Evolutionary sequence of spacetime/intrinsic spacetime and associated sequence of geometries in a metric force field. Part I.

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Having isolated a four-world picture in which four symmetrical universes in different spacetime domains coexist and in which an isolated two-dimensional intrinsic spacetime underlies the four-dimensional spacetime in each universe, and having shown that the special theory of relativity rests on a four-world background elsewhere, we review the geometry of spacetime/intrinsic spacetime in a long-range metric force field within the four-world picture in the four parts of this paper. We show within an elaborate programme that the four-dimensional metric spacetime and its underlying two-dimensional intrinsic metric spacetime undergo two stages of evolution in the sequence of absolute spacetime/absolute intrinsic spacetime $\rightarrow$ proper spacetime/proper intrinsic spacetime $\rightarrow$ relativistic spacetime/relativistic intrinsic spacetime in all finite neighborhood of a long-range metric force field and that these are supported by a sequence of spacetime/intrinic spacetime geometries. The programme takes off in this first paper by isolating two classes of three-dimensional Riemannian metric space namely, the conventional three-dimensional Riemannian metric space and a new 'three-dimensional' absolute intrinsic Riemannian metric space.

1 Introduction

There is perhaps no better place to start a fundamental theory of physics than a discourse of the underlying space(s) and geometry(ies). We have started this by isolating the proper (or classical) four-dimensional spacetimes of classical mechanics (CM) and their underlying flat two-dimensional proper (or classical) intrinsic spacetimes of intrinsic classical mechanics (ϕCM) of co-existing four symmetrical universes, referred to as positive (or our) universe, negative universe, positive time-universe and negative time-universe in the previous papers [1-4]. The four universes exhibit perfect symmetry of natural laws and perfect symmetry of state among themselves. Lorentz transformation/intrinsic Lorentz transformation (LT/ϕLT) and their inverses were derived with a new set of affine spacetime/intrinsic affine spacetime diagrams within the pertinent four-world picture.

The immutability of Lorentz invariance is shown to be a consequence of perfect symmetry of state among the four universes in section 2 of [4], where perfect symmetry of state implies that the four members of every quartet of symmetry-partner particles or objects in the four universes have perfectly identical magnitudes of masses, perfectly identical shapes and perfectly identical sizes and that they are involved in perfectly identical relative motions at all times. The flat two-dimensional proper (or classical) intrinsic metric spacetime of intrinsic classical mechanics (ϕCM) that underlies the flat four-dimensional proper (or classical) metric spacetime of classical mechanics (CM) in each universe, introduced as *ansatz* in sub-section 4.3 of [1], were derived formally in sub-section 1.2 of [4]. There is essentially no outstanding issue in [1-4] that could prevent the description of the isolation of the four-world picture in those papers as having attained a close-form.

Now, as discussed in section 5 of [3], the special theory of relativity/intrinsic special theory of relativity (SR/ϕSR) operate on extended flat proper (or classical) metric spacetimes/underlying extended flat proper (or classical) intrinsic metric spacetimes of the four universes in the absence of gravity. However, since SR/ϕSR involve affine spacetime/intrinsic affine spacetime (or affine spacetime/intrinsic affine spacetime geometry) in each universe, SR/ϕSR cannot alter the extended flat four-dimensional proper metric spacetime/extended flat proper intrinsic metric spacetime on which they operate in the absence of gravity.

It is the presence of a long-range metric force field, such as the gravitational field, that can change the extended flat proper metric spacetimes and its underlying flat two-dimensional proper intrinsic metric spacetimes to four-dimensional relativistic metric spacetimes and its underlying flat two-dimensional relativistic intrinsic metric spacetimes in all finite neighborhoods of the sources of symmetry-partner long-range metric force fields in the four universes. The two-dimensional intrinsic metric spacetime is unknown and the relativistic four-dimensional spacetime that evolves from the flat proper (or classical) spacetime is prescribed to be curved in a gravitational field within the existing one-world picture in the context of the general theory of relativity (GR) [5, see pp. 111-149].

The next natural step in the further development of the spaces and geometrical foundation for a fundamental theory of physics in addition to the affine spacetime/affine intrinsic spacetime geometry for SR/ϕSR in the four-world picture developed in [1-4], is to develop the counterpart metric spacetime/intrinsic metric spacetime geometry, which will convert extended flat proper (or classical) metric spacetimes and their underlying flat proper intrinsic metric spacetimes to relativistic metric spacetimes and their underlying relativistic intrinsic metric spacetimes in all finite neighborhoods of symmetry-partner long-range metric force fields in the four universes.

More often than not, there arises the need to adapt a subject from its sophisticated form in pure mathematics to an applicable form in an applied field. The reason being that, guided by logical and mathematical consistency only, a pure mathematical subject can be pursued to any level of generalization and sophistication. In application, on the other hand, the requirement for mathematics to describe physical reality, that is, to model physical situations and concepts and to satisfy physical constraints, often leads to a lowering of the levels of sophistication and generalization of a mathematical subject in its applicable form.

It is therefore the responsibility of a physicist to marry the underlying concepts and constraints of a physical theory to the conceptual foundation of a mathematical subject to be applied and, in the process, as is often possible, evolve the applicable form of the mathematical subject. Sometimes the applicable form, having lost all sophistication in the process of putting on a physical or an application face, bears only a crude resemblance to the original subject. However whatever beauty is lost in mathematics is usually gained in terms of ease of interpretation and transparency of connection to reality of the resulting physical theory.

One subject of pure mathematics that is of direct relevance to fundamental physics is Riemann geometry. Riemann geometry evolved from elementary differential geometry of surfaces in the Euclidean space by the usual process of mathematical abstraction. Although Albert Einstein applied Riemann geometry to the problem of gravity, the link of the subject to physics was not formally established prior to this. A formal link of Riemann geometry to physics would entail a marriage of the relevant concepts and principles of physics to the concepts and principles of Riemann geometry and, according to the preceding paragraph, such an exercise should yield the form of Riemann geometry to be applied in physics.

The concepts of absolute spacetime, absolutism and observers in physics are incorporated into Riemann geometry and a ‘two-dimensional’ absolute intrinsic Riemann geometry on certain curved ‘two-dimensional’ absolute intrinsic metric spacetime (which should support absolute intrinsic metric theory of physics), is isolated at the first stage of evolution of spacetime/intrinsic spacetime in a long-range metric force field. Then the concepts of relative spacetime, relativity and observers in physics are brought into play in developing a local Lorentzian spacetime/intrinsic spacetime geometry on a curved proper intrinsic spacetime, within the four-world picture, (which should support the theory of relativity/intrinsic theory of relativity associated with the presence of a long-range metric force field/long-range intrinsic metric force field) at the second (and final) stage of evolution of spacetime/intrinsic spacetime in a long-range metric force field. The long-range metric force field of gravity shall ultimately be linked to the geometries developed elsewhere in making connection to physics. Division of this paper into a number of parts is inevitable.

2 On the incorporation of the time dimension into Riemann geometry

Friederich Bernhard Riemann in his famous lecture of June 10, 1854, at the Götingen University entitled, “On the Hypotheses Which Lie at the Foundation of Geometry”, as translated in [6], evolved the geometry that is now named after him. With a prophetic vision, Riemann had raised issues during this lecture that would have far-reaching consequences in physics. For example, he wrote in the paper he presented at the lecture, “... the basis of the metric relation of a manifold must be sought outside the manifold in the binding forces that act upon it.”

It would be a disservice to describe Riemann lesser than a precursor of the various metric theories of physics, with the general theory of relativity being the leading member. However the time dimension and the significant role it plays in linking Riemann geometry to physics, as developed by Albert Einstein [5, see pp. 111-149], was unknown to Riemann. Riemann simply generalized Gauss’s theory of surfaces in the Euclidean 3-space to general curved $n-$dimensional spaces (without time dimension), where points are characterized by n coordinates as follows:

$$u^\nu = f^\nu(x^1, x^2, x^3, ..., x^n);\ \nu = 1, 2, 3, ..., n \qquad (1)$$

The distance element ds between two indefinitely close points in this general n-dimensional curved space is given as follows:

$$ds^2 = \sum_{\mu,\nu=1}^{n} g_{\mu\nu}(x^1, x^2, x^3, ..., x^n)dx^\mu dx^\nu \qquad (2)$$

where the metric tensor $g_{\mu\nu}$ is defined as,

$$g_{\mu\nu}(x^1, x^2, x^3, ..., x^n) = \sum_{\alpha=1}^{n} \frac{\partial f^\alpha}{\partial x^\mu}\frac{\partial f^\alpha}{\partial x^\nu} = \sum_{\alpha=1}^{n} \frac{\partial u^\alpha}{\partial x^\mu}\frac{\partial u^\alpha}{\partial x^\nu} \qquad (3)$$

Albert Einstein introduced the time dimension, $ct \equiv x^0$, into Riemann geometry in a direct manner somewhat. Having successfully added ct to the three dimensions x^1, x^2 and x^3 of the Euclidean 3-space to have the flat four-dimensional spacetime, (the Minkowski space), in the special theory of relativity [5, see pp. 37-65], he considered the four-dimensional spacetime to be curved, thereby yielding a four-dimensional Riemannian spacetime manifold in a gravitational field in the general theory of relativity [5, see pp. 111-149] and [7, see chap. 5].

Albert Einstein applied Riemann geometry in an unaltered form to the proposed curved four-dimensional spacetime in a gravitational field [5, see pp. 111-149;7, see chap. 5]. The only significant difference in Riemann geometry without time dimension (that is, manifolds of type M^p) and Riemann geometry with time dimension (that is, manifolds of the type M^{p+q}) is in the structure of the metric tensor. While the metric tensor is elliptical with signature $(++++)$ in a four-dimensional Riemann space with a sub-Riemannian metric tensor (without time dimension), it is hyperbolic with signature $(+---)$ or $(---+)$ on a curved four-dimensional spactime with a sub-Riemannian metric tensor. As a matter of fact, it is at the point of solving Einstein's field equations, that K. Schwarzschild introduced the hyperbolic metric tensor, so that the metric tensor obtained could reduce to the Lorentzian metric tensor at infinity [7, see pp. 185-186].

The important point to note in the foregoing is that Albert Einstein introduced the time dimension into Riemann geometry by allowing the time dimension and the three dimensions of space to be curved at once (or simultaneously) to form a curved four-dimensional spacetime continuum. He then applied Riemann geometry (for four-dimensional Riemann space without time dimension) in an unaltered form to the curved four-dimensional spacetime continuum thus obtained. This approach of introducing the time dimension into Riemann geometry by Albert Einstein has been referred to as direct approach earlier.

However quite apart from the direct approach of Einstein, there is another approach, (which shall be referred to as indirect approach), towards introducing the time dimension into Riemann geometry, which leads to a kind of Riemannian spacetime geometry that is different from the conventional Riemannian spacetime geometry of Einstein's direct approach. The first two parts of this paper shall be devoted to the development of the new kind of Riemannian spacetime geometry.

3 Isolating two classes of three-dimensional Riemannian metric spaces

Let us start by considering the proper (or classical) Euclidean 3-space, denoted by Σ' in [1-4], but which shall be denoted by E'^3 in the three parts of this paper, with dimensions x'^1, x'^2 and x'^3 and the absolute time 'dimension' to be denoted by $\hat{c}\hat{t} \equiv \hat{x}^0$. The proper Euclidean 3-space and the absolute time 'dimension' constitutes the Galileo space $(E'^3; \hat{c}\hat{t})$. Let us assume that due to a yet unspecified phenomenon, the proper Euclidean 3-space becomes a curved space to be denoted by M^3 within a region of the universal 3-space, while the absolute time coordinate remains not curved.

Let us give a graphical illustration of the Galileo space $(E'^3; \hat{c}\hat{t})$ and the curved space $(M^3; \hat{c}\hat{t})$. In doing this, we shall consider E'^3 as an hyper-surface, $\hat{c}\hat{t} =$ const, and represent it by a plane surface along the horizontal and the absolute time 'dimension' $\hat{c}\hat{t}$ by a vertical normal line to the hyper-surface, as illustrated in Fig. 1a.

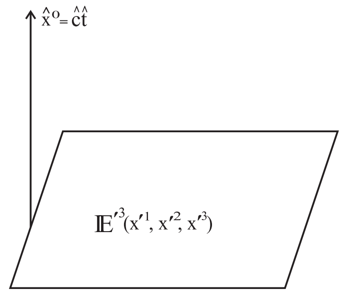

Fig. 1: (a) Graphical representation of the Galileo space.

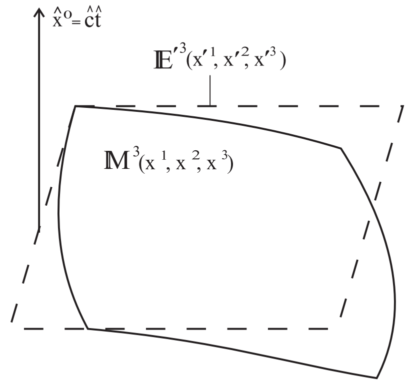

Fig. 1: (b) The Euclidean 3-space E'^3 of the Galileo space evolves into a curved 3-dimensional (Riemannian) metric space M^3, such that none of the coordinates of M^3 spans the absolute time coordinate $\hat{c}\hat{t}$ along the vertical.

In the case of graphical representation of $(M^3; \hat{c}\hat{t})$, there are two possibilities. The first is obtained by letting the hyper-surface E'^3 along the horizontal in Fig. 1a to become a curved hyper-surface M^3 still on the horizontal plane, so that none of the coordinates x^1, x^2 and x^3 of M^3 spans the absolute time coordinate $\hat{c}\hat{t}$ along the vertical, as illustrated in Fig. 1b.

The coordinates of the curved space M^3 span the coordinates of the proper (or classical) Euclidean 3-space E'^3 only. Actually the proper Euclidean 3-space E'^3 has evolved into the curved space M^3 within the region of 3-space being considered. Hence the proper Euclidean space does not exist along with M^3 within the region. Nevertheless the curved metric space M^3 is embedded in the global proper Euclidean 3-space E'^3 and the coordinates x'^i of E'^3 serve as cartesian coordinates for points on M^3, while x^i are the coordinates of M^3.

The second possibility (or case) is obtained by allowing the coordinates x^1, x^2 and x^3 of the curved space M^3 to span the absolute time coordinate $\hat{c}\hat{t}$ along the vertical solely, so that M^3 is curved towards $\hat{c}\hat{t}$ as illustrated in Fig. 1c. Intermediate cases in which some coordinates of M^3 span the absolute time coordinate, while others do not, are actually possible. However such cases must be considered as generic forms of the second case illustrated in Fig. 1c.

Since a vacuum cannot be created along the horizontal, the curved space M^3 will project a new hyper-surface − a new Euclidean 3-space − to be denoted by E^3, with coordinates x^1, x^2 and x^3 along the horizontal, as shown in Fig. 1c. In other words, the curved space M^3 will be underlied by its projective Euclidean 3-space E^3 in this second case. The concept of underlying projective space does not arise in the first case, (Fig. 1b), since the curved hyper-surface M^3 lies along the horizontal in that case. We shall now investigate the two cases of curved space formed from the Galileo space (of Fig. 1a) described above in order to show the essential difference that may exist between them.

Case I: Conventional Riemannian metric 3-space

The first case of curved metric space formed from the Galileo space, in which each coordinate of the curved space M^3 spans one, two or all the coordinates of the proper Euclidean 3-space E'^3 that evolved into it and none spans the absolute time coordinate along the vertical, illustrated in Fig. 1b, is a conventional Riemannian metric 3-space. It must be noted that two metric spaces namely, the proper Euclidean 3-space E'^3, (with Euclidean metric tensor), and the curved metric space M^3, (with a Riemannian metric tensor), do not co-exist in Fig. 1b, since E'^3 has evolved into M^3 within the given region of the proper Euclidean 3-space.

Eqs. (1) through (3) in conventional Riemann geometry are applicable in this case. We must simply let $n = 3$ in them to have as follows:

$$x'^{\nu} = f^{\nu}(x^1, x^2, x^3);\ \nu = 1, 2, 3. \tag{4}$$

where x'^{ν} are the coordinates of the three-dimensional proper (or classical) space E'^3 of the Galileo space that evolved into M^3, but which still serve as the cartesian coordinates for points on the curved space M^3, and x^{ν} are the coordinates of M^3. The distance element is given on M^3 as follows:

$$ds^2 = \sum_{\mu,\nu=1}^{3} g_{\mu\nu}(x^1, x^2, x^3)dx^{\mu}dx^{\nu} \tag{5}$$

where

$$g_{\mu\nu}(x^1, x^2, x^3) = \sum_{\alpha=1}^{3} \frac{\partial f^{\alpha}}{\partial x^{\mu}} \frac{\partial f^{\alpha}}{\partial x^{\nu}} = \sum_{\alpha=1}^{3} \frac{\partial u^{\alpha}}{\partial x^{\mu}} \frac{\partial u^{\alpha}}{\partial x^{\nu}} \tag{6}$$

Since we have identified the first case of curved space that evolved from the Galileo space, illustrated in Fig. 1b, as a conventional Riemannian metric space of type M^p; $p = 3$, there is nothing new to know about it. We shall therefore proceed to investigate the second case illustrated in Fig. 1c.

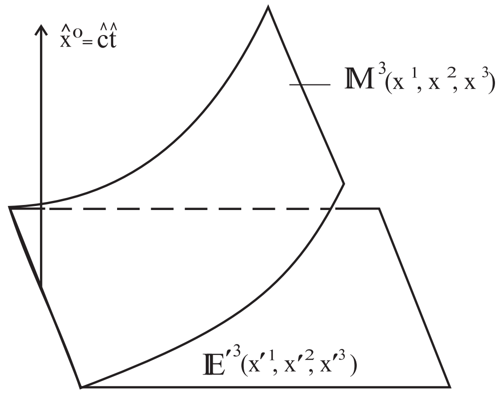

Fig. 1: (c) The proper (or classical) Euclidean 3-space E'^3 of the Galileo space evolves into a curved 3-dimensional (Riemannian) metric space M^3, such that the coordinates of M^3 span the absolute time coordinate $\hat{c}\hat{t}$ the vertical solely. The curved space M^3 (as curved hyper-surface) projects a new Euclidean 3-space E^3, with coordinates x^1, x^2 and x^3 underneath it, lying as a flat hyper-surface along the horizontal.

Case II: A new kind of Riemannian metric 3-space

The curved space M^3 in Fig. 1c has evolved from the proper (or classical) Euclidean 3-space E'^3. Hence Eqs. (4) through (6) of conventional Riemann geometry are equally valid for the curved space M^3 in Fig. 1c. There is however a necessary further step to be taken in this second case, which consists in obtaining the projection of the Riemannian metric space M^3 into the horizontal to obtain the underlying new Euclidean 3-space E^3 in Fig. 1c.

The second case of curved metric space M^3 that evolves from the Galileo space, illustrated in Fig. 1c, shall undergo extensive modification with further development in this paper. We shall be led in a consistent manner to the identification of certain curved 'one-dimensional' absolute intrinsic space for it, instead of the physical (or relative) 3-space in Fig. 1c.

4 Isolating absolute intrinsic Riemannian metric spaces and absolute intrinsic Riemann geometry

Now two observers located at two distinct positions P_1 and P_2 in the Riemannian metric space M^3 in Fig. 1b or 1c are located at positions of different Riemannian curvatures K_1 and K_2 respectively, where K_1 and K_2 are determined relative to the reference Euclidean space E'^3. These observers will therefore observe different curvatures K_{31} and K_{32} respectively, of a third position P_3 on the curved space M^3. Consequently these observers will observe different metric tensors and construct different Riemann geometries for the third position.

Since observers within the region of space being considered are necessarily located on the curved space M^3 in the first case (Fig. 1b), there is no way of resolving the problem of the non-uniqueness of Riemann geometry derived by ob-

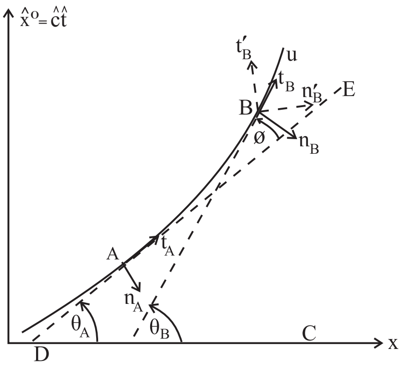

Fig. 2: A one-dimensional metric space curving onto the absolute time 'dimension' along the vertical, projects a straight line one-dimensional metric space along the horizontal.

servers located at different positions in a Riemannian metric space discussed in the preceding paragraph in the first case. On the other hand, Riemann geometry of the curved space M^3 can be formulated uniquely with respect to observers located at different positions in the underlying Euclidean space E^3 in the second case, (Fig. 1c), as explained below.

Let us consider a plane curve u, which is curved onto the absolute time 'dimension' and underneath which lies a straight line coordinate x along the horizontal, (which the curve u projects along the horizontal), as illustrated in Fig. 2. The curved u and its projection x shall be taken to be one-dimensional metric spaces.

The curvatures k_A and k_B at points A and B respectively of the one-dimensional curved metric space u are given by definition [5, see chap. 1], as follows:

$$\frac{d\theta}{du}|_A = \frac{dt_A}{du} = k_A \qquad (7a)$$

and

$$\frac{d\theta}{du}|_B = \frac{dt_B}{du} = k_B \qquad (7b)$$

The angle θ is measured relative to the one-dimensional straight line metric space x along the horizontal in Fig. 2. It can thus be said that the curvatures k_A and k_B at points A and B respectively of the curve u are valid relative to 'one-dimensional observer' at point C that can be anywhere in the coordinate x along the horizontal.

Now let us consider the curvature of u at point B relative to a 'one-dimensional observer' at point A on the curve u. The projective one-dimensional metric space x along the horizontal on which the 'one-dimensional observer' at point C is located must be replaced by the tangent DE on which the 'one-dimensional observer' at point A is located. The curvature of u must be defined in terms of a different angle ϕ measured relative to the line DE with respect to the 'one-dimensional observer' at A. Hence the curvature k_{BA} at point B relative to point A of the curve u is given as follows:

$$\frac{d\phi}{du}|_B = \frac{dt'_B}{du} = k_{BA} \qquad (8)$$

where t'_B and n'_B are the unit tangent vector and unit normal vector to the curve u with respect to the 'one-dimensional observer' at A, which correspond to t_B and n_B respectively (shown in Fig. 2) with respect to 'one-dimensional observer' located anywhere along x.

We find from the above that the curvature at a given point on a plane curve u on the vertical $x\hat{c}\hat{t}-$plane depends on the position of the 'one-dimensional observer' located along the curve, but is the same relative to 'one-dimensional observers' located at different positions in the one-dimensional straight line metric space x, which the one-dimensional curved metric space u projects along the horizontal. The curvatures k_A and k_B of equations (7a) and (7b) are valid relative to a 'one-dimensional observer' located at point C that may be anywhere along the one-dimensional space x. Hence the position C of such 'observer' does not appear as a label on k_A and k_B. On the other hand, the position A of the 'observer' located along the curve u appears as a label on the curvature k_{BA} at position B of the curve u in Eq. (8).

Now the curve u in Fig. 2 is a one-dimensional Riemann metric space M^1, as mentioned above. It is a member of the second case of Riemannian metric spaces illustrated in Fig. 1c, which can be generated from the Galileo space of Fig. 1a. Fig. 2 and the discussion on it above can be generalized to the case of the 3-dimensional metric space M^3, (with dimensions u^1, u^2 and u^3), which is curved towards the absolute time 'dimensions' $\hat{c}\hat{t}$, and which is curved relative to its projective 3-dimensional Euclidean space E^3, (with dimensions x^1, x^2 and x^3), (also a metric space), in Fig. 1c, re-illustrated as Fig. 3.

One observes that there are two co-existing metric spaces of different metric tensors namely, the curved space M^3 with Riemannian metric tensor and the underlying flat space E^3 with Euclidean metric tensor in Fig. 3. However only singular metric spaces are known in Riemann geometry. This paradox raised by Fig. 3 shall be resolved with further development of this paper. The first class of Riemannian metric spaces M^3 illustrated in Fig. 1b, which evolves from the proper (or classical) Euclidean 3-space, does not raise the paradox raised by Fig. 1c or Fig. 3 mentioned above, since the curved hypersurface M^3 lies along the horizontal, thereby precluding any projective space in Fig. 1b of conventional Riemannian metric space. There is no duplication of metric spaces in the first case (or in conventional Riemann geometry).

The metric manifold M^3 in Fig. 3 is locally Euclidean at every point of it relative to an observer at that point. On the other hand, M^3 possesses a Riemannian curvature K_{AC} and Riemannian metric tensor $g_{ik}^{(AC)}$ at point A on it relative to the observer at point C in E^3, and Riemannian curvature

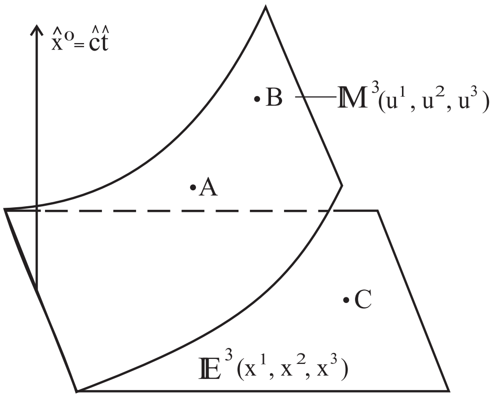

Fig. 3: A 3-dimensional Riemannian metric space curving onto the absolute time 'dimension' along the vertical (as a curved hyper-surface) and its underlying projective Euclidean 3-space (as a flat hyper-surface) along the horizontal.

K_{BA} and Riemannian metric tensor $g_{ik}^{(BA)}$ at point B relative to the observer at point A on it (on M^3) in Fig. 3. Thus the Riemannian curvature and metric tensor at a given point on a curved metric space M^3, underneath which lies its projective Euclidean space E^3, illustrated in Fig. 3, depends on the location of the observer in general.

Now the Riemannian curvature K_{BC}, and hence the metric tensor $g_{ik}^{(BC)}$ at point B on M^3 are the same for different positions C, (or for different 3-observers or frames), in the underlying Euclidean space E^3. Thus the label C of the position of the 3-observer in E^3 is redundant and does not have to appear in the curvature and metric tensor at any point in the curved space M^3. In other words, $g_{ik}^{(B)}$ and $g_{ik}^{(A)}$ are unique or invariant metric tensors at points B and A respectively on M^3 with respect to 3-observers located at different positions in E^3.

On the other hand, the curvature and metric tensor at a given point on the curved space M^3 relative to an observer at another point on M^3 depends on the position of the observer. Thus the curvature K_{BA} and the metric tensor $g_{ik}^{(BA)}$ at point B on M^3 relative to an observer at position A on M^3 in Fig. 3, contains the position A of the observer as a label.

We shall sometimes refer to the 3-observer in the underlying projective Euclidean space E^3 as Euclidean observers, while observers at different positions on the curved (or Riemann) space M^3 shall be referred to as Riemannian observers. The foregoing two paragraphs simply state that the metric tensor at any given point on the curved metric manifold M^3 is the same with respect to all Euclidean observers (or in all frames) in the underlying Euclidean space E^3, but depends on the location (or the local frame) of a Riemannian observer. We shall be concerned with the Riemann geometry of the curved metric manifold M^3 in the second case illustrated in Fig. 1c or Fig. 3, relative to Euclidean observers mainly.

As mentioned previously, the proper Riemannian 3-observer at position B observes Euclidean metric tensor δ_{ik} locally on the curved manifold M^3 at his position. Hence this observer writes the Gaussian line element locally at his position as follows:

$$ds^2 = \hat{c}^2 d\hat{t}^2 - \sum_{i,k=1}^{3} \delta_{ik} du^i du^k \tag{9}$$

On the other hand, the curved manifold M^3 possesses a unique Riemannian metric tensor $g_{ik}^{(B)}$ at point B relative to all observers in the underlying Euclidean 3-space E^3, (or relative to all Euclidean observers). Hence Euclidean observers will formulate Riemann geometry on M^3 in Fig. 1c or Fig. 3 in the context of conventional Riemann geometry by writing general coordinate transformations like system (4), which shall be re-written as follows because of a certain point to be made:

$$x'^{\nu} = f^{\nu}(u^1, u^2, u^3);\ \nu = 1, 2, 3. \tag{10a}$$

Hence

$$g_{ik}(u^1, u^2, u^3) = \sum_{\alpha=1}^{3} \frac{\partial f^\alpha}{\partial u^\mu}\frac{\partial f^\alpha}{\partial u^\nu} = \sum_{\alpha=1}^{3} \frac{\partial x'^\alpha}{\partial u^\mu}\frac{\partial x'^\alpha}{\partial u^\nu} \tag{10b}$$

The point to note is that x'^{ν} are the coordinates of the original proper (or classical) Euclidean 3-space E'^3 in Fig. 1a that evolved into the curved space M^3 in Fig. 1c or Fig. 3, while u^ν are the coordinates of M^3. The Euclidean observers in E'^3 will then write a unique Gaussian line element at point B on M^3 as follows:

$$ds^2 = \hat{c}^2 d\hat{t}^2 - \sum_{i,k=1}^{3} g_{ik}^{(B)}(u^1, u^2, u^3) du^i du^k; \tag{11}$$

$(\text{w.r.t. } 3-\text{observers in } E^3)$.

The Euclidean 3-observers will construct Riemann geometry in the context of conventional Riemann geometry, (equations (10b) and (10)), uniquely on the curved manifold M^3 in terms of coordinate u^1, u^2 and u^3 of M^3. They will also derive the projection of M^3 into the horizontal to form the Euclidean space E^3, as shall be done shortly.

Now let us change local coordinate set from (u^1, u^2, u^3) of one local frame to another local coordinate set (v^1, v^2, v^3) of another local frame at the same position B on the curved manifold M^3, (in Fig. 3), in Eq. (11) to have the following

$$d\tilde{s}^2 = \hat{c}^2 d\hat{t}^2 - \sum_{i,k=1}^{3} \tilde{g}_{ik}^{(B)}(v^1, v^2, v^3) dv^i dv^k; \tag{12}$$

$(\text{w.r.t } 3-\text{observers in } E^3)$.

The line element is invariant with re-parametrization (or with change of local frame). By applying this between equations (11) and (12) we have the following

$$g_{ik}^{(B)}(u^1, u^2, u^3) du^i du^k = \tilde{g}_{ik}^{(B)}(v^1, v^2, v^3) dv^i dv^k$$

Hence

$$\tilde{g}_{ik}^{(B)} = g_{ik}^{(B)} \frac{\partial u^i}{\partial v^i} \frac{\partial u^k}{\partial v^k} \tag{13}$$

Now the Riemannian curvature K_B at point B on the manifold M^3 relative to the underlying Euclidean 3-space E^3 is the same for all local frames at that point. This is so because all local frames lie on the curved hyper-surface M^3 at the given point B and thereby possess the same unique curvature K_B relative to E^3 as the curved hyper-surface M^3 itself. It follows then that the metric tensor at point B on M^3 is unchanged as one changes from the local frame (u^1, u^2, u^3) to the local frame (v^1, v^2, v^3) at this point, with respect to 3-observers at different positions (or in different frames) in the underlying Euclidean 3-space E^3. In other words, $\tilde{g}_{ik}^{(B)}(v^1, v^2, v^3)$ is the same as $g_{ik}^{(B)}(u^1, u^2, u^3)$, with respect to 3-observers at different positions (or in different frames) in the underlying Euclidean 3-space E^3.

The foregoing paragraph states a significant difference between Riemann geometry of a curved metric space M^3 of the second case, in which the curved space (as a curved hyper-surface) lies above its projective Euclidean space E^3 (as s flat hyper-surface along the horizontal) in which the observers are located, illustrated in Fig. 1c or Fig. 3, and the conventional Riemann geometry of the first case in which the curved metric space M^3 is embedded in the global Euclidean 3-space E'^3, as illustrated in Fig. 1b. There is no projective Euclidean space in the first case, and observers are necessarily located in the curved metric space M^3 within the region covered by M^3. The significant difference between Riemann geometries for the two cases is that both the line element and metric tensor are invariant with re-parametrization, ($ds^2 = d\tilde{s}^2$ and $g_{ik} = \tilde{g}_{ik}$), in the second case (of Fig. 1c or Fig. 3), while the line element is invariant but the metric tensor transforms as Eq. (13) with re-parametrization in the first case (of Fig. 1b). Riemann geometry for the first case (of Fig. 1b) is obviously the conventional Riemann geometry, as identified earlier.

The necessary invariance with re-parametrization of both the metric tensor and the line element in the second case of a curved metric space, which lies above its projective Euclidean space in which the observers are located, (in Fig. 3), allows us to write the following from Eq. (13):

$$\tilde{g}_{ik}^{(B)} = g_{ik}^{(B)} \frac{\partial u^i}{\partial v^i} \frac{\partial u^k}{\partial v^k} = g_{ik}^{(B)} \tag{14}$$

Hence

$$\frac{\partial u^i}{\partial v^i} \frac{\partial u^k}{\partial v^k} = \delta_{ik} \tag{15}$$

Equation (15) is valid for every pair of local coordinate sets (or local frames) at any given point on the the curved manifold M^3 in the Riemann geometry of the second case illustrated in Fig. 1(c) or Fig. 3. It simply states that all local coordinate sets at a given point on the curved manifold M^3 are identical with respect to 3-observers in the underlying projective Euclidean 3-space E^3, and this is true at every point of M^3, in the Riemann geometry of the second case.

It follows from the foregoing that all local coordinate sets (u^1, u^2, u^3), (v^1, v^2, v^3), (w^1, w^2, w^3), etc, at any point on the curved manifold M^3 are identical to a singular local coordinate set (or frame) with coordinates to be denoted by (ξ^1, ξ^2, ξ^3) with respect to all 3-observers in the underlying Euclidean space E^3. Thus natural laws formulated in terms of the singular local coordinate set (ξ^1, ξ^2, ξ^3) at any position on M^3 is valid for every local coordinate set (u^1, u^2, u^3), (v^1, v^2, v^3), (w^1, w^2, w^3), etc, at that position, with respect to all 3-observers (or local frames) in the underlying Euclidean 3-space E^3. It then follows that laws of nature are naturally covariant [7, see pg. 117; 5, see pg. 117] on the curved space M^3 with respect to all observers (or frames) in the underlying Euclidean 3-space E^3.

Now a space in which all local coordinate sets (or local frames) are identical to a singular coordinate set (or a singular local frame) at each point of it is an absolute space, an absolute space being a distinguished coordinate set (or a distinguished frame) [7, see pg. 2]. Thus the curved M^3 in the second class of Riemannian metric spaces illustrated in Fig. 1c of Fig. 3 is an absolute space with respect to observers in the underlying Euclidean 3-space E^3. It shall be re-denoted by $\hat{M}^3$ with curved global absolute 'dimensions' $\hat{\eta}^1, \hat{\eta}^2$ and $\hat{\eta}^3$. The different local coordinate sets in the absolute space $\hat{M}^3$ shall likewise be denoted by $(\hat{u}^1, \hat{u}^2, \hat{u}^3), (\hat{v}^1, \hat{v}^2, \hat{v}^3)$, $(\hat{w}^1, \hat{w}^2, \hat{w}^3)$, etc. A hat label shall be used to denote absolute coordinates/absolute intrinsic coordinates and absolute parameters/absolute intrinsic parameters uniformly in the three parts of this paper. The curved absolute space $\hat{M}^3$ introduced here is different from the controversial Newtonian absolute space [7, see pg. 2; 8, 9], as shall become clear with further development in this paper.

Now, the curved absolute space $\hat{M}^3$ will project a flat hyper-surface − a flat three-dimensional space − to be denoted by σ'^3 temporarily along the horizontal, such that the extended curved global 'dimensions' $\hat{\eta}^1, \hat{\eta}^2$ and $\hat{\eta}^3$ of $\hat{M}^3$ become projected as extended straight line global dimensions η'^1, η'^2 and η'^3 respectively of σ'^3 and the singular local coordinate sets $(\hat{\xi}_A^1, \hat{\xi}_A^2, \hat{\xi}_A^3)$, $(\hat{\xi}_B^1, \hat{\xi}_B^2, \hat{\xi}_B^3)$, $(\hat{\xi}_C^1, \hat{\xi}_C^2, \hat{\xi}_C^3)$, etc, at different positions A, B, C, etc, on the curved absolute space $\hat{M}^3$, become projected as singular local coordinate sets $(\xi_A'^1, \xi_A'^2, \xi_A'^3)$, $(\xi_B'^1, \xi_B'^2, \xi_B'^3)$, $(\xi_C'^1, \xi_C'^2, \xi_C'^3)$, etc, at the corresponding positions A', B', C', etc, in σ'^3. In other words, the different local coordinate sets $(\hat{u}_A^1, \hat{u}_A^2, \hat{u}_A^3)$, $(\hat{v}_A^1, \hat{v}_A^2, \hat{v}_A^3)$, $(\hat{w}_A^1, \hat{w}_A^2, \hat{w}_A^3)$, etc, all of which are equivalent to a singular coordinate set $(\hat{\xi}_A^1, \hat{\xi}_A^2, \hat{\xi}_A^3)$ at a point A on $\hat{M}^3$, are projected as local coordinate sets $(u_A'^1, u_A'^2, u_A'^3)$, $(v_A'^1, v_A'^2, v_A'^3)$, $(w_A'^1, w_A'^2, w_A'^3)$, etc, all of which are equivalent to a singular local coordinate set $(\xi_{A'}'^1, \xi_{A'}'^2, \xi_{A'}'^3)$ at the corresponding point A' in σ'^3 and this is true at every other position in σ'^3.

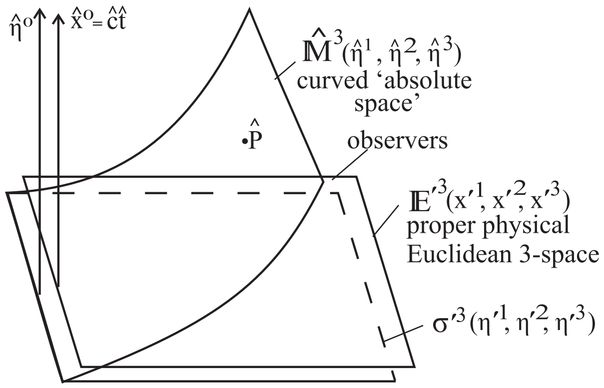

Fig. 4:

The projective space σ'^3 of $\hat{M}^3$ in which all local coordinate sets (or local frames) at any given point of it are identical to a singular coordinate set at the given point, is certainly different from the observed physical Euclidean 3-space E^3 with uncountable number of possible distinct local coordinate sets at every point of it. The projective space σ'^3 is not the space in which 3-observers are located but the physical Euclidean 3-space E^3. It is therefore mandatory for us to prescribe both the physical Euclidean 3-space E^3 in which 3-observers are located and the flat projective 3-space σ'^3 of the curved absolute space $\hat{M}^3$ along the horizontal, such that σ'^3 lies underneath E^3.

Contrary to Fig. 1c or Fig. 3 that we started with, in which a curved physical (or relative) 3-dimensional space M^3 that is curved towards the absolute time 'dimension' along the vertical, lies above its projective Euclidean space E^3, the flatness of the original physical proper (or classical) Euclidean 3-space E'^3 (in Fig. 1a) shall be left unaffected by the evolution of the curved absolute space $\hat{M}^3$, which lies above its projective flat space σ'^3, where σ'^3 underlies E'^3. Consequently Fig. 3 shall be modified as Fig. 4 temporarily, where the curved absolute space lies above its projective flat space σ'^3 that lies underneath the original proper Euclidean space E'^3 in this situation.

The flat space σ'^3 projected along the horizontal by the curved absolute space $\hat{M}^3$ has been given a prime label like the proper physical Euclidean 3-space E'^3 in which observers are located, lying over it along the horizontal. The prime label shall be used to indicate proper (or classical) spaces, coordinates and parameters uniformly in this paper and later parts of it. Consequently σ'^3 is a proper (or classical) space like E'^3. The reference to the projective σ'^3 as proper is educated by the fact that an absolute coordinate evolves into a proper coordinate and a proper coordinate evolves into a relativistic coordinate in the evolutionary sequence of coordinates in mechanics. For instance, the absolute time $\hat{t}$ evolves into the proper time t' (or τ), and the proper time evolves into the relativistic time t, as known in relativity.

Now, as noted earlier, two distinct observable metric spaces of different metric tensors in Fig. 1c or Fig. 3 have evolved from the singular Galileo space of Fig. 1a, whereas such duplication of observable metric spaces is not observed in nature. This has been remarked as a paradox raised by Fig. 1c or Fig. 3 earlier. The duplication of metric spaces in Fig. 1c or Fig. 3 has now become a triplet of metric spaces in Fig. 4. These are the curved absolute space $\hat{M}^3$ with absolute Riemannian metric tensor, the proper physical Euclidean 3-space E'^3 (with Euclidean metric tensor) and the projective flat proper space σ'^3 also with Euclidean metric tensor in Fig. 4.

In order for the 3-observers to observe only the proper physical Euclidean 3-space E'^3 in which they are located in Fig. 4, so that the paradox noted above is resolved, the projective underlying proper space σ'^3 must be an intrinsic (i.e, a non-observable and non-detectable) space to observers in E'^3. Thus σ'^3 shall be referred to as proper intrinsic space. The curved absolute space $\hat{M}^3$ that projects the flat proper intrinsic space σ'^3 along the horizontal must itself be an intrinsic space. It shall be renamed absolute intrinsic space consequently. Thus the non-observable absolute intrinsic space $\hat{M}^3$ projects the non-observable proper intrinsic space σ'^3 along the horizontal, leaving the proper physical Euclidean 3-space as the only observable space to observers in it in Fig. 4.

It is natural to associate an absolute intrinsic time 'dimension' $\hat{\eta}^0$ with the proper intrinsic space σ'^3, which lies parallel to the absolute time 'dimension' $\hat{c}\hat{t}$ along the vertical, as done in Fig 4.

Thus one consequence of the fact deduced earlier, that the metric tensor and line element are both invariant with reparametrization in Riemann geometry in which a curved 3-space, (as a curved hyper-surface) is curved onto the absolute time 'dimension' along the vertical, lies above its projective Euclidean 3-space (as a flat hyper-surface) along the horizontal, in which the observers are located, illustrated in Fig. 1c or Fig. 3, is that such Riemann geometry is realizable on a curved non-observable and non-detectable absolute intrinsic space $\hat{M}^3$, in which all local coordinate sets are equivalent to singular local absolute intrinsic coordinate sets, $(\hat{\eta}^1_A, \hat{\eta}^2_A, \hat{\eta}^3_A)$, $(\hat{\eta}^1_B, \hat{\eta}^2_B, \hat{\eta}^3_B)$, $(\hat{\eta}^1_C, \hat{\eta}^2_C, \hat{\eta}^3_C)$, etc, at different positions A, B, C, etc, on it. The curved absolute intrinsic space lies above its projective flat proper intrinsic space σ'^3, in which all local coordinate sets (or local frames) are equivalent to singular local coordinate sets (or local frames) $(\eta'^1_{A'}, \eta'^2_{A'}, \eta'^3_{A'})$, $(\eta'^1_{B'}, \eta'^2_{B'}, \eta'^3_{B'})$, $(\eta'^1_{C'}, \eta'^2_{C'}, \eta'^3_{C'})$, etc, at different positions A', B', C', etc, in it, with respect to 3-observers in the proper physical Euclidean 3-space E'^3 that lies above σ'^3 along the horizontal. The Riemann geometry on the curved absolute intrinsic space $\hat{M}^3$ with respect to 3-observers in the underlying proper physical Euclidean 3-space E'^3 shall be entitled absolute intrinsic Riemann geometry.

As the next step, we shall adopt more appropriate notations and representations for the intrinsic spaces and the associated intrinsic time coordinates than used above. The no-

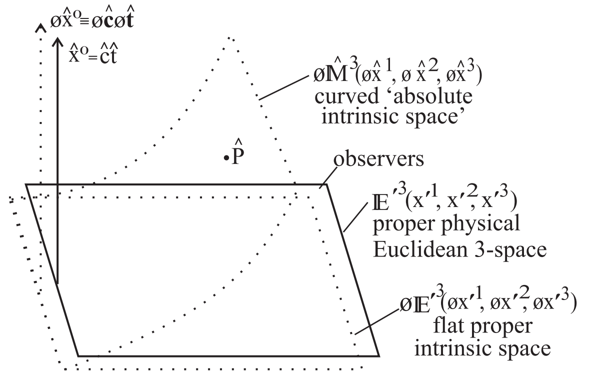

Fig. 5: The '3-dimensional' absolute intrinsic space curving towards the absolute intrinsic time 'dimension' along the vertical, projects flat 3-dimensional proper intrinsic space, which lies underneath the flat proper physical 3-space along the horizontal.

tation $\hat{M}^3(\hat{\eta}^1, \hat{\eta}^2, \hat{\eta}^3)$ for the curved absolute intrinsic space shall be replaced by $\phi\hat{M}^3(\phi\hat{x}^1, \phi\hat{x}^2, \phi\hat{x}^3)$, and the the projective flat proper intrinsic space $\sigma'^3(\eta'^1, \eta'^2, \eta'^3)$ shall be replaced by $\phi E'^3(\phi x'^1, \phi x'^2, \phi x'^3)$. Also the absolute intrinsic time 'dimension' $\hat{\eta}^0$ shall be replaced by, $\phi\hat{x}^0 \equiv \phi\hat{c}\phi\hat{t}$. By effecting these new notations in Fig. 4, we have Fig. 5. The non-observable and non-detectable (or hidden) intrinsic spaces have been shown with dotted boundaries in Fig. 5, as shall be done henceforth.

Since different local proper intrinsic coordinate sets or local intrinsic frames $(\phi u'^1_{A'}, \phi u'^2_{A'}, \phi u'^3_{A'})$, $(\phi v'^1_{A'}, \phi v'^2_{A'}, \phi v'^3_{A'})$, $(\phi w'^1_{A'}, \phi w'^2_{A'}, \phi w'^3_{A'})$, etc, at a position A', say, in the projective proper intrinsic space $\phi E'^3$ are equivalent to a singular intrinsic local coordinate set $(\phi\xi'^1_{A'}, \phi\xi'^2_{A'}, \phi\xi'^3_{A'})$ with respect to 3-observers in the physical proper Euclidean 3-space E'^3, natural laws in $\phi E'^3$ are naturally covariant with respect to 3-observers in E'^3. The fact that natural laws on the curved absolute intrinsic space $\phi\hat{M}^3$ that projects $\phi E'^3$ along the horizontal are naturally covariant with respect to 3-observers in E'^3 has been deduced in a similar manner earlier.

The following features of the new notations in Fig. 5 make them more appropriate than those in Fig. 4:

1. Apart from the attachment of the symbol ϕ to the usual coordinates, no new symbol has been introduced to represent the intrinsic coordinates. This minimizes the number of symbols that enters into the theory, which is aesthetically desirable.
2. The fact that the observed physical space is outward manifestation of the underlying non-observable intrinsic space can be seen from the new notations. For if we remove the symbol ϕ from $\phi E'^3(\phi x'^1, \phi x'^2, \phi x'^3)$ we obtain $E'^3(x'^1, x'^2, x'^3)$, which must be interpreted as E'^3 is the outward manifestation of $\phi E'^3$; x'^1 is the outward manifestation of $\phi x'^1$; etc. Likewise if we remove the symbol ϕ from $\phi\hat{x}^0 \equiv \phi\hat{c}\phi\hat{t}$ we obtain $\hat{x}^0 \equiv \hat{c}\hat{t}$. The fact that the proper physical 3-space E'^3 is the outward (or physical) manifestation of the proper intrinsic space $\phi E'^3$, which is clear from the foregoing, cannot be easily seen or demonstrated with other notations, such as the one adopted initially, illustrated in Fig. 4.
3. Following the formal derivation of the two-dimensional proper intrinsic spacetime (or proper nospace-notime) that underlies the flat four-dimensional proper spacetime (Σ', ct') in sub-section 1.2 of [4], the symbol ϕ attached to the intrinsic coordinates in the new notations has the meaning of 'void' or 'null'. Thus ϕspace can be referred to as 'void-space' or 'null-space', but 'nospace' has been preferred, as discussed in sub-section 1.2 of [4]. Any distance $\phi d'$ of proper intrinsic space (or proper nospace) $\phi E'^3$ is equivalent to zero distance of the pro- per physical Euclidean 3-space E'^3. This can be seen directly from the symbol ϕ attached to $\phi d'$, with the meaning of 'void' or 'null', whereas the fact that an interval of intrinsic space $\Delta\eta'$ in the notation of Fig. 4 is equivalent to zero distance of the physical 3-space cannot be seen directly. The fact that any interval of intrinsic space (or nospace) is equivalent to zero interval of physical space makes it non-detectable to observers in the physical space. The symbol ϕ attached to a space or coordinate or a physical parameter is used to indicate that the space or coordinate or parameter is intrinsic, that is, non-detectable (or hidden) to observers in the proper physical Euclidean 3-space E'^3.
4. The intrinsic coordinates $\phi x'^1, \phi x'^2, \phi x'^3$ and $\phi c\phi t'$ of the proper intrinsic spacetime must be deemed to have been formally derived following the formal derivation of the two-dimensional proper intrinsic spacetime (or proper nospace-notime) that underlies the flat four-dimensional proper spacetime (Σ', ct') in sub-section 1.2 of [4], from which it is clear that these new intrinsic coordinates and their notations are not arbitrary creations. The new notations in Fig. 5 for the new intrinsic spacetime coordinates are the natural notations.

What we end up having as the new kind of Riemannian metric space in which the coordinates of a curved three-dimensional space M^3 span the absolute time coordinate along the vertical solely, which projects a flat 3-space E'^3 underneath it, illustrated in Fig. 1c or Fig. 3, is a curved absolute intrinsic space (or curved absolute nospace) $\phi\hat{M}^3(\phi\hat{x}^1, \phi\hat{x}^2, \phi\hat{x}^3)$, an absolute intrinsic Riemann space, whose all its absolute intrinsic 'dimensions' span the absolute intrinsic time 'dimension' along the vertical solely, which projects a flat proper intrinsic space (or flat proper nospace) $\phi E'^3$ underneath the proper physical Euclidean 3-space E'^3 along the horizontal, illustrated in Fig. 5. The observers with respect to whom the new geometry is valid are all 3-observers located in

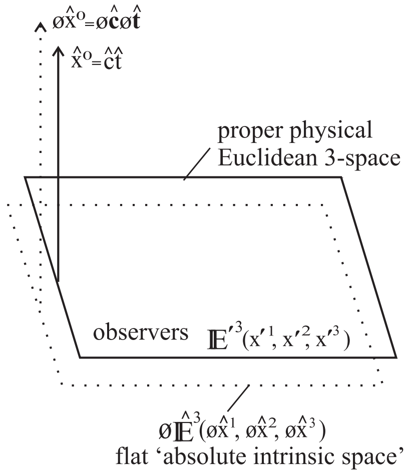

Fig. 6: The flat proper physical 3-space - absolute time, (the Galileo space), underlied by flat '3-dimensional' absolute intrinsic space - absolute intrinsic time, (an impossible situation).

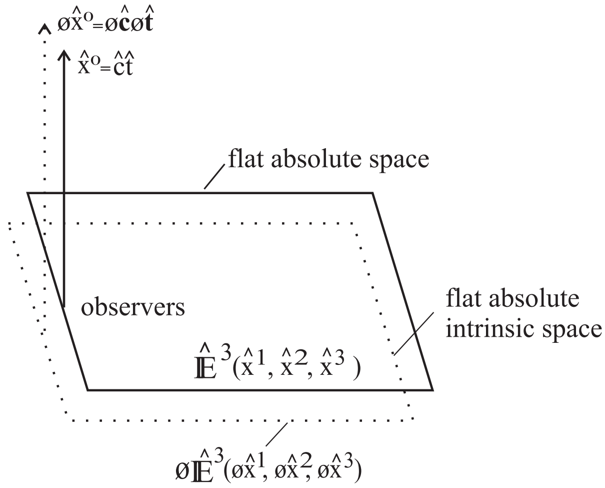

Fig. 7: Flat '3-dimensional' absolute space - absolute time, underlied by flat '3-dimensional' absolute intrinsic space - absolute intrinsic time, (a possible situation).

the underlying physical proper Euclidean 3-space E'^3, which is the outward (or physical) manifestation of its underlying proper intrinsic space $\phi E'^3$.

True to the title of this section, we can only talk of a new absolute intrinsic Riemannian metric space (or absolute Riemannian metric nospace) and the associated new absolute intrinsic Riemann geometry (or absolute Riemannian nospace geometry), which are being unearthed in this paper. As said above, the observers with respect to whom the absolute intrinsic Riemann geometry is valid are all 3-observers in the physical proper Euclidean 3-space E'^3.

Now in the absence of absolute intrinsic Riemann geometry, the curved absolute intrinsic space $\phi\hat{M}^3$ in Fig. 5 becomes a flat absolute intrinsic space $\phi\hat{E}'^3$ underlying the flat proper physical Euclidean 3-space E'^3 along the horizontal, as illustrated in Fig. 6. Now the proper physical 3-space E'^3 lies above the proper intrinsic space $\phi E'^3$ in Fig. 5. That is a correct situation. Indeed the proper physical 3-space $E'^3(x'^1, x'^2, x'^3)$ is the outward (or physical) manifestation of the proper intrinsic space $\phi E'^3(\phi x'^1, \phi x'^2, \phi x'^3)$ in Fig. 5, since the removal of symbol ϕ from $\phi E'^3$ converts it to E'^3, as discussed earlier.

On the other hand, the flat proper physical 3-space E'^3 lies above the flat absolute intrinsic space $\phi\hat{E}^3$ in Fig. 6. This is an incorrect situation because the flat proper physical 3-space E'^3 cannot be the outward (or physical) manifestation of the absolute intrinsic space $\phi\hat{E}^3$ underlying it. The removal of the symbol ϕ from $\phi\hat{E}^3$ converts it to $\hat{E}^3$, which implies that $\hat{E}^3$ is the correct outward manifestation of $\phi\hat{E}^3$. Let us therefore replace the flat proper physical 3-space $E'^3(x'^1, x'^2, x'^3)$ by flat absolute space $\hat{E}^3(\hat{x}^1, \hat{x}^2, \hat{x}^3)$ in Fig. 6 to have Fig. 7. Fig. 7 must replace Fig. 6 as the reference geometry, (in the absence of absolute intrinsic Riemann geometry). Fig. 7 will exist everywhere in a universe that is hypothetically devoid of a long-range metric force field, as shall be explained formally elsewhere with further development.

Now let us introduce the source of a long-range absolute metric force field at a point in the flat absolute space $\hat{E}^3$ in Fig. 7, which implies that the source of a long-range absolute intrinsic metric force field is automatically introduced into the absolute intrinsic space $\phi\hat{E}^3$, directly underneath the source of the long-range absolute metric force field in $\hat{E}^3$ in that figure. As shall be explained with further development elsewhere, this action will cause the flat absolute intrinsic space $\phi\hat{E}^3$ in Fig. 7 to evolve into curved absolute intrinsic space $\phi\hat{M}^3$ in Fig. 5. The curved $\phi\hat{M}^3$ will then project flat proper intrinsic space $\phi E'^3$ along the horizontal, which will be made manifest outwardly in flat proper physical 3-space E'^3, as illustrated in Fig. 5. It then follows that the flat absolute space $\hat{E}^3$ in Fig. 7 automatically transforms into flat proper physical space E'^3 in Fig. 5 within a long-range absolute metric force field without any need to prescribe the curvature of absolute space. Only the absolute intrinsic space is required to be curved.

We have described the first stage of evolution of space and its underlying intrinsic space within a long-range metric force field. Thus the product of the first stage of evolution of space/intrinsic space is the extended curved absolute intrinsic metric space $\phi\hat{M}^3$, its projective extended flat proper intrinsic metric space $\phi E'^3$ and the outward manifestation of $\phi E'^3$ namely, the proper physical Euclidean 3-space E'^3, in which the observers are located. The absolute intrinsic time 'dimension' is not curved from its vertical position simultaneously with the 'three-dimensional' absolute intrinsic metric space in the context of the absolute intrinsic metric phenomena that give rise to curved absolute intrinsic metric spaces. Consequently the curvature of the absolute intrinsic time 'dimension' is not considered in this first paper. We shall proceed to a robust graphical analysis of the geometry of the curved absolute intrinsic metric space $\phi\hat{M}^3$ (an absolute intrinsic Riemannian metric space) in Fig. 5 with respect to 3-observers in the underlying proper physical Euclidean 3-space E'^3 in the second part of this paper.

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