Reflections on the Future of Particle Theory

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Abstract

Experimental observations of recent years suggest that developing the theory beyond the Standard Model (SM) may require a careful revision of conceptual foundations of quantum field theory (QFT) and its consistency conditions. As it is known, QFT describes interaction of stable or quasi-stable fields whose evolution is deterministic and time-reversible. By contrast, behavior of strongly coupled fields or nonlinear dynamics of the Terascale sector is prone to become unstable and chaotic. A specific signature of this transient regime is the onset of long-range dynamic correlations in space-time, the emergence of strange attractors in phase space and transition from smooth to fractal topology. In this report we explore the impact of fractal topology on physics unfolding above the electroweak (EW) scale. Arguments are given for perturbative renormalization of field theory on fractal space-time, breaking of discrete symmetries, hierarchical generation of parameters as well as the potential for exotic phases of matter that are ultra-weakly coupled to SM. A surprising implication of this approach is that classical gravity emerges as dual description of field theory on fractal space-time.

1. INTRODUCTION

Time and again, experimental observations have confirmed that the SM is a robust theoretical framework for the description of elementary particle physics up to the scale of EW interaction. Experiments have covered a wide range of direct searches at particle accelerators, as well as precision tests of EW parameters.

It is known that relativistic QFT represents the backbone of SM and is built in compliance with a number of postulates called *consistency conditions*. They define the range of applicability of SM. The remarkable success of SM can be attributed to a unitary, local, renormalizable, gauge invariant and anomaly-free formulation of QFT [ ].

Since SM is based on a renormalizable gauge field theory, the prevailing opinion among theorists is that it can be extrapolated to energies above the EW scale. The underlying assumption is that QFT *stays compliant* to consistency conditions throughout all energy scales.

Despite being confirmed in many independent tests, SM remains an incomplete framework. The root cause of EW symmetry breaking (EWSB) is still unknown. We lack compelling evidence for the Higgs boson that is alleged to break the electroweak $SU(2)_L \times U(1)_Y$ symmetry to its smaller electromagnetic $U(1)_{EM}$ subgroup. The search for the source of EWSB has been one of the main drivers in both experimental and theoretical high-energy physics for the past 25 years.

Beyond our ignorance on the mechanism of EWSB, there are expectations that new phenomena will surface at the Large Hadron Collider (LHC) and other detector sites in the not-so-distant future [ ]:

- A fundamental scalar Higgs boson is not the only way to induce EWSB. What is certain is that a *light Higgs boson* is consistent with precision EW data, but this does not generally preclude other EWSB scenarios.
- The mass parameter of the Higgs boson — which is closely tied to the scale of EWSB — is extremely sensitive to quantum corrections. As a result, attempts to extrapolate SM to energies much higher than the EW scale lead to the *gauge hierarchy problem*, where an extreme fine tuning is required to maintain the EW scale at its observed value. Although this is not inconsistent with the underlying principles of QFT, it appears to be contrived.

- SM is unable to account for the presence of *dark matter*. In many theories beyond SM, dark matter consists of stable and weakly-coupling states whose existence protects the EW scale.
- SM is unable to account for the asymmetry of visible matter over antimatter. New physics near or above the EW scale can potentially explain the fundamental baryon asymmetry of the universe.

Among other challenges facing SM, we list the origin of fermion replication, a quantum description of gravity, an explanation for the cosmological constant, the source of broken discrete symmetries, the sources of flavor mixing and neutrino masses [ ]. It is believed that these open questions are likely to be solved by new physics above the EW scale. Irrespective of the particular nature of new physics, it is also generally believed that the outcome at the LHC would contain an excess of observed leptons, photons, jets and missing transverse energy in some combination. Searches for new physics and SM-related phenomenology at the LHC and other detector sites include, but are not limited to, the following items:

- Supersymmetry (SUSY), leptoquarks, hidden valley states, unparticles, extra-dimensions and strings.
- CP violation in the B-meson sector.
- Top quark physics.
- Z^0 physics and ultra-heavy gauge bosons.
- Probing the origin of neutrino mass.
- Understanding the phase diagram of deconfined high-temperature quantum chromodynamics (QCD). The goal is explaining the behavior and properties of

quark-gluon plasma (QGP) and color condensates (GLASMA) resulting from collisions of heavy ions.

- Probing for the fourth family quark and the existence of sterile neutrino.
- Probing for exotic phases of matter including dark matter.

Inspired by the ubiquity of nonlinear dynamics and complex behavior in natural phenomena [ ], we follow here a less explored path to physics beyond SM. To this end, we start by recalling that many *anomalies* and *broken symmetries* appear to be related to few-body or multi-body physics near or above the EW scale: mass generation via gauge symmetry breaking, violation of CP and chiral symmetries [ ], absence of flavor transitions between charged leptons and their anomalous magnetic moments [ ], non-unitarity of lepton mixing matrix due to neutrino oscillations [ ], the alleged symmetry violation between neutrinos and anti-neutrinos in Mini-BooNE data [ ], QGP and GLASMA in collisions of heavy nuclei [ ], the CDF anomaly [ ], the PAMELA excess of positrons [ ] and so on.

Tying all these hints together leads to the conjecture that *time-asymmetric* and *non-local* field theories are among the most likely candidates for physics beyond SM. In particular, developing the theory beyond SM may require a careful revision of conceptual foundations of QFT and its consistency conditions.

It is known that QFT describes interaction of stable or quasi-stable fields whose evolution is deterministic and time-reversible. Divergence cancellation in UV is tantamount for a successful description of physics beyond SM. By contrast, behavior of strongly coupled fields or dynamics in the Terascale sector is prone to become unstable and chaotic. Non-renormalizable interactions are likely to proliferate and prevent full cancellation of ultraviolet divergences. As a result of incessant fluctuations, any system of fields in nonlinear interaction much above the EW scale is bound to

- Become inherently statistical and dissipative,
- Migrate from stationary to out-of-equilibrium conditions.

A transient regime in nonlinear dynamics opens the door for the emergence of strange attractors in phase space and transition from smooth to fractal topology [ ]. Drawing from these premises, the goal of this report is to evaluate the likely impact of *fractal topology* on physics unfolding above the EW scale.

Ideas introduced in this chapter are gradually built in self-contained steps. For the sake of concision and clarity, the presentation is often times formatted in a "bulleted" style. Next section develops the motivation for model building using fractional dynamics. A brief review of what fractional dynamics stands for and its array of current applications is outlined in section 3. Section 4 focuses on a series of hints for fractional dynamics stemming from the theoretical structure of SM. The remainder of the report discusses the connection between physics beyond SM and fractional dynamics. Summary, conclusions and a list of future challenges are presented in the last section.

We caution from the outset that ideas discussed in this chapter are preliminary. Since, by construction, SM is an "effective" theoretic framework, any proposed extensions beyond its realm must be approached with a healthy dose of skepticism. At this stage, many controversial issues remain unsettled and successful theoretical developments are yet to come. We believe that the intricate nature of topics and incomplete knowledge from the experimental side preclude a comprehensive and definitive analysis. Model building efforts as well as concurrent testing data are needed to refute, confirm or expand these tentative findings.

2. FOUNDATIONAL QUESTIONS

In our view, there are three foundational questions that need to be answered prior to develop the theory beyond SM:

- **Are Terascale phenomena in dynamic equilibrium?**

By dynamic equilibrium we mean a condition in which all processes act simultaneously to maintain the system of interacting fields in an overall steady state. Consider a few-body system of interacting classical fields. Its steady state follows from minimization of the interaction energy and is described as stable if sufficiently small perturbations away from it damp out in time. Perturbations may be *internal* to the system or *external*, the latter case describing *open* systems coupled to their environment. The replica of equilibrium states in nonlinear dynamics are the fixed point solutions of evolution equations [ ].

- **Are Terascale phenomena quantum or classical?**

Take an isolated system of interacting quantum fields whose Hamiltonian factors out into three independent contributions,

$$H = H_0 + H_P + H_I \qquad (1)$$

H_0 is the term associated with the fields, H_P describes internal perturbations and H_I the coupling between fields and perturbations. Decoherence represents the inherent *loss of phase information* induced by H_I and is responsible for suppressing the quantum nature of fields [ ]. The time it takes a generic system of quantum oscillators to decohere is on the order of

$$t_d = \frac{1}{\gamma \langle E \rangle T \langle \Delta E_P \rangle^2} \quad (2)$$

Here, γ encodes the dissipative effects produced by perturbations, $\langle E \rangle$ is the average overall energy of the system, T its temperature and $\langle \Delta E_P \rangle$ the average energy spacing in the perturbation spectrum. Since Terascale physics is characterized by large values of parameters appearing in the denominator, transition to classical behavior is bound to occur extremely fast. A similar scenario applies to interacting quantum fields whose dynamics exhibits spontaneous symmetry breaking [ ]. It is instructive to note that:

- Decoherence enables non-abelian gauge fields to undergo transition to chaos as classical fields [ ].
- Erasing phase information encoded in the quantum description of phenomena is an inherent source of entropy increase [ ].

It follows from these considerations that at very large temperatures, commensurate with probing the near and deep Terascale sector, many quantum phenomena are likely to decohere almost instantaneously and become unstable [ ][1]. On account of previous points, we adopt the foundational view of [ ] that unstable few-body quantum processes are *intrinsically time-asymmetric* and favor the onset of *non-equilibrium dynamics* [ ]. This conjecture has been reinforced in recent years by the observation that *complex behavior* in the form of bifurcations and chaos, fractal geometry and random-looking evolution in time and space can occur in low-dimensional as well as in few-body systems [ ]. Because chaos is ubiquitous at the level of microscopic dynamics of single particles it should also determine to a large extent the macroscopic behavior of interacting fields.

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[1] It is unclear if this conjecture stays valid regardless of the energy scale. Fast thermalization of QGP may provide a valid counter-argument, if it stands at transition temperatures well above 175 MeV.

- **What constraints need to be applied to phenomenological models of the Terascale sector?**

A successful model of the Terascale sector must be able to recover the physics of SM in its low-energy limit. [ ]. In particular:

- Has to be compatible with EW precision data,
- Has to convincingly resolve the unitarity problem at the SM scale**,**
- Has to maintain gauge invariance and renormalizability at the SM scale.

Next sections indicate how fractional dynamics has the potential of meeting all these constraints as the departure from equilibrium dynamics goes to zero. In a nut-shell, transition to equilibrium at low-energies decouples fractional dynamics from the physics of SM.

3. WHAT IS FRACTIONAL DYNAMICS?

Fractional dynamics studies the behavior of nonlinear physical systems that are [ ]

- Out-of-equilibrium *and*
- Described by differential and integral operators of non-integer orders (fractal operators).

Equations containing such operators are used to analyze the behavior of systems characterized by

- Power-law nonlinearity,
- Power-law long-range spatial correlations or long-term memory,
- Fractal or multi-fractal properties.

In the last decade, the number of applications of fractional dynamics in science and engineering has been steadily growing [ ]. They include models of fractional-relaxation effects, anomalous transport in fluids and plasma, wave propagation in complex media, viscoelastic materials, universal response in dielectric media, non-Markovian evolution of quantum fields, networks of fractional oscillators, dynamics of non-extensive statistical systems and so on. The reader is referred to [ ] and [ ] for a comprehensive review of fractional calculus and fractional dynamics.

For the sake of convenience and to fix notation, we introduce next few definitions and properties of fractal operators that are relevant to our context. Let $f(x,\lambda) \in L_p(E^1)$ an arbitrary function of x defined on a one-dimensional Euclidean space E^1 where λ is a parameter and $1 < p < 1/\alpha$. Fractional integration of order α on $(-\infty, y)$ and $(y, +\infty)$ is described by [ ]

$$(I_+^\alpha f)(y,\lambda) = \frac{1}{\Gamma(\alpha)} \int_{-\infty}^{y} \frac{f(x,\lambda)\,dx}{(y-x)^{1-\alpha}}, \qquad (I_-^\alpha f)(y,\lambda) = \frac{1}{\Gamma(\alpha)} \int_{y}^{+\infty} \frac{f(x,\lambda)\,dx}{(x-y)^{1-\alpha}} \tag{3}$$

There is a close connection between *fractals* and *fractional dynamics* [ ]. Fractals are metric sets with non-integer dimensionality. Integration over an axially-symmetric fractal space W with *Hausdorff dimension* D is defined as

$$\int_W f(x)\,d\mu_H(x) = \frac{2\pi^{D/2}}{\Gamma(D/2)} \int_0^\infty f(r)\, r^{D-1} dr \tag{4}$$

in which $d\mu_H(x)$ stands for the differential *Hausdorff measure* of W [ ]. It satisfies the scale-invariance property

$$d\mu_H(x/s) = s^{-D} d\mu_H(x) \tag{5}$$

The same property applies to (4) on account of (5)

$$\int_W f(sx)\,d\mu_H(x) = s^{-D} \int_W f(x)\,d\mu_H(x) \tag{6}$$

The Hausdorff dimension for a subset $E \subset W$ is given by

$$D = \dim_H(E) \tag{7a}$$

such that, for any non-negative number α ,

$$\begin{aligned} \mu_H(E) &= \infty \text{ if } 0 \le \alpha < D \\ \mu_H(E) &= 0 \text{ if } D < \alpha < \infty \end{aligned} \tag{7b}$$

In section 9 we introduce quantum charges associated with non-abelian gauge theory. In anticipation of that discussion, consider an arbitrary charge distribution on W defined by dimension D. Let $\rho(\mathbf{r},t)$ describe the charge density function. The total charge enclosed within the fractal volume V_D is described by [ ]

$$q_D(W) = \int_W \rho(\mathbf{r},t)\,dV_D\,, \quad dV_D = c_3(D,\mathbf{r})\,dV_3 \tag{8a}$$

where V_3 represents the ordinary volume of space and

$$c_3(D,\mathbf{r}) = \frac{2^{3-D}\,\Gamma(3/2)}{\Gamma(D/2)}\left|\mathbf{r}\right|^{D-3} \tag{8b}$$

4. HINTS FOR FRACTIONAL DYNAMICS IN HIGH-ENERGY PHYSICS

Extreme quantum regimes such as ultra-relativistic nucleus-nucleus collisions, quark-gluon plasma and the emergence of color condensates, decays of heavy resonances, strong-coupling in infrared QCD, behavior of non-Fermi liquids, fractional quantum Hall effect, non-extensive behavior of high-temperature or large-density QCD, spin glasses, high-momentum scattering of longitudinally polarized vector bosons are few representative examples of out-of-equilibrium processes[2].

Recent years have consistently shown that fractional dynamics is an indispensable tool for modeling such processes [ ]. A natural question to ask is: *What leads are there that suggest using fractional dynamics for model building beyond SM?* Answering this question is our next objective.

- **Hints from dimensional regularization**

Theoretical challenges associated with divergences of perturbative QFT were first recognized by Heisenberg and Pauli in 1929 and 1930. A viable solution had to wait until 1949 when Dyson realized that divergences can be reabsorbed in a countable number of parameters defining the theory [ ]. Models that accommodate this procedure were called "renormalizable". It was later determined that typical non-renormalizable theories contain coupling coefficients having dimensions of inverse powers of mass [ ].

Standard renormalization in QFT is conceived as a two-step program: regularization and subtraction. One first controls the divergence present in momentum integrals by inserting a suitable "regulator", and then brings in a set of "counter-terms" to cancel out the divergence. Momentum integrals in perturbative QFT have the generic form

$$I = \int_0^\infty d^4 q \, F(q) \tag{9}$$

[2] We mention here the pioneering work of Prigogine who conjectured that non-equilibrium microscopic processes cannot be properly described by S-matrix theory and require moving beyond the conventional Hilbert space of quantum theory [ ].

Two regularization techniques are frequently employed to manage (9), namely "momentum cutoff" and "dimensional regularization". In the momentum cutoff scheme, the upper limit of (9) is replaced by a finite mass scale M ,

$$I \to I_M = \int_0^M d^4 q F(q) \tag{10}$$

Explicit calculation of the convergent integral (10) amounts to a sum of three polynomial terms (to be checked against QFT courses in the folder) [ ]

$$I_M = A(M) + B + C(1/M) \tag{11}$$

Dimensional regularization proceeds instead by shifting the momentum integral (9) from a four-dimensional space to a continuous D - dimensional space

$$I \to I_D = \int_0^\infty d^D q F(q) \tag{12}$$

Introducing the parameter $\varepsilon = 4 - D$ leads to

$$I_D \to I_\varepsilon = A'(\varepsilon) + B' + C'(1/\varepsilon) \tag{13}$$

It is known that M and ε are not independent regulators and relate to each other via the approximate connection [ ]

$$\varepsilon = 4 - D \approx \frac{1}{\log(M/M_0)} \tag{14}$$

where M_0 stands for an arbitrary and finite reference scale.

(11), (12) and (13) may be interpreted in two different ways:

a) In the asymptotic limit $M \to \infty$ and $\varepsilon \to 0$, C and A' vanish whilst A and C' become singular.

b) Let E denote the energy scale of phenomena described by a given field theory. If the regulator is chosen to stay finite or non-zero (that is, either $M < \infty$ or $\varepsilon \neq 0$), the theory is no longer meaningful for any $E \geq M$ or for any $\varepsilon' \leq \varepsilon$.

Renormalizability goes along with a) and boils down to the requirement that all momentum integrals (1) are convergent and independent of the regulator as $M \to \infty$ or $\varepsilon \to 0$. For a number of years, this criterion was regarded as a necessary consistency condition that any trustworthy QFT must satisfy [ ]. The modern point of view has now shifted to b). According to this interpretation, a field theory that is non-renormalizable represents a *valid low-energy approximation* to a more comprehensive theoretical framework. To understand why this is the case, consider a non-renormalizable theory with a single generic coupling g whose mass dimension is M^{-2}. The renormalized perturbative expansion of an N - point amplitude up to the order $(g^2)^n$ reads [ ]

$$A_N(E) = A_N^0(E) \sum_{i=0}^{n} c_i \left(\frac{E}{M}\right)^{2i} \qquad (15)$$

Here $c_0 = 1$ and all coefficients c_i, $i = 2, 3, ..., n-1$ are fixed once renormalization has been carried out for amplitudes with less than N points. Since new divergences may develop at order n, the last coefficient in the series (c_n) cannot be derived from theory. This lack of predictivity on c_n becomes however irrelevant if $E \ll M$ due to the small contribution arisen from the corresponding term in (15). Higher-order divergences can be safely ignored as long as $E \ll M$ or $\varepsilon' \geq \varepsilon$ and the chosen built-in scale M or *continuous dimension* ε sets the limit of validity of the underlying theory.

- **Hints from effective field theory**

Any effective Lagrangian can be presented as [ ]

$$L_{EFF} = \sum_i g_i O_i \tag{16}$$

where O_i are *local* operators built with the light fields, and the information on any heavy fields is contained in the couplings g_i. The operators O_i are usually organized according to their dimension (d_i) which fixes the dimension of their coefficients:

$$[O_i] = d_i \rightarrow g_i \sim \frac{1}{\Lambda^{d_i - 4}} \tag{17}$$

with Λ some characteristic heavy scale of the system. At energies below this scale ($E < \Lambda$), the behavior of the different operators is determined by their dimension. There are three types of operators: relevant ($d_i < 4$), marginal ($d_i = 4$) and irrelevant ($d_i > 4$). The effect of irrelevant operators is weak at low energies because it is suppressed by powers of E/Λ. Irrelevant operators usually contain interesting information about the underlying dynamics at higher scales. For example, the SM Lagrangian without the Higgs and Yukawa sectors assumes the generic form

$$L_{SM} = \sum_\alpha \overline{\Psi}^\alpha [i\gamma^\mu (D_\mu \Psi)^\alpha] - \frac{1}{4} F^a_{\mu\nu} F^{a\mu\nu} = L_M[\Psi^\alpha, (D_\mu \Psi)^\alpha] - \frac{1}{4} F^a_{\mu\nu} F^{a\mu\nu} \tag{18}$$

Using (16) and (17) we can write (18) as

$$L_{EFF} = L_{SM\,(d_i<4)} + \sum_{d_i>4} \frac{g_i O_i}{\Lambda^{d_i - 4}} \tag{19}$$

where corrections induced by non-renormalizable interactions $d_i > 4$ are highly suppressed by powers of E/Λ at energies $E < \Lambda$. For example, the dependence of matter Lagrangian L_M on $F^{a\mu\nu}$ as well as higher covariant derivatives $D_\nu D_\mu \Psi$ creates non-renormalizable terms that are absent below the scale of EW interaction.

A basic premise of effective field theory is that *non-local* heavy–particle exchanges can be replaced by a tower of *local* and non-renormalizable interactions among light particles [ ]. There are two ways in which this assumption can be violated at large energies:

- Heavy fields that yield relevant interactions near Λ cannot be integrated out and remain coupled to light fields,
- The onset of out-of-equilibrium dynamics prevents non-local heavy particles to be replaced by local interactions among light particles.

It is apparent from this discussion that effective field theory may cease to remain a good metric for what happens at energies far beyond the EW scale.

- **Hints from the requirement of scale invariance**

The Lagrangian density for classical massless electrodynamics reads

$$L = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \overline{\psi} i \gamma^{\mu} D_{\mu} \psi \tag{20}$$

An arbitrary change in coordinate scale $x \to x' = \lambda x$ along with the corresponding field transformations

$$\psi(x) \to \psi'(x) = \lambda^{3/2} \psi(x), \quad A_{\mu}(x) \to A'_{\mu}(x) = \lambda A_{\mu}(x) \tag{21}$$

can be shown to leave the action unchanged [ ]. The Noether current associated with the change of scale is given by

$$J^{\mu}_{scale} = x_{\nu} \theta^{\mu\nu} \tag{22}$$

in which $\theta^{\mu\nu}$ represents the conserved energy-momentum tensor of the theory, $\partial_{\mu} \theta^{\mu\nu} = 0$. The conservation of scale current (22) amounts to the vanishing of the trace of the energy-momentum tensor, that is,

$$\partial_{\mu} J^{\mu}_{scale} = \theta^{\mu}_{\mu} = 0 \tag{23}$$

In D space-time dimensions the trace of massive theory can be cast in the form

$$\theta_{\mu}^{\mu} = \frac{\varepsilon}{4} F^{\eta\sigma} F_{\eta\sigma} + m\overline{\psi}\psi + R(\varepsilon, m, \psi, \overline{\psi}, F^{\eta\sigma}, F_{\eta\sigma}) \tag{24}$$

where the first two terms explicitly highlight the contribution of electron mass and the deviation from four-dimensionality of underlying space-time. All terms vanish in the limiting case $m = 0$ and $\varepsilon = 0$.

It is known that *scale invariance* of the theory can be interpreted as the independence of the action functional from the choice of measurement units. Scale invariance represents a fundamental symmetry of covariant field theories and is broken in SM by the presence of fermion masses or the mass scale of QCD [ ]. Enforcing scale invariance defined by a vanishing trace in (24) implies that electrons gain mass *on account of* deviations from $D = 4$. Since ε is related to the mass scale of the theory M and $\varepsilon \to 0$ is equivalent to $M \to \infty$, the relationship between m and ε amounts to a non-perturbative Renormalization Group flow. The flow equation can be presented as

$$\frac{dm}{d\varepsilon} = \beta_m(m) \tag{25a}$$

Unlike the electromagnetic field tensor, the field tensor of Yang-Mills theory ($F_{\mu\nu}$) depends explicitly on the coupling charge g_{YM}. The "pure" Yang-Mills term in (24) vanishes in four-dimensional space-time ($\varepsilon \to 0$). This means that, when considering *free* Yang-Mills theories in four-dimensional space-time, there are no grounds to invoke a flow equation similar to (25a). This is no longer the case when $\varepsilon \neq 0$ and gauge fields interact with fermions. In this situation, g_{YM} plays a dynamic role similar to m in (24). One is led to a flow equation for g_{YM} having the form

$$\frac{dg_{YM}}{d\varepsilon} = \beta_g(g_{YM}) \tag{25b}$$

5. FROM QUANTUM FIELD THEORY TO FRACTIONAL DYNAMICS

For the sake of clarity, it is instructive to consolidate all arguments developed so far in a mnemonic flowchart. Its purpose is to enable a "bird's eye view" of how description of the Terascale sector of particle physics may evolve from QFT to a framework based on fractional dynamics [ ]. This transition may uncover a new layer of reality with its own set of concepts and rules and it may very well emerge in a variety of unexpected ways.

NONLINEAR QUANTUM FIELDS

⇓

DECOHERENCE

⇓

NONLINEAR CLASSICAL FIELDS

⇓

TRANSITION TO CHAOS AND FRACTALS

⇓

SELF-SIMILARITY

⇓

NON-LOCALITY AND NON-EQUILIBRIUM CONDITIONS

⇓

FRACTIONAL DYNAMICS

The meaning of the flowchart is as follows: SM describes quantum interaction of non-linear gauge fields with matter fields. Decoherence turns quantum fields into their classical counterparts and triggers the irreversible transition to chaos and fractal topology of underlying space-time and phase-space. Self-similarity associated with fractal structures blurs the traditional distinction between "locality" and "non-locality": fractals are identical objects living on infinitely many observation scales. Physical processes on fractals are no longer in stationary conditions but in an ever-evolving and random state of change. Adequate modeling of such processes requires use of fractional dynamics and fractal operators.

6. FRACTIONAL DYNAMICS AND CONSISTENCY CONDITIONS

As it is known, u*nitarity* and *locality* are two fundamental principles that ensure internal consistency of both QFT and SM [ ]. Perturbative QFT relies on a unitary S-matrix formulation, regularization of quantum corrections is required to preserve consistency by suppressing infrared or ultraviolet divergences, introduction of unphysical "ghost" states is mandatory for internal consistency of local gauge field theories. Likewise, since QFT is a manifestly relativistic field theory, locality is mandatory to ensure compliance with Lorentz invariance. In a nut-shell,

- *Unitarity* enforces conservation of probability. It excludes transitions that fail to be norm-preserving as well as negative-norm solutions of field theory.
- *Locality* precludes the possibility of action-at-a distance. Lagrangian is forbidden to contain terms depending on two spatially separated points, for example

$$L_{NL} = \int \varphi(x)\varphi(y)\, d^3x d^3y \quad \text{or} \quad L_{NL} = \int \varphi(x \pm y)\, d^3x d^3y \tag{26}$$

The object of this section is to elaborate upon the relationship between fractional dynamics and these two principles of QFT. To fix ideas, consider the scattering of longitudinally polarized W bosons. The tree-level scattering amplitude computed in SM without the Higgs boson grows with the square of scattering energy and it threatens to violate unitarity around 1 TeV [ ]. The contribution from the Higgs exchange cancels the dangerously growing terms and the full amplitude is well behaving for arbitrary high energies.

The unitarity issue in *WW* scattering at large energies can be, however, approached from a standpoint that goes beyond S-matrix theory. To this end we proceed in two steps:

- We first follow [ ] and indicate the difference between "*transient*" and "*persistent*" scattering. The latter leads to violation of unitarity condition.
- Next, we show how fractional dynamics can be used to restore unitarity of persistent scattering upon a suitable re-definition of probability distribution function.

The probability distribution function $\rho(\mathbf{x},\mathbf{p},t)$ in S-matrix theory is localized in phase space and can be normalized to unity

$$\int d\mathbf{p}\int d\mathbf{x}\,\rho(\mathbf{x},\mathbf{p},t) = (const)\int d\mathbf{p}\int d\mathbf{k}\,\rho_{\mathbf{k}}(\mathbf{p},t)\,\delta(\mathbf{k}) = 1 \tag{27}$$

where $\rho_{\mathbf{k}}(\mathbf{p},t)$ represents the Fourier transform of $\rho(\mathbf{x},\mathbf{p},t)$

$$\rho_{\mathbf{k}}(\mathbf{p},t) = \int \rho_{\mathbf{k}}(\mathbf{x},\mathbf{p},t)e^{-i\mathbf{p}\mathbf{x}}d\mathbf{x} \tag{28}$$

Unitarity can be alternatively expressed as

$$\int d\mathbf{x}\int d\mathbf{p}\,\rho(\mathbf{x},\mathbf{p},t) = (const)\int d\mathbf{x}\int d\ \ \rho_{\lambda}(\ ,t)\,\delta(\ ) = 1 \tag{29}$$

with

$$\rho_{\lambda}(\mathbf{x},t) = \int \rho_{\lambda}(\mathbf{x},\mathbf{p},t)e^{i\mathbf{p}\mathbf{x}}d\mathbf{p} \tag{30}$$

Relations (27) to (30) describe "transient" scattering. Consider now the situation where $\rho(\mathbf{x},\mathbf{p},t)$ is a function which is delocalized in phase space. For example, it fails to vanish either in the infrared limit $|\mathbf{x}| \to \infty$ or in the ultraviolet limit $|\mathbf{p}| \to \infty$. In these asymptotic cases, the Fourier component of $\rho(\mathbf{x},\mathbf{p},t)$ becomes singular at $\mathbf{k}=0$ and $\boldsymbol{\lambda}=0$, respectively, with a delta function singularity. Consider the first case, that is,

$$\lim_{|x|\to\infty} \rho(\mathbf{x},\mathbf{p},t) > 0 \tag{31}$$

The scattering is now "persistent". The Fourier component of the distribution function is singular at $\mathbf{k} = 0$ with a delta-function singularity

$$\rho_{\mathbf{k}}(\mathbf{p},t) = \rho_0(\mathbf{p},t)\,\delta(\mathbf{k}) + \rho_{\mathbf{k}}^{NS}(\mathbf{p},t) \tag{32}$$

in which $\rho_{\mathbf{k}}^{NS}$ is the non-singular part of the distribution function at $\mathbf{k} = 0$. This distribution function cannot be normalized to unity as the square of the delta function and not the delta function enters (27) [ ].

One can employ to the tools of fractional calculus to restore unitarity [ ]. Consider a generic probability distribution function $\rho(x,\lambda)$ depending on parameter λ and defined on one-dimensional Euclidean space E^1, $\rho(x,\lambda) \in L_1(E^1)$. The standard normalization condition corresponding to (27) is given by

$$\int_{-\infty}^{+\infty} \rho(x,\lambda)\,dx = 1 \tag{33}$$

Using (3) we can generalize (33) as follows

$$(I_+^{\alpha}\rho)(y,\lambda) + (I_-^{\alpha}\rho)(y,\lambda) = 1 \tag{34}$$

Fractional equivalent of the normalization condition reads

$$\int_{-\infty}^{+\infty} \overline{\rho}(x,\lambda)\,d\mu_{\alpha}(x) = 1 \tag{35}$$

where

$$\overline{\rho}(x,\lambda) = \frac{1}{2}[\rho(y-x,\lambda) + \rho(y+x,\lambda)] \tag{36}$$

and the Hausdorff measure introduced in section 3 is

$$d\mu_{\alpha}(x) = \frac{|x|^{\alpha-1}}{\Gamma(\alpha)}\,dx \tag{37}$$

Comparing of (36) with (26) shows that the price paid for restoring unitarity in (35) is a manifest loss of *locality*. To restore locality, we note that *self-similarity* of fractals blurs the distinction between observation scales. Taking advantage of this property, one can simply rescale the distance $|x-y|$ below the spatial measurement resolution Δ with no consequence on results. By definition, coordinates x, y are indistinguishable from each other if and only if

$$|(y-x)-(y+x)| = 2|x| \leq \Delta \tag{38}$$

Divide each term in (36) by an arbitrary large scale $s \gg 1$ such that

$$\frac{2|x|}{s} \ll \Delta \tag{39}$$

Using (5) and (6) leads to a local normalization condition, that is

$$\overline{\rho}(\frac{x}{s},\lambda) = \frac{1}{2}[\rho(\frac{y-x}{s},\lambda) + \rho(\frac{y+x}{s},\lambda)] \tag{40}$$

whose outcome is

$$\int_{-\infty}^{+\infty} \overline{\rho}(x/s,\lambda)\, d\mu_{\alpha}(x/s) = \int_{-\infty}^{+\infty} \overline{\rho}(x,\lambda)\, d\mu_{\alpha}(x) = 1 \tag{41}$$

8. PERTURBATIVE RENORMALIZATION ON FRACTAL SPACE-TIME

The concept of space-time endowed with a non-integer metric can be used for perturbative renormalization of QFT. Here we follow [ ] and reproduce a method for renormalization of low-order radiative corrections in quantum electrodynamics (QED) defined on fractal space-time. Consider the full momentum-space propagator S of electron

$$S = \frac{1}{(\gamma p - m_0 - \Sigma + i\varepsilon)} \tag{42}$$

where m_0 stands for the bare electron mass and Σ the proper self-energy. Replacing for Σ its lowest-order contribution yields

$$S(p) = \frac{Z_2}{(\gamma p - m + i\varepsilon)}[1 + Z_2(\gamma p - m)\sigma(p)]^{-1} \tag{43}$$

in which the physical electron mass m and its renormalization constant are given by

$$\begin{aligned} m &= Z_2 m_0 \\ Z_2 &= 1 + \frac{3}{2\pi}\frac{\alpha_{EM}}{(4-D)} \end{aligned} \tag{44}$$

In (43) $\sigma(p)$ is a function defined by (A1.5b) in [ ], α_{EM} is the fine-structure constant and it is assumed that the departure from four space-time dimensionality is small ($4 - D \ll 1$). Expanding the vacuum polarization $\Pi(q^2)$ around the mass-shell $q^2 = 0$, we obtain

$$(Z_3)^{-2} = 1 - \Pi(q^2 = 0) = 1 + \frac{2}{\pi}\frac{\alpha_{EM}}{(4-D)} \tag{45}$$

$$(\alpha_{EM})_0 = \frac{e_0^2}{4\pi} = \alpha_{EM} Z_3^{-2} = \alpha_{EM}[1 + \frac{2}{\pi}\frac{\alpha_{EM}}{(4-D)}] \tag{46}$$

Here, $(\alpha_{EM})_0$ represents the bare fine-structure constant and Z_3 the charge renormalization factor. It can be also shown that, based on the degree of divergence of QED diagrams, singular behavior of some radiative corrections tends to attenuate or vanish for $0 < D \leq 4$.

We close this section with the general observation that, embedding perturbative field theory on fractal space-time, helps reducing or eliminating divergence of momentum integrals. Place (9) on fractal space-time support characterized by the Hausdorff measure in momentum space $\mu_H(E)$ and Hausdorff dimension D. According to definition (7b), singular behavior of the integrand $f(q)$ can be dampened by choosing $D < \alpha < \infty$ which automatically leads to a vanishing Hausdorff measure, that is, $d\mu_H(E) = 0$.

9. GAUGE BOSONS AND FERMIONS ON FRACTALS

This section explores the consequences of placing classical SM fields on fractal space-time supports. This setting may be well-suited to describe conditions developing near or above the EW scale.

9.1) Gauge fields on fractals

One of the most counter-intuitive properties of fractal space-time supports is that they carry a topological form of internal *energy*. This contribution stems from the ability of fractal topology to *polarize space-time* and can be quantified in terms of continuous parameter (14). The net result is that fractal space-time can be modeled as an *effective medium* departing from the passive properties of classical vacuum. For example, classical electrodynamics action on fractals built from effective field quantities reads [ ]

$$S_{eff} = -\int d^4x\left(\frac{1}{4}F_{eff,\mu\nu}F_{eff}^{\mu\nu} + J_{eff}^{\mu}A_{eff,\mu}\right) \tag{47}$$

Action (47) is invariant to local gauge transformations $A_{eff,\mu} \to A_{eff,\mu} - \partial_\mu \theta$ if and only if the fractional continuity equation holds true, that is, if

$$\partial_\mu J_{eff}^{\mu} = \partial_\mu (c_\mu J^\mu) = 0 \tag{48}$$

Here, c_μ are coefficients depending on fractal dimension, as listed in [ ]. Using the language of effective quantities, Lagrangian of the free Maxwell fields on fractals can be presented as

$$L_{eff} = -\frac{1}{4}F_{eff,\mu\nu}F_{eff}^{\mu\nu} = \frac{1}{2}(\mathbf{E}_{eff}^2 - \mathbf{B}_{eff}^2) = \frac{1}{2}(\mathbf{E}^2 - \mathbf{B}^2) + \Delta L \tag{49}$$

where

$$\mathbf{E}^{eff}(\mathbf{r},t) = c_1(\gamma,\mathbf{r})\,\mathbf{E}(\mathbf{r},t) \tag{50}$$

$$\mathbf{B}^{eff}(\mathbf{r},t) = c_2(d,\mathbf{r})\,\mathbf{B}(\mathbf{r},t) \tag{51}$$

$$c_1(\gamma,\mathbf{r}) = \frac{2^{1-\gamma}\,\Gamma(1/2)}{\Gamma(\gamma/2)}\,|\mathbf{r}|^{\gamma-1} \tag{52}$$

Hence the differential contribution of fractality to the Lagrangian is given by

$$\Delta L = L - L_{eff} = \frac{1}{2}[(c_1^2 - 1)\mathbf{E}^2 + (c_3^2 - 1)\mathbf{B}^2] \tag{54}$$

(54) vanishes on smooth space-time $\varepsilon = 0$ however, near $\varepsilon \ll 1$, it may emerge in various forms: it can produce an excess of charges or currents, change the magnitude of the fine-structure constant, generate new particles or make photons massive. The ability of fractal space-time to impart mass to Maxwell fields follows from identification of (54) with the mass term of the Proca Lagrangian [ ]

$$\Delta L = \frac{1}{2}M^2 A_\mu A^\mu = \frac{1}{2}M^2(A_0 A^0 + A_i A^i) \tag{55}$$

Since Proca Lagrangian describes dynamics of a spin-1 massive field, (54) and (55) lead to a novel mechanism of mass generation in the EW sector arising from polarization attributes of fractal space-time. Unlike Proca model which fails gauge invariance, (47) - (49) lead to a gauge invariant theory containing massive gauge bosons.

Using effective quantities, Maxwell's equations on fractals can be cast in their traditional condensed form,

$$\partial_\mu (F_{eff})^{\mu\nu} = (J_{eff})^\nu\,, \quad \partial_\mu (\tilde{F}_{eff})^{\mu\nu} = 0 \tag{56}$$

where [ ]

$$(\tilde{F}_{eff})^{\mu\nu} = \frac{1}{2}\varepsilon^{\mu\nu\rho\sigma}(F_{eff})_{\rho\sigma}$$

$$(F_{eff})^{0i} = -(E_{eff})^{i}, \quad (F_{eff})^{ij} = -\varepsilon^{ijk}(B_{eff})^{k} \tag{57}$$

Let us next generalize these findings and consider coupling of two-component massless Weyl fermions to Yang-Mills fields on fractals. We posit that, near $\varepsilon \approx 0$, fermions pick up infinitesimal corrections from fractal topology which convert massless states into *nearly* massless states, i.e.

$$\Psi_L = \begin{pmatrix} \psi_L(1+\varepsilon) \\ \varepsilon\psi_L \end{pmatrix}, \qquad \Psi_R = \begin{pmatrix} \varepsilon\psi_R \\ \psi_R(1+\varepsilon) \end{pmatrix} \tag{58}$$

The interaction of Yang-Mills fields with a system of massless fermions in four-dimensional space-time is represented by [ ]

$$L = \sum_{\alpha} \overline{\Psi}^{\alpha}[i\gamma^{\mu}(D_{\mu}\Psi)^{\alpha}] - \frac{1}{4}F^{a}_{\mu\nu}F^{a\mu\nu} \tag{59}$$

Using again the language of effective quantities on fractals, we define effective chromo-electric and chromo-magnetic fields as [ ]

$$(E_{eff})^{i}_{a} = (F_{eff})^{i0}_{a}, \ (B_{eff})^{i}_{a} = -\frac{1}{2}\varepsilon_{ijk}(F_{eff})^{jk}_{a}, \quad i,j,k = 1,2,3 \tag{60}$$

The effective Yang-Mills Lagrangian assumes the form

$$(L_{eff})_{YM} = \frac{1}{2}\sum_{a}[(\mathbf{E}_{eff})_{a}\cdot(\mathbf{E}_{eff})_{a} - (\mathbf{B}_{eff})_{a}\cdot(\mathbf{B}_{eff})_{a}] \tag{61}$$

It is seen that (61) contains an extra term due to fractal corrections that can be formally attributed to the emergence of massive gauge fields

$$\boxed{\frac{1}{2}\sum_{a} M_{a}^{2}(A^{a\mu}A^{a}_{\mu}) \to \Delta L_{YM} = (L_{eff})_{YM} - L_{YM}} \tag{62}$$

The coupling term between effective gauge field and Weyl fermions becomes [ ]

$$(L_{eff})_{\text{int}} = g_{eff}\ (A_{eff})^a_\mu [\overline{\Psi}^\alpha \gamma^\mu (T^a)_{\alpha\beta} \Psi^\beta] \tag{63}$$

The difference in interaction terms may be attributed to the emergence of massive fermions, that is

$$\boxed{-m\Psi\overline{\Psi} \to \Delta L_{\text{int}} = (L_{eff})_{\text{int}} - L_{\text{int}}} \tag{64}$$

in which the scalar $\overline{\Psi}\Psi$ is built from the nearly massless Weyl fields introduced in (58).

9.2) Fermions on fractals

We now turn to a model building strategy that highlights how conserved quantities arise on fractals. To this end, consider one of the many fractional generalizations of the free Dirac equation, namely [ ]

$$(A\partial_t^\alpha + B\partial_x)\Psi(t,x) = 0\,, \quad \Psi(t,x) = \begin{pmatrix} \psi_L(t,x) \\ \psi_R(t,x) \end{pmatrix} \tag{65}$$

Here, $0 < \alpha < 1$, I stands for the identity operator and A and B are 2×2 matrices obeying Pauli's algebra

$$A^2 = I\,, \quad B^2 = -I\,, \quad \{A, B\} = AB + BA = 0 \tag{66}$$

Both components of the Dirac field satisfy the fractional evolution equation

$$\partial_t^{2\alpha}\psi_{L,R}(t,x) - \partial_{xx}\psi_{L,R}(t,x) = 0 \tag{67}$$

Lagrangian of the free Dirac field corresponds to $\alpha = 1$ and is given by

$$L_D = \overline{\Psi}A\partial_t\Psi + \overline{\Psi}B\partial_x\Psi \tag{68}$$

or by its equivalent conventional form [ ]

$$L_D = \overline{\Psi}(i\gamma^\mu\partial_\mu)\,\Psi \tag{69}$$

Dirac Lagrangian is invariant under parity transformation since the parity operator turns ψ_L into ψ_R, ∂_x into $-\partial_x$ and $\overline{\sigma}^{\mu}\partial_{\mu}$ into $\sigma^{\mu}\partial_{\mu}$, in which

$$\sigma^{\mu} = (1, \sigma^{i}), \quad \overline{\sigma}^{\mu} = (1, -\sigma^{i}) \tag{70}$$

Here, σ^{i} represent Pauli matrices. Despite the non-local nature of the fractional time operator ∂_t^{α}, it can be shown that (68) leads to a conserved analogue of the Dirac Hamiltonian defined as

$$H_{\alpha}(t, x) = -\int_{-\infty}^{+\infty} \Psi^{T}(AB)\,\partial_x \Psi\, dx \tag{71}$$

<u>10. BREAKING OF DISCRETE SYMMETRIES</u>

Non-local properties of fractal operators prevent invariance under discrete symmetries: in general, fractional Dirac equation (67) fails to stay invariant under space-time transformations [ ]. This property is consistent with the well-known breaking of P and CP symmetries in weak interactions [ ].

Consider for example the Galileian transformation of space-time coordinates

$$t' = t, \quad x' = x + \mathrm{v}t \tag{72}$$

This transformation can be explicitly formulated as

$$\Psi(t, x') = W\,\Psi[t, x(x', t)] \tag{73}$$

where W is a 2×2 operator,

$$W = \begin{pmatrix} \mathrm{w}_{11} & \mathrm{w}_{12} \\ \mathrm{w}_{21} & \mathrm{w}_{22} \end{pmatrix} \tag{74}$$

Invariance of (67) to (72) is preserved if and only if [ ]

$$\psi_L(t, x) = -\left(\frac{\mathrm{w}_{22}}{\mathrm{w}_{21}}\right)\psi_R(t, x) + c(t) \tag{75}$$

in which $c(t)$ represents a constant function of x.

As it is known, the free Dirac equation for massive fermions is given by [ ]

$$(\gamma^{\mu} p_{\mu} - m)\Psi = 0 \tag{76}$$

or, in terms of chiral components

$$\begin{pmatrix} -m & E + \vec{\sigma} \cdot \vec{p} \\ E - \vec{\sigma} \cdot \vec{p} & -m \end{pmatrix} \begin{pmatrix} \psi_R \\ \psi_L \end{pmatrix} = 0 \tag{77}$$

(77) implies that the mass parameter m induces a linear mixing between left and right spinors and a corresponding violation of chiral symmetry, that is,

$$\psi_R = \left(\frac{E + \vec{\sigma} \cdot \vec{p}}{m} \right) \psi_L \tag{78a}$$

$$\psi_L = \left(\frac{E - \vec{\sigma} \cdot \vec{p}}{m} \right) \psi_R \tag{78b}$$

Comparing (78b) with (75) for the trivial case $c(t) = 0$ leads to the identification

$$\frac{\mathrm{w}_{22}}{\mathrm{w}_{21}} = -\frac{E - \vec{\sigma} \cdot \vec{p}}{m} \tag{79}$$

We conclude from (75) to (79) that, imposing Galileian invariance of Dirac equation on fractal space-time (67), yields a *massive* Dirac equation in standard space-time. This finding is consistent with (62) and (64) which show that *massivation* is a direct consequence of placing massless field theories on fractal space-time. As stated, the underlying cause of massivation is that fractals tend to polarize classical space-time vacuum and convert it in an effective medium.

We close this section by recalling that breaking of parity and chiral symmetry violation are related to each other [ ]. Consider the operation of parity in ordinary three-dimensional space,

$$x = (t, \mathbf{x}) \to x_P = (t, -\mathbf{x}) \quad (80)$$

Parity violation is seen to be closely related to breaking of chiral symmetry since

$$\psi_{L,R} \to P\psi_{L,R}P^{-1} = \gamma^0 \psi_{R,L}(x_P) \quad (81)$$

11. EFFECTIVE CHARGES ON FRACTALS AND ANOMALOUS PROPERTIES

Section (9) has shown that all physical quantities become effective on fractal space-time. This includes not only the electric charge but also $SU(2)$ and $SU(3)$ charges associated with gauge field theories. The conserved fermion current on fractals can be written as [ ]

$$j^{\mu}_{eff}(D, \mathbf{r}) = c^{\mu}(D, \mathbf{r})\overline{\psi}\gamma^{\mu}\psi \quad (82)$$

where

$$c^0(D, \mathbf{r}) = c_3(D, \mathbf{r}), \quad c^i(D, \mathbf{r}) = c_2(D, \mathbf{r}), \quad i = 1, 2, 3 \quad (83)$$

(82) leads to a conserved charge

$$\boxed{g_{eff}(D, \mathbf{r}) = \int d^3x c_3(D, \mathbf{r})\overline{\psi}\gamma^0\psi} \quad (84)$$

Effective charge (84) has a different magnitude than its value on ordinary space-time $(D = 4)$. Since the square of gauge charge gives the probability for emission or absorption of virtual particles, it follows that g^2_{eff} may be able to explain, at least in principle, the *excess* of observed leptons, photons, jets that are anticipated to surface in some combination at LHC and other detector sites.

(84) may be also able to fully account for the source of anomalous magnetic moment of massive leptons (AMM). It is known that the magnetic moment $\boldsymbol{\mu}$ of a particle with mass m and charge e is related to the particle spin $\mathbf{S}$ by the gyro-magnetic ratio g [ ]:

$$\boldsymbol{\mu} = g\left(\frac{e}{2m}\right)\mathbf{S} \quad (85)$$

At the tree level, QED predicts the result $g = 2$ for all elementary fermions. Quantum effects produced by QED loop diagrams, from strong and weak interactions or from contributions arising above the electroweak scale lead to a deviation

$$a = \frac{1}{2}(g-2) \tag{86}$$

which measures the magnitude of AMM. Loop corrections from heavy particles with mass M are generally suppressed by a factor $({}^{m}\!/\!_{M})^2$. Therefore the effect of quantum corrections to AMM scales quadratically with the mass of charged leptons. The SM prediction for the muon anomaly, for example, is typically factored into a QED, EW and hadronic (leading and higher order) contributions [ ]

$$a_{\mu}^{SM} = a_{\mu}^{QED} + a_{\mu}^{EW} + (a_{\mu}^{HLO} + a_{\mu}^{HHO}) \tag{87}$$

The difference between a_{μ}^{SM} computed with (87) and the most updated experimental value amounts to $\Delta a_{\mu} = +302(88)\times 10^{-11}$, which is on the order of 3, 4 standard deviations with all errors added in the quadrature. This numerical discrepancy can be attributed to two main sources: an erroneous determination of leading-order hadronic contributions (a_{μ}^{HLO}) or possible corrections induced by physics beyond SM. It is plausible, in this context, that the effective charge (84) may provide an appealing explanation for the muon anomaly. Along the same line of arguments, (84) may alter conventional cross sections and decay rates and favor new phase transitions beyond SM predictions. It appears likely that this scenario plays an important role in the phenomenology of QGP and GLASMA, the multi-muon CDF anomaly, PAMELA excess of positrons, formation of hadronic and leptonic jets in relativistic proton-proton (pp) collisions and so on.

12. GENERATION STRUCTURE OF PARAMETERS

The table shown below is a summary of results published in [ ]. It contains a side-by-side comparison of estimated versus actual mass ratios for charged leptons and quarks, massive gauge bosons and ratios of interaction strengths. All masses are reported in MeV and evaluated at the energy scale set by the top quark mass (m_t). Using recent results issued by the Particle Data Group [ ], we take

$$m_u = 2.12, \quad m_d = 4.22, \quad m_s = 80.9$$

$$m_c = 630, \quad m_b = 2847, \quad m_t = 170{,}800$$

Coupling strengths are evaluated at the scale set by the mass of the "Z" boson, namely

$$\alpha_{EM} = 1/128\ , \quad \alpha_W = 0.0338\ , \quad \alpha_s = 0.123$$

Here, "u", "d", "s", "c", "b" and "t" stand for the six quark flavors, "e", "μ" and "τ" represent the three flavors of charged leptons, "W" and "Z" the two flavors of massive gauge bosons and "α_{EM}", "α_W", "α_s" the coupling strengths associated with the electromagnetic, weak and strong interactions.

Based on the above table, one can nicely recover Koide's formula [ ]

$$3(m_e + m_\mu + m_\tau) = 2(\sqrt{m_e} + \sqrt{m_\mu} + \sqrt{m_\tau})^2 \tag{88}$$

Replacing $m_e = m_\mu \overline{\delta}^{-4}$ and $m_\mu = m_\tau \overline{\delta}^{-2}$ leads to the polynomial equation

$$(1 + \overline{\delta}^{-2} + \overline{\delta}^{-6}) = 4(\overline{\delta}^{-1} + \overline{\delta}^{-3} + \overline{\delta}^{-4}) \tag{89}$$

This equation is solved by a Feigenbaum constant whose numerical value falls near its characteristic value for hydrodynamic flows, namely ($\overline{\delta} = 3.9$).

Parameter ratio	Behavior	Actual	Predicted
m_u / m_c	$\bar{\delta}^{-4}$	3.365×10^{-3}	4.323×10^{-3}
m_c / m_t	$\bar{\delta}^{-4}$	3.689×10^{-3}	4.323×10^{-3}
m_d / m_s	$\bar{\delta}^{-2}$	0.052	0.066
m_s / m_b	$\bar{\delta}^{-2}$	0.028	0.066
m_e / m_μ	$\bar{\delta}^{-4}$	4.745×10^{-3}	4.323×10^{-3}
m_μ / m_τ	$\bar{\delta}^{-2}$	0.061	0.066
M_W / M_Z	$(1-\frac{1}{\bar{\bar{\delta}}})^{1/2}$	0.8823	0.8623
$(\alpha_{EM} / \alpha_W)^2$	$\bar{\delta}^{-2}$	0.053	0.066
$(\alpha_{EM} / \alpha_s)^2$	$\bar{\delta}^{-4}$	4.034×10^{-3}	4.323×10^{-3}

Tab 1: Actual versus predicted ratios of SM parameters

13. GRAVITATION AS DUAL TO FIELD THEORY ON FRACTALS

Consider the fractional analog of a free non-relativistic Hamiltonian system [ ]

$$H(\alpha,\beta)=\frac{\alpha}{2(\alpha+\beta)}\eta_{\mu\nu}(p^{\mu})^{\frac{\alpha+\beta}{2}}(p^{\nu})^{\frac{\alpha+\beta}{2}} \tag{90}$$

A typical embodiment of (90) is the Hamiltonian describing the dynamics of classical free fields on fractal space-time. The case $\beta = \alpha = 1$ recovers the familiar expression for kinetic energy density, namely

$$T_{1,1} = \frac{1}{2} p^{\mu} p^{\nu} \tag{91}$$

Hamiltonian (90) may be cast in the equivalent form

$$H(\alpha, \beta) = \tilde{g}_{\mu\nu}(\alpha, \beta) p^{\mu} p^{\nu} \tag{92}$$

in which

$$\tilde{g}_{\mu\nu}(\alpha, \beta) = \frac{\alpha}{2(\alpha + \beta)} (p^{\mu})^{\frac{\alpha+\beta}{2}-1} (p^{\nu})^{\frac{\alpha+\beta}{2}-1} \tag{93}$$

The action of a minimally coupled classical field in curved space-time is defined as [ ]

$$S = \frac{1}{2} \int \sqrt{-g}\, d^4x\, g^{\mu\nu} p^{\mu} p^{\nu} \tag{94}$$

Direct comparison of (93) and (94) yields the straightforward identification

$$(\sqrt{-g}) g^{\mu\nu} \to 2\tilde{g}^{\mu\nu}(\alpha, \beta) \tag{95}$$

<u>REFERENCES</u> (to be included in the revised paper).