The Dirac Equation in Accelerating Frames

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Abstract

I predict a new translational-rotational coupling in rotating frames that may have been missed hitherto. This eq. 1.8 below should give rise to new physics of clamped charges in rotating capacitors for example. Since accelerating frames are also *locally* equivalent to Newton's gravity force there may be some new quantum mechanical effects here as well.

Einstein eliminated Newton's gravity force and replaced it by force-free motion in curved space-time. This is the meaning of the Einstein equivalence principle and it explains why physicists are confused and stumped when they naively try to unify gravity with electromagnetism and the weak-strong forces.

The theory of symmetry called group theory in mathematics shows that gravity is universal. It is the compensating local gauge field of the symmetries of spacetime. In contrast, electromagnetism and the weak-strong forces are the compensating local gauge fields of the symmetries of the extra dimensions of string-brane theory.

The basic idea of relativity is to compute local invariant observable numbers that are the same for different detector[1] measurements of the actual same events.

SR only works for the above measurements of the same actual events by detectors in unaccelerated motions in a globally flat 4D spacetime. Each detector will not feel g-forces. Of course, the test particles measured may be accelerating.

GR works now for all detectors that are now allowed to accelerate feeling g-forces. However, the local invariant observable numbers are limited to locally coincident detectors. "Coincident" here means that the spacetime separations of a pair of detectors measuring the same actual events must be small compared to the scale of radii of curvature.

The tetrad fields describe the relationships between an unaccelerated "local inertial frame" (LIF) and a coincident accelerating "local non-inertial frame" (LNIF).

Einstein's "general coordinate transformations" called "diffeomorphisms"[2] by the mathematicians are the relationships between two coincident accelerating LNIFs.

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[1] aka "frames of reference"

[2] The mathematician's idea here has excess formal baggage not needed by experimental physicists.

SR works in the LIF. The objective observables must be tensors or spinors of the relevant symmetry-invariance groups connecting locally coincident detectors. A spinor is the square root of a tensor. Each tensor index gets two spinor indices. For example[3]

$$\begin{aligned} &A_I = \sigma_I^{ii'} A_{ii'} \\ &i, i' = 1,2 \\ &I, \mu = 0,1,2,3 \end{aligned} \tag{1.1}$$

Consider a first rank-tensor A^I in the unaccelerated LIF. A coincident accelerating LNIF[4] measures this same tensor observable as A^μ where

$$A^\mu\left(LNIF\right) = e_I^\mu A^I\left(LIF\right) \tag{1.2}$$

The 16 tetrad components are

$$\begin{aligned} &e_I^\mu = \delta_I^\mu + \Xi_I^\mu \\ &\delta_I^\mu \equiv \begin{matrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{matrix} \end{aligned} \tag{1.3}$$

The fundamental gravity fields are really the spin 1 Lorentz group vector fields

$$\Xi^I \equiv \Xi_\mu^I e^\mu \tag{1.4}$$

For an LNIF basis e^μ. The relation to the spin 2 metric tensor gravity waves is

$$\begin{aligned} &ds^2 = \eta_{IJ}\left(LIF\right)e^I\left(LIF\right)e^J\left(LIF\right) = g_{\mu\nu}\left(LNIF\right)e^\mu\left(LNIF\right)e^\nu\left(LNIF\right) \\ &g_{\mu\nu} \equiv \eta_{IJ} e_\mu^I e_\nu^J = \eta_{\mu\nu} + \eta_{IJ}\left(\delta_\mu^I \Xi_\nu^J + \Xi_\mu^I \delta_\nu^J + \Xi_\mu^I \Xi_\nu^J\right) \\ &\eta_{IJ} = \eta_{\mu\nu} = \begin{matrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{matrix} \end{aligned} \tag{1.5}$$

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[3] Of course summation convention on identical pairs of upper and lower indices.

[4] A static LNIF in a spherically symmetric metric field outside the event horizon of a non-rotating black hole is accelerating with $g = c^2 r_s / r^2 \sqrt{1 - r_s/r} \underset{r \to r_s +}{\to} \infty$ in order to stand still at constant Schwarzschild radius r.

where Einstein's fundamental local spacetime differential invariant is ds^2. This is beautiful and it works passing all experimental tests to extraordinary accuracy and precision. Let no mere mortal tear asunder God's design. ☺

SR is based on the 10-parameter Poincare Lie group whose generators of the Lie algebra of their commutators are the total 4-momentum P_I and the spacetime rotations $P_{IJ} = -P_{JI}$. The Poincare group is a subgroup of the 15-parameter conformal group of the invariant light cone with five more generators, i.e. one dilation and four "constant" accelerations.

The 4-momentum operator in the accelerating LNIF of GR is

$$P_\mu = e^I_\mu P_I + \omega^{IJ}_\mu P_{IJ} \tag{1.6}$$

where $\omega^{IJ}_\mu = -\omega^{JI}_\mu$ are the twenty four spin connection coefficients. We have here a qualitatively new physical effect – *the six spacetime rotations induce translational motions*!

When we do particle quantum mechanics, the generators of the Poincare group are matrices operating on the qubit Hilbert spaces that form David Bohm's quantum potential Q. In SR quantum field theory the generators of the Poincare group are integrals of field operators and their gradients over spacelike slices of globally flat Minkowski spacetime. In the case of the electron particle Dirac equation we use the Lorentz invariant scalar

$$\begin{aligned} \not{P} &\equiv \gamma^I P_I \\ \not{P}\psi &= 0 \end{aligned} \tag{1.7}$$

for a zero rest mass charge neutral Dirac 4-spinor ψ and the 4x4 Dirac matrices γ^I.

We get a new effect in the accelerating LNIFs

$$\not{P}(LIF) \to \not{P}(LNIF) = \not{P}(LIF) + e^\mu_{I'}\gamma^{I'}\omega^{IJ}_\mu P_{IJ} + e^\mu_{I'}P^{I'}\omega^{IJ}_\mu [\gamma_I, \gamma_J] \tag{1.8}$$