Automorphic Functions And Fermat's Last Theorem（3）(Fermat's Proof of FLT)

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Abstract

In 1637 Fermat wrote: "*It is impossible to separate a cube into two cubes, or a biquadrate into two biquadrates, or in general any power higher than the second into powers of like degree: I have discovered a truly marvelous proof, which this margin is too small to contain.*"

This means: $x^n + y^n = z^n (n > 2)$ has no integer solutions, all different from 0(i.e., it has only the trivial solution, where one of the integers is equal to 0). It has been called Fermat's last theorem (FLT). It suffices to prove FLT for exponent 4 and every prime exponent P. Fermat proved FLT for exponent 4. Euler proved FLT for exponent 3.

In this paper using automorphic functions we prove FLT for exponents $4P$ and P, where P is an odd prime. We rediscover the Fermat proof.The proof of FLT must be direct. But indirect proof of FLT is disbelieving.

In 1974 Jiang found out Euler formula of the cyclotomic real numbers in the cyclotomic fields

$$\exp\left(\sum_{i=1}^{4m-1} t_i J^i\right) = \sum_{i=1}^{4m} S_i J^{i-1}, \qquad (1)$$

where J denotes a $4m$ th root of unity, $J^{4m} = 1$, t_i are the real numbers.

S_i is called the automorphic functions(complex hyperbolic functions) of order $4m$ with $4m-1$ variables [2,5,7].

$$S_i = \frac{1}{4m}\left[e^{A_1} + 2e^H \cos\left(\beta + \frac{(i-1)\pi}{2}\right) + 2\sum_{j=1}^{m-1} e^{B_j} \cos\left(\theta_j + \frac{(i-1)j\pi}{2m}\right)\right]$$

$$+ \frac{(-1)^{(i-1)}}{4m}\left[e^{A_2} + 2\sum_{j=1}^{m-1} e^{D_j} \cos\left(\phi_j - \frac{(i-1)j\pi}{2m}\right)\right] \qquad (2)$$

where $i = 1, \ldots, 4m$;

$$A_1=\sum_{\alpha=1}^{4m-1} t_\alpha,\quad A_2=\sum_{\alpha=1}^{4m-1} t_\alpha(-1)^\alpha,\qquad H=\sum_{\alpha=1}^{2m-1} t_{2\alpha}(-1)^\alpha,\quad \beta=\sum_{\alpha=1}^{2m} t_{2\alpha-1}(-1)^\alpha,$$

$$B_j=\sum_{\alpha=1}^{4m-1} t_\alpha\cos\frac{\alpha j\pi}{2m},\quad \theta_j=-\sum_{\alpha=1}^{4m-1} t_\alpha\sin\frac{\alpha j\pi}{2m},$$

$$D_j=\sum_{\alpha=1}^{4m-1} t_\alpha(-1)^\alpha\cos\frac{\alpha j\pi}{2m},\quad \phi_j=\sum_{\alpha=1}^{4m-1} t_\alpha(-1)^\alpha\sin\frac{\alpha j\pi}{2m},$$

$$A_1+A_2+2H+2\sum_{j=1}^{m-1}(B_j+D_j)=0. \qquad (3)$$

From (2) we have its inverse transformation[5,7]

$$e^{A_1}=\sum_{i=1}^{4m} S_i,\quad e^{A_2}=\sum_{i=1}^{4m} S_i(-1)^{1+i}$$

$$e^H\cos\beta=\sum_{i=1}^{2m} S_{2i-1}(-1)^{1+i},\quad e^H\sin\beta=\sum_{i=1}^{2m} S_{2i}(-1)^i,$$

$$e^{B_j}\cos\theta_j=S_1+\sum_{i=1}^{4m-1} S_{1+i}\cos\frac{ij\pi}{2m},\quad e^{B_j}\sin\theta_j=-\sum_{i=1}^{4m-1} S_{1+i}\sin\frac{ij\pi}{2m},$$

$$e^{D_j}\cos\phi_j=S_1+\sum_{i=1}^{4m-1} S_{1+i}(-1)^i\cos\frac{ij\pi}{2m},\quad e^{D_j}\sin\phi_j=\sum_{i=1}^{4m-1} S_{1+i}(-1)^i\sin\frac{ij\pi}{2m}. \qquad (4)$$

(3) and (4) have the same form.

From (3) we have

$$\exp\left[A_1+A_2+2H+2\sum_{j=1}^{m-1}(B_j+D_j)\right]=1 \qquad (5)$$

From (4) we have

$$\exp\left[A_1+A_2+2H+2\sum_{j=1}^{m-1}(B_j+D_j)\right]=\begin{vmatrix} S_1 & S_{4m} & \cdots & S_2\\ S_2 & S_1 & \cdots & S_3\\ \cdots & \cdots & \cdots & \cdots\\ S_{4m} & S_{4m-1} & \cdots & S_1\end{vmatrix}$$

$$=\begin{vmatrix} S_1 & (S_1)_1 & \cdots & (S_1)_{4m-1}\\ S_2 & (S_2)_1 & \cdots & (S_2)_{4m-1}\\ \cdots & \cdots & \cdots & \cdots\\ S_{4m} & (S_{4m})_1 & \cdots & (S_{4m})_{4m-1}\end{vmatrix} \qquad (6)$$

where

$$(S_i)_j = \frac{\partial S_i}{\partial t_j} [7]$$

From (5) and (6) we have circulant determinant

$$\exp\left[A_1 + A_2 + 2H + 2\sum_{j=1}^{m-1} (B_j + D_j)\right] = \begin{vmatrix} S_1 & S_{4m} & \cdots & S_2 \\ S_2 & S_1 & \cdots & S_3 \\ \cdots & \cdots & \cdots & \cdots \\ S_{4m} & S_{4m-1} & \cdots & S_1 \end{vmatrix} = 1 \qquad (7)$$

Assume $S_1 \neq 0, S_2 \neq 0, S_i = 0$, where $i = 3,...,4m.$ $S_i = 0$ are $(4m-2)$ indeterminate equations with $(4m-1)$ variables. From (4) we have

$$e^{A_1} = S_1 + S_2, \quad e^{A_2} = S_1 - S_2, \quad e^{2H} = S_1^2 + S_2^2$$

$$e^{2B_j} = S_1^2 + S_2^2 + 2S_1S_2 \cos\frac{j\pi}{2m}, \quad e^{2D_j} = S_1^2 + S_2^2 - 2S_1S_2 \cos\frac{j\pi}{2m} \qquad (8)$$

Example [2]. Let $4m = 12$. From (3) we have

$A_1 = (t_1 + t_{11}) + (t_2 + t_{10}) + (t_3 + t_9) + (t_4 + t_8) + (t_5 + t_7) + t_6$,

$A_2 = -(t_1 + t_{11}) + (t_2 + t_{10}) - (t_3 + t_9) + (t_4 + t_8) - (t_5 + t_7) + t_6$,

$H = -(t_2 + t_{10}) + (t_4 + t_8) - t_6$,

$B_1 = (t_1 + t_{11})\cos\frac{\pi}{6} + (t_2 + t_{10})\cos\frac{2\pi}{6} + (t_3 + t_9)\cos\frac{3\pi}{6} + (t_4 + t_8)\cos\frac{4\pi}{6} + (t_5 + t_7)\cos\frac{5\pi}{6} - t_6$,

$B_2 = (t_1 + t_{11})\cos\frac{2\pi}{6} + (t_2 + t_{10})\cos\frac{4\pi}{6} + (t_3 + t_9)\cos\frac{6\pi}{6} + (t_4 + t_8)\cos\frac{8\pi}{6} + (t_5 + t_7)\cos\frac{10\pi}{6} + t_6$,

$D_1 = -(t_1 + t_{11})\cos\frac{\pi}{6} + (t_2 + t_{10})\cos\frac{2\pi}{6} - (t_3 + t_9)\cos\frac{3\pi}{6} + (t_4 + t_8)\cos\frac{4\pi}{6} - (t_5 + t_7)\cos\frac{5\pi}{6} - t_6$,

$D_2 = -(t_1 + t_{11})\cos\frac{2\pi}{6} + (t_2 + t_{10})\cos\frac{4\pi}{6} - (t_3 + t_9)\cos\frac{6\pi}{6} + (t_4 + t_8)\cos\frac{8\pi}{6} - (t_5 + t_7)\cos\frac{10\pi}{6} + t_6$,

$$A_1 + A_2 + 2(H + B_1 + B_2 + D_1 + D_2) = 0, \quad A_2 + 2B_2 = 3(-t_3 + t_6 - t_9). \qquad (9)$$

From (8) and (9) we have

$$\exp[A_1 + A_2 + 2(H + B_1 + B_2 + D_1 + D_2)] = S_1^{12} - S_2^{12} = (S_1^3)^4 - (S_2^3)^4 = 1. \qquad (10)$$

From (9) we have

$$\exp(A_2 + 2B_2) = [\exp(-t_3 + t_6 - t_9)]^3. \qquad (11)$$

From (8) we have

$$\exp(A_2 + 2B_2) = (S_1 - S_2)(S_1^2 + S_2^2 + S_1S_2) = S_1^3 - S_2^3. \qquad (12)$$

From (11) and (12) we have Fermat's equation

$$\exp(A_2+2B_2)=S_1^3-S_2^3=[\exp(-t_3+t_6-t_9)]^3. \qquad (13)$$

Fermat proved that (10) has no rational solutions for exponent 4 [8].

Therefore we prove we prove that (13) has no rational solutions for extponent 3. [2]

Theorem . Let $4m=4P$, where P is an odd prime, $(P-1)/2$ is an even number.

From (3) and (8) we have

$$\exp[A_1+A_2+2H+2\sum_{j=1}^{P-1}(B_j+D_j)]=S_1^{4P}-S_2^{4P}=(S_1^P)^4-(S_2^P)^4=1. \qquad (14)$$

From (3) we have

$$\exp[A_2+2\sum_{j=1}^{\frac{P-1}{4}}(B_{4j-2}+D_{4j})]=[\exp(-t_P+t_{2P}-t_{3P})]^P. \qquad (15)$$

From (8) we have

$$\exp[A_2+2\sum_{j=1}^{\frac{P-1}{4}}(B_{4j-2}+D_{4j})]=S_1^P-S_2^P. \qquad (16)$$

From (15) and (16) we have Fermat's equation

$$\exp[A_2+2\sum_{j=1}^{\frac{P-1}{4}}(B_{4j-2}+D_{4j})]=S_1^P-S_2^P=[\exp(-t_P+t_{2P}-t_{3P})]^P. \qquad (17)$$

Fermat proved that (14) has no rutional solutions for exponent 4 [8]. Therefor we prove that (17) has no vational solutions for prime exponent P.

Remark. Mathematicians said Fermat could not possibly had a proof, because they do not understand FLT.In complex hyperbolic functions let exponent n be $n=\Pi P$, $n=2\Pi P$ and $n=4\Pi P$. Every factor of exponent n has Fermat's equation [1-7]. Using modular elliptic curves Wiles and Taylor prove FLT [9,10]. This is not the proof that Fermat thought to have had. The classical theory of automorphic functions,created by Klein and Poincare, was concerned with the study of analytic functions in the unit circle that are invariant under a discrete group of transformation. Automorphic functions are the generalization of trigonometric, hyperbolic,elliptic, and certain other functions of elementary analysis. The complex trigonometric functions and complex hyperbolic functions have a wide application in mathematics and physics.

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