Aotomorphic Functions And Fermat's Last Theorem(4)

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In 1637 Fermat wrote: "*It is impossible to separate a cube into two cubes, or a biquadrate into two biquadrates, or in general any power higher than the second into powers of like degree: I have discovered a truly marvelous proof, which this margin is too small to contain.*"

This means: $x^n + y^n = z^n (n > 2)$ has no integer solutions, all different from 0(i.e., it has only the trivial solution, where one of the integers is equal to 0). It has been called Fermat's last theorem (FLT). It suffices to prove FLT for exponent 4. and every prime exponent P. Fermat proved FLT for exponent 4. Euler proved FLT for exponent 3.

In this paper using automorphic functions we prove FLT for exponents $3P$ and P, where P is an odd prime. We find the Fermat proof. The proof of FLT must be direct. But indirect proof of FLT is disbelieving..

In 1974 Jiang found out Euler formula of the cyclotomic real numbers in the cyclotomic fields

$$\exp\left(\sum_{i=1}^{n-1} t_i J^i\right) = \sum_{i=1}^{n} S_i J^{i-1} \qquad (1)$$

where J denotes a n th root of negative unity, $J^n = -1$, n is an odd number, t_i are the real numbers.

S_i is called the automorphic functions(complex trigonometric functions) of order n with $n-1$ variables [1-7].

$$S_i = \frac{(-1)^{i-1}}{n}[e^A + 2\sum_{j=1}^{\frac{n-1}{2}} (-1)^{(i-1)j} e^{B_j} \cos(\theta_j + (-1)^j \frac{(i-1)j\pi}{n})] \qquad (2)$$

where

$$A = \sum_{\alpha=1}^{n-1} t_\alpha (-1)^\alpha, \qquad B_j = \sum_{\alpha=1}^{n-1} t_\alpha (-1)^{(j-1)\alpha} \cos\frac{\alpha j\pi}{n}, \qquad (3)$$

$$\theta_j = (-1)^{j+1}\sum_{\alpha=1}^{n-1} t_\alpha (-1)^{(j-1)\alpha} \sin\frac{\alpha j\pi}{n}, \qquad A + 2\sum_{j=1}^{\frac{n-1}{2}} B_j = 0$$

(2) may be written in the matrix form

$$\begin{bmatrix} S_1 \\ S_2 \\ S_3 \\ \cdots \\ S_n \end{bmatrix} = \frac{1}{n} \begin{bmatrix} 1 & 1 & 0 & \cdots & 0 \\ -1 & \cos\frac{\pi}{n} & \sin\frac{\pi}{n} & \cdots & \sin\frac{(n-1)\pi}{2n} \\ 1 & \cos\frac{2\pi}{n} & \sin\frac{2\pi}{n} & \cdots & -\sin\frac{(n-1)\pi}{n} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 1 & \cos\frac{(n-1)\pi}{n} & \sin\frac{(n-1)\pi}{n} & \cdots & -\sin\frac{(n-1)^2\pi}{2n} \end{bmatrix}$$

$$\begin{bmatrix} e^A \\ 2e^{B_1}\cos\theta_1 \\ 2e^{B_1}\sin\theta_1 \\ \cdots \\ 2\exp B_{\frac{n-1}{2}} \sin\theta_{\frac{n-1}{2}} \end{bmatrix} \quad (4)$$

where $(n-1)/2$ is an even number.

From (4) we have its inverse transformation

$$\begin{bmatrix} e^A \\ e^{B_1}\cos\theta_1 \\ e^{B_1}\sin\theta_1 \\ \cdots \\ \exp(B_{\frac{n-1}{2}})\sin(\theta_{\frac{n-1}{2}}) \end{bmatrix} = \begin{bmatrix} 1 & -1 & 1 & \cdots & 1 \\ 1 & \cos\frac{\pi}{n} & \cos\frac{2\pi}{n} & \cdots & \cos\frac{(n-1)\pi}{n} \\ 0 & \sin\frac{\pi}{n} & \sin\frac{2\pi}{n} & \cdots & \sin\frac{(n-1)\pi}{n} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & \sin\frac{(n-1)\pi}{2n} & -\sin\frac{(n-1)\pi}{n} & \cdots & -\sin\frac{(n-1)^2\pi}{2n} \end{bmatrix} \begin{bmatrix} S_1 \\ S_2 \\ S_3 \\ \cdots \\ S_n \end{bmatrix} \quad (5)$$

From (5) we have

$$e^A = \sum_{i=1}^{n} S_i(-1)^{i+1}, \qquad e^{B_j}\cos\theta_j = S_1 + \sum_{i=1}^{n-1} S_{1+i}(-1)^{(j-1)i}\cos\frac{ij\pi}{n}$$

$$e^{B_j}\sin\theta_j = (-1)^{j+1}\sum_{i=1}^{n-1} S_{1+i}(-1)^{(j-1)i}\sin\frac{ij\pi}{n}, \quad (6)$$

In (3) and (6) t_i and S_i have the same formulas. (4) and (5) are the most critical formulas of proofs for FLT. Using (4) and (5) in 1991 Jiang invented that every factor of exponent n has the Fermat equation and proved FLT [1-7] Substituting (4) into (5) we prove (5).

$$\begin{bmatrix} e^{A} \\ e^{B_1}\cos\theta_1 \\ e^{B_1}\sin\theta_1 \\ \cdots \\ \exp(B_{\frac{n-1}{2}})\sin(\theta_{\frac{n-1}{2}}) \end{bmatrix} = \frac{1}{n}\begin{bmatrix} 1 & -1 & 1 & \cdots & 1 \\ 1 & \cos\frac{\pi}{n} & \cos\frac{2\pi}{n} & \cdots & \cos\frac{(n-1)\pi}{n} \\ 0 & \sin\frac{\pi}{n} & \sin\frac{2\pi}{n} & \cdots & \sin\frac{(n-1)\pi}{n} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & \sin\frac{(n-1)\pi}{2n} & -\sin\frac{(n-1)\pi}{n} & \cdots & -\sin\frac{(n-1)^2\pi}{2n} \end{bmatrix} \times$$

$$\begin{bmatrix} 1 & 1 & 0 & \cdots & 0 \\ -1 & \cos\frac{\pi}{n} & \sin\frac{\pi}{n} & \cdots & \sin\frac{(n-1)\pi}{2n} \\ 1 & \cos\frac{2\pi}{n} & \sin\frac{2\pi}{n} & \cdots & -\sin\frac{(n-1)\pi}{n} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 1 & \cos\frac{(n-1)\pi}{n} & \sin\frac{(n-1)\pi}{n} & \cdots & -\sin\frac{(n-1)^2\pi}{2n} \end{bmatrix} \begin{bmatrix} e^{A} \\ 2e^{B_1}\cos\theta_1 \\ 2e^{B_1}\sin\theta_1 \\ \cdots \\ 2\exp(B_{\frac{n-1}{2}})\sin(\theta_{\frac{n-1}{2}}) \end{bmatrix}$$

$$= \frac{1}{n}\begin{bmatrix} n & 0 & 0 & \cdots & 0 \\ 0 & \frac{n}{2} & 0 & \cdots & 0 \\ 0 & 0 & \frac{n}{2} & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & \cdots & \frac{n}{2} \end{bmatrix} \begin{bmatrix} e^{A} \\ 2e^{B_1}\cos\theta_1 \\ 2e^{B_1}\sin\theta_1 \\ \cdots \\ 2\exp(B_{\frac{n-1}{2}})\sin(\theta_{\frac{n-1}{2}}) \end{bmatrix}$$

$$= \begin{bmatrix} e^{A} \\ e^{B_1}\cos\theta_1 \\ e^{B_1}\sin\theta_1 \\ \cdots \\ \exp(B_{\frac{n-1}{2}})\sin(\theta_{\frac{n-1}{2}}) \end{bmatrix}, \qquad (7)$$

where $1+\sum_{j=1}^{n-1}(\cos\frac{j\pi}{n})^2 = \frac{n}{2}$, $\sum_{j=1}^{n-1}(\sin\frac{j\pi}{n})^2 = \frac{n}{2}$.

From (3) we have

$$\exp(A+2\sum_{j=1}^{\frac{n-1}{2}} B_j) = 1. \qquad (8)$$

From (6) we have

$$\exp(A+2\sum_{j=1}^{\frac{n-1}{2}}B_j)=\begin{vmatrix} S_1 & -S_n & \cdots & -S_2 \\ S_2 & S_1 & \cdots & -S_3 \\ \cdots & \cdots & \cdots & \cdots \\ S_n & S_{n-1} & \cdots & S_1 \end{vmatrix}=\begin{vmatrix} S_1 & (S_1)_1 & \cdots & (S_1)_{n-1} \\ S_2 & (S_2)_1 & \cdots & (S_2)_{n-1} \\ \cdots & \cdots & \cdots & \cdots \\ S_n & (S_n)_1 & \cdots & (S_n)_{n-1} \end{vmatrix}, \tag{9}$$

where $(S_i)_j=\dfrac{\partial S_i}{\partial t_j}$ [7].

From (8) and (9) we have the circulant determinant

$$\exp(A+2\sum_{j=1}^{\frac{n-1}{2}}B_j)=\begin{vmatrix} S_1 & -S_n & \cdots & -S_2 \\ S_2 & S_1 & \cdots & -S_3 \\ \cdots & \cdots & \cdots & \vdots \\ S_n & S_{n-1} & \cdots & S_1 \end{vmatrix}=1 \tag{10}$$

If $S_i\neq 0$, where $i=1,2,\cdots,n$, then (10) has infinitely many rational solutions.

Assume $S_1\neq 0$, $S_2\neq 0$, $S_i=0$ where $i=3,4,\cdots,n$. $S_i=0$ are $n-2$ indeterminate equations with $n-1$ variables. From (6) we have

$$e^A=S_1-S_2,\ \ e^{2B_1}=S_1^2+S_2^2+2S_1S_2(-1)^{(j-1)i}\cos\frac{j\pi}{n}. \tag{11}$$

From (10) and (11) we have the Fermat equation

$$\exp(A+2\sum_{i=1}^{\frac{n-1}{2}}B_j)=(S_1-S_2)\prod_{i=1}^{\frac{n-1}{2}}(S_1^2+S_2^2+2S_1S_2(-1)^{j-1}\cos\frac{j\pi}{n})=S_1^n-S_2^n=1 \tag{12}$$

Example[1]. Let $n=15$. From (3) we have

$$A=-(t_1-t_{14})+(t_2-t_{13})-(t_3-t_{12})+(t_4-t_{11})-(t_5-t_{10})+(t_6-t_9)-(t_7-t_8)$$

$$B_1=(t_1-t_{14})\cos\frac{\pi}{15}+(t_2-t_{13})\cos\frac{2\pi}{15}+(t_3-t_{12})\cos\frac{3\pi}{15}+(t_4-t_{11})\cos\frac{4\pi}{15}$$
$$+(t_5-t_{10})\cos\frac{5\pi}{15}+(t_6-t_9)\cos\frac{6\pi}{15}+(t_7-t_8)\cos\frac{7\pi}{15},$$

$$B_2=-(t_1-t_{14})\cos\frac{2\pi}{15}+(t_2-t_{13})\cos\frac{4\pi}{15}-(t_3-t_{12})\cos\frac{6\pi}{15}+(t_4-t_{11})\cos\frac{8\pi}{15}$$
$$-(t_5-t_{10})\cos\frac{10\pi}{15}+(t_6-t_9)\cos\frac{12\pi}{15}-(t_7-t_8)\cos\frac{14\pi}{15},$$

$$B_3=(t_1-t_{14})\cos\frac{3\pi}{15}+(t_2-t_{13})\cos\frac{6\pi}{15}+(t_3-t_{12})\cos\frac{9\pi}{15}+(t_4-t_{11})\cos\frac{12\pi}{15}$$
$$+(t_5-t_{10})\cos\frac{15\pi}{15}+(t_6-t_9)\cos\frac{18\pi}{15}+(t_7-t_8)\cos\frac{21\pi}{15},$$

$$B_4=-(t_1-t_{14})\cos\frac{4\pi}{15}+(t_2-t_{13})\cos\frac{8\pi}{15}-(t_3-t_{12})\cos\frac{12\pi}{15}+(t_4-t_{11})\cos\frac{16\pi}{15}$$
$$-(t_5-t_{10})\cos\frac{20\pi}{15}+(t_6-t_9)\cos\frac{24\pi}{15}-(t_7-t_8)\cos\frac{28\pi}{15},$$

$$B_5 = (t_1 - t_{14})\cos\frac{5\pi}{15} + (t_2 - t_{13})\cos\frac{10\pi}{15} + (t_3 - t_{12})\cos\frac{15\pi}{15} + (t_4 - t_{11})\cos\frac{20\pi}{15}$$

$$(t_5 - t_{10})\cos\frac{25\pi}{15} + (t_6 - t_9)\cos\frac{30\pi}{15} + (t_7 - t_8)\cos\frac{35\pi}{15},$$

$$B_6 = -(t_1 - t_{14})\cos\frac{6\pi}{15} + (t_2 - t_{13})\cos\frac{12\pi}{15} - (t_3 - t_{12})\cos\frac{18\pi}{15} + (t_4 - t_{11})\cos\frac{24\pi}{15}$$

$$-(t_5 - t_{10})\cos\frac{30\pi}{15} + (t_6 - t_9)\cos\frac{36\pi}{15} - (t_7 - t_8)\cos\frac{42\pi}{15},$$

$$B_7 = (t_1 - t_{14})\cos\frac{7\pi}{15} + (t_2 - t_{13})\cos\frac{14\pi}{15} + (t_3 - t_{12})\cos\frac{21\pi}{15} + (t_4 - t_{11})\cos\frac{28\pi}{15}$$

$$+(t_5 - t_{10})\cos\frac{35\pi}{15} + (t_6 - t_9)\cos\frac{42\pi}{15} + (t_7 - t_8)\cos\frac{49\pi}{15},$$

$$A + 2\sum_{j=1}^{7} B_j = 0, \qquad A + 2B_3 + 2B_6 = 5(-t_5 + t_{10}). \tag{13}$$

Form (12) we have the Fermat equation

$$\exp(A + 2\sum_{j=1}^{7} B_j) = S_1^{15} - S_2^{15} = (S_1^5)^3 - (S_2^5)^3 = 1. \tag{14}$$

From (13) we have

$$\exp(A + 2B_3 + 2B_6) = [\exp(-t_5 + t_{10})]^5. \tag{15}$$

From (11) we have

$$\exp(A + 2B_3 + 2B_6) = S_1^5 - S_2^5. \tag{16}$$

From (15) and (16) we have the Fermat equation

$$\exp(A + 2B_3 + 2B_6) = S_1^5 - S_2^5 = [\exp(-t_5 + t_{10})]^5. \tag{17}$$

Euler proved that (14) has no rational solutions for exponent 3[8]. Therefore we prove that (17) has no rational solutions for exponent 5[1].

Theorem 1. Let $n = 3P$,where $P > 3$ is odd prime. From (12) we have the Fermat's equation

$$\exp(A + 2\sum_{j=1}^{3P-1} B_j) = S_1^{3P} - S_2^{3P} = (S_1^P)^3 - (S_2^P)^3 = 1. \tag{18}$$

From (3) we have

$$\exp(A + 2\sum_{j=1}^{\frac{P-1}{2}} B_{3j}) = [\exp(-t_P + t_{2P})]^P. \tag{19}$$

From (11) we have

$$\exp(A + 2\sum_{j=1}^{\frac{P-1}{2}} B_{3j}) = S_1^P - S_2^P. \tag{20}$$

From (19) and (20) we have the Fermat equation

$$\exp(A+2\sum_{j=1}^{\frac{P-1}{2}} B_{3j}) = S_1^P - S_2^P = [\exp(-t_P + t_{2P})]^P . \quad (21)$$

Euler proved that (18) has no rational solutions for exponent 3[8]. Therefore we prove that (21) has no rational solutions for $P > 3$ [1, 3-7].

Theorem 2. We consider the Fermat's equation

$$x^{3P} - y^{3P} = z^{3P} \quad (22)$$

we rewrite (22)

$$(x^P)^3 - (y^P)^3 = (z^P)^3 \quad (23)$$

From (24) we have

$$(x^P - y^P)(x^{2P} + x^P y^P + y^{2P}) = z^{3P} \quad (24)$$

Let $S_1 = \frac{x}{z}$, $S_2 = \frac{y}{z}$. From (20) and (24) we have the Fermat's equation

$$(x^{2P} + x^P y^P + y^{2P} = z^{2P}[\exp(t_P - t_{2P})]^P \quad (25)$$

$$x^P - y^P = [z \times \exp(-t_P + t_{2P})]^P \quad (26)$$

Euler proved that (23) has no integer solutions for exponent 3[8]. Therefore we prove that (26) has no integer solutions for prime exponent P.

Fermat Theorem. It suffices to prove FLT for exponent 4. We rewrite (22)

$$(x^3)^P - (y^3)^P = (z^3)^P \quad (27)$$

Euler proved that（23）has no integer solutions for exponent 3 [8]. Therefore we prove that (27) has no integer solutions for all prime exponent P [1-7].

We consider Fermat equation

$$x^{4P} - y^{4P} = z^{4P} \quad (28)$$

We rewrite (28)

$$(x^P)^4 - ((y^P)^4 = (z^P)^4 \quad (29)$$

$$(x^4)^P - (y^4)^P = (z^4)^P \quad (30)$$

Fermat proved that (29) has no integer solutions for exponent 4 [8]. Therefore we prove that (30) has no integer solutions for all prime exponent P [2,5,7].This is the proof that Fermat thought to have had.

Remark. It suffices to prove FLT for exponent 4. Let $n = 4P$, where P is an odd prime. We have the Fermat's equation for exponent $4P$ and the Fermat's equation for exponent P [2,5,7]. This is the proof that Fermat thought to have had. In complex hyperbolic functions let exponent n be $n = \Pi P$, $n = 2\Pi P$ and $n = 4\Pi P$. Every factor of exponent n has the Fermat's equation [1-7]. In complex trigonometric functions let exponent n be $n = \Pi P$, $n = 2\Pi P$ and $n = 4\Pi P$. Every factor of exponent n has Fermat's equation [1-7]. Using modular elliptic

Curves Wiles and Taylor prove FLT[9,10]. This is not the proof that Fermat thought to have had. The classical theory of automorphic functions,created by Klein and Poincare, was concerned with the study of analytic functions in the unit circle that are invariant under a discrete group of transformation. Automorphic functions are the generalization of trigonometric, hyperbolic,elliptic, and certain other functions of elementary analysis. The complex trigonometric functions and complex hyperbolic functions have a wide application in mathematics and physics.

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