

we have $\gamma_A(x_\circ \gamma y_\circ) < \alpha_\circ < \gamma_A(y_\circ)$. It follows that $y_\circ \in \gamma^{\geq}_{A,\alpha}$ and $x_\circ \in S$, $\gamma \in \Gamma$ but $x_\circ \gamma y_\circ \notin \gamma^{\geq}_{A,\alpha}$, which is again a contradiction. Thus

$$\gamma_A(x_\circ \gamma y_\circ) \leq \gamma_A(y_\circ).$$

Hence $A = \langle \mu_A, \gamma_A \rangle$ an intuitionistic fuzzy $\Gamma-$ ideal of $\Gamma-$LA-semigroup S.

Conversely, suppose $A = \langle \mu_A, \gamma_A \rangle$ is an intuitionistic fuzzy left $\Gamma-$ ideal of $\Gamma-$LA-semigroup S, and let $\alpha \in [0, 1]$ and for any $x \in S$, $\gamma \in \Gamma$ and $y \in \mu^{\geq}_{A,\alpha}$. Then

$$\begin{aligned} \mu_A(x\gamma y) &\geq \mu_A(y) \geq \alpha \\ \mu_A(x\gamma y) &\geq \alpha \end{aligned}$$

$x\gamma y \in \mu^{\geq}_{A,\alpha}$ for all $x \in S$, $\gamma \in \Gamma$ and $y \in S$. Hence $\mu^{\geq}_{A,\alpha}$ is left $\Gamma-$ ideal of $\Gamma-$LA-semigroup. Now $x \in S$, $\gamma \in \Gamma$ and $y \in \gamma^{\geq}_{A,\alpha}$. Then

$$\gamma_A(x\gamma y) \leq \ \gamma_A(y) \leq \alpha$$

$x\gamma y \in \gamma^{\geq}_{A,\alpha}$ for all $x \in S$, $\gamma \in \Gamma$ and $y \in S$. Hence $\gamma^{\geq}_{A,\alpha}$ is left $\Gamma-$ ideal of $\Gamma-$LA-semigroup S. □

Example 2. *Let $S = \{1, 2, 3, 4, 5\}$ with binary operation "$*$". Then $(S, *)$ is an LA-semigroup by the following table*

*	1	2	3	4	5
1	2	2	2	2	2
2	2	2	2	2	2
3	2	2	2	2	2
4	2	2	2	2	2
5	2	3	3	2	2

Now let $S = \{1, 2, 3, 4, 5\}$ and $\Gamma = \{1\}$ and define a mapping $S \times \Gamma \times S \longrightarrow S$, by $a1b = a * b$ for all $a, b \in S$. Then it is easy to see that S is a $\Gamma-$LA-semigroup. Let $A = \langle \mu_A, \gamma_A \rangle$ be an intuitionistic fuzzy set defined by $\mu_A(1) = \mu_A(2) = \mu_A(3) = 0.7$, $\mu_A(4) = 0.5$, $\mu_A(5) = 0.2$. and $\gamma_A(1) = \gamma_A(2) = \gamma_A(3) = 0.2$, $\gamma_A(4) = 0.4$, $\gamma_A(1) = 0.7$. Now we find its level sets $\mu^{\geq}_{A,\alpha}$ and $\gamma^{\leq}_{A,\alpha}$of A.

$$\mu^{\geq}_{A,\alpha}(x) = \begin{cases} S & \text{If } \alpha \in (0, 0.2] \\ \{1, 2, 3, 4\} & \text{If } \alpha \in (0.2, 0.5] \\ \{1, 2, 3\} & \text{If } \alpha \in (0.5, 0.7] \\ \Phi & \text{If } \alpha \in (0.7, 1] \end{cases}$$

$$\gamma_A(x) = \begin{cases} \Phi & \text{If } \alpha \in [0, 0.2) \\ \{1, 2, 3, \} & \text{If } \alpha \in [0.2, 0.5) \\ \{1, 2, 3, 4\} & \text{If } \alpha \in [0.4, 0.7) \\ S & \text{If } \alpha \in [0.7, 1) \end{cases}$$

By using Theorem **??**, $A = \langle \mu_A, \gamma_A \rangle$ is an intuitionistic fuzzy $\Gamma-$ideal of $\Gamma-$LA-semigroup S. By routine calculation $A = \langle \mu_A, \gamma_A \rangle$ is an intuitionistic fuzzy bi$-\Gamma-$ideal of $\Gamma-$LA-semigroup S.

Theorem 5. *Every intuitionistic fuzzy left(right),$\Gamma-$ideals of $\Gamma-$LA-semigroup S is an intuitonistic fuzzy bi-$\Gamma-$ideals of $\Gamma-$LA-semigroup S.*

Proof. Let $A = (\mu_A, \gamma_A)$ be an intuitionistic fuzzy left,$\Gamma-$ideals of $\Gamma-$LA-semigroup S. And $w, x, y \in S$ and $\alpha, \gamma \in \Gamma$ then

$$\begin{aligned} \mu_A((x\alpha w)\gamma y) &\geq \mu_A(y) \\ \mu_A((x\alpha w)\gamma y) &= \mu_A((y\alpha w)\gamma x) \geq \mu_A(y) \\ \mu_A((x\alpha w)\gamma y) &\geq \min\{\mu_A(z), \mu_A(y)\} \end{aligned}$$

and

$$\begin{aligned} \gamma_A((x\alpha w)\gamma y) &\leq \gamma_A(y) \\ \gamma_A((x\alpha w)\gamma y) &= \gamma_A((y\alpha w)\gamma x) \leq \gamma_A(x) \\ \gamma_A((x\alpha w)\gamma y) &\leq \max\{\gamma_A(x), \gamma_A(y)\} \end{aligned}$$

for all $x, w, y \in S$. Hence $A = (\mu_{A,}\gamma_A)$ is an intuitonistic fuzzy bi-$\Gamma-$ideals of $\Gamma-$LA-semigroup S. □

Theorem 6. *Let IF(S) denote the set of all intuitionistic fuzzy left(right) $\Gamma-$ideal of $\Gamma-$LA-semigroup S.Then $(IF(S), \subseteq, U, \cap)$ is lattice.*

Proof. For all $A, B, C \in IF(S)$ then we have satisfied the foolowing conditions

1) Reflexive: Since

$$\mu_A(x) \leq \mu_A(x) and \gamma_A(x) \geq \gamma_A(x)$$

always then $A \subseteq B$

2) Antisymmetric: For all $A, B \in IF(S)$ we have $A \subseteq B$ and $B \subseteq A$ then

$$\mu_A(x) \leq \mu_B(x), \gamma_A(x) \geq \gamma_B(x)$$

and

$$\mu_B(x) \leq \mu_A(x), \gamma_B(x) \geq \gamma A(x)$$

for all $x \in S$. Thus $A = B$

3) Transitive For all $A, B, C \in IF(S)$ Such that

$$A \subseteq B \text{ and } B \subseteq C$$

then

$$\begin{aligned} \mu_A(x) &\leq \mu_B(x), \gamma_A(x) \geq \gamma_B(x) \\ \mu_B(x) &\leq \mu_C(x), \gamma_B(x) \geq \gamma_C(x) \end{aligned}$$

it follows that

$$\mu_A(x) \leq \mu_C(x), \gamma_A(x) \geq \gamma_C(x)$$

Thus $A \subseteq C$ Hence $(IF(S), \subseteq)$ is Poset.Now for lattice we have see that sup and inf of any two intuitionistic fuzzy set $A, B \in (IF(S)$

Inf: For any two $A, B \in (IF(S)$ Inf$\{A, B\} = A \cap B$

$$A \cap B = \{\mu_A \wedge \mu_B, \gamma_A \vee \gamma_B$$

Now we show that $A \cap B$ is an intuitionistic fuzzy right $\Gamma-$ideal of $\Gamma-$LA-semigroup S. For any $x, y \in S$ and $\alpha \in \Gamma$

$$\begin{aligned} (\mu_A \wedge \mu_B)(x\alpha y) &= \mu_A(x\alpha y) \wedge \mu_A(x\alpha y) \\ &\geq \mu_A(x) \wedge \mu_A(x) = (\mu_A \wedge \mu_B)(x) \\ (\mu_A \wedge \mu_B)(x\alpha y) &\geq (\mu_A \wedge \mu_B)(x) \end{aligned}$$

and

$$\begin{aligned} (\gamma_A \vee \gamma_B)(x\alpha y) &= \gamma_A(x\alpha y) \vee \gamma_A(x\alpha y) \\ &\leq \gamma_A(x) \vee \gamma_A(x) = (\gamma_A \vee \gamma_B)(x) \\ (\gamma_A \vee \gamma_B)(x\alpha y) &\leq (\gamma_A \vee \gamma_B)(x) \end{aligned}$$

$A \cap B$ is intuitionistic fuzzy right $\Gamma-$ideal of $\Gamma-$LA-semigroup S. This mean $A \cap B \in IF(S)$, inf$\{A, B\}$ exist in $IF(S)$.

Inf For any two $A, B \in (IF(S)$ Sup $\{A, B\} = A \cup B$

$$\begin{aligned} A \cup B &= \{\mu_A \vee \mu_B, \gamma_A \wedge \gamma_B\} \\ (\mu_A \vee \mu_B)(x\alpha y) &= \mu_A(x\alpha y) \vee \mu_A(x\alpha y) \\ &\geq \mu_A(x) \vee \mu_A(x) = (\mu_A \vee \mu_B)(x) \\ (\mu_A \vee \mu_B)(x\alpha y) &\geq (\mu_A \vee \mu_B)(x) \end{aligned}$$

and

$$\begin{aligned}(\gamma_A \wedge \gamma_B)(x\alpha y) &= \gamma_A(x\alpha y) \wedge \gamma_A(x\alpha y) \\ &\leq \gamma_A(x) \wedge \gamma_A(x) = (\gamma_A \wedge \gamma_B)(x) \\ (\gamma_A \wedge \gamma_B)(x\alpha y) &\leq (\gamma_A \wedge \gamma_B)(x)\end{aligned}$$

$A \cup B$ is intuitionistic fuzzy right $\Gamma-$ideal of $\Gamma-$LA-semigroup S. This mean $A \cup B \in IF(S)$, Sup$\{A, B\}$ exist in $IF(S)$. Hence $(IF(S), \subseteq, U, \cap)$ is lattice. □

Definition 11. *Let f be mapping from a set X to Y and μ be fuzzy set in Y, then the pre-image of μ under f denoted by $f^{-1}(\mu)$ and define as*

$$f^{-1}(\mu(x)) = \mu(f(x)) \qquad \text{for all } x \in S$$

Definition 12. *Let $f : S \longrightarrow S_1$ be homomorphism from $\Gamma-LA$-semigroup S to $\Gamma-LA$-semigroup S_1 and $h : \Gamma \longrightarrow \Gamma_1$.If $A = \langle \mu_A, \gamma_A \rangle$ an intuitionistic fuzzy set in S_1then the preimage of $A = \langle \mu_A, \gamma_A \rangle$ is denoted by $f^{-1}(A) = \langle f^{-1}(\mu_A), f^{-1}(\gamma_A) \rangle$ and define as $f^{-1}(\mu_A(x)) = (\mu_A(f(x)))$ and $f^{-1}(\gamma_A(x)) = (\gamma_A(f(x)))$*

Theorem 7. *Let the pair of mappings $f : S \longrightarrow S_1$, $h : \Gamma \longrightarrow \Gamma_1$ be homomorphism of $\Gamma-LA$-semigroup. $A = \langle \mu_A, \gamma_A \rangle$ is an intuitonistic fuzzy left(resp, right) $\Gamma-$ideal of $\Gamma-LA$-semigroup S_1. Then $f^{-1}(A) = \langle f^{-1}(\mu_A), f^{-1}(\gamma_A) \rangle$ is an intuitonistic fuzzy left(resp, right) $\Gamma-$ideal of $\Gamma-LA$-semigroup S.*

Proof. Let $x, y \in S$ and $\alpha \in \Gamma$ and let $A = \langle \mu_A, \gamma_A \rangle$ is an intuitonistic fuzzy left $\Gamma-$ideal of $\Gamma-$LA-semigroup S_1. Then

$$\begin{aligned}f^{-1}(\mu_A(x\alpha y) &= (\mu_A(f(x\alpha y))) = (\mu_A(f(x)h(\alpha)f(y))) \\ f^{-1}(\mu_A(x\alpha y) &\geq \mu_A(f(y)) = f^{-1}\mu_A(y)\end{aligned}$$

and

$$\begin{aligned}f^{-1}(\gamma_A(x\alpha y)) &= (\gamma_A(f(x\alpha y))) = (\gamma_A(f(x)h(\alpha)f(y))) \\ f^{-1}(\gamma_A(x\alpha y)) &\leq \gamma_A(f(y)) = f^{-1}(\gamma_A(y))\end{aligned}$$

for all $x, y \in S$ and $\alpha \in \Gamma$. Hence $f^{-1}(A) = \langle f^{-1}(\mu_A), f^{-1}(\gamma_A) \rangle$ is an intuitonistic fuzzy left $\Gamma-$ideal of $\Gamma-$LA-semigroup S. And similarly for an intuitonistic fuzzy right $\Gamma-$ideal of $\Gamma-$LA-semigroup S. □

Definition 13. *Let $f : [1, 0] \longrightarrow [1, 0]$ is an increasing function and $A = (\mu_{A,}\gamma_A)$ be an IFS of $\Gamma-LA$-semigroup S. Then $A_f = (\mu_{A_f}, \gamma_{A_f})$ be an IFS of $\Gamma-LA$-semigroup S, define as $\mu_{A_f}(x) = f(\mu_A(x))$ and $\gamma_{A_f}(x) = f(\gamma_A(x))$ for all $x \in S$.*

Proposition 1. *Let S be $\Gamma-LA$-semigroup. If $A = (\mu_{A,}\gamma_A)$ is an intuitonistic fuzzy left(resp, right) $\Gamma-$ideal of S, then $A_f = (\mu_{A_f}, \gamma_{A_f})$ an intuitonistic fuzzy left(resp, right) $\Gamma-$ideal of S.*

Proof. Let $A = (\mu_{A,}\gamma_A)$ is an intuitonistic fuzzy left $\Gamma-$ideal of S. Let for any $x, y \in S$ and $\alpha \in \Gamma$ and $A_f = (\mu_{A_f}, \gamma_{A_f})$ be IFS of S. then

$$\mu_{A_f}(x\alpha y) = f(\mu_A(x\alpha y)) \geq f(\mu_A(y))$$

and

$$\begin{aligned}\gamma_{A_f}(x\alpha y) &= f(\gamma_A(x\alpha y)) \leq f(\gamma_A(y)) \\ \mu_{A_f}(x\alpha y) &\geq f(\mu_A(y)) \text{ and } \gamma_{A_f}(x\alpha y) \leq f(\gamma_A(y))\end{aligned}$$

for all $x, y \in S$. Hence $A_f = (\mu_{A_f}, \gamma_{A_f})$ is an intuitonistic fuzzy left $\Gamma-$ideal of S. □

Proposition 2. *Let $A = (\mu_{A,} \gamma_A)$ be an intutionistic fuzzy left $\Gamma-$ideal of left zero $\Gamma-$LA-semigroup S. Then $A(x) = A(z)$ for all $x, z \in S$.*

Proof. Let $x, z \in S$ and $\alpha \in \Gamma$. Since S is left zero $\Gamma-$LA-semigroup S then $x\alpha z = x$ and $z\alpha x = z$ then we have

$$\begin{aligned} \mu_A(x) &= \mu_A(x\alpha z) \geq \mu_A(z) \Longrightarrow \mu_A(x) \geq \mu_A(z) \\ \mu_A(z) &= \mu_A(z\alpha x) \geq \mu_A(x) \Longrightarrow \mu_A(z) \geq \mu_A(x) \\ \mu_A(x) &= \mu_A(z) \end{aligned}$$

and

$$\begin{aligned} \gamma_A(x) &= \gamma_A(x\alpha z) \leq \gamma_A(z) \Longrightarrow \gamma_A(x) \leq \gamma_A(z) \\ \gamma_A(z) &= \gamma_A(z\alpha x) \leq \gamma_A(x) \Longrightarrow \gamma_A(z) \leq \gamma_A(x) \\ \gamma_A(x) &= \gamma_A(z) \end{aligned}$$

For all $x, z \in S$.Hence $A(x) = A(z)$ for all $x, z \in S$. □

Proposition 3. *Let I be left $\Gamma-$ideal of $\Gamma-$LA-semigroup S. Then $A = (x_{I,} \bar{x_I})$ is an intuitonistic fuzzy left $\Gamma-$ideal of $\Gamma-$La-semigroup S. Where x_I is characteristic functions.and $\bar{x_I} = 1 - x_I$*

Proof. Let $y, z \in S$ and $\alpha \in \Gamma$ and $A = (x_{I,} \bar{x_I})$ be IFS of S. Since I left $\Gamma-$ideal of $\Gamma-$LA-semigroup S, then we have two case's i) if $y \in I$ and ii) $y \notin I$

case i) if $y \in I$ then $y\alpha z \in I$ then

$$x_I(y) = 1 \text{ and } x_I(y\alpha z) = 1$$

and also

$$x_I(y\alpha z) = 1 = x_I(y)$$

ii) if $y \notin I$ then

$$\begin{aligned} x_I(y) &= 0 \text{ and } x_I(y\alpha z) \geq 0 \\ x_I(y\alpha z) &\geq 0 = x_I(y) \Longrightarrow x_I(y\alpha z) \geq x_I(y) \end{aligned}$$

if $y \in I$

$$\begin{aligned} 1 - x_I(y) &= 1 - 1 = 0 \text{ and } 1 - x_I(y\alpha z) = 1 - 1 = 0 \\ \bar{x_I}(y\alpha z) &= \bar{x_I}(y) \end{aligned}$$

if $y \notin I$ then

$$\begin{aligned} \bar{x_I}(x) &= 1 - x_I(y) = 1 - 0 = 1 \\ \bar{x_I}(y\alpha z) &\leq \bar{x_I}(x) \end{aligned}$$

Hence $A = (x_{I,} \bar{x_I})$ is an ntuitonistic fuzzy left $\Gamma-$ideal of $\Gamma-$La-semigroup S. □

Definition 14. *Let $A = (\mu_{A,} \gamma_A)$ and $B = (\mu_{B,} \gamma_B)$ two an intuitonistic fuzzy left(resp, right) $\Gamma-$ideal of $\Gamma-$LA-semigroup S. then product of $A = (\mu_{A,} \gamma_A)$ and $B = (\mu_{B,} \gamma_B)$ is denoted by $A\Gamma B$ and defined as*

$$\begin{aligned} \mu_{A\Gamma B}(x) &= \vee_{x=y\alpha z}\{\mu_A(y) \wedge \mu_B(x)\} \\ \gamma_{A\Gamma B}(x) &= \wedge_{x=y\alpha z}\{\gamma_A(y) \vee \gamma_A(z) \end{aligned}$$

Lemma 1. *$A = (\mu_{A,}\gamma_A)$ $B = (\mu_{B,}\gamma_B)$ be any two intuitionistic fuzzy right(left) ideal of $\Gamma-LA$-semigroup S with left identity.Then $A\Gamma B$ is also intuitionistic fuzzy right(left) ideal of S*

Theorem 8. *Let $IF(S)$ denote the set of all intuitionistic fuzzy left(right) ideal of $\Gamma-LA$-semigroup S with left identity. Then $(IF(S),\Gamma)$ is $\Gamma-LA$-semigroup*

Proof. let $IF(S)$ denote the set of all intuitionistic fuzzy left(right) ideal of S then clearly $(IF(S),\Gamma)$ is closed by Lemma 1. Now for any $A = (\mu_{A,}\gamma_A)$ $B = (\mu_{B,}\gamma_B)$ $C = (\mu_{C,}\gamma_C) \in S$, then

$$\begin{aligned}
\mu_{(A\Gamma B)\Gamma C}(x) &= \vee_{x=y\alpha z}\{\mu_{A\Gamma B}(y) \wedge \mu_C(z)\} \\
&= \vee_{x=y\alpha z}\{\vee_{y=p\beta q}\{\mu_A(p) \wedge \mu_B(q)\} \wedge \mu_C(z)\} \\
&= \vee_{x=(p\beta q)\alpha z}\{\mu_A(p) \wedge \mu_B(q) \wedge \mu_C(z)\} \\
&= \vee_{x=(z\beta q)\alpha p}\{\mu_C(z) \wedge \mu_B(q) \wedge \mu_A(p)\} \\
&\leq \vee_{x=w\alpha p}\{\vee_{w=z\beta q}\{\mu_C(z) \wedge \mu_B(q)\} \wedge \mu_C(p)\} \\
&= \vee_{x=w\alpha p}\{\mu_{C\Gamma B}(w) \wedge \mu_A(p)\} = \mu_{(C\Gamma B)\Gamma A}(x)
\end{aligned}$$

This implies $\mu_{(A\Gamma B)\Gamma C}(x) \leq \mu_{(C\Gamma B)\Gamma A}(x)$

Similarly $\mu_{(C\Gamma B)\Gamma A}(x) \leq \mu_{(A\Gamma B)\Gamma C}(x)$ and thus $\mu_{(A\Gamma B)\Gamma C}(x) = \mu_{(C\Gamma B)\Gamma A}(x)$

and

$$\begin{aligned}
\gamma_{(A\Gamma B)\Gamma C}(x) &= \wedge_{x=y\alpha z}\{\gamma_{A\Gamma B}(y) \vee \gamma_C(z)\} \\
&= \wedge_{x=y\alpha z}\{\wedge_{y=m\beta n}\{\gamma_A(m) \vee \gamma_B(n)\} \vee \gamma_C(z)\} \\
&= \wedge_{x=(m\beta n)\alpha z}\{\gamma_A(m) \vee \gamma_B(n) \vee \gamma_C(z)\} \\
&= \wedge_{x=(z\beta n)\alpha m}\{\gamma_C(z) \vee \gamma_B(n) \vee \gamma_A(m)\} \\
&\geq \wedge_{x=l\alpha m}\{\wedge_{x=z\beta n}\{\gamma_C(z) \vee \gamma_B(n)\} \vee \gamma_A(m)\} \\
&= \wedge_{x=l\alpha m}\{\gamma_{A\Gamma B}(l) \vee \gamma_C(m)\} = \gamma_{(C\Gamma B)\Gamma A}(x) \\
\gamma_{(A\Gamma B)\Gamma C}(x) &\geq \gamma_{(C\Gamma B)\Gamma A}(x)
\end{aligned}$$

Similarly $\gamma_{(C\Gamma B)\Gamma A}(x) \geq \gamma_{(A\Gamma B)\Gamma C}(x)$ and thus $\gamma_{(A\Gamma B)\Gamma C}(x) = \gamma_{(C\Gamma B)\Gamma A}(x)$

Hence

$$(A\Gamma B)\Gamma C = (C\Gamma B)\Gamma A$$

Thus $(IF(S),\Gamma)$ is $\Gamma-$LA-semigroup S. □

Proposition 4. *Let S be a $\Gamma-LA$-semigroup with left identity, if $A = \langle\mu_A, \gamma_A\rangle$ is an intuitionistic fuzzy right $\Gamma-$ideal of $\Gamma-LA$-semigroup S. Then $A\Gamma A$ is an intuitionistic fuzzy $\Gamma-$ideal of S.*

Proof. Since $A = \langle\mu_A, \gamma_A\rangle$ is an intuitionistic fuzzy right $\Gamma-$ideal of S, then $A = \langle\mu_A, \gamma_A\rangle$ is an intuitionistic fuzzy left $\Gamma-$ideal of S. Let for all $a, b \in S$ and $\alpha, \gamma \in \Gamma$ if $a \neq x\gamma y$ then

$$\mu_{A\Gamma A}(a) = 0 \text{ and } \mu_{A\Gamma A}(a\alpha b) \geq \mu_{A\Gamma A}(a)$$

and

$$\gamma_{A\Gamma A}(a) = 0 \text{ and } \gamma_{A\Gamma A}(a\alpha b) \leq \gamma_{A\Gamma A}(a)$$

otherwise

$$\begin{aligned}
\mu_{A\Gamma A}(a) &= \bigvee_{a=x\gamma y}\{\mu_A(x)\wedge\mu_A(y)\} \\
\text{if } a &= x\gamma y \text{ then } a\alpha b=(x\gamma y)\alpha b=(b\gamma y)\alpha x \text{ by left invertible law.} \\
\mu_{A\Gamma A}(a) &= \bigvee_{a=x\gamma y}\{\mu_A(y)\wedge\mu_A(x)\} \\
\mu_{A\Gamma A}(a) &\leq \bigvee_{a=x\gamma y}\{\mu_A(b\gamma y)\wedge\mu_A(x)\} \text{ since } A \text{ is IF left } \Gamma-\text{ideal} \\
&\leq \bigvee_{a\alpha b=(b\gamma y)\alpha x}\{\mu_A(b\gamma y)\wedge\mu_A(x)\}=\mu_{A\Gamma A}(a\alpha b) \\
\mu_{A\Gamma A}(a\alpha b) &\geq \mu_{A\Gamma A}(a) \\
\text{and } \gamma_{A\Gamma A}(a) &= \bigwedge_{a=x\gamma y}\{\gamma_A(x)\vee\gamma_A(y)\} \\
\gamma_{A\Gamma A}(a) &= \bigwedge_{a=x\gamma y}\{\gamma_A(y)\vee\gamma_A(x)\} \\
\gamma_{A\Gamma A}(a) &\geq \bigwedge_{a=x\gamma y}\{\gamma_A(b\gamma y)\vee\gamma_A(x)\} \text{ since } A \text{ is IF left } \Gamma-\text{ideal} \\
&\geq \bigwedge_{a\alpha b=(b\gamma y)\alpha x}\{\gamma_A(b\gamma y)\vee\gamma_A(x)\}=\gamma_{A\Gamma A}(a\alpha b) \\
\gamma_{A\Gamma A}(a\alpha b) &\leq \gamma_{A\Gamma A}(a)
\end{aligned}$$

Hence $A\Gamma A=\langle\mu_{A\Gamma A},\gamma_{A\Gamma A}\rangle$ is an intuitionistic fuzzy right $\Gamma-$ideal of S, and by Theorem 1 $A\Gamma A=\langle\mu_{A\Gamma A},\gamma_{A\Gamma A}\rangle$ is an intuitionistic fuzzy left $\Gamma-$ideal of S. □

Theorem 9. *Let S be a $\Gamma-$LA-semigroup with left identity. Then for any A,B,C IFS of S. $A\Gamma(B\Gamma C)=B\Gamma(A\Gamma C)$*

Proof. Let $x\in S$ and $A=\langle\mu_A,\gamma_A\rangle, B=\langle\mu_B,\gamma_B\rangle, C=\langle\mu_C,\gamma_C\rangle$ be any IFS of S. Then

$$\begin{aligned}
\mu_{A\Gamma(B\Gamma C)}(x) &= \bigvee_{x=y\alpha z}\{\mu_A(y)\wedge\mu_{B\Gamma C}(z)\} \\
&= \bigvee_{x=y\alpha z}\{\mu_A(y)\wedge[\bigvee_{z=s\beta t}\{\mu_B(s)\wedge\mu_C(t)\}]\} \\
&= \bigvee_{x=y\alpha(s\beta t)}\{\mu_A(y)\wedge\mu_B(s)\wedge\mu_C(t)\} \\
&= \bigvee_{x=s\alpha(y\beta t)}\{\mu_B(s)\wedge\mu_A(y)\wedge\mu_C(t)\} \\
\text{since } \mu_A(y)\wedge\mu_C(t) &\leq \bigvee_{y\alpha t=a\gamma b}\{\mu_A(a)\wedge\mu_C(b)\} \\
\text{so} \quad &\leq \bigvee_{x=s\alpha(y\beta t)}\{\mu_B(s)\wedge[\bigvee_{y\beta t=a\gamma b}\{\mu_A(a)\wedge\mu_C(b)\}]\} \\
&= \bigvee_{x=s\alpha(y\beta t)}\{\mu_B(s)\wedge\mu_{A\Gamma C}(y\beta t)\} \\
&\leq \bigvee_{x=p\alpha q}\{\mu_B(p)\wedge\mu_{A\Gamma C}(q)\}=\mu_{B\Gamma(A\Gamma C)}(x) \\
\mu_{A\Gamma(B\Gamma C)}(x) &\leq \mu_{B\Gamma(A\Gamma C)}(x)\Longrightarrow\mu_{A\Gamma(B\Gamma C)}\leq\mu_{B\Gamma(A\Gamma C)} \\
\text{Similarly } \gamma_{A\Gamma(B\Gamma C)}(x) &\geq \gamma_{B\Gamma(A\Gamma C)}(x)\Longrightarrow\gamma_{A\Gamma(B\Gamma C)}\geq\gamma_{B\Gamma(A\Gamma C)}
\end{aligned}$$

and

$$\begin{array}{rcl} \gamma_{A\Gamma(B\Gamma C)}(x) & = & \bigwedge\limits_{x=y\alpha z}\{\gamma_A(y)\vee\gamma_{B\Gamma C}(z)\} \\ & = & \bigwedge\limits_{x=y\alpha z}\{\gamma_A(y)\vee[\bigwedge\limits_{z=s\beta t}\{\gamma_B(s)\vee\gamma_C(t)\}]\} \\ & = & \bigwedge\limits_{x=y\alpha(s\beta t)}\{\gamma_A(y)\vee\gamma_B(s)\vee\gamma_C(t)\} \\ & = & \bigwedge\limits_{x=s\alpha(y\beta t)}\{\gamma_B(s)\vee\gamma_A(y)\vee\gamma_C(t)\} \\ \text{since } \gamma_A(y)\wedge\gamma_C(t) & \geq & \bigwedge\limits_{y\alpha t=a\gamma b}\{\gamma_A(a)\wedge\gamma_C(b)\} \\ & \geq & \bigwedge\limits_{x=s\alpha(y\beta t)}\{\gamma_B(s)\vee[\bigwedge\limits_{y\beta t=a\gamma b}\{\gamma_A(a)\vee\gamma_C(b)\}]\} \\ \text{so} & = & \bigwedge\limits_{x=s\alpha(y\beta t)}\{\gamma_B(s)\vee\gamma_{A\Gamma C}(y\beta t)\} \\ & \geq & \bigwedge\limits_{x=p\alpha q}\{\gamma_B(p)\vee\gamma_{A\Gamma C}(q)\} = \gamma_{B\Gamma(A\Gamma C)}(x) \end{array}$$

Thus $A\Gamma(B\Gamma C)\leq B\Gamma(A\Gamma C)$ and similarly $A\Gamma(B\Gamma C)\geq B\Gamma(A\Gamma C)$. Hence $A\Gamma(B\Gamma C)=B\Gamma(A\Gamma C)$. □

Lemma 2. *Let S be $\Gamma-LA$-semigroup and $A=\langle\mu_A,\gamma_A\rangle$ be an intuitionistic fuzzy right $\Gamma-$ideal of S and $B=\langle\mu_B,\gamma_B\rangle$ be an intuitionistic fuzzy left $\Gamma-$ideal of S. Then $A\Gamma B\subseteq A\cap B$*

Proof. Let for any $x\in S$ and $\alpha\in\Gamma$. If $x\neq y\alpha z$ for any $y,z\in S$, then

$$\mu_{A\Gamma B}(x)=0\leq\mu_{A\cap B}(x)=\mu_A\wedge\mu_B(x)$$

otherwise

$$\begin{array}{rcl} \mu_{A\Gamma B}(x) & = & \bigvee\limits_{x=y\alpha z}\{\mu_A(y)\wedge\mu_B(z)\} \\ & \leq & \bigvee\limits_{x=y\alpha z}\{\mu_A(y\alpha z)\wedge\mu_B(y\alpha z)\} \\ & = & \bigvee\limits_{x=y\alpha z}\{\mu_A(x)\wedge\mu_B(x)\} \\ \mu_{A\Gamma B}(x) & \leq & (\mu_A\wedge\mu_B)(x)\Longrightarrow\mu_{A\Gamma B}\leq(\mu_A\wedge\mu_B) \end{array}$$

and If $x\neq y\alpha z$ for any $y,z\in S$, then

$$\gamma_{A\Gamma B}(x)=0\geq\gamma_{A\cap B}(x)=\gamma_A\vee\gamma_B(x)$$

otherwise

$$\begin{array}{rcl} \mu_{A\Gamma B}(x) & = & \bigwedge\limits_{x=y\alpha z}\{\gamma_A(y)\vee\gamma_B(z)\} \\ & \leq & \bigwedge\limits_{x=y\alpha z}\{\gamma_A(y\alpha z)\vee\gamma_B(y\alpha z)\} \\ & = & \bigwedge\limits_{x=y\alpha z}\{\gamma_A(x)\vee\gamma_B(x)\} \\ \gamma_{A\Gamma B}(x) & \leq & (\gamma_A\vee\gamma_B)(x)\Longrightarrow\gamma_{A\Gamma B}\leq(\gamma_A\vee\gamma_B) \end{array}$$

Hence $A\Gamma B=\langle\mu_{A\Gamma B},\gamma_{A\Gamma B}\rangle\subseteq\langle\mu_A\wedge\mu_B,\gamma_A\vee\gamma_B\rangle=A\cap B$. □

Corollary 2. *Let S be $\Gamma-LA$-semigroup and $A=\langle\mu_A,\gamma_A\rangle, B=\langle\mu_B,\gamma_B\rangle$ be any intuitionistic fuzzy $\Gamma-$ideal of S. Then $A\Gamma B\subseteq A\cap B$*

Remark 1. *If S is a $\Gamma-LA$-semigroup with left identity e and $A = \langle \mu_A, \gamma_A \rangle$ and $B = \langle \mu_B, \gamma_B \rangle$ are intuitionistic fuzzy right $\Gamma-$ideal of S. Then $A\Gamma B \subseteq A \cap B$*

Remark 2. *If S is a $\Gamma-LA$-semigroup and $A = \langle \mu_A, \gamma_A \rangle$ be an intuitionistic fuzzy $\Gamma-$ideal of S. Then $A\Gamma A \subseteq A$*

Definition 15. *A $\Gamma-LA$-semigroup $S$is called regular if for every $a \in S$, there exists x in S and $\alpha, \beta \in \Gamma$ such that $a = (a\alpha x)\beta a$, or equivalently, $a \in (a\Gamma S)\Gamma a$.*

For regular $\Gamma-$LA-semigroup it is easy to see that $S\Gamma S = S$

Proposition 5. *Every intuitionistic fuzzy right $\Gamma-$ideal of regular $\Gamma-LA$-semigroup S is an intuitionistic fuzzy left $\Gamma-$ideal of S.*

Proof. Let $A = \langle \mu_A, \gamma_A \rangle$ be an intuitionistic fuzzy right $\Gamma-$ideal of S and $a, b \in S$ and $\gamma \in \Gamma$. Since S is regular, there exist $x \in S$, and $\alpha, \beta \in \Gamma$ such that $a = (a\alpha x)\beta a$. Then

$$\begin{aligned} \mu_A(a\gamma b) &= \mu_A(((a\alpha x)\beta a)\gamma b) \\ &= \mu_A((b\beta a)\gamma(a\alpha x)) \geq \mu_A(b\beta a) \\ \mu_A(a\gamma b) &\geq \mu_A(b) \end{aligned}$$

and

$$\begin{aligned} \gamma_A(a\gamma b) &= \gamma_A(((a\alpha x)\beta a)\gamma b) \\ &= \gamma_A((b\beta a)\gamma(a\alpha x)) \geq \gamma_A(b\beta a) \\ \gamma_A(a\gamma b) &\geq \gamma_A(b) \end{aligned}$$

Hence $A = \langle \mu_A, \gamma_A \rangle$ be an intuitionistic fuzzy left $\Gamma-$ideal of S. □

Corollary 3. *In a regular $\Gamma-LA$-semigroup S, every intuitionistic fuzzy right $\Gamma-$ideal of S is an intuitionistic fuzzy $\Gamma-$ideal of S.*

Proposition 6. *If $A = \langle \mu_A, \gamma_A \rangle$ and $B = \langle \mu_B, \gamma_B \rangle$ be any intuitionistic fuzzy right $\Gamma-$ideal of regular $\Gamma-LA$-semigroup S, then $A\Gamma B = A \cap B$*

Proof. Since S regular, by proposition 5, Every intuitionistic fuzzy right $\Gamma-$ideal of regular $\Gamma-$LA-semigroup S is an intuitionistic fuzzy left $\Gamma-$ideal of S. By Lemma 2 $A\Gamma B \subseteq A \cap B$.

On the other hand, let $a \in S$, then there exist $x \in S$ and $\alpha, \beta \in \Gamma$ such that $a = (a\alpha x)\beta a$. Thus

$$\begin{aligned} (\mu_A \wedge \mu_B)(a) &= \mu_A(a) \wedge \mu_B(a) \\ &\leq \mu_A(a\alpha x) \wedge \mu_B(a) \\ &\leq \bigvee_{a=(a\alpha x)\beta a} \mu_A(a\alpha x) \wedge \mu_B(a) \\ (\mu_A \wedge \mu_B)(a) &\leq \mu_{A\Gamma B}(a) \Longrightarrow \mu_A \wedge \mu_B \leq \mu_{A\Gamma B} \end{aligned}$$

and

$$\begin{aligned} (\gamma_A \vee \gamma_B)(a) &= \gamma_A(a) \vee \gamma_B(a) \\ &\geq \gamma_A(a\alpha x) \vee \gamma_B(a) \\ &\geq \bigwedge_{a=(a\alpha x)\beta a} \gamma_A(a\alpha x) \vee \gamma_B(a) \\ (\gamma_A \vee \gamma_B)(a) &\geq \gamma_{A\Gamma B}(a) \Longrightarrow \gamma_A \vee \gamma_B \geq \gamma_{A\Gamma B} \end{aligned}$$

Thus $A \cap B \subseteq A\Gamma B$, therefore

$$A\Gamma B \subseteq A \cap B \text{ and } A \cap B \subseteq A\Gamma B \Longrightarrow A \cap B = A\Gamma B.$$

□

Definition 16. *A $\Gamma-LA$-semigroup S is called $\Gamma-LA$ band if all of its elemnets are idempotent i.e for all $x \in S$, there exist $\alpha \in \Gamma$, such that $x\alpha x = x$.*

Theorem 10. *The concept of intuitionistic fuzzy right and left $\Gamma-$ideal in a $\Gamma-LA$ band are coincide.*

Proof. Let $A = \langle \mu_A, \gamma_A \rangle$ be an intuitionistic fuzzy right $\Gamma-$ideal in a $\Gamma-$LA band S and $x, y \in S$ and $\alpha, \beta, \gamma \in \Gamma$. Then

$$\begin{aligned} \mu_A(x\alpha y) &= \mu_A((x\beta x)\alpha y) \\ &= \mu_A((y\beta x)\alpha x) \text{ by left invertible law} \\ &\geq \mu_A(y\beta x) \geq \mu_A(y) \\ \mu_A(x\alpha y) &\geq \mu_A(y) \end{aligned}$$

and

$$\begin{aligned} \gamma_A(x\alpha y) &= \gamma_A((x\beta x)\alpha y) \\ &= \gamma_A((y\beta x)\alpha x) \text{ by left invertible law} \\ &\leq \gamma_A(y\beta x) \leq \gamma_A(y) \\ \mu_A(x\alpha y) &\leq \mu_A(y) \end{aligned}$$

Therefore $A = \langle \mu_A, \gamma_A \rangle$ be an intuitionistic fuzzy left $\Gamma-$ideal in a $\Gamma-$LA band S

Conversely suppose that $A = \langle \mu_A, \gamma_A \rangle$ be an intuitionistic fuzzy left $\Gamma-$ideal in a $\Gamma-$LA band S and $x, y \in S$ and $\alpha, \beta, \gamma \in \Gamma$. Then

$$\begin{aligned} \mu_A(x\alpha y) &= \mu_A((x\beta x)\alpha y) \\ &= \mu_A(((y\beta x)\alpha y)) \geq \mu_A(y\beta x) \\ &\Longrightarrow \mu_A(x\alpha y) \geq \mu_A(x) \end{aligned}$$

and

$$\begin{aligned} \gamma_A(x\alpha y) &= \gamma_A((x\beta x)\alpha y) \\ &= \gamma_A((y\beta x)\alpha y) \geq \gamma_A(y\beta x) \\ &\Longrightarrow \gamma_A(x\alpha y) \geq \gamma_A(x) \end{aligned}$$

Thererfore $A = \langle \mu_A, \gamma_A \rangle$ be an intuitionistic fuzzy right $\Gamma-$ideal in a $\Gamma-$LA band. S □

References

[1] K.T. Atanassov, Intuitionistic fuzzy sets,Fuzzy sets and Systems 20 (1986), no,1,87-96.

[2] K.T. Atanassov, New operations defined over the intuitionistic fuzzysets, Fuzzy Sets and Systems 61 (1994), 2,137-142.

[3] K.H.Kim and Y.B.Jun, Intuitionistic fuzzy interiror ideals of semigroups, Int. J. Math. Sci. 27 (2001), no. 5, 261-142.

[4] M.A. Kazim and M. Naseerudin, On almost semigroups, Alig.Bull.Math. 2 (1972), 1-7.

[5] L. A. Zadeh, Fuzzy sets, Information and Control 8 (1965), 338-353.

[6] T. Shah and I. Rehman, On Γ−ideals and Γ−Bi-ideals in Γ−AG-groupoid, Int. J. Algebra, 4 (2010), no. 267-276.

[7] , Intuitionistic fuzzy Ideal of semigroups, Indian. J. Pure Appl. Math. 33 (2002), no. 4, 443-449.

[8] M. Uckun, M. A. Ozturk and Y. B. Jun, Intuitionistic fuzzy sets in $\Gamma-$semigroups, Bull. Korean. Math. Soc. 44 (2007), no. pp. 359-367.
[9] T. Shah, I. Rehman and A.Khan, Fuzzy $\Gamma-$ideals in $\Gamma-$AG groupoids, (Submitted).
[10] N. Palaniappan, P.S. Veerappan and M. Ramachandran, Characterizations of Intutitionistic fuzzy ideals of $\Gamma-$Rings, Appl. Math. Sciences, 4 (2019), no.23, 1107-1117.

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