Contents

On a Generalized Theory *of* Relativity

October 6, 2010

Abstract. The General Theory *of* Relativity (GTR) is essentially a theory of gravitation. It is built on the Principle *of* Relativity. It is bonafide knowledge, known even to Einstein the founder, that the GTR violets the very principle upon which it is founded *i.e.*, it violets the Principle *of* Relativity; because a central equation *i.e.*, the geodesic law which emerges from the GTR, is well known to be in contempt of the Principle *of* Relativity because the geodesic law, must in complete violation of the Principle *of* Relativity, be formulated in special (or privileged) coordinate systems *i.e.*, Gaussian coordinate systems. The Principle *of* Relativity clearly and strictly forbids the existence/use of special (or privileged) coordinate systems in the same way the Special Theory *of* Relativity forbids the existence of privileged and or special reference systems. In the pursuit of a more Generalized Theory *of* Relativity *i.e.*, an all-encampusing unified field theory to include the Electromagnetic, Weak & the Strong force, Einstein and many other researchers, have successfully failed to resolve this problem. In this reading, we propose a solution to this dilemma faced by Einstein and many other researchers *i.e.*, the dilemma of obtaining a more Generalized Theory *of* Relativity. Our solution brings together the Gravitational, Electromagnetic, Weak & the Strong force under a single roof *via* an extension of Riemann geometry to a new hybrid geometry that we have coined the Riemann-Hilbert Space (RHS). This geometry is a fusion of Riemann geometry and the Hilbert space. Unlike Riemann geometry, the RHS preserves both the length and the angle of a vector under parallel transport because the affine connection of this new geometry, is a tensor. This tensorial affine leads us to a geodesic law that truly upholds the Principle *of* Relativity. It is seen that the unified field equations derived herein are seen to reduce to the well known Maxwell-Procca equation, the non-Abelian nuclear force field equations, the Lorentz equation of motion for charged particles and the Dirac equation.

Keywords: Principle *of* Equivalence, Generalized Principle *of* Relativity, reference system, coordinate system, coordinate transformation, gauge transformation, reference transformation.

PACS numbers, (2010): 04.60.-m, 04.90.+e, 11.00.00, 04.60.-m, 04.60.-m, 98.80.-k, 98.80.Cq

"I am not interested in the spectrum of this and that atom ...
I want to know whether God had a choice in building the Universe."

– **Albert Einstein** (1879-1955)

1 Introduction

THIS reading is a second edition, or better, a successor to the reading Nyambuya (2007) where the present *Unified Field Theory* (UFT) was first presented. With better insight gained through the passage of time – in our view; this reading is a significant improvement on its predecessor. It thus replaces it altogether. Over the intervening years, we have sagaciously reflected on this UFT, each time endeavouring to get a complete picture of the *World* that it paints. We believe this picture has now been clearly understood and captured, and we lay it down in the present; in the clearest manner that we possibly can, thus we hope we will be understood by our readers. Further, we are of the view that this latest version of the UFT, is a perdurable one, it is one that can be considered to be standard for work of its kind.

Clearly – it goes without saying that, from a philosophical level, unification of all the forces of *Nature* imply beauty, simplicity and a purpose of design. The dream of unification of all the forces of *Nature* in its present pursuit probably began in 1849 in the Royal Astronomical Society in London with Michael Faraday (1791 – 1867) soon after his great works in electrodynamics when he tried to experimentally find a relationship between the electromagnetic and gravitational force – for obvious reasons that need not be stated here, he succeeded in failing (*e.g.* see Thomas 1991). In his book[1] – *GRAVITY*, George Gamow wrote: In the laboratory of Michael Faraday, who made many important contributions to the knowledge of electricity and magnetism, there is an interesting entry in 1849. It reads:

> *"Gravity. Surely this force must be capable of an experimental relation to electricity, magnetism, and other forces, so as to build it up with them in reciprocal action and equivalent effect. Consider for a moment how to go about touching this matter by facts and trial."*

Gawmow continues: But the numerous experiments this famous British physicist undertook to discover such a relation were fruitless, and he concluded this section of his diary with these words:

> *"Here end my trials for the present. The results are negative. They do not shake my strong feeling of the existence of a relation between gravity and electricity, though they give no proof that such a relation exists."*

As evidenced from the last entry in his laboratory notebook, despite the failure to find this intimate relationship between gravitation and electromagnetism, Michael Faraday unshakably believed that all the forces of *Nature* were but manifestations of a single *universal force* and ought therefore to be inter-convertible into one another in much the same manner as electricity and magnetism. Inspired by Albert Einstein's success to bring to altar (and marry) the Principle *of* Relativity and gravitation, the pursuit to achieve this seemingly elusive dream of unification of the forces of *Nature* remains much alive to the present day and is the theme of the present reading. If what is presented herein is viable and or anything to go-by; or a correct description of physical and natural reality – as we would like to believe; then this reading may well be a significant contribution toward the attainment of this dream.

Regarding the forces of *Nature* as described above, a Unified Field Theory in the physics literature is a theory that proposes to bring any of the four interactions or forces into one coherent and consistent theoretical framework that conforms with experience. A Grand Unified Theory (GUT) is a theory that proposes to bring all the forces with the exception of the gravitational force, into one coherent and consistent theoretical framework and a Theory *of* Everything (TOE) is a theory that proposes to bring all the four forces into one "giant", coherent and consistent theoretical framework which is consistent with basic facts and natural reality. The present attempt is the ambitious attempt on the so-called TOE. The title of the reading clearly suggests that this reading is about a UFT and not about a TOE. We have chosen the modest title "Generalized Theory *of* Relativity" for philosophical[2] reasons that are not necessary to clarify here. We thus persuade the reader to accept this modest title.

[1]George Gamow, "Gravity", Dover Publications; Dover Ed. edition (January 23, 2003), ISBN: 0486425630

[2]We feel that calling a theory a "Theory *of* Everything" is too ambitious, one ought to safe guard themselves from being shipwrecked by the laughter of the gods. A TOE must include everything physics, chemistry, biology *etc*, *i.e.* all known and unknown, how can this be?

Since the renaissance of the dream of a UFT was set-forth in 1925 by Albert Einstein (1879 – 1955) after the emergence of his General Theory *of* Relativity (GTR) and this being a result of Herman Weyl's beautiful, elegant but failed attempt, which was one of the first such on a unification of electromagnetism and gravitation (Weyl 1918), great progress has been made in the effort to achieving a better understanding of the natural *World* on this footing. As said, Herman Weyl embarked on his grandiose work in 1918 after inspiration from Einstein's great works in GTR. The GTR is an elegant and beautiful but incomplete unification theory of the spacetime and matter paradigm. Weyl achieved his theory by pure mathematical reasoning and his effort brought-forth and into being the powerful gauge concept without which the current efforts of unification could not be. To this day, the two forces (gravity and electromagnetism) theoretically stand side-by-side independent of each other and the attempts to bring them together has since been abandoned if not forgotten as a historical footnote.

The GTR is one of the pillars of modern physics and it has not only revolutionised our way of viewing space, time and matter but has also greatly advanced our knowledge insofar as unity of *Nature* is concerned. The search for a unified theory of all the forces of nature has largely continued on a theoretical front and as already mentioned, beginning with Herman Weyl (1918, 1927*a*, *b*) and thereafter followed by Theodore Kaluza (1921), Albert Einstein (1919, 1920, 1921*a*, *b*, 1923, 1928, 1930*a*, *b*, 1945), Oscar Klein (1926), Erwin Schrödinger (1948), Sir Arthur S. Eddington (1921) and many others. These authors sought a unified theory of the gravitational and electromagnetic force because gravitation and electromagnetism – then, were the only forces known to humankind. Latter, with the discovery of the nuclear and sub-nuclear forces, the attempts to unify gravitation with electromagnetism were abandoned by the mainstream physicists with the simple remark that this was a fruitless adventure for the reason that the subatomic forces needed to be taken into account.

The emergence of, or the discovery of the existence of sub-atomic forces marked a new era in the history of physics bringing forth another pillar of modern physics – Quantum Field Theory (QFT). The effort of unification now largely depended on both observations and theoretical insight because the quantum phenomena must be taken into account and this requires counter-intuitive pondering & delicate observations of the quantum phenomena since it is alien to our everyday experience in that it defies common sense. Despite the fact that we don't understand the deeper meaning of the quantum phenomena well over 80 years after the emergence of Quantum Mechanics (QM); unremitting and unwavering attempts on the unification of all the known forces of *Nature* has proceeded undaunted and unabated. Further, this is despite the fact that most if not all efforts to apply the rules applicable to the quantum phenomena to the gravitational phenomena that apply well to the other forces, has brought nothing frustration to the pine-ing physicist.

In the effort of unification, it is believed or supposed that the two key pillars of modern physics – QM and the GTR – behold the secrets to the "final unification program" and these must fuse into one consistent theory but much to the chagrin of the esoteric and curious practitioners in this field, these two bodies of knowledge appear to be fragmently disjoint in that they seem little adapted to fusion into one harmonious, coherent and consistent unified theoretical system. They do not directly contradict – though they have taken physics to the terrains of philosophy and religion because of their adamant refusal to come to the altar and marry. Their marriage is thought to be absolutely essential because it is generally agreed that a complete, unified, and deeper understanding of the natural *World* lies in bringing the two theoretical systems together into one coherent and consistent unified structure since each describe a different *World* – for there to be unity, it is logical that there must be one *World*. It is thus the *dream* of most if not all practising theoretical physicist to find such a system if it is exist to begin with. The belief and faith is that such a system ought to exist in order to preserve beauty, simplicity, an independent reality and harmony in the natural *World*.

The first ever successful UFT was that by the Scottish physicist James Clerk Maxwell (1831 – 1879). He brought the electric and magnetic forces into one theoretical framework (Maxwell 1973). Amongst others, Maxwell's theory showed that light is part and parcel of electricity and magnetism. Maxwell's theory was however not consistent with Newtonian mechanics – a very successful theory at that time. The inconsistency between Maxwellian and Newtonian *World* views lead Einstein to ponder deeper into the intimate relationship between space and time, and by so doing he [Einstein] arrived at a new theory now known as the Special Theory *of* Relativity (STR) (Einstein 1905). Preserving the Maxwellian *World* view, the STR asymptotically overturned the Newtonian doctrine of absolute space and time by proposing that time and space were not absolute as Newton had wanted or postulated, but relative – different observers measure different time lapses and length depending on their relative states of motion. We will elaborate further in §(2) on this. The STR applies to inertial observers and Einstein did not stop there but proceeded to generalize the STR to include non-inertial observers thus arriving at the simple, elegant and all-time beautiful GTR which as presently understood is essentially is a theory of the gravitational phenomena.

Naturally, after the achievement of the GTR, the next task is to bring the other forces within the framework of the GTR or the GTR into the framework of the other forces, which is to bring the GTR into the QM paradigm or the QM paradigm into the GTR. To achieving this, the main thrust amongst the majority of the present day physicist is to seek a GUT, where upon it is thought that ideas to finding a TOE will dawn and shade light on the way forward[3]. Currently, the only successful unification of forces in the micro-*World* is the $1967-68$ theory by Sheldon Lee Glashow, Steven Weinberg & Abdus Salam. They succeeded in showing that the Weak & Electromagnetic force can be brought together into one theoretical framework. Since then, no satisfactory attempts (*i.e.*, experience and theory are in harmony) have come forth. The promising Standard Model *of* Particle Physics is also a good unification of the Weak, the Strong and the Electromagnetic force but many questions, largely theoretical ones, remain unanswered.

According to the popular science media, the most promising theoretical attempts made to date that bring the sub-nuclear forces together including the gravitational force are the theories that embrace the notion of extra dimensions beyond the known four of space and time such as String Theory. It is said by string theory's foremost proponents that this theory offers the best yet clues about a unified theory that en-campuses all the forces of *Nature* and at the sametime it is not understood (*e.g.* Witten 2005). One learn from the popular science press of the present year, that, Edward Witten has moved into including the long forgotten Twister Theory of Roger Penrose into string theory while Stephen W. Hawking is of the view that supergravity may be the way to go. Despite its grandeur and beauty, this shows amongst others, that string theory may not be the paradigm *Nature* has chose to vehicle natural reality.

It is our view and the view shared by many that, the draw-back of theories that employ extra-dimensions is that they do not submit themselves to observations and experience, hence there is little room if any at all, to know whether these theories conform with natural reality. We are of the opinion that no matter how beautiful, elegant and appealing or seductive a theory may be or may appear to be, it ought only to be accepted as a truly physical theory *if and only if* it successfully submits itself to experience, otherwise it remains but an elegant piece of mathematics that is yet to make contact with experience (if at all). Although mathematics is its unquestionable backbone, physics is an experimental science and all its theories must make reasonable contact with observations and experience.

From a purely physical stand-point, there is not much one can say (if anything at all) about ideas based on the notion of higher dimensions since they do not naturally submit themselves to observations & experience and the reason given is that "our collective technology as a human-race has not reached that level where we can submit these theories to experience" or that "the conditions of experience to test these ideas are only found at the unique moment of birth of space and time." As someone that wishes to fathom the mysteries of the natural *World*, we so much would love that string theory be the right theory given its supposed exquisite beauty, elegance and far reaching imagination but at the sametime, we find it hard to forever keep our heard stuck in the superstring sands thereof knowing that there is no way to verifying the theory.

Adding further to highlight the discontentness and or frustration with string theory, Smolin (2006) a leading theoretical physicist, who is a founding member and researcher at the Perimeter Institute for Theoretical Physics is of the opinion that string theory is at a dead end and openly encourages young physicists to investigate new alternatives because there is not much chance that string theory will be verified in the foreseeable future. In fact, he and others argue convincingly that string theory is not even a fully formed theory in the true sense and spirit of a scientific theory but is just but a conjecture because the theory has not been able to prove any of the exotic ideas posited by it.

The discovery of darkenergy and darkmatter he [Smolin] says is not even explained by string theory and is proving troublesome for the theory's foremost advocates. Further, Smolin (2006) writes in his book "*The*

[3]see *e.g.* SALAM A., 1981, Einstein's Last Dream: The Space-Time Unification of the Fundamental Forces, *Physics News*, 12, No 2: Visit http://www.iisc.ernet.in/academy/resonance/Dec2005/pdf/Dec2005p246-253.pdf

Trouble with Physics", that he believes that physicists are making the mistake of searching for a theory that is "beautiful" and "elegant" like string theory but instead they should seek falsifiable theories that can be backed up by experiments. Seeking beauty and elegance in a theory is a philosophy at the centre of Paul Dirac's work (see *e.g.* Kragh 1990) – this is a philosophy which we follow with the important difference that we believe that all ideas that purport to describe the true physical *World*, no matter how elegant and beautiful they may appear, they must naturally submit themselves to experience well within the premises on which these ideas are founded. With regard to beauty, this is what Dirac had to say:

> *"One should allow oneself to be led in the direction which the mathematics suggests ... [and] ... one must follow up a mathematical idea and see what its consequences are, even though one gets led to a domain which is completely foreign to what one started with. Mathematics can lead us in a direction we would not take if we only followed up physical ideas by themselves."*

Furthermore, he went on to say:

> *"I [have] learned to distrust all physical concepts as the basis for a theory. Instead one should put one's trust in a mathematical scheme, even if the scheme does not appear at first to be connected with physics. One should concentrate on getting interesting mathematics [out and into the measurable World]."*

In the spirit of or on the advice of Smolin (2006), we seek a new avenue of thought. We demanded of all the Laws *of* Physics to ***absolutely*** remain invariant and or covariant under both the change of the coordinate system and reference system and more importantly that the physics under a change of the coordinate system remains ***absolutely*** invariant. In this way, we seek to realize fully the Principle *of* Equivalence by extending it to include the physical description of events in any given coordinate system and reference system. Before leaving this section, it is important to mention here that this reading is directed to a specialized audience of "professionals" in the field of unification. We assume the reader has a good access to the STR, the GTR, Quantum Electrodynamics (QED), Quantum Flavor-dynamics (QFD) and Quantum Chromodynamics (QCD).

Further, it is important that we mention that the present work will be followed by two monographs each spanning about 200 pages. These monographs are set to be published as books and the prospective publisher is *NOVA Science Publishers*[4]. The first monograph, hereafter Monograph (I), is a monograph dealing with particle physics in-accordance with the present UFT. In this monograph, a new paradigm of particle physics able to explain a number of open problems in particle physics is ushered in. Parts of Monograph (I) are: Nyambuya (2008, 2009a, b, 2010f). The second monograph, hereafter Monograph (II), is a monograph dealing with a new gravitational paradigm and as in Monograph (I), this paradigm championed in Monograph (II), is in-accordance with the present UFT. Monograph (I) is expected to be completed in the current year while Monograph (II) is expected to be complete in the first quarter of 2011. Parts of Monograph (II) are: Nyambuya (2008, 2010a, b, c, d, e).

So, when the reader goes through this reading, they should consider the present reading as laying down the foundations of new physics and this new physics is to be dealt with at length in Monographs (I & II). What we present here is the essence of the UFT in a nutshell. It is our submission to the reader that the most important task in reviewing this work is that they check thoroughly its mathematical consistency and as as-well the physical legitimacy *vis* that the emergent equations do have resemblance and bearing with physical reality as we know it. As to ourself, we feel rather strongly that this work is a significant contribution to the program of the unification of the forces of *Nature*.

Inclosing this section, the synopsis of this reading is as follows. In the subsequent section, we give a brief exposition of the special and general theory of relativity. In §(3), we highlight one of the greatest setbacks of the GTR and this problem is used as a strong excuse to seek a more generalised theory of relativity. In §(4), make a revision of our treatment of time *vis* our treatment of it when dealing with reference systems

[4] I have approached *NOVA Science Publishers* on this matter and they have agreed to consider my work for publication.

and coordinate systems. In §(5), the generalised theory of relativity is delivered. In §(6), we briefly explore the nature of the symmetries that emerge from the the present UFT. In §(7), we argue that the present UFT does embody within its anatomy the there nuclear forces, the Electromagnetic force, the Weak force and the Strong force. In §(8), we show that the present UFT leads to a geodesic law that does not require that it be formulated in geodesic systems. It is shown that this geodesic law leads to the Lorentz force law. In §(9), we give a general discussion and some of the conclusions drawn from this reading.

2 A Brief Exposition of the Theory of Relativity

In the present section, we give a brief exposition of the theory of relativity, namely, the Special and the General Theory *of* Relativity.

2.1 Special Theory of Relativity

In all history of physics (perhaps in all history of human thought), down from the era of the great thinkers of antiquity such as, Aristotle, Pythagoras, *etc*, to the present day, no other development in physics has radically revolutionised our view of the world more than Einstein's 1905 STR. This theory is based on the idea that motion of a body cannot be defined absolutely, but only relative with respect to others. In his theory, Einstein concluded that the Laws *of* Physics should look alike in any two reference systems, moving with respect to each other with constant velocity. Together with the experimentally confirmed fact that the speed of light is independent of the reference system, this led to the conclusion that time cannot be an absolute quantity, but that time and space constitute a *4*-dimensional unit. Temporal and spatial distances both must depend on the reference system. In essence, the STR was developed so as to iron-out the inconsistencies between Newtonian mechanics and Maxwellian electrodynamics concerning absolute motion and the Laws *of* Physics. The problem at hand was as follows:

(1) After a careful study of the great works of Galileo Galilee (1542−1642), Sir Isaac Newton (1642−1727) founded a body of knowledge that beheld that in moving from one inertial reference system to another time preserved its nature absolutely. That is to say, given the three space dimensions and also that of time – suppose we have two inertial observers (the primed and un-primed) whose space-time coordinates are $(x,\ y,\ z,\ t)$ and $(x',\ y',\ z',\ t')$ respectively, with one moving along the $x-axis$ relative to the other at a speed v, then, the two observers' coordinates intervals are related:

$$\begin{pmatrix} \Delta x' \\ \Delta y' \\ \Delta z' \\ ic\Delta t' \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & -iv/c \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \Delta x \\ \Delta y \\ \Delta z \\ ic\Delta t \end{pmatrix} \tag{1}$$

declared Galileo in his great-works; c here and after denotes the speed of light. Essentially this is the entire conceptual constitution of Newtonian spacetime and the above transformation laws are known as the Galilean Transformation Law (GTL). What the law implies is that (assuming that the law is fundamentally true) all objects in the Universe move relative to one another – there is no such thing as absolute motion. On the other hand, the GTL predicts that, like time $(t' = t)$, acceleration $(a' = d^2x'/dt'^2 = d^2x/dt^2 = a)$ is an absolute quantity. This means that motion is both absolute and relative. This apparent contradiction bothered Newton and lead to many philosophical debates between him and some of his contemporaries – How can motion be relative while acceleration is absolute, is acceleration not some kind of motion or is it a special kind of motion? they pondered in wonderment. Newton proposed that accelerations be measured relative to the immovable absolute space which he identified with the background of the "fixed" stars. We shall not go into this difficult philosophical subject.

(2) In excellent agreement with Thomas Young's famous double split experiment, Maxwell's theory however predicted that light was a wave and in addition to this, Maxwell's theory predicated that the speed of this wave was a fundamental physical constant; this was and is in fragment contradiction with the Newtonian doctrine of motion as understood from, and embodied and encapsulated, in the Galilean transformation law. While this clashed with the Newtonian doctrine, it solved another problem; that of the existence of absolute space (or reference system). That is, if the speed of light were absolute; it [light] ought [in accordance with the Galilean Principle *of* Relativity] to move relative to some universal reference system that is at absolute rest. Also, light being a wave meant it ought to move through some medium – this medium would then naturally explain Newton's doctrine of absolute space (and time as well), so it was thought. This hypothetical medium was then postulated to exist and it was coined the *Aether*. Attempts to detect this aether by measuring the speed of the Earth through its passage suggested that there is no such thing as an aether medium. With the aether having escaped detection by one the finest and most beautiful experiment ever carried out by humankind – *i.e.* the Michelson-Mosley Experiment (MM-Experiment) (Michelson 1881, 1887), theoretical attempts to save the aether paradigm were championed by notable figures such as the great Dutch physicist Hendrick Lorentz (1853 – 1928) amongst others. Lorentz's theory (Lorentz 1895) preserved the aether hypothesis by proposing that the lengths of objects underwent physical length contraction relative to the stationery aether (Lorentz-Fitzgerald contraction) and a change in the temporal rate (time dilation). At that time, this appeared to reconcile electrodynamics and Newtonian physics by replacing the GTL with a new set of transformation laws which came to be known as the Lorentz Transformation Law (LTL). If $\triangle t$, $\triangle x$, $\triangle y$, $\triangle z$ are the time and space separations relative to the absolute, all-stationery and immovable aether and $\triangle t'$, $\triangle x'$, $\triangle y'$, $\triangle z'$ the time and space separations in the moving frame (speed v), then:

$$\begin{pmatrix} \Delta x' \\ \Delta y' \\ \Delta z' \\ ic\Delta t' \end{pmatrix} = \begin{pmatrix} \Gamma & 0 & 0 & iv\Gamma/c \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -iv\Gamma/c & 0 & 0 & \Gamma \end{pmatrix} \begin{pmatrix} \Delta x \\ \Delta y \\ \Delta z \\ ic\Delta t \end{pmatrix} \tag{2}$$

where: $\Gamma = \left(1 - v^2/c^2\right)^{-1/2}$. The above are the LTL. Indirectly and, after much careful and sagacious pondering on the negative result of the MM-Experiment: by considering the apparent contradictions between Newtonian and Maxwellian electrodynamics; with a leap of *faith* and **boldness**, Einstein cut the Gordian knot and then untied it thereafter by the following reasoning; *If* – he asked; we accept the Laws *of* Electromagnetism as fundamental and also we accepted Newtonian Laws of motion as fundamental, then there ought not to be a contradiction when Newtonian Laws of motion are applied to inertia reference system in which the Electrodynamic Laws hold good and when these laws are transformed to an equivalent reference system within the framework of Newtonian mechanics. Either of the two must be at fault or both. Newtonian mechanics, then had stood the test of time – for nearly 250 years it passed all the experimental tests to which it was submitted to, and it was almost taken for granted as a self evident truth, an axiom of science and a natural tautology of *Nature*, so much that the celebrated British physicist and philosopher, Lord Kelvin, was amongst other prominent and highly esteemed thinkers of his time; so confident of Newtonian mechanics that he proclaimed before the turn of the past century that:

"There is nothing new to be discovered in physics now.
All that remains is more and more precise measurement."

We know now, he was not right – Einstein was to soon unequivocally demonstrate this durable fact which is now well anchored in the annals of science history as one of the major landmarks of twentieth century physics. Also, the emergence of the quanta made the words of Lord Kelvin fall flat on their face, they crumbled under the weight of their own short sight. This demonstrates that the ocean of knowledge is too vast to make all-sweeping conclusions about physical reality. Sure, when the horizons of knowledge look well understood, some new knew knowledge must be eminent.

On the other hand electrodynamics was a new field where more elaborate experiments to confirm it where yet to be carried out. It is here that Einstein boldly & faithfully cut sharply though the thick dark clouds hovering over the horizon of science, chopping and un-tieing the Gordian knot by upholding electrodynamics as more fundamental than Newtonian mechanics and thus went on to replace it with a new mechanics by putting forward the following two postulates:

(1). *The Laws of Physics are the same for all inertial frames of reference in uniform relative motion.*

(2). *The speed of light in free space is the same for all inertial observers.*

The first postulate, known also as the Principle *of* Relativity, dispels the notion that there is such a thing as a preferred or absolute reference system. The Laws *of* Physics must be the same in equivalent reference systems. Inertial reference systems have the same status of motion in that Newton's first Law holds good in them. If the first postulate were true and Maxwell's theory were a fundamental theory of *Nature*, then the second postulate follows immediately since Maxwell's theory predicts explicitly that the speed of light has a definite numerical value. The constancy of the speed of light predicted here lead us *via* Einstein's great insight to rethink our view of space and time. Time for different frames of reference runs at different rates and lengths are not absolute but depend on the observers state of motion. The LTL follow immediately from these two postulates but with the important difference that the aether hypothesis is not any longer necessary.

This is the entire conceptual content of the STR. Einstein was not satisfied with the STR because it only dealt with observers in uniform relative motion and he wanted to know how the Laws *of* Nature manifest themselves in the case of non-inertial observers and the quest for an answer to this question culminated in the GTR (Einstein 1915).

2.2 General Theory of Relativity

The problem with non-inertial observers is that gravitation becomes a problem since it is an all pervading "non-vanishing force". By analysing the motion of a body in free-fall in a gravitational field, Einstein was able to overcome the problem of gravitation by noting that if gravitational mass (m_g) and inertia mass (m_i) were equal or equivalent, then gravitation and acceleration are equivalent too (Einstein 1907). Because of the importance of this, it came to be known as the Principle *of* Equivalence. This meant that the effect(s) of acceleration and gravitation are the same – one can introduce or get rid of the gravitational field by introducing acceleration into the system. The deep rooted meaning of the Principle *of* Equivalence is that Physical Laws should remain the same in a local reference system in the presence of a gravitational field as they do in an inertial reference system in the absence of gravitation. In Einstein's own words:

Principle *of* Equivalence: *"We shall therefore assume the complete physical equivalence of a gravitational field and the corresponding acceleration of the reference system. This assumption extends the Principle of Relativity to the case of uniformly accelerated motion of the reference system."*

A consequence of this is that no mechanical or optical experiment can locally distinguish between a uniform gravitational field and uniform acceleration. It is here that we would like to point out that the Principle *of* Equivalence as used in the formulation of the GTR does not demand that the physics must remain invariant. By "***the physics***" we mean that the description of a physical event ought to remain invariant unlike for example in black-hole physics – depending on the coordinate system employed (and not the reference system – **this is important**), a particle can be seen to pass or not pass through the Schwarzschild sphere for the same observer supposedly under the same conditions of experience. Also the chronological ordering of events is violated – *i.e.*, the Law *of* Causality is not upheld. For example, in a rotating Universe as first pointed-out by the great mathematician and philosopher, Kant Gödel (1949); it is possible to travel back in time meaning to say it is possible in principle to violate the Second Law *of* Thermodynamics. Though the idea of time travel is very fascinating and appealing to the mind, it is difficult to visualize by means of binary logical reasoning how it can work in the Physical *World* as we know it. From intuition, the Laws *of* Nature must somehow have it deeply engraved and embedded in them the non-permissibility of time travel.

Therefore, we must demand that the physics, that to say, the physical state and chronological ordering of events, must remain invariant – *i.e.*, extend the Principle *of* Equivalence to include the physical state or physical description of events and the Law *of* Causality. Because this must be universal and important, let us call the extended Principle *of* Equivalence to what we shall coin the Generalized Principle *of* Relativity:

Generalized Principle *of* Relativity: *Physical Laws have the same form in all equivalent reference systems independently of the coordinate system used to express them and the complete physical state or physical description of an event emerging from these laws in the respective reference systems must remain absolutely and independently unaltered – i.e. invariant and congruent; by the transition to a new coordinate system.*

This forms the basic guiding principle of the present theory. The deeper meaning of the Generalized Principle *of* Relativity is that if one is describing the same physical event in spacetime *e.g.* a blackhole, it should not be permissible to transform a singularity by employing a different set of coordinates as is common place in the study of the Schwarzchild metric of spacetime. If the singularity exists, it exists independently of the coordinate system and reference system used – it is intrinsic and permanent. Therefore if we are to have no singularities, the theory itself must be free of these. If a particle is seen not to pass through the event horizon, it will not be seen to pass the event horizon no matter the coordinate system employed and the reference system to which the current situation is transformed into.

Now, back to the main vein; the Principle *of* Equivalence is in the context of Riemann geometry, mathematically embodied in the mathematical expression:

$$g_{\mu\nu;\sigma} = g_{\mu\nu,\sigma} + \Gamma^{\lambda}_{\sigma\mu} g_{\lambda\nu} + \Gamma^{\lambda}_{\sigma\nu} g_{\mu\lambda} = 0, \tag{3}$$

where $g_{\mu\nu}$ is the metric tensor describing the geometry of space-time and:

$$\Gamma^{\lambda}_{\mu\nu} = \frac{1}{2} g^{\lambda\alpha} \left\{ g_{\alpha\mu,\nu} + g_{\nu\alpha,\mu} - g_{\mu\nu,\alpha} \right\}, \tag{4}$$

are the affine connections or the Christoffel symbols (first defined in the reading Christoffel 1869). The affine connections play an important role in that they relate tensors between different reference systems and coordinate systems. Its draw back insofar as Physical Laws are concerned is that it is not a tensor. It transforms as:

$$\Gamma^{\lambda'}_{\mu'\nu'} = \frac{\partial x^{\mu}}{\partial x^{\mu'}} \frac{\partial x^{\nu}}{\partial x^{\nu'}} \frac{\partial x^{\lambda'}}{\partial x^{\lambda}} \Gamma^{\lambda}_{\mu\nu} + \frac{\partial x^{\lambda'}}{\partial x^{\lambda}} \frac{\partial^2 x^{\lambda}}{\partial x^{\mu'} \partial x^{\nu'}}. \tag{5}$$

The extra term on the right makes it a non-tensor and without it, the Christophel symbol would be a tensor. Most of the problems facing the GTR can be traced back to the non-tensorial nature of the affine connections – some of the problems will be highlighted in the succeeding section. Due to the nature of these affinities, the real problem is that in its bare form, Riemann geometry does not provide a way to determine permissible and non-permissible coordinate and reference system transformations. The new hybride geometry on which the UFT being championed is built, does have a way to determine permissible and non-permissible coordinate and reference system transformations and this will be seen in §(6).

Now, both the invariance and covariance of Physical Laws under a change of the coordinate system and or reference system transformation is, in Riemann geometry encoded and or expressed through the invariance of the line element:

$$ds^2 = g_{\mu\nu} dx^{\mu} dx^{\nu}. \tag{6}$$

The line element is a measure of the distance between points in spacetime and remains invariant under anykind of transformation of the reference system and or the coordinate system. This is the essence of the GTR. From this, Einstein was able to deduce that gravitation is and or can be described by the metric tensor $g_{\mu\nu}$, thus, according to the Einstein doctrine of gravitation, it [gravitation] manifests itself as the curvature of space-time. Through his [Einstein] own intuition & imagination, he was able to deduce that the curvature of space-time ought to be proportional to the amount of matter-energy present – a fact that has been verified by numerous experiments. The resulting law emerging from Einstein's thesis is:

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = \kappa T_{\mu\nu} + \Lambda g_{\mu\nu}, \tag{7}$$

which is the well known Einstein's Field Equation *of* Gravitation where:

$$R_{\mu\nu} = \Gamma^{\lambda}_{\mu\nu,\lambda} - \Gamma^{\lambda}_{\mu\lambda,\nu} + \Gamma^{\lambda}_{\mu\sigma}\Gamma^{\sigma}_{\lambda\nu} - \Gamma^{\lambda}_{\nu\sigma}\Gamma^{\sigma}_{\lambda\mu}, \tag{8}$$

is the contracted Riemann curvature tensor and $T_{\mu\nu} = \varrho v_\mu v_\nu + p g_{\mu\nu}$ is the stress and energy tensor where ϱ is the density of matter, p is the pressure and v_μ the four velocity, $\kappa = 8\pi G/c^4$ is the Einstein constant of gravitation with G being Newton's universal constant of gravitation, c the speed of light and Λ is the controversial and so-called cosmological constant term *ad hoc-ly* added by Einstein so as to stop the Universe from expanding (Einstein 1917). Einstein was motivated to include the cosmological constant because of the strong influence from the astronomical wisdom of his day that the Universe appeared to be static and thus was assumed to be so. Besides this, the cosmological constant fulfilled Mach's Principle (Mach 1893), a principle that had inspired Einstein to search for the GTR and he thus thought that the GTR will have this naturally embedded in it – to his dissatisfaction, the GTR did not exactly fullfil this in the manner Einstein had envisaged. Mach's principle forbids the existence of a truly empty space and at the sametime supposes that the inertia of an object is due to the induction effect(s) of the totality of all-matter in the Universe.

Einstein's equation of gravitation, *i.e.* (7), for a given mass-energy distribution, it tells spacetime how to curve, and vise-versa *i.e.*, for a given spacetime geometry, it tells how mass-energy must be distributed. To complete Einstein's theory, we need to have a law that tells particles how to move on any given geometry of spacetime. This laws is called the geodesic law, it will be introduced in the next section. Equation (7) and the geodesic law make up Einstein's GTR. As will be argued in the next section, the geodesic law violets the Principle *of* Relativity, a principle upon which the GTR is founded. Our quest is to find a geodesic law that does not violet the Principle *of* Relativity, and this means seeking a more generalized theory of relativity where the affine has a tensor form.

3 Problem & Quest

In our view, the major problem that the GTR faces is that it is based on pure Riemann geometry – a geometry that is well known to violate the Principle *of* Equivalence at the affine level because the affine connections are not tensors. If pure Riemannian geometry is to be the true geometry to describe the natural *World*, then, no Laws *of* Physics should exist at the affine level of Riemann geometry. However, this is not so, since the Geodesic Law:

$$\frac{d^2x^\lambda}{ds^2} + \Gamma^{\lambda}_{\mu\nu}\frac{dx^\mu}{ds}\frac{dx^\nu}{ds} = 0, \tag{9}$$

that describes the path and motion of particles in spacetime emerges at the affine level. Thus accepting Riemann geometry as a true geometry of *Nature* means we must accept contrary to the Principle *of* Relativity that there exists in *Nature* preferred reference system and coordinate system because the above Geodesic Law leads us to formulating the equation of motion in prefaced reference systems and coordinate systems, namely, geodesic coordinate systems also know as Gaussian coordinate systems. Gaussian coordinate systems are those coordinate systems such that $g_{\mu\nu,\sigma} = 0$. It can be shown for example that given a flat space-time in which say the rectangular coordinate system (where $g_{\mu\nu,\sigma} = 0$ holds) are used to begin with; where [in the rectangular coordinate system] the affine vanish identically in this system and changing the coordinate system to spherical, the affinities do not vanish. Further, the scalar $v_\lambda \dot{v}^\lambda$ is not a scalar in the GTR. The dot over the four velocity, *i.e.* $\dot{v}^\lambda$ represents the time derivative hence $\dot{v}^\lambda$ is the four acceleration. One can verify that $v_\lambda \dot{v}^\lambda$ is not a scalar by taking the term involving the affine in (9) to the otherside of the equality sign and then multiplying bothsides by v_α and thereafter contracting the indices ($\lambda = \alpha$). After the said operations, we will have on the left-handside of the equation a scalar and on the right a pseudo-scalar – how can this be? This is a serious desideratum, akin to the Newton-Maxwell conundrum prior to Einstein's STR – a conundrum of how to reconcile or comprehend the apparent contradiction of the prediction of Maxwell's theory's that demanded that the speed of light be a universal and absolute speed and the Galilean philosophy of relativity that there is no such thing as a universal and absolute speed in the Universe.

Given for example, that the affinities represent forces as is the case in the GTR, this means a particle could be made to pass from existence into non-existence (or vise-versa) by simply changing the coordinate system. This on its own violates the Laws *of* Logic and the need for *Nature* to preserve an independent reality devoid of magic. For this reason, there is a need to ask:

> *"What exactly do we mean by a coordinate system and reference system and what relationship should these have to Physical Laws so that the Generalized Principle of Relativity is upheld?"*

This shall constitute the subject of the next section. Clearly, the only way out of this conundrum is to seek – as Einstein, Schrödinger *etc* have done; a theory in which the affinities have a tensor form hence in the present approach, the first and most important guide is to seek tensorial affinities. Einstein, Schrödinger *etc* have made attempts along these lines only to fail. The reason for their failure may perhaps stem from the fact that theirs was a pure mathematical exercise to try to find a set of tensorial affinities from within the framework of the classical spacetime of Riemannian geometry.

4 Nature of Time

> *"Absolute, true, and mathematical time, of itself, and from its own nature, flows equable without relation to anything external ..."*
>
> – **Sir Isaac Newton** (1642 – 1727)

We already know from the STR that time does not transform absolutely when dealing with different reference systems and this was Einstein's radical new idea that changed forever our view of time and the order of the natural *World*. We should say, we are not about to change this but simple "*clip the wings*" of our use of this idea when dealing with *coordinate systems* (not *reference systems*). We ask here the question whether or not the time coordinate is invariant under a change of the coordinate system? The answer to this question shall provide an answer to the question paused in the preceding section namely, whether or not it is right that the change of a coordinate system should lead to a change in the physics as happens in blackhole physics. In conclusion, we shall establish that time – *viz* when transforming between different coordinate systems – is a scalar quantity and this manifests itself as a self-evident-truth beyond any doubt whatsoever. In order that we accomplish our mission in this part of this reading, it is necessary that we begin by defining succinctly what we mean by reference system and coordinate system – these two are used interchangeably in most textbooks of physics. While this is a trivial thing, the understanding of what is a reference system and coordinate system is key to the present presentation.

For example, starting with the Schwarzschild metric; Stephani (2004), in his effort of trying to describe events near and at the event horizon of in blackhole, goes on to say "We seek coordinate systems which are better adapted to the description of physical processes ...". This is nothing more than an admission that physics in different coordinate systems will be different – there exist coordinate systems that are unsuitable for the description of physical events. Why should this be so? Physics and or physical processes should never be dependent on the choice of coordinates – at the very least, this is in-contempt of the sacrosanct Principle *of* Relativity. Let us devote some little time to understanding what is a coordinate system and a reference system and thereafter look deeper into the meaning of what these really are. *Prima facie*, this exercise to make an introspection of what a coordinate system and a reference system really are may appear naïve, nonetheless, we believe it is a necessary exercise.

Coordinate System: When thinking about space, it is extremely useful to think of it as constituting of points, each labelled so that one can distinguish one point from another – each point is and must be unique. These labels are called coordinates. One must choose these labels in such a way that it is easy to manipulate. In practice, numbers are used because we understand and can manipulate them. To manipulate these labels, a universal and well defined rule must be set out so as to label and manipulate the labels and this is what is called the *Coordinate System*. One ought to be free to choose any coordinate systems of their choice provided the labelling scheme makes each point to be unique because any space exists independent of the coordinate systems used. Examples of coordinate systems are the spherical coordinates (denoted: r, θ, ϑ), rectangular (denoted: x, y, z), cylindrical (denoted: r, θ, z) and curvilinear (denoted: x_1, x_2, x_3) to mention but a few. The coordinate itself is thought to have no physical significance but only its relative distance from other coordinates is what is of physical significance. Due to Minkowski's brilliant insight, we must add a forth dimension (t) in order to label the arena where physical events take place *i.e.*, for spacetime where spherical coordinates are used to label space, we have (r, θ, ϑ, t), and likewise for rectangular spacetime coordinates we have (x, y, z, t) *etc*. The question is, for example when we have to make a transition from say rectangular spacetime coordinates to say spherical spacetime coordinates (r, θ, ϑ, t), do we have the right to alter the forth dimension? We shall provide an answer to this in a short-while.

Reference System: After having chosen a system of coordinates of our liking, suppose we station an observer at every-point of space. For any given coordinate systems (rectangular, spherical, curvilinear *etc*) there exists a point that one can call the point of origin, this point can be any-point, there ought not to be a preferred point. In the usual three dimensions of space, this point is the point (0, 0, 0) – this choice gives the easiest way to manipulate the coordinates. Once the observer has set the $(0, 0, 0)$ point, they will set up about this point $(0, 0, 0)$, their axis and the set of axis then constitutes the *Reference System*. The observer that has declared their point of origin and has set their reference system "sees" every other point relative to the (0, 0, 0) point thus this point is their point of reference which together with the set of axis is in the usual language of STR is the reference system. The reference system thus provides one with a reference point $(0, 0, 0)$ and a set of axes relative to which the observer can measure the position and motion of all other points in spacetime as seen in other reference systems.

The above defines a reference system and we hope the reader is able to make a clear distinction between the two – that is a coordinate system and reference system. It follows that the STR is concerned with nature of Physical Laws under a change of the reference system, *i.e.*, from one-point of spacetime to another depending on these points's state of motion while the GTR is concerned with nature of Physical Laws under both a change of the coordinate system and reference system. The STR posits that the Laws *of* Physics remain the same for observers in uniform relative motion with the GTR positing through the Principle *of* Equivalence that even for observers in uniform relative acceleration the Laws *of* Physics remain the same and these are the same as for those observers in uniform relative motion. The GTR goes further and extends this to encamps different coordinate system by maintaining that the Laws *of* Physics remain invariant under a change of coordinate system. We will point out here a logical flew in the GTR in its endeavours to be a beacon and paradigm that describes Natural Laws under general coordinate and reference system transformations. This is deeply rooted in its treatment of time under a change of the coordinate system. The logical flew lies in the equal-footing treatment of the space and time coordinates applicable to the STR or to transformations between different but equivalent reference systems being unconsciously extended to describe natural processes under a change of the coordinate system. Let us look closely at the coordinate transformation law:

$$\Delta x^{\mu'} = \left(\frac{\partial x^{\mu'}}{\partial x^{\mu}}\right)\Delta x^{\mu}. \tag{10}$$

Lets pluck out the time coordinate, *i.e.,* $\mu' = \mu = 0$. From pure logic, one must know that changing the way we label points in space should not alter the geometry of space nor the flow of time between the previous labels and the new labels – this is not debatable. For example, changing the name of a street should not physically alter the street itself. However, from (10), it follows that a time difference of $\triangle t'$ in the primed coordinate system is related to the time lapse $\triangle t$ in the un-primed coordinate system by:

$$\triangle t' = \left(\frac{\partial x^{0'}}{\partial x^0}\right)\triangle t + \left(\frac{\partial x^{0'}}{\partial x^1}\right)\triangle x^1 + \left(\frac{\partial x^{0'}}{\partial x^2}\right)\triangle x^2 + \left(\frac{\partial x^{0'}}{\partial x^3}\right)\triangle x^3. \tag{11}$$

Clearly, if:

$$\left(\frac{\partial x^{0'}}{\partial x^0}\right) \equiv 1, \ \left(\frac{\partial x^{0'}}{\partial x^1}\right) \equiv \left(\frac{\partial x^{0'}}{\partial x^2}\right) \equiv \left(\frac{\partial x^{0'}}{\partial x^3}\right) \equiv 0, \tag{12}$$

then, $\triangle t' \equiv \triangle t$, thus time will flow equably between the two coordinate systems, but if (12) does not hold identically, then this means for different coordinate systems, time moves at different rates! That is to say, if say one is somewhat fade up of say the way time behaves in their rectangular spacetime coordinates (x, y, z, t), they can choose say a spherical system of coordinates $(r, \theta, \vartheta, \tilde{t})$ where time $(t, \tilde{t})$ between these two coordinate systems flows un-equably and in a manner that best suits the desideratum of their own heart?!

A very good example of this is in blackhole physics where the Schwarzchild singularity is treated not as a physical singularity, but as a mathematical singularity. Eddington (1924) and latter Finkelstein (1958), proposed that the Schwarzchild singularity can be transformed away by moving from one set of spherical coordinates $(r, \theta, \vartheta, t)$ to a new set of spherical coordinates $(r, \theta, \vartheta, \tilde{t})$, if we transformed the time coordinate from t to $\tilde{t}$ *i.e.*:

$$\tilde{t} = t + \left(\frac{R_s}{c}\right)\ln\left(\frac{r}{\mathcal{R}_s} - 1\right), \tag{13}$$

where $\mathcal{R}_s > 0$ is the Schwarzchild radius of the blackhole. The above time coordinate leads to the time interval transforming as:

$$d\tilde{t} = dt - \frac{1}{c}\left(1 - \frac{r}{\mathcal{R}_s}\right)^{-1} dr \Longrightarrow dt = d\tilde{t} + \frac{1}{c}\left(1 - \frac{r}{\mathcal{R}_s}\right)^{-1} dr, \tag{14}$$

and in-turn this leads to the usual Schwarzchild metric which when written in spherical coordinates is:

$$ds^2 = \left(1 - \frac{\mathcal{R}_s}{r}\right)c^2dt^2 - \left(1 - \frac{\mathcal{R}_s}{r}\right)^{-1} dr^2 - r^2 d\theta^2 - r^2 \sin^2\theta d\vartheta^2, \tag{15}$$

to transform as:

$$ds^2 = \left(1 - \frac{\mathcal{R}_s}{r}\right)c^2d\tilde{t}^2 - \left(1 + \frac{\mathcal{R}_s}{r}\right)dr^2 - r^2 d\theta^2 - r^2 \sin^2\theta d\vartheta^2 - \left(\frac{2\mathcal{R}_s}{r}\right)cd\tilde{t}dr, \tag{16}$$

where now, *prima facie*, the singularity does not seem to be present at $r = \mathcal{R}_s$; it has been concealed in the apparent mist of the anti-labyrinth of the mathematics thereof. It has been said of such treatments, that they provide a good basis to study the spacetime structure and around the blackhole. By changing the coordinate systems, as we do in space when we change from rectangular coordinates (x, y, z) to spherical coordinates (r, θ, ϑ), the transformation (13) is not a different way of labelling the time axis, it is a physical change, a change of the actual geometry of spacetime. In (13), we have *time dilation* intimately associated with the way in which we label points in spacetime?! This transformation is conducted under the guise of a coordinate transform yet it is a change of the geometry. Herein lies one of central problems of the GTR:

This means a photon can be blue or red-shifted by just changing the system of coordinates!

Red or blue-shifting is a physical process but changing of the system of coordinates is not a physical process at all! Here we have it - this is the source of our problems in our endeavours to completely understand *Nature* from the current GTR view-point especially when it comes to blackholes, we alter the time-coordinate so as to read ourself of singularities under the guise of legitimate coordinate transformations; but in so doing we are making a physical alteration and not a an alteration of the way we label spacetime. Clearly, the only way in which a photon's physical state will remain invariant is if time preserved its nature under a change of the coordinate systems. This could mean time is not a vector but a scalar when it comes to coordinate transformations. If time behaved as predicted by equation (11) with $\partial x^{0'}/\partial x^0 \neq 1$ and $\partial x^{0'}/\partial x^j \neq 0$, it could mean all physical events in spacetime are affected by a change of the coordinate systems and as already stated it means the way in which we label points does has a realizable physical significance?! This on its own makes no physical or logical sense, it constitute a serious desideratum – it allows for *pure magic* to occur in physics *i.e.*, one would choose at will a coordinate systems of their liking and they would give a different description from that of another observer that employs a different set of coordinates of the same physical phenomena or event in spacetime. A priori to this analysis and also a posteriori justified execution, is that, it is absolutely necessary that we put forward the following **Protection Postulate** so as to uphold the Generalized Principle *of* Relativity:

Postulate I: *In order to preserve the physical state and the chronological evolution of a physical system when making a transition from one coordinate system to another, of itself, and from its own nature, time must flow equable without relation to anything external, it must remain invariant under any kind of transformation of the coordinate system.*

It is not difficult to show that if a particular or all spatial coordinates where to transform in a non-linear manner with respect to the corresponding coordinate, events and or points in spacetime will cease to be unique and also the physics is altered just by changing the coordinate system! In order to strictly preserve the physics and second to preserve the uniqueness of events when a transition to a new coordinate system is made, it is necessary to put forward another protection postulate:

Postulate II: *In order to preserve the physics when a transition to a new coordinate system is made and for this same transition to preserve the uniqueness of physical events in spacetime, the points in the new coordinate system for a non-periodic coordinate system, must be linear and have a one-to-one relation with the old one and in the case of a periodic coordinate system the periodicity is ignorable.*

Linearity has a two-fold meaning here: (1) suppose in a transformation of the coordinate system from A to B a point in the coordinate system A has more than one corresponding coordinate for a non periodic coordinate system like spherical coordinate system (this periodicity can be ignored because it does not physically place the point to another point in the same space), then in such a coordinate transformation, events cease to be unique and this must be guarded against – hence the second postulate.

Mathematically speaking, the first postulate means that when it comes to coordinate transformations, time is a scalar quantity, *i.e.*, **for** a coordinate transformation and **not** a transformation of the reference system:

$$\left(\frac{\partial x^{0'}}{\partial x^0}\right) \equiv 1, \text{ and } \left(\frac{\partial x^{j'}}{\partial x^0}\right) \equiv \left(\frac{\partial x^{j}}{\partial x^{0'}}\right) \equiv \left(\frac{\partial x^{j'}}{\partial x^{0'}}\right) \equiv \left(\frac{\partial x^{j}}{\partial x^0}\right) \equiv 0. \tag{17}$$

We thus have established here that time must behave as a scalar when transforming from one system of spacetime coordinates to another and this is not so when transforming from one reference system to another. Because of this, let us adopt the terminology **coordinate scalar** or **coordinate vector** to mean a quantity behaves as a scalar under a coordinate transformation and likewise we will have a **frame scalar** and **frame vector** to mean a quantity that transforms as a scalar or vector when transforming from one reference system to the other.

5 Theory

We shall [successfully] seek a geometry whose affinities are tensors. This geometry is a union of the respective geometries on which quantum and classical physics are founded. Quantum Mechanics is defined on a Hilbert space (or Hilbert geometry) while classical physics is founded on the classical spacetime of Riemannian geometry. The resultant theory that we shall set-forth can be viewed as an improvement on Weyl's failed attempt (Weyl 1918, 1927a, b). Weyl added a scalar (ϕ) to the Riemannian metric ($g_{\mu\nu}$) such that $g_{\mu\nu} \longmapsto \phi g_{\mu\nu}$. Weyl's scalar is defined as $\phi = \oint A_\mu dx^\mu$ where A_μ is what Weyl identified with the electromagnetic vector potential. This scalar gave raise to a supplementary affine connection:

$$W^{\lambda}_{\mu\nu} = \frac{1}{2}\left(\delta^{\lambda}_{\mu}A_{\nu} + \delta^{\lambda}_{\nu}A_{\mu} - \delta_{\mu\nu}A^{\lambda}\right). \tag{18}$$

This supplementary affinity is tensorial in nature thus it did not solve the non-tensorial nature of the Christophel symbol. However, what was deeply profound and all-inspiring about Weyl's theory is that like Maxwell's theory of electromagnetism, it was invariant under the transformation:

$$A_\mu \longmapsto A_\mu + \frac{\partial\chi}{\partial x^\mu}, \tag{19}$$

where $\chi = \chi(x)$ is some arbitrary scalar field. This very fact is what lead Weyl to believe that his theory was a union of the gravitation and electromagnetism. Other than the fact that Weyl's theory had no correspondence with reality, its greatest draw back was that the resultant affinities retained the old non-tensorial nature of the Christophel symbol. Instead of making ϕ a scalar, we here are going to demand that it be not a scalar but a mathematical object that leads to the resultant affinities that are tensors. This is a constraint to be met and fulfilled.

As pointed out by his [Weyl's] great contemporaries Einstein, Wolfgang Pauli and others, that Weyl's theory was beautiful but had no resemblance with the measured world but went against it, Weyl was at pains to look away from the beauty he saw in his theory and this perhaps can best be understood from one of his famous words:

> *"My work always tried to unite the truth with the beautiful, but when I had to choose one or the other, I usually chose the beautiful."*

Actually, though he did not further publish on his theory after 1929, Weyl never really looked away from it, but passionately believed that it contained a beauty that oneday would blossom and one of the blossoms was the gauge concept without which all modern efforts at a unification can not have been without Weyl's gauge concept.

Now, let us begin by defining these two geometries.

Hilbert Space: Given the object $\psi = \sum_{j=0}^{\infty} c_j\psi_j$, then, every inner product $\langle,\rangle$ on a real or complex vector space $\mathcal{H}$ gives rise to a norm:

$$ds^2_H = \langle\psi,\psi\rangle = \psi^\dagger\psi = \sum_{j=0}^{\infty} c^\dagger_j c_j, \tag{20}$$

where $\psi^\dagger_i\psi_j = \delta_{ij}$ and the space $\mathcal{H}$ is said to be a Hilbert space if it is complete with respect to this norm. Completeness in this context means that any cauchy sequence of elements of the space converges to an element in the space, in the sense that the norm of differences approaches zero.

Riemann Spacetime: A space is said to be Riemannian if the norm is invariant under a coordinate transformation such that the metric of the space satisfies the fundamental theorem of Riemann geometry, that is the covariant derivative equation (3) resulting in the definition of the affine connection as given by equation (4).

From these spaces as defined above, one can by a closer inspection of the Riemann geometry imagine a union of both the Riemann and Hilbert space. Let us coin this space the Riemann-Hilbert Space (RHS). This space is some-kind of a Riemann Space in its formulation with it embedded the Hilbert objects that gives the space the necessary machinery to overcome the criticism levelled earlier against pure Riemann geometry that of the affinities being non-tensorial.

Riemann-Hilbert Spacetime: If the metric tensor is defined $g_{\mu\nu} = \boldsymbol{e}_\mu \cdot \boldsymbol{e}_\nu$ then, for the ordinary flat spacetime geometry of Minkowski where $g_{\mu\nu} = \eta_{\mu\nu}$, the unit vectors that would give this metric are the four objects:

$$\boldsymbol{e}_0 = i\begin{pmatrix} -i \\ 0 \\ 0 \\ 0 \end{pmatrix}, \quad \boldsymbol{e}_1 = i\begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \quad \boldsymbol{e}_2 = i\begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}, \quad \boldsymbol{e}_3 = i\begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}. \tag{21}$$

Notice that the components or length of the axis unit vectors are all constants – why is this so? Is it really necessary that they become constants and at the sametime is it really necessary that the significant component of these unit vectors be equal? Just for a minute, suppose we set up a 3D system of coordinates in the usual space that we inhabit with three orthogonal axis. Let each of these axes have an observer, say X monitors the $x - axis$ and Y monitors the $y - axis$ and like wise Z monitors the $z - axis$. Along each of these axis the observer can define a unit length and it need not be equal to that of the others. Having defined their unit length to compare it with that of the others, they will have to measure the resultant vector which is the magnitude of the vector sum of the three "unit" vectors along their respective axis. This setting does not affect anything in the physical *World* for as long as one commits to mind that the unit vectors along each of the axis are different and they have in mind the length of the resultant unit vector. This little picture tells us we can have variable unit vectors along each of the axis *i.e.*:

$$\boldsymbol{e}_0 = i\begin{pmatrix} \psi_0 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \quad \boldsymbol{e}_1 = i\begin{pmatrix} 0 \\ \psi_1 \\ 0 \\ 0 \end{pmatrix}, \quad \boldsymbol{e}_2 = i\begin{pmatrix} 0 \\ 0 \\ \psi_2 \\ 0 \end{pmatrix}, \quad \boldsymbol{e}_3 = i\begin{pmatrix} 0 \\ 0 \\ 0 \\ \psi_3 \end{pmatrix}, \tag{22}$$

where $\psi_j = \psi_j(x)$ for $j = 0, 1, 2, 3$ real variable functions. If as usual the position vector in this space is given by $\mathbf{X} = x^\mu \boldsymbol{e}_\mu$ where x^μ is the usual spacetime coordinate in Riemann geometry, then, it is not difficult for one to see that the resulting metric from the above set of unit vectors will be diagonal, the meaning of which is that all the off-diagonal terms will equal zero. We must in general be able to obtain a metric with non-zero components and not only diagonal as is the case if the unit vectors are as given in equation (22).

The important and general idea to be drawn from the above thesis, is that there is no reason to stick to static unit vectors, we can have them as space and time variables. The kind of variable unit vectors that we seek here are ones that will at the end of the day, enable us to explain from a purely geometric standpoint, the forces and particles of *Nature*. There is no way for me to explain to the reader how we have arrived at the choice of the unit vectors that we shall soon present. All we can say is that, we noted that in quantum mechanics, particles are described by the spinor ψ, and in the Standard Model, the force fields are described by the vector A_μ and gravitation can be described by a scalar potential ϕ. All this information we in-cooperated into the unit vector. We should say that, we did not just randomly do this, but over a time spanning well over about ten years or so, we meditated on these matters and, intuition guided me to this final choice. At the end of the day, one must be able to come up with a model that fits measured facts and it strongly appears to me that the choice that we arrived at, does just that. This choice of the unit vectors that we discovered is:

$$\boldsymbol{e}_{\mu}^{(a)} = \frac{1}{2} i\phi A_{\mu}\gamma_{\mu}^{(a)} \begin{pmatrix} \psi_0 \\ \psi_1 \\ \psi_2 \\ \psi_3 \end{pmatrix} = \frac{1}{2} i\phi A_{\mu}\gamma_{\mu}^{(a)}\psi, \tag{23}$$

where $\gamma_{\mu}^{(a)}$ is a set of three (hence the index $a = 1, 2, 3$) 4×4 matrices with $\gamma_{\mu}^{(1)} = \gamma_{\mu}$ being the usual 4×4 Dirac matrices and:

$$\gamma_0^{(2)} = \begin{pmatrix} \mathrm{I} & 0 \\ 0 & -\mathrm{I} \end{pmatrix}, \quad \gamma_i^{(2)} = \tfrac{1}{2} \begin{pmatrix} 2\mathrm{I} & i\sqrt{2}\sigma_i \\ -i\sqrt{2}\sigma_i & -2\mathrm{I} \end{pmatrix}, \tag{24}$$

and:

$$\gamma_0^{(3)} = \pm \begin{pmatrix} \mathrm{I} & 0 \\ 0 & -\mathrm{I} \end{pmatrix}, \quad \gamma_i^{(3)} = \mp\tfrac{1}{2} \begin{pmatrix} 2\mathrm{I} & i\sqrt{2}\sigma_i \\ -i\sqrt{2}\sigma_i & -2\mathrm{I} \end{pmatrix}, \tag{25}$$

and $\phi \neq 0$ is a zero-rank scalar and $A_{\mu} \neq 0$ is a component of a four vector, I is the 2×2 identity matrix, and σ_i are the usual 2×2 Pauli matrices.

Now, from the above, we have: $\left(ds^{(a)}\right)^2 = \boldsymbol{e}_{\mu}^{(a)\dagger}\boldsymbol{e}_{\nu}^{(a)} dx^{\mu}dx^{\nu}$, it follows that: $\left(ds^{(a)}\right)^2 = \rho\varphi g_{\mu\nu}^{(a)} dx^{\mu}dx^{\nu}$ where:

$$g_{\mu\nu}^{(a)} = \frac{1}{\rho\varphi} \left\{\boldsymbol{e}_{\mu}^{(a)\dagger}, \boldsymbol{e}_{\nu}^{(a)}\right\}, \tag{26}$$

and $\{,\}$ is the usual anti-commutation bracket and this anti-commutation is in the indices, $\rho = \psi^{\dagger}\psi$ and $\varphi = \phi^{\dagger}\phi$. From all the above, it follows that the metric tensor is given by:

$$g_{\mu\nu}^{(a)} = A_{\mu}^{(a)\dagger} A_{\nu}^{(a)}, \tag{27}$$

and if this metric tensor is to be symmetric as it must, then $A_{\mu}^{(a)} = A_{\mu}^{(a)\dagger}$, hence $A_{\mu}^{(a)}$ must be a real function. Written in full the three metrics are:

$$\left[g_{\mu\nu}^{(1)}\right] = \begin{pmatrix} A_0^{(1)}A_0^{(1)} & 0 & 0 & 0 \\ 0 & -A_1^{(1)}A_1^{(1)} & 0 & 0 \\ 0 & 0 & -A_2^{(1)}A_2^{(1)} & 0 \\ 0 & 0 & 0 & -A_3^{(1)}A_3^{(1)} \end{pmatrix}, \tag{28}$$

and:

$$\left[g_{\mu\nu}^{(2)}\right] = \begin{pmatrix} A_0^{(2)}A_0^{(2)} & A_0^{(2)}A_1^{(2)} & A_0^{(2)}A_2^{(2)} & A_0^{(2)}A_3^{(2)} \\ A_1^{(2)}A_0^{(2)} & -A_1^{(2)}A_1^{(2)} & A_1^{(2)}A_2^{(2)} & A_1^{(2)}A_3^{(2)} \\ A_2^{(2)}A_0^{(2)} & A_2^{(2)}A_1^{(2)} & -A_2^{(2)}A_2^{(2)} & A_2^{(2)}A_3^{(2)} \\ A_3^{(2)}A_0^{(2)} & A_3^{(2)}A_1^{(2)} & A_3^{(2)}A_2^{(2)} & -A_3^{(2)}A_3^{(2)} \end{pmatrix}, \tag{29}$$

and:

$$\left[g_{\mu\nu}^{(3)}\right] = \begin{pmatrix} A_0^{(3)}A_0^{(3)} & -A_0^{(3)}A_1^{(3)} & -A_0^{(3)}A_2^{(3)} & -A_0^{(3)}A_3^{(3)} \\ -A_1^{(3)}A_0^{(3)} & -A_1^{(3)}A_1^{(3)} & -A_1^{(3)}A_2^{(3)} & -A_1^{(3)}A_3^{(3)} \\ -A_2^{(3)}A_0^{(3)} & -A_2^{(3)}A_1^{(3)} & -A_2^{(3)}A_2^{(3)} & -A_2^{(3)}A_3^{(3)} \\ -A_3^{(3)}A_0^{(3)} & -A_3^{(3)}A_1^{(3)} & -A_3^{(3)}A_2^{(3)} & -A_3^{(3)}A_3^{(3)} \end{pmatrix}. \tag{30}$$

It is seen that the metric $g^{(3)}_{\mu\nu}$ is simple the metric $g^{(2)}_{\mu\nu}$ under the transformation $A_k \longmapsto -A_k$. Also, we note that the metric $g^{(a)}_{\mu\nu}$ is invariant under $A_\mu \longmapsto -A_\mu$.

The line element equation, $\left(ds^{(a)}\right)^2 = \rho\varphi g^{(a)}_{\mu\nu} dx^\mu dx^\nu$; is similar in form to that for the scalar-tensor theories of gravity in which ρ is a pure scalar quantity (Brans 1961) or in Wely's brilliant but failed attempt on a unified of 1918. scalar tensor theories are an alternative theory to Einstein's GTR whose endeavour is similar to the present, *i.e.*, incorporate or unify quantum phenomena with the gravitational phenomena.

Unlike scalar-tensor theories, the object ρ shall here be chosen such that it is not a scalar as in Brans-Dicke Theory or Weyl's 1918 UFT. This choice of ρ affords us the opportunity and the economy to un-chain ourself from the bondage of non-tensorial affinities (as will be seen shortly) because we can forcefully choose this object in such a way that the resultant affine connections are tensors. Comparing the RHS with Riemann geometry and demanding that in the limiting case, *i.e.,* $\rho = 1$, the RHS reduces to the well known Riemann space – this would require, that we make the substitution $g_{\mu\nu} \longmapsto \tilde{g}^{(a)}_{\mu\nu} = \rho\varphi g^{(a)}_{\mu\nu}$ into equation (3), *i.e.*:

$$g^{(a)}_{\mu\nu;\sigma} = \rho\varphi\left(g^{(a)}_{\mu\nu,\sigma} + \bar{\Gamma}^\lambda_{\sigma\mu} g^{(a)}_{\lambda\nu} + \bar{\Gamma}^\lambda_{\sigma\nu} g^{(a)}_{\mu\lambda} + Q_\sigma g^{(a)}_{\mu\nu} + G_\sigma g^{(a)}_{\mu\nu}\right) = 0, \tag{31}$$

where $Q_\mu = \partial_\mu \ln\rho$ and $G_\mu = \partial_\mu \ln\varphi$ and $\bar{\Gamma}^\lambda_{\mu\nu}$ is the new affine connection. From this equation, one can deduce that:

$$\bar{\Gamma}^\lambda_{\sigma\nu} = \Gamma^\lambda_{\sigma\nu} + \mathrm{Q}^\lambda_{\sigma\nu} + \mathrm{G}^\lambda_{\sigma\nu}, \tag{32}$$

where $\Gamma^\lambda_{\sigma\nu}$ is the usual Christophel affine connection and $\mathrm{G}^\lambda_{\sigma\nu}$ is a new tensorial connection given by:

$$\mathrm{G}^\lambda_{\sigma\nu} = \frac{1}{2} g^{(\lambda\alpha}_{(a)}\left\{g^{(a)}_{\alpha\sigma} G_\nu + g^{(a)}_{\nu\alpha} G_\sigma - g^{(a)}_{\sigma\nu} G_\alpha\right\}, \tag{33}$$

while $\mathrm{Q}^\lambda_{\sigma\nu}$ is also a new but non-tensorial affine connection given by:

$$\mathrm{Q}^\lambda_{\sigma\nu} = \frac{1}{2} g^{\lambda\alpha}_{(a)}\left\{g^{(a)}_{\alpha\sigma} Q_\nu + g^{(a)}_{\nu\alpha} Q_\sigma - g^{(a)}_{\sigma\nu} Q_\alpha\right\}. \tag{34}$$

Now, taking advantage of the fact that the liberty is wholly ours to make a proper choice of ψ, let us seize the moment and demand (here and now) as set out in §(3) that the affine connection ($\bar{\Gamma}^\lambda_{\mu\nu}$) be a tensor. We will achieve this by making a suitable choice of ρ. We shall also require that our choice be such that the object ψ be defined on the Hilbert space – the subtle aim being to identify this object with the quantum mechanical spinor wavefunction. First things first, it is clear that if we envisage the material field to be defined by the Dirac wavefunction, then ρ can not be a scalar. If it is a scalar, this reduces the theory to a theory much akin to Weyl's un-successful unified theory (Weyl 1918, 1927*a*, *b*) and at the same time, the inclusion of the scalar field φ will be rendered void.

We note that if Q_μ is chosen such that it transformations:

$$Q_{\mu'} = \frac{\partial x^\mu}{\partial x^{\mu'}} Q_\mu - 2\frac{\partial^2 x^\lambda}{\partial x^\lambda \partial x^{\mu'}} \tag{35}$$

this would lead to $Q^\lambda_{\mu\nu}$ to transform as:

$$Q^{\lambda'}_{\mu'\nu'} = \frac{\partial x^\mu}{\partial x^{\mu'}}\frac{\partial x^\nu}{\partial x^{\nu'}}\frac{\partial x^{\lambda'}}{\partial x^\lambda} Q^\lambda_{\mu\nu} - \frac{\partial x^{\lambda'}}{\partial x^\lambda}\frac{\partial^2 x^\lambda}{\partial x^{\mu'}\partial x^{\nu'}}. \tag{36}$$

The above transformation law clearly and immediately verifies the fact that the affine connection, $\bar{\Gamma}^\lambda_{\mu\nu}$, is indeed a tensor. At this point, we have achieved with relative ease to obtain tensorial affinities and thus the task now is to obtain physically meaningful field equations that conform with natural reality. Before leaving this section, we must find the transformation properties of the object ρ and this will have to be done from (36). From this we see that if $\psi' = S\psi$ where S is some 4×4 transformation matrix; then, from this transformation equation ($\psi' = S\psi$) and (36) we will have to have: $\rho' = \Phi\rho$, where:

$$\Phi = \exp\left[-2\int\left(\frac{\partial^2 x^\lambda}{\partial x^\lambda \partial x^{\mu'}}\right)dx^{\mu'}\right]. \tag{37}$$

For $\rho' = \Phi\rho$ to hold, this would require that: $S^\dagger S = \Phi\mathcal{I}$, where here and after $\mathcal{I}$ is the 4×4 identity matrix. This means a constraint is placed on how the object ψ can transform. We shall argue in §(6) that $\Phi \equiv 1$, leading to the fact that S can only be a unitary hermitian matrix.

Before leaving this section, we must ask the question "What kind of geometric object would submit to the transformation (35)?" It is not difficult to see that if $Q_\mu = -\partial^\nu g_{\mu\nu}$, then, the transformation (35) is satisfied hence the geometric object $Q_\mu = -\partial^\nu g_{\mu\nu}$ submits to the transformation (35). If $Q_\mu = -\partial^\nu g_{\mu\nu}$, then it follows that:

$$\rho = \exp\left(-\int\frac{\partial g_{\mu\nu}}{\partial x^\nu}dx^\mu\right). \tag{38}$$

Since $\rho = \psi^\dagger\psi$ is a function of the wavefunction, what (38) means is that the wavefunction has here been described as a geometric object. Much in line with the above, it is important that we mention that it was Einstein's desideratum (to turn wood into marble), that in the final theory, matter must submit to a geometric description *i.e.*, the quantum mechanical wavefunction must be a function of the metric.

5.1 Force Fields

Given the metric: $g^{(a)}_{\mu\nu} = A^{(a)}_\mu A^{(a)}_\nu$, and plugging this into the Christoffel symbol, one obtains:

$$\Gamma^{(a)\lambda}_{\mu\nu} = \frac{1}{2}\left[\partial_\nu\left(A^{(a)\lambda}A^{(a)}_\mu\right) + \partial_\mu\left(A^{(a)}_\nu A^{(a)\lambda}\right) - \partial^\lambda\left(A^{(a)}_\mu A^{(a)}_\nu\right)\right]. \tag{39}$$

Now, differentiating the products in the brackets and rearranging, one obtains:

$$\begin{array}{rcl} 2\Gamma^{(a)\lambda}_{\mu\nu} & = & A^{(a)}_\mu\partial_\nu A^{(a)\lambda} + A^{(a)\lambda}\partial_\nu A^{(a)}_\mu + A^{(a)\lambda}\partial_\mu A^{(a)}_\nu + \\ & & A^{(a)}_\nu\partial_\mu A^{(a)\lambda} - A^{(a)}_\nu\partial^\lambda A^{(a)}_\mu - A^{(a)}_\mu\partial^\lambda A^{(a)}_\nu \end{array}, \tag{40}$$

and further re-arranging, one will have:

$$\begin{array}{rcl} \Gamma^{(a)\lambda}_{\mu\nu} & = & \frac{1}{2}A^{(a)}_\mu\left(\partial_\nu A^{(a)\lambda} - \partial^\lambda A^{(a)}_\nu + \left(A^{(a)}_\mu\right)^{-1}\left[A^{(a)\lambda}\partial_\nu A^{(a)}_\mu\right]\right) + \\ & & \frac{1}{2}A^{(a)}_\nu\left(\partial_\mu A^{(a)\lambda} - \partial^\lambda A^{(a)}_\mu + \left(A^{(a)}_\nu\right)^{-1}\left[A^{(a)\lambda}\partial_\mu A^{(a)}_\nu\right]\right) \end{array}. \tag{41}$$

The above can be written as:

$$\Gamma^{(a)\lambda}_{\mu\nu} = \frac{1}{2}A^{(a)}_\mu F^{(a\nu)\lambda}{}_\nu + \frac{1}{2}A^{(a)}_\nu F^{(a\nu)}{}_\mu{}^\lambda, \tag{42}$$

where:

$$F^{(a\nu)}_{\mu\nu} = \overbrace{\partial_\mu A^{(a)}_\nu - \partial_\nu A^{(a)}_\mu}^{\textbf{Abelien-term}} + \overbrace{A^{(a)}_\mu \left(A^{(a)}_\lambda\right)^{-1} \partial_\nu \left(A^{(a)}_\lambda\right)}^{\textbf{non-Abelien term}}, \tag{43}$$

which has the form of a non-Abelian field – *i.e.*, the *Yang-Mills Field*. The λ is – just like the a in the bracket in the superscript; not an active index but a label informing us that, in the partial derivative ∂_μ appearing in the non-linear term, the vector being differentiated is the one with this index-λ, *i.e.*, $A^{(a)}_\lambda$.

In-order for (43) to have the exact form of the Yang-Mills field, there is need to introduce a gauge constraint. There are two such gauge constraints and the first is:

$$\partial_\mu A^{(a)}_\nu = g_{(\mu)} A^{(a)}_\mu A^{(a)}_\nu, \tag{44}$$

where $g_{(\mu)}$ is a constant parameter. In $g_{(\mu)}$, we have put the Greek index in the brackets to symbolize that it is not an active but a dummy index. The above gauge choice leads directly to:

$$F^{(a)}_{\mu\nu} \quad = \quad \partial_\mu A^{(a)}_\nu - \partial_\nu A^{(a)}_\mu + g_{(\nu)} A^{(a)}_\mu A^{(a)}_\nu \quad . \tag{45}$$

Notice that in the above, λ drops out from $F^{(a\lambda)}_{\mu\nu}$. The second choice of the gauge constraint is:

$$\partial_\mu A^{(a)}_\nu = g_{(\nu)} A^{(a)}_\mu A^{(a)}_\nu, \tag{46}$$

which leads to:

$$F^{(a\lambda)}_{\mu\nu} \quad = \quad \partial_\mu A^{(a)}_\nu - \partial_\nu A^{(a)}_\mu + g_{(\lambda)} A^{(a)}_\mu A^{(a)}_\nu \quad . \tag{47}$$

Notice again; in the above, λ does not drop out from $F^{(a\lambda)}_{\mu\nu}$, this invariably means there are four different configurations of this field and each is identified by the parameter $g_{(\lambda)}$. With regard to the two fields *i.e.*, (45) and (47), at this moment, we do not know for sure which gauge corresponds to reality and why. One thing is clear though, these gauge choices lead us to a field that bare a very close resemblance with the Yang-Mills field.

Clearly, Yang Mills theory appears here to arises naturally on the Riemann-Hilbert spacetime, and its origins is intimately connected with the nature of the spacetime geometry and more the fact that $g_{\mu\nu} = A_\mu A_\nu$. From this, we are lead to believe that the origins of Yang-Mills theory can be sought and perhaps explained from the nature of the Riemann-Hilbert Spacetime and the nature of the metric $g_{\mu\nu} = A_\mu A_\nu$.

To the above question of which of the two fields (45) and (47) corresponds to reality, in an effort to close on this question, we will argue in favour of (47). The choice (45) allows the particle to have just one configuration of the Yang-Mills field. This means, exclusively, it can have either the electromagnetic force field, the weak force field, or the strong force field. This does not appear to fit reality because for example, the Proton does carry the pure Abelien electromagnetic field and along with this field, it also carries the strong force field. If we choose (45), one can not explain this, but the choice (47) can, since it allows a particle to carry four fields.

The second reason for advocating for the choice (46), is as follows. Let us assume the Lorentz gauge $\partial^\mu A^{(a)}_\mu = 0$. First, if $\partial^\mu A^{(a)}_\mu = 0$, then, from (46), we will have:

$$\partial^\mu A^{(a)}_\mu = g^{(\mu)} A^\mu_{(a)} A^{(a)}_\mu = g^{(0)} A^0_{(a)} A^{(a)}_0 + g^{(1)} A^1_{(a)} A^{(a)}_1 + g^{(2)} A^2_{(a)} A^{(a)}_2 + g^{(3)} A^3_{(a)} A^{(a)}_3 = 0. \tag{48}$$

We had to write down the full expression of $g^{(\mu)} A^\mu_{(a)} A^{(a)}_\mu$ so as to avoid one mistakenly thinking that $g^{(\mu)} A^{(a)\mu} A^{(a)}_\mu = 0$ means $g^{(\mu)} A^{(a)\mu} = A^{(a)}_\mu = 0$. Second is that, if the Lorentz gauge $\partial^\mu A^{(a)}_\mu = 0$ is given, then differentiating (46) by ∂^μ, one arrives at:

$$\Box A_{\mu}^{(a)} = \kappa_{(\mu)}^{2} A_{\mu}^{(a)}, \tag{49}$$

where $\kappa_{(\mu)}^{2} = g_{(\mu)}^{2} A_{(a)}^{\alpha} A_{\alpha}^{(a)} = 4g_{(\mu)}^{2}$. It is clear that comparing the above with the standard equations of electromagnetism, $g_{(\mu)}$ is a mass term for the particle carrying the field $A_{\mu}^{(a)}$. If one where to choose the gauge (44), then, given the Lorentz gauge $\partial^{\mu} A_{\mu}^{(a)} = 0$, they will be unable to arrive at (49). From these simple arguments which off cause require much more strong grounds to stand on, we believe this should be substantial motivation for our choice of gauge (46).

Now, at this point, we have managed to obtain the 'tensor' field normally associated with the nuclear forces, the task from hereon is to find the field equations that correspond to reality – *i.e.*, equations that describe what we know about the forces of *Nature*.

If we are permitted to, then we should say that, the task of tracking the many indices in our search for the resultant field equations will perhaps appear tedious, but this could never compare to the task of arriving at the idea of how to arrive at the non-Riemann geometry that is capable of yielding this very result of a metric whose components are described by the four potential $A_{\mu}^{(a)}$. Given that in the successful and dominate paradigm of gravitation – *i.e.* Einstein's GTR, the 10 metric components all describe the gravitational field, to leap from this and conceive (anew) of this very metric being described by just four objects and these objects themselves describing not the gravitational field, but the electromagnetic field and other nuclear forces, it is not an easy leap as one first has to unchain themselves from the bondage of the highly successful and dominate paradigm of old. Having arrived at this point ourself, it took us quite a while to finally move on from the Einsteinian view of the metric tensor. As Dirac advised *i.e.* "One should allow oneself to be led in the direction which the mathematics suggests . . . [and] . . . one must follow up a mathematical idea and see what its consequences are". Finding strength and wisdom in Dirac's words, we moved on and followed the mathematics to where it would lead us. We did this with the great hope that Einstein's wood would soon be turned into marble. Whether or not this has been achieved, we leave it up to the reader to decide.

5.2 Metric Tensor of the Riemann-Hilbert Space

Let B_{α}, C_{β}, D_{γ} *etc*, be tensors and let $T_{\alpha\beta\gamma} = B_{\alpha} C_{\beta} D_{\gamma}$ be a product tensor as defined. We can lower and raise the indices of the tensor $T_{\alpha\beta\gamma}$ using the metric $g_{\mu\nu}$, that is:

$$T^{\alpha}_{\beta\gamma} = g^{\alpha\nu} T_{\nu\beta\gamma}. \tag{50}$$

Notice that: $g^{\alpha\nu} T_{\nu\beta\gamma} = A^{\alpha} A^{\nu} B_{\nu} C_{\beta} D_{\gamma} = A^{\alpha} \left(A^{\nu} B_{\nu}\right) C_{\beta} D_{\gamma}$. What this means is the metric vector A^{ν} in $g^{\alpha\nu}$ has gone onto the remove the index ν and the A^{α} in $g^{\alpha\nu}$ has gone into restoring the dimensionality of the tensor *albeit*, with a raised index. What this means is that the metric vector A^{ν} can be used to contract a tensor and as-well, increase its dimensions, *i.e.*:

$$T_{\beta\gamma} = A^{\alpha} T_{\alpha\beta\gamma}, \tag{51}$$

$$T_{\beta\gamma\gamma\lambda} = T_{\alpha\beta\gamma} A_{\lambda}. \tag{52}$$

We do not need the whole metric tensor to do the raising of an index and then latter contracting; all we need is just the metric vectors A_{μ}.

Further, since A_{μ} is a vector its magnitude can be fixed to unity i.e $A_{\mu} A^{\mu} \equiv 1$. Now given that $A_{\mu} = A_{\mu} A_{\nu} A^{\nu} = g_{\mu\nu} A^{\mu} = A_{\mu} \left(A_{\nu} A^{\nu}\right)$, it follows that:

$$A_{\mu} = A^{\mu}. \tag{53}$$

It is important to keep these facts that we have presented in this section at hand, because in some of the cases, we will assume them without any mention or notice.

5.3 Lorentz Gauge

In the derivations of the field equations, we will need the Lorentz gauge in-order that the equations are symmetric in the indices. It is therefore necessary to address this issue here and now, once and for altime. The Lorentz gauge is defined:

$$\partial^{\mu} A_{\mu} \equiv \partial_{\mu} A^{\mu} \equiv 0. \tag{54}$$

In Maxwellian Electrodynamics, the Lorentz gauge is necessitated by the need to justify the Law *of* Conservation of electronic current, without it, Maxwellian Electrodynamics would violate the Law *of* Conservation of electronic current. In the present, we could institute the same reason for having it, but we seek it on different grounds, namely that for the RHS to be symmetric under the interchange of the Greek indices where the partial derivative ∂^{μ} is involved, it is necessary that the Lorentz gauge hold identically. For example, the Riemann tensor involves double partial derivatives $\partial^{\mu}\partial^{\nu}$.

Clearly, $\partial^{\mu}\partial_{\mu} = \partial_{\mu}\partial^{\mu}$ is always assumed to be scalar. Strictly, this will not hold unless the Lorentz gauge holds. If strictly $\partial^{\mu}\partial_{\mu} = \partial_{\mu}\partial^{\mu}$ is a scalar, then $\partial^{\mu}\left(A_{\mu}^{(a)} A_{\alpha}^{(a)} \partial^{\alpha}\right) - \left(A_{\mu}^{(a)} A_{\alpha}^{(a)} \partial^{\alpha}\right)\partial^{\mu} \equiv 0$. This can only be so *if and only if* $\partial^{\mu} A_{\mu}^{(a)} \equiv 0$ holds identically. This is the Lorentz gauge. We thus need the Lorentz gauge to maintain the scalar properties of $\partial^{\mu}\partial_{\mu} = \partial_{\mu}\partial^{\mu}$.

Now, what we believe is an important relationship follows from the Lorentz gauge and the gauge (44) *i.e.* $\partial_{\mu} A_{\nu}^{(a)} = g_{(\mu)} A_{\mu}^{(a)} A_{\nu}^{(a)}$. This important relationship is:

$$\left(g^{(\alpha)} A^{(a)\alpha}\right) A_{\alpha}^{(a)} = 0. \tag{55}$$

We believe this equation is important insofar as it will aid in fixing the parameter $g^{(\mu)}$. In the end, when all the details of the theory have been worked out, particle masses will be fixed by the theory and the gauge conditions of this theory is what will give the values of the masses and other properties of the particles thereof.

5.4 Field Equations

Riemann geometry is built on the idea of parallel transport of vectors along a given path. A good intuitive description of parallel transport is perhaps that by John Baez[5]. Following him [*i.e.* John Baez]; say one starts at the north pole holding a javelin that points horizontally in some direction, and they carry this javelin to the equator always keeping the javelin pointing ‘in as same a direction as possible’, subject to the constraint that it points horizontally, *i.e.*, tangent to the Earth. In so doing, we see that the idea is that we are taking ‘space’ to be the *2*-dimensional surface of the Earth and the javelin is the ‘little arrow’ or ‘tangent vector’, which must remain tangent to ‘space’. After marching down to the equator and making a 90° turn at the equator and then marching along the equation until some-point along the equator where another 90° turn toward the north pole is made thus marching back up to the north pole, always keeping the javelin pointing horizontally and ‘in as same a direction as possible’. Obviously, because the surface of the Earth is curved, by the time one gets back to the north pole, the javelin will be pointing in a different direction. The javelin is said to have been parallel transported from its initial starting point to the final end point.

[5] Baez J., May 28 2009, General Relativity Tutorial - Parallel Transport: http://math.ucr.edu/home/baez/gr/parallel.transport.html

Parallel transport is an operation that takes a tangent vector and moves it along a path in space without turning it (relative to the space) or changing its length akin to the a person that carries a javelin as described above. In flat space, we can say that the transported vector is parallel to the original vector at every point along the path. In curved space as described above, the original and final vector after the parallel transport operation are not coincident and the change in this can be computed as will be done below.

If say we have a vector v^λ and we parallel transport this vector along a closed circuit ABCD in the order A $\longrightarrow$ B then B $\longrightarrow$ C then C $\longrightarrow$ D and then finally D $\longrightarrow$ A. The changes of this vector along these paths are:

$$\begin{aligned} dv^\lambda_{AB} &= -\Gamma^\lambda_{\mu\nu}(x)v^\nu(x)da^\mu \\ dv^\lambda_{BC} &= -\Gamma^\lambda_{\mu\nu}(x+da)v^\nu(x+da)da^\mu \\ dv^\lambda_{CD} &= +\Gamma^\lambda_{\mu\nu}(x+db)v^\nu(x+da)da^\mu \\ dv^\lambda_{DA} &= +\Gamma^\lambda_{\mu\nu}(x)v^\nu(x)db^\mu \end{aligned} \tag{56}$$

where $\Gamma^\lambda_{\mu\nu}$ and v^μ are evaluated at the location indicated in the parenthesis and the vector da^μ is the vector along $\boldsymbol{AB}$ and likewise, the vector db^μ is the vector along $\boldsymbol{BC}$. Collecting these terms (*i.e.* $dv^\lambda_{AB} + dv^\lambda_{BC} + dv^\lambda_{CD} + dv^\lambda_{DA}$) yields the overall change (dv^λ) suffered by v^λ, *i.e.*:

$$dv^\lambda = \frac{\partial(\Gamma^\lambda_{\mu\nu}v^\nu)}{\partial x^\alpha}db^\alpha da^\mu - \frac{\partial(\Gamma^\lambda_{\mu\nu}v^\nu)}{\partial x^\beta}db^\beta da^\mu, \tag{57}$$

and this further reduces to:

$$dv^\lambda = \left(\Gamma^\lambda_{\mu\nu,\alpha}v^\nu - \Gamma^\lambda_{\mu\nu}\Gamma^\lambda_{\sigma\alpha}v^\sigma\right)db^\alpha da^\mu - \left(\Gamma^\lambda_{\mu\nu,\beta}v^\nu - \Gamma^\lambda_{\mu\delta}\Gamma^\delta_{\sigma\beta}v^\sigma\right)db^\beta da^\mu, \tag{58}$$

and using the identities $da^\mu\Gamma^\lambda_{\mu\nu,\sigma} = da^\alpha\Gamma^\lambda_{\alpha\nu,\sigma}$, one arrives at:

$$dv^\lambda = \left(\Gamma^\lambda_{\mu\nu,\alpha} - \Gamma^\lambda_{\mu\alpha,\nu} + \Gamma^\lambda_{\delta\alpha}\Gamma^\delta_{\mu\nu} - \Gamma^\lambda_{\delta\nu}\Gamma^\delta_{\mu\alpha}\right)v^\mu db^\alpha da^\nu, \tag{59}$$

and this can be written compactly as:

$$dv^\lambda = R^\lambda_{\mu\sigma\nu}v^\mu da^\sigma db^\nu, \tag{60}$$

where $R^\lambda_{\mu\sigma\nu}$ is the curvature tensor (see *e.g.* Kenyon 1990; or any good book on GTR). The above result is the important reason why we have gone through all the above calculation, namely to find (*via* this exposition) the mathematical relationship that informs us of the change that occurs for a any given vector after parallel transport. In Riemann geometry, the affinities are not tensors and this leads to a vector altering its direction as it is parallel transported.

For a moment, let us shy-away from the abstract *World* of mathematics and pause a perdurable question to the reader. Suppose one is in a freely falling laboratory and this laboratory moves in a gravitational field in a closed circuit such that the laboratory leaves a given point and latter it returns to the same-point and throughout its path at all points it is in free-fall. The best scenario is a laboratory orbiting a central massive body. If in this laboratory we have a stationery object – do we (or does one) expect that after a complete orbit this object will have its motion altered? Or, does one expect that an object (inside the laboratory) that – say, has a specific momentum (relative to the laboratory) will after a complete circuit alter its momentum without any external force being applied to the free-falling system?

If this did happen, then Newton's first Law *of* Motion that defines inertia systems of reference is violated and it would mean that there is no such thing as an inertial system of reference; actually this renders the Principle *of* Equivalence obsolete. Surely, something must be wrong because the Principle *of* Equivalence can not be found in this wanting-state. We argue the reader to carefully go through the above argument to convince themselves of its correctness (or its incorrectness thereof). Whatever conclusion the reader will reach, it does not affect the final thesis being advanced namely that the affinities must be tensors. If they disagree with the above, it really does not matter as long as they agree that tensorial affinities preserve both the angle and the magnitude of a vector under parallel transport.

In the above, we say this renders the Principle *of* Equivalence obsolete because for a system in free-fall like the laboratory above, according to the Principle *of* Equivalence; it is an inertial system throughout its journey thus we do not expect an object in an inertial system to alter its momentum without a force being applied to it. The non-preservation of angles during parallel transport in Riemann geometry is in violation of the Principle *of* Equivalence if it is understood that parallel transport takes place in a geodesic system of reference *i.e.*, inertial systems of reference.

Naturally, we expect that for an observer inside the laboratory, they should observe a zero net change in the momentum. This, in the context of parallel transport of vectors, means that, such a spacetime will parallel transport vectors (in free-falling frames) in a manner such that after a complete circuit the parallel transported vector and the original vector, will still have the same magnitude and direction *i.e.*, $dv^{\lambda} = 0$. Actually, this means that throughout its parallel transport, the magnitude and direction of the vector must be preserved. Riemann geometry does not preserve the angles but only the length of the vector under parallel transport. The only way to have both the angles and the length preserved is if the affinities are tensors and the curvature tensor of such a spacetime will be identically equal to zero. We have already discovered a geometry whose affines are tensors. All we need to do now is to make the transformation: $\Gamma^{\lambda}_{\mu\nu} \longmapsto \bar{\Gamma}^{\lambda}_{\mu\nu}$, so that:

$$dv^{\lambda} = \bar{R}^{\lambda}_{\mu\sigma\nu} v^{\mu} da^{\sigma} db^{\nu}, \tag{61}$$

where:

$$\bar{R}^{\lambda}_{\mu\sigma\nu} = \overbrace{\bar{\Gamma}^{\sigma}_{\mu\nu,\lambda} - \bar{\Gamma}^{\lambda}_{\mu\sigma,\nu}}^{\textbf{Linear terms}} + \overbrace{\bar{\Gamma}^{\lambda}_{\mu\alpha}\bar{\Gamma}^{\alpha}_{\sigma\nu} - \bar{\Gamma}^{\lambda}_{\nu\alpha}\bar{\Gamma}^{\alpha}_{\sigma\mu}}^{\textbf{non-Linear terms}}. \tag{62}$$

and the fact that $dv^{\lambda} = 0$ implies $\bar{R}^{\lambda}_{\mu\sigma\nu} = 0$, because $(v^{\mu}, da^{\sigma}, db^{\nu}) \neq 0$. Also, the fact that $\bar{R}^{\lambda}_{\mu\sigma\nu} = 0$ does not necessary imply $\bar{\Gamma}^{\lambda}_{\mu\sigma} = 0$. Actually, now is our time *i.e.*, it is time to take the fullest advantage of the tensorial nature of the affinities. We have the mathematical and physical prerogative, legitimacy and liberty to choose a spacetime where the non-linear terms vanish identically *i.e.*, a spacetime such that $\bar{\Gamma}^{\lambda}_{\mu\sigma} \neq 0$ and $\bar{\Gamma}^{\lambda}_{\mu\alpha}\bar{\Gamma}^{\alpha}_{\sigma\nu} \equiv 0$. Clearly and without any doubt, this fact that we have chosen $\bar{\Gamma}^{\lambda}_{\mu\alpha}\bar{\Gamma}^{\alpha}_{\sigma\nu} \equiv 0$ means that "in a single and triumphant moment of joy", we have just reed ourself of the "monstrous" and "troublesome" non-linear terms in the Riemann tensor (62) because with this beautiful and elegant choice $\bar{\Gamma}^{\lambda}_{\mu\alpha}\bar{\Gamma}^{\alpha}_{\sigma\nu} \equiv 0$, they now vanish identically "to become but footnotes of history". In summary, we will have:

$$\bar{R}^{\lambda}_{\mu\sigma\nu} = 0 \quad \text{and} \quad (\bar{\Gamma}^{\lambda}_{\mu\sigma} \neq 0 : \; \bar{\Gamma}^{\lambda}_{\mu\alpha}\bar{\Gamma}^{\alpha}_{\sigma\nu} \equiv 0), \tag{63}$$

as the field equations that we sought and these field equations, as we will seen shortly; describe in a unified manner, the fields $\left(\psi, \phi, A^{(a)}_{\mu}\right)$. It will be seen in Monograph (I), that the fields $\left(\psi, \phi, A^{(a)}_{\mu}\right)$ describe, Leptons, Mesons and Baryons.

Now, for the "new" Riemann tensor (63), one sees that contracting the λ and σ indices, one gets $\bar{R}_{\mu\nu} = 0$, and for $\bar{\Gamma}^{\lambda}_{\mu\alpha}\bar{\Gamma}^{\alpha}_{\sigma\nu} \equiv 0$, one obtains $\bar{\Gamma}^{\alpha}_{\mu\alpha}\bar{\Gamma}^{\alpha}_{\alpha\nu} \equiv 0$, which invariably means $\bar{\Gamma}_{\mu}\bar{\Gamma}_{\nu} \equiv 0$, hence $\bar{\Gamma}_{\mu} \equiv 0$ thus, in summary we will have:

$$\bar{R}_{\mu\nu} = \bar{R}^{\alpha}_{\mu\nu\alpha} = 0 \quad \text{and} \quad \bar{\Gamma}_{\mu} = \bar{\Gamma}^{\alpha}_{\mu\alpha} = 0, \tag{64}$$

and further raising the μ-index and then contracting it with ν to get the equivalent of the Ricci scalar, we will obtain:

$$\bar{R} = \bar{R}^{\alpha}_{\alpha} = \bar{R}^{\alpha\beta}_{\alpha\beta} = 0. \tag{65}$$

Equations (63), (64) and (65) are the source coupled field equations. As a starting point, equation (64) *i.e.* $\bar{\Gamma}_{\mu} = 0$, leads to the equation:

$$A^{(a)}_{\nu} F^{(a\lambda)\nu}_{\mu} + G_{\mu} + Q_{\mu} = 0, \tag{66}$$

and multiplying this throughout by $A^{(a)\mu}$, and remembering that $g^{(a)\mu}{}_{\nu} = A^{(a)\mu} A^{(a)}_{\nu}$, we will have: $A^{(a)\mu}(G_{\mu} + Q_{\mu}) + g^{(a)\mu}{}_{\nu} F^{(a\lambda)\nu}_{\mu} = 0$, and knowing that: $g^{(a)\mu}{}_{\nu} F^{(a\lambda)\nu}_{\mu} = F^{(a\lambda)\nu}_{\nu} = 0$, it follows that:

$$A^{(a)\mu}(G_{\mu} + Q_{\mu}) = 0. \tag{67}$$

What this means is that if we are to envisage G_{μ} and Q_{μ} as (some sort of) currents, then the vector sum of the currents in spacetime will *always* be orthogonal to the vector $A^{(a)}_{\mu}$.

5.5 Source Coupled Field Equations

Now, equation (65) written in terms of the fields: $F^{(a\lambda)}_{\mu\nu}$, Q_{μ} and G_{μ} is:

$$\frac{1}{2} A^{(a)}_{\mu} D^{(a)}_{\alpha} F^{(a\lambda)\alpha}{}_{\nu} + \frac{1}{2} A^{(a)}_{\nu} D^{(a)}_{\alpha} F^{(a\lambda)}{}_{\mu}{}^{\alpha} - \frac{1}{2} A^{(a)}_{\alpha} D^{(a)}_{\nu} F^{(a\lambda)}{}_{\mu}{}^{\alpha} + \partial_{\mu} G_{\nu} + \partial_{\mu} Q_{\nu} + \left(G^{\lambda}\partial_{\lambda} + Q^{\lambda}\partial_{\lambda}\right) g^{(a)}_{\mu\nu} = 0, \tag{68}$$

and multiplying this by $A^{(a)\mu}$ and remembering that $A^{(a)\mu} A^{(a)}_{\nu} = g^{(a)\mu}{}_{\nu}$, the resultant equation is: $D^{(a)}_{\mu} F^{(a\lambda)\mu}{}_{\nu} - J^{(a)}_{\nu} - \hat{V} A^{(a)}_{\nu} = 0$ and this can be written more neatly as:

$$D^{(a)\nu} F^{(a\lambda)}_{\mu\nu} - J^{(a)}_{\mu} - \hat{V} A^{(a)}_{\mu} = 0, \tag{69}$$

where $D^{(a)}_{\mu} = \partial_{\mu} + g_{(\mu)} A^{(a)}_{\mu}$ and $J^{(a)}_{\mu} = J^{(a)}_{(1)\mu} + J^{(a)}_{(2)\mu}$ where $J^{(a)}_{(1)\mu} = -A^{(a)\nu}\partial_{\nu}\partial_{\mu} \ln\varphi$ is a vector current, $J^{(a)}_{(2)\mu} = -A^{(a)\nu}\partial_{\nu}\partial_{\mu} \ln\rho$ is a pseudo-vector current and $\hat{V} = -\partial^{\mu}(\ln\rho\varphi)\partial_{\mu}$ an potential operator operating on $A^{(a)}_{\mu}$. Other than the appearance of the pseudo-vector current, clearly equation (70) is the Maxwell-Proca equation! We can write $\hat{V} A^{(a)}_{\mu} = -\kappa^2_{(\mu)} A^{(a)}_{\mu}$, where $\kappa^2 = \kappa_{(\alpha)}\partial^{\alpha}(\ln\rho\varphi)$ where in this case we are summing over the dummy index of $\kappa_{(\alpha)}$, thus we can now write (69) as:

$$D^{(a)\nu} F^{(a\lambda)}_{\mu\nu} - J^{(a)}_{\mu} - \kappa^2 A^{(a)}_{\mu} = 0. \tag{70}$$

Now, moving further, contracting the indices μ and ν of (68), and then making use of (66), one arrives at:

$$\partial^{\mu} G_{\mu} + \partial^{\mu} Q_{\mu} = \Box \ln\varphi + \Box \ln\rho = 0. \tag{71}$$

This equation is in actual fact an equation of conservation of the quantity $Q_{\nu} + G_{\nu}$. When we come to the derivation of the geodesic equation of motion, we shall see that if this equation is to match with reality, we will have to identify $\Phi = c^2 \ln\varphi$ with the classical gravitational potential and $\Psi = c^2 \ln\rho$ with the quantum gravitational potential or the Klein-Gordon Wave function. These quantities (Φ, Ψ) are mutually independent quantities, it follows that equation (71) must reduce to: $\Box(c^2 \ln\varphi) = -\Box(c^2 \ln\rho) = k$ where k an absolute physical constant. If $k \neq 0$, the equation $\Box(\ln\varphi) = k$, would for spherically symmetric gravitation, in addition to inverse square law, this would lead to an extra term whose force increases as the square of the distance. Clearly, this will not match with experience, and the only way to lead to conformity, is by setting $k \equiv 0$, hence, we will have:

$$\Box\Phi = 0 \qquad \text{and} \qquad \Box\Psi = 0. \tag{72}$$

As Einstein did when he discovered his gravitational field equations, he made sure that these equations reduced to Poisson's well know result in the first order approximation. One may ask, for some low order approximation, do the equations (75) reduce to Poisson's equation? Our answer is yes, actually the equation $\Box\Phi = 0$ is Poisson's equations for gravitation. We know that Poisson's equation for gravitation is $\boldsymbol{\nabla}^2\Phi = 4\pi G\varrho$. So the question is, "Where is the density term in this equation: $\Box\Phi = 0$?".

To answer this we need to take note of the fact that the operator $\Box$ is actually given as $\Box^{(a)} = g^{(a)}_{\mu\nu}\partial^\mu\partial^\nu$: this operator can be separated into its linear and non-linear parts, *i.e.*:

$$\Box^{(a)} = \sum_{\mu=\nu} g^{(a)}_{\mu\nu}\partial^\mu\partial^\nu + \sum_{\mu\neq\nu} g^{(a)}_{\mu\nu}\partial^\mu\partial^\nu, \tag{73}$$

where we can now choose to write:

$$\bar{\Box}^{(a)} = \sum_{\mu=\nu} g^{(a)}_{\mu\nu}\partial^\mu\partial^\nu \qquad \text{and} \qquad \tilde{\Box}^{(a)} = \sum_{\mu\neq\nu} g^{(a)}_{\mu\nu}\partial^\mu\partial^\nu, \tag{74}$$

so that $\Box^{(a)} = \bar{\Box}^{(a)} + \tilde{\Box}^{(a)}$. In Nyambuya (2010*c*), it has been argued that the non-linear part of $\Box\Phi$, *i.e.* $\tilde{\Box}\Phi$ should be identified with the mass density term *i.e.*, $\tilde{\Box}^{(a)}\Phi \equiv -4\pi G\varrho$. This same argument is easily extendable to the case $\Box\Psi = 0$. The important question is what to identify the non-linear term with? If we think of Ψ as being the Schrödinger-Klein-Gordon wavefunction, then, the identification/conjecture $\tilde{\Box}^{(a)}\Psi \equiv -(m_0c/\hbar)^2\Psi$ leads us to a more familiar quantum mechanical wave equation. As demonstrated in Nyambuya (2010*c*), the choice $\tilde{\Box}^{(a)}\Phi \equiv -4\pi G\varrho$ leads to desirable gravitational equations and this is good enough motivation for the choice, but the same can not be said of the choice $\tilde{\Box}^{(a)}\Psi \equiv -(m_0c/\hbar)^2\Psi$. Intuition rather that facts is what leads us to this conjecture/identification. We only hope that further work will vindicate this choice. With this, equation (75) then becomes:

$$\bar{\Box}\Phi = 4\pi\lambda G\varrho \qquad \text{and} \qquad \bar{\Box}\Psi = \lambda\left(\frac{m_0c}{\hbar}\right)^2\Psi. \tag{75}$$

Where we have removed the superscript on the operator $\bar{\Box}^{(a)}$ and it is now represented by $\lambda = \pm1, 0$ *i.e.*: for $a = 1$ we have $\lambda = 0$;for $a = 2$ we have $\lambda = +1$ and for $a = 3$ we have $\lambda = -1$.

The above equations *i.e.* (75), are the present UFT's equations of gravitation on the astronomical and quantum scale respectively. For static or miniature time varying gravitational potentials, the equation $\bar{\Box}\Phi = 4\pi\lambda G\varrho$ gives the usual gravitational Poisson. It is important to note that equations (75) are Lorentz invariant. The meaning of the above is that the presence of mass in spacetime is a manifestation of a non-linear distribution of the gravitational potentials. In the treatise on gravitation *i.e.* Monograph (II), we shall explore some of the richness of the gravitational equation $\bar{\Box}\Phi = 4\pi G\varrho$. *Prima facie*, it appears as a well known equation, contrary to this view, it will be demonstrated in Monograph (II), that we have taken the Poisson equation for granted and part of this work can be seen in Nyambuya (2010*a*, *c*, *d*, *e*).

If all the above proves correct, then, it should not be difficult for one to deduce that the Dirac spinor ψ and the Schrödinger-Klein-Gordon wavefunction Ψ are related by the equation:

$$\psi^\dagger\psi = \exp\left[\lambda\left(\frac{\Psi}{c^2}\right)\right]. \tag{76}$$

5.6 Source-Free Field Equations

For the source-free field equations, we know that the curvature tensor (65) satisfies the identity:

$$\bar{R}^{\sigma}_{\alpha\mu\nu;\lambda} + \bar{R}^{\sigma}_{\alpha\lambda\mu;\nu} + \bar{R}^{\sigma}_{\alpha\nu\lambda;\mu} = 0 \tag{77}$$

and contracting the indices σ and α of this equation, it is not difficult to see that one arrives at:

$$D^{(a\lambda)}_{\sigma} F^{(a\lambda)}_{\mu\nu} + D^{(a\lambda)}_{\nu} F^{(a\lambda)}_{\sigma\mu} + D^{(a\lambda)}_{\mu} F^{(a\lambda)}_{\nu\sigma} = 0, \tag{78}$$

which is the source free field equation. We still can obtain another source free field equation and this is by differentiating (70) with respect to ∂^{ν}; so doing we obtain:

$$D^{\mu}_{(a\lambda)} D^{\nu}_{(a\lambda)} F^{(a\lambda)}_{\mu\nu} - V\partial^{\nu} A^{(a)}_{\nu} = 0, \tag{79}$$

and if we take into account the Lorentz gauge $\partial^{\nu} A^{(a)}_{\nu} = 0$, then this equation reduces to:

$$D^{\mu}_{(a\lambda)} D^{\nu}_{(a\lambda)} F^{(a\lambda)}_{\mu\nu} = 0. \tag{80}$$

With equations (70) and (78), we have arrived at the desired field equations.

We note that, if we consider $\left(\phi, \psi, A^{(a)}_{\mu}\right)$ as giving the complete description of a fundamental particle, then, this fundamental particle will carry four fields, *i.e.*: $F^{(a0)}_{\mu\nu}$, $F^{(a1)}_{\mu\nu}$, $F^{(a2)}_{\mu\nu}$ and $F^{(a3)}_{\mu\nu}$. These fields are not independent entities, but an integral part of the system of the particle $\left(\phi, \psi, A^{(a)}_{\mu}\right)$; they can not be separated from the particle $\left(\phi, \psi, A^{(a)}_{\mu}\right)$. As will be seen in §(7), we can have an arrangement where one of these fields is an Abelien field while the other three are non-Abelien fields and also a setting where two of these fields are Abelien fields while the other two are non-Abelien fields.

5.7 Dirac Equations

We show here that under certain conditions of experience, the present theory yields the curved space Dirac equations already derived in Nyambuya (2008). If say $\mathbf{e}_{\mu}$ is any general unit vector, then $\partial^{\mu}\mathbf{e}_{\mu} = \cos\vartheta$ where ϑ is the angle between that unit vector and the tangent surface at the point where this unit vector is located. Given this definition and that of the unit vector of the RHS: $\boldsymbol{e}^{(a)}_{\mu}$; then $\partial^{\mu}\boldsymbol{e}^{(a)}_{\mu} = \cos\vartheta$. For this spacetime (RHS), clearly the angle ϑ must be a 4×1 scalar object (*i.e.*, rank one scalar), *i.e.*:

$$\vartheta = \begin{pmatrix} \vartheta_0 \\ \vartheta_1 \\ \vartheta_2 \\ \theta_3 \end{pmatrix} \text{ and } \cos\vartheta = \begin{pmatrix} \cos\vartheta_0 \\ \cos\vartheta_1 \\ \cos\vartheta_2 \\ \cos\vartheta_3 \end{pmatrix}. \tag{81}$$

We know that under natural conditions where for a given point, the normal line from at any given point, meets the target surface at right angles, thus one would expect in general that for any space, that the unit vector at any given point, be perpendicular to the tangent surface at that point, this means $\cos\vartheta = 0$ thus, we will have:

$$iA_{\mu}\gamma^{(a)}_{\mu}\partial^{\mu}\psi + \left(\phi^{-1}\frac{\partial\left(\gamma^{(a)}_{\mu} iA_{\mu}\phi\right)}{\partial x_{\mu}}\right)\psi = 0. \tag{82}$$

Let us set:

$$\mathcal{M} = \left(\phi^{-1}\frac{\partial\left(\gamma_\mu^{(a)} iA_\mu\phi\right)}{\partial x_\mu}\right), \tag{83}$$

where $\mathcal{M}$ is a 4×4 matrix, thus (82) reduces to:

$$iA_\mu\gamma_\mu^{(a)}\partial^\mu\psi - \mathcal{M}\psi = 0. \tag{84}$$

What we shall do now is to argue from experience that $\mathcal{M} = \lambda_c\mathcal{I}$ where $\mathcal{I}$ is a 4×4 identity matrix and λ_c is a Lorentz invariant. The equation: $iA_\mu\gamma_\mu^{(a)}\partial^\mu\psi - \mathcal{M}\psi = 0$, can be written: $\left[iA_\mu\gamma_\mu^{(a)}\partial^\mu - \mathcal{M}\right]\psi = 0$. Now multiplying this equation from the left with $\left[iA_\mu\gamma_\mu^{(a)}\partial^\mu - \mathcal{M}\right]^\dagger$, which is a transposed complex conjugate of $\left[iA_\mu\gamma_\mu^{(a)}\partial^\mu - \mathcal{M}\right]$, we will have: $g_{\mu\nu}\partial^\mu\partial^\nu\psi = \left(iA_\mu\gamma_\mu^{(a)\dagger}\partial^\mu\mathcal{M} + iA_\mu\mathcal{M}^\dagger\gamma_\mu^{(a)}\partial^\mu + \mathcal{M}^\dagger\mathcal{M}\right)\psi$. In arriving at this result, one must not forget the Lorentz gauge condition $\partial^\mu A_\mu \equiv 0$. This equation $g_{\mu\nu}\partial^\mu\partial^\nu\psi = \left(iA_\mu\gamma_\mu^{(a)\dagger}\partial^\mu\mathcal{M} + iA_\mu\mathcal{M}^\dagger\gamma_\mu^{(a)}\partial^\mu + \mathcal{M}^\dagger\mathcal{M}\right)\psi$ is written in a Lorentz covariant form. We know from the special theory of relativity that the term in the brackets [*i.e.* $\left(iA_\mu\gamma_\mu^{(a)\dagger}\partial^\mu\mathcal{M} + iA_\mu\mathcal{M}^\dagger\gamma_\mu^{(a)}\partial^\mu + \mathcal{M}^\dagger\mathcal{M}\right)$], must be a Lorentz invariant quantity. If we set $\mathcal{M} = \lambda_c\mathcal{I}$ where λ_c is in general a complex constant quantity, one obtains both the curved spacetime Dirac equations and as well the corresponding Klein-Gordon equation. Experience strongly suggests that $\lambda_c = m_0c/\hbar$ where m_0 is the rest mass of the particle and $\hbar$ is Planck's normalised constant. With this, it follows that:

$$iA_\mu\gamma_\mu^{(a)}\partial^\mu\psi - \left(\frac{m_0c}{\hbar}\right)\psi = 0. \tag{85}$$

$$g_{\mu\nu}\partial^\mu\partial^\nu\psi = \left(\frac{\sqrt{2}m_0c}{\hbar}\right)^2\psi \tag{86}$$

Now, from the choice $\mathcal{M} = \lambda_c\mathcal{I}$, it follows that:

$$i\partial^\mu\left(\gamma_\mu^{(a)}A_\mu\right) + i\phi^{-1}\gamma_\mu^{(a)}A_\mu\partial^\mu\phi \equiv \left(\frac{m_0c}{\hbar}\right)\mathcal{I} \tag{87}$$

and now imposing a the gauge condition:

$$\partial^\mu\left(\gamma_\mu^{(a)}A_\mu\right) \equiv 0, \tag{88}$$

this leads to equation (87) to reduce to:

$$iA_\mu\gamma_\mu^{(a)}\partial^\mu\phi = \left(\frac{m_0c}{\hbar}\right)\phi. \tag{89}$$

Now, due to the gauge condition (44) *i.e.* $\partial_\mu A_\nu^{(a)} = g_{(\mu)}A_\mu^{(a)}A_\nu^{(a)}$, the gauge condition (88) reduces to $\left(g^{(\mu)}A^{(a)\mu}\right)\left(\gamma_\mu^{(a)}A_\mu^{(a)}\right) \equiv 0$, which must further reduce to:

$$g^{(\mu)}\gamma_\mu^{(a)} \equiv 0. \tag{90}$$

In conjunction with (55), the above puts further constraints on the parameter $g_{(\mu)}$.

Equation (85) is the curved spacetime Dirac equation (first) proposed in the reading Nyambuya (2007). We have dropped the superscript a in A_μ because this is present in $\gamma_\mu^{(a)}$. This equation has been derived in a different manner in the reading Nyambuya (2008).

Question: why did we decided to include in this section the curved spacetime Dirac equations which are already part of the reading Nyambuya (2008)? Other than the fact that showing that this equation does under certain conditions arise from the present theory from a completely different vantage point and also other than that this gives the theory some credence and some ground to stand on; we want to discuss an extension of the curved spacetime Dirac equations, namely, Nyambuya (2009). Also, I wanted to point out that, it strongly appears from this method of derivation of the Dirac equations, that the mysterious foundations of the Dirac equations can now be sort from the RHS as being intimately linked to the properties of the unit vectors of the RHS.

In Nyambuya (2009*a*), we did show that the Dirac equation can be generalized to describe both Bosons and Fermions. According to our present understanding *viz*, from the accepted literature, Bosons are described by a zero-rank scalar function while Fermions are described by the four component Dirac function ψ. The present theory requires that Bosons and Fermions be described using not just the Dirac four component function ψ, but the same equation. So we want to clear this here and now. We direct the reader to the reading Nyambuya (2009*a*) for this. In its bare formulation, the Dirac Equation describes only spin-1/2 particles but in Nyambuya (2009*a*), we did show that it [Dirac Equation] can be written in a more general form to describe in general any spin particle (*i.e.*, $s = 1/2, 1, 3/2, 2, \ldots, n/2 \ldots$: $n = 1, 2, 3, \ldots$ *etc*) and this generalization extends to (85) as-well. In Monograph (I), we will give a more robust proof that the Dirac equation can indeed be written as has been done in Nyambuya (2009*a*), so that it becomes a general spin equation. This proof to be given in Monograph (I) is a proof by mathematical induction.

Additionally, equation (85) admits both negative and positive energy solutions. Unlike Dirac, we are not going to try and get reed of these negative energy solutions as these negative energy solutions will prove to be vital in defining a whole invisible *World* of darkmatter and darkenergy. We are not going to give the details of this work here, but merely give the reader our insurance that this work is at an advanced stage of completion in Monograph (I).

6 Fundamental Symmetries of the Riemann-Hilbert Spacetime

Just like the Riemannian line element, the Riemann-Hilbert line element: $\left(ds^{(a)}\right)^2 = \rho\varphi g^{(a)}_{\mu\nu} dx^\mu dx^\nu$, must be invariant under any kind of coordinate and or frame transformation. The question is:

1. What are the coordinate and or frame transformations that leave the line element: $\left(ds^{(a)}\right)^2 = \rho\varphi g^{(a)}_{\mu\nu} dx^\mu dx^\nu$, invariant?
2. What symmetries do these coordinate and or frame transformations obey?

The coordinate transformations that leave the Riemann-Hilbert line element, invariant shall determine the permissible coordinate transformations and on the same footing, the frame transformations that leave this same line element invariant, shall determine the permissible frame transformations.

6.1 Linear Transformations

In arguments leading to Postulate II – in a rather hand-waving manner, we substantiated why we need linear coordinate transformations. Now we shall show from a geometric vantage point, how these linear coordinate transformation come to be. In arriving at (37), we argued that under the transformation $\psi' = S\psi$ the function ρ will transform as $\rho' = \Phi\rho$. In order that the physics is preserved or remains unaltered, we must have $\Phi \equiv 1$ and that if $\Phi \not\equiv 1$, we are describing a different geometry with a different physics altogether. When we make a coordinate transformation we expect that the equations remain the same *i.e.* $\Phi \equiv 1$. Also, when we make a frame transformation, we expect the equations to remain the same *i.e.*

$\Phi \equiv 1$. When a change of the geometry, which is a change of the physics occurs, we expect the equations to change *i.e.* $\Phi \not\equiv 1$, the original equations and the resulting equations after the transformation will be different.

From the above arguments, if we are to have $\Phi \equiv 1$, then, it follows that:

$$\int \left(\frac{\partial^2 x^\lambda}{\partial x^\lambda \partial x^{\mu'}} \right) dx^{\mu'} \equiv 0, \tag{91}$$

hence:

$$\frac{\partial^2 x^\lambda}{\partial x^\lambda \partial x^{\mu'}} = \partial_\lambda \delta^\lambda_{\mu'} \equiv 0, \tag{92}$$

where $\delta^\lambda_{\mu'} = \partial_{\mu'} x^\lambda$. The Lorentz transformations (2), which are transformations from one frame to another, readily meet this condition. Also, the coordinate transformations:

$$\begin{pmatrix} icdt' \\ dx \\ dy \\ dz \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\vartheta\cos\theta & r\cos\vartheta\sin\theta & -r\sin\vartheta\cos\theta \\ 0 & \cos\vartheta\sin\theta & r\cos\vartheta\cos\theta & -r\sin\vartheta\sin\theta \\ 0 & \sin\vartheta & r\cos\vartheta & 0 \end{pmatrix} \begin{pmatrix} icdt \\ dr \\ d\theta \\ d\vartheta \end{pmatrix}, \tag{93}$$

which are coordinate transformations of the differentials from rectangular (x, y, z, t) to spherical $(r, \theta, \vartheta, t)$ do meet this condition (here: $x = r\cos\vartheta\sin\theta$, $y = r\cos\vartheta\cos\theta$ and $z = r\sin\vartheta$). In the above coordinate transformation, note that because the point (r, θ, ϑ) is the same point $(r, \theta, -\vartheta)$, this means $\cos\vartheta = -\cos\vartheta$ and $\cos\theta \neq -\cos\theta$. In principle $\cos\theta = -\cos\theta$, but if we are to take this, it would have a different meaning on the (r, θ, ϑ) grid; it would mean the point (r, θ, ϑ) is the same point as $(r, -\theta, \vartheta)$, which is not true.

For the coordinate transformations, note that $dt' \equiv dt$, which implies $t' \equiv t$. If one takes the Eddington-Finkelstein coordinate transformations, which when written in matrix notion as in (93), are:

$$\begin{pmatrix} icdt' \\ dr' \\ d\theta' \\ d\vartheta' \end{pmatrix} = \begin{pmatrix} 1 & -\frac{i}{c(1-r/\mathcal{R}_s)} & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} icdt \\ dr \\ d\theta \\ d\vartheta \end{pmatrix}, \tag{94}$$

one finds that $\Phi \not\equiv 1$, the meaning of which is that the Eddington-Finkelstein transformations lead to a different physical condition. From what we have presented, it is clear that linear transformations are necessary if our transformations are to describe the same physical phenomena. According to our current understanding of physics, linear transformations are always assumed and their necessity is justified on the grounds that if we are to describe the same physical phenomenon, it is necessary to have them. Here, we have derived their necessity from a geometrical vantage point. The preservation of the geometry, which is the preservation of the physics, requires that any transformations made should be linear. We expect that when a particle undergoes a physical change, we must have $\Phi \not\equiv 1$. As to what this physical change may be or what this physical change may do to the particle, we have no idea.

6.2 Global Symmetries

Since $\varphi g^{(a)}_{\mu\nu} dx^\mu dx^\nu$ is a scalar, the question "What are the coordinate and or frame transformations that leave the line element: $\left(ds^{(a)}\right)^2 = \rho\varphi g^{(a)}_{\mu\nu} dx^\mu dx^\nu$, invariant?", amounts to asking what are the transformations that leave ρ invariant? Since $\rho = \psi^\dagger\psi$ and ψ is a spinor which transforms as $\psi' = S\psi$ such that $\rho' = \Phi\rho$ where we have argued that $S^\dagger S = \Phi\mathcal{I}$. If ρ is to be invariant, then $\Phi \equiv 1$. From this, it follows that S can only be a unitary 4×4 matrix. This kind of transformation transforms the wavefunction on a global scale in such a way that the initial geometry is preserved. It shall be demonstrated in Monograph (I) that these global transformations is what leads to the decay of particles such as the reaction when a Neutron transforms into a Proton, Electron and anti-Electron-neutrino: $\mathrm{n} \longmapsto \mathrm{p}^{+} + \mathrm{e}^{-} + \bar{\nu}_{\mathrm{e}}$.

6.3 Internal Symmetries

The metric $\tilde{g}^{(a)}_{\mu\nu} = \rho\varphi A^{(a)\dagger}_{\mu} A^{(a)}_{\nu}$ can be decomposed by decomposing its parts *i.e.*:

$$\begin{array}{lcll} \phi & = & \sum_j \mathcal{T}_j \phi_{(j)} & \ldots\ldots\ldots \quad (\mathbf{a}) \\ \psi & = & \sum_j \mathcal{T}_j \psi_{(j)} & \ldots\ldots\ldots \quad (\mathbf{b}) \\ A^{(a)}_{\mu} & = & \sum_j \mathcal{T}_j A^{(aj)}_{\mu} & \ldots\ldots\ldots \quad (\mathbf{c}) \end{array}, \tag{95}$$

where $\left(\phi_{(j)}, \psi_{(j)}, A^{(aj)}_{\mu}\right)$ are generators of the $\mathrm{U}(1,4)$ or the $\mathrm{SU}(2,4)$ group and the $\mathcal{T}$-matrices are 4×4 orthogonal hermitian matrices that obey the Clifford algebra: $\left[\mathcal{T}_i, \mathcal{T}_j\right] = i f_{ijl} \mathcal{T}_l$, and f_{ijl} are the suitable structural constants for that particular gauge group. The $\mathrm{U}(1,4)$ and $\mathrm{SU}(2,4)$ group is the $\mathrm{U}(1)$ and $\mathrm{SU}(2)$ gauge group in four dimensions, we are not going to define this group here; we will do this in Monograph (I). To give an idea to the reader, we shall list the $\mathrm{SU}(2,4)$ matrices. There are three sets of these matrices, *i.e.*:

For the first set, we have:

$$\mathcal{T}_1 = \begin{pmatrix} \sigma_1 & 0 \\ 0 & -\sigma_1 \end{pmatrix}, \quad \mathcal{T}_2 = \begin{pmatrix} \sigma_2 & 0 \\ 0 & -\sigma_2 \end{pmatrix}, \quad \mathcal{T}_3 = \begin{pmatrix} \sigma_3 & 0 \\ 0 & -\sigma_3 \end{pmatrix}. \tag{96}$$

For the second set, we have:

$$\mathcal{T}_1 = \begin{pmatrix} 0 & \sigma_1 \\ \sigma_1 & 0 \end{pmatrix}, \quad \mathcal{T}_2 = \begin{pmatrix} 0 & \sigma_2 \\ \sigma_2 & 0 \end{pmatrix}, \quad \mathcal{T}_3 = \begin{pmatrix} 0 & \sigma_3 \\ \sigma_3 & 0 \end{pmatrix}, \tag{97}$$

and for the third set, we have:

$$\mathcal{T}_1 = \frac{1}{\sqrt{2}}\begin{pmatrix} \sigma_1 & \sigma_1 \\ \sigma_1 & -\sigma_1 \end{pmatrix}, \quad \mathcal{T}_2 = \frac{1}{\sqrt{2}}\begin{pmatrix} \sigma_2 & \sigma_2 \\ \sigma_2 & -\sigma_2 \end{pmatrix}, \quad \mathcal{T}_3 = \frac{1}{\sqrt{2}}\begin{pmatrix} \sigma_3 & \sigma_3 \\ \sigma_3 & -\sigma_3 \end{pmatrix}. \tag{98}$$

where $\sigma_j : \ j = 1, 2, 3$ are as usual, the three 2×2 Pauli matrices. These matrices span the $\mathrm{SU}(2,4)$ space and as will be seen in Monograph (I), these matrices do explain the there color charges of quarks.

Now, with this set of matrices, the metric can be decomposed as:

$$\tilde{g}^{(a)}_{\mu\nu} = \rho\varphi g^{(a)}_{\mu\nu} = \sum_j \rho_{(j)}\varphi_{(j)} A^{(aj)\dagger}_{\mu} A^{(aj)}_{\nu} = \sum_j \rho_{(j)}\varphi_{(j)} g^{(aj)}_{\mu\nu}, \tag{99}$$

where $\rho_{(j)} = \psi^{\dagger}_{(j)}\psi_{(j)}$ and $\varphi_{(j)} = \phi^{\dagger}_{(j)}\phi_{(j)}$. Now, the components $\left(\phi_{(j)}, \psi_{(j)}, A^{(aj)}_{\mu}\right)$ can undergo unitary transformations such that they leave $\left(\phi, \psi, A^{(aj)}_{\mu}\right)$ invariant, *i.e.*:

$$\left.\begin{array}{lcl} \phi_{(j)} & \longmapsto & \mathcal{U}_j \phi_{(j)} \\ \psi_{(j)} & \longmapsto & \mathcal{U}_j \psi_j \\ A^{(aj)}_{\mu} & \longmapsto & \mathcal{U}_j A^{(aj)}_{\mu} \end{array}\right\} \Longrightarrow \begin{array}{lcl} \phi & \longmapsto & \phi \\ \psi & \longmapsto & \psi \\ A^{(a)}_{\mu} & \longmapsto & A^{(a)}_{\mu} \end{array}, \tag{100}$$

and as stated, these transformations are such that $\left(\phi, \psi, A_{\mu}^{(a)}\right)$ remains invariant after the above transformations have taken place *i.e.*:

$$\left.\begin{array}{lll}\phi & \longmapsto & \phi \\ \psi & \longmapsto & \psi \\ A_{\mu}^{(a)} & \longmapsto & A_{\mu}^{(a)}\end{array}\right\} \Longrightarrow \tilde{g}_{\mu\nu}^{(a)} \longmapsto \tilde{g}_{\mu\nu}^{(a)}, \tag{101}$$

where $\mathcal{U}_{\mathrm{j}}$ a is a set of 4×4 matrices such that $\mathcal{U}_j^{\dagger}\mathcal{U}_j = \mathcal{I}$. These transformations are internal transformations, hence internal symmetries, they affect the subsystems of the particle and not the particle itself. Now, just as $\left(\phi, \psi, A_{\mu}^{(a)}\right)$ is a complete system making up a particle, $\left(\phi_{(j)}, \psi_{(j)}, A_{\mu}^{(aj)}\right)$ is also a system making up a particle. The system $\left(\phi, \psi, A_{\mu}^{(a)}\right)$ thus can be thought of as comprising of j non-interacting particles each acting such that as a whole, they appear as a single particle just as the three quarks found in the Proton and Neutron act as independent particles such that all their actions contribute to the wholesome properties of the Proton and Neutron respectively. Actually, we shall demonstrate in the Monograph (I) that the particles $\left(\phi_{(j)}, \psi_{(j)}, A_{\mu}^{(aj)}\right)$ are quarks. We shall further demonstrate therein that one can derive exactly the fractional electronic charges of quarks, their color charges, the existence of the generations *etc*.

7 Nuclear Forces

At the nuclear level, the Electromagnetic, the Electroweak and the Strong force are the forces known to operate. The Electromagnetic force is driven by a U(1) Abelian gauge field while the other forces (the Electroweak and the Strong forces) are driven by SU(2) and SU(3) non-Abelian gauge fields (respectively). The Neutron, which is an electronically neutral particle, has Electromagnetic properties such as a non-zero gyromagnetic ratio. The Neutron certainly carries an electromagnetic field and it decays *via* the SU(2) Electroweak interaction to form the Proton, the Electron and the Electron-antineutrino. Clearly, the Neutron should carry both the U(1) Electromagnetic and SU(2) Electroweak fields. Because the Neutron is comprised of three quarks, it must carry the SU(3)-Strong force field. From the foregoing, the Neutron must carry at least three fields, *i.e.* the U(1) Electromagnetic, the SU(2) Electroweak and the SU(2)-Strong force fields.

The dream of unification has always been that all these forces (the Electromagnetic, the Electroweak and the Strong force, including the gravitational force) be explained by a single field. From the present attempt, excluding the gravitational force, we find this is possible. The vector gauge field A_{μ}, gives raise to four Yang-Mills fields $F_{\mu\nu}^{(a\lambda)} = \left(F_{\mu\nu}^{(a0)}, F_{\mu\nu}^{(a1)}, F_{\mu\nu}^{(a2)}, F_{\mu\nu}^{(a3)}\right)$ which are distinguished by the parameter $g^{(\lambda)}$. This parameter has four components and if each of the four are unique, then, there should be at most four distinct tensor fields $F_{\mu\nu}^{(a\lambda)}$. There will be as many distinct tensor fields as they are unique $g^{(\lambda)}$ parameters. Given the argument in the previous paragraph that the pedestrian Neutron should carry three force fields, it means that there should exist out of the four tensor fields $F_{\mu\nu}^{(a\lambda)}$, only three that are unique. The Proton also carries three force fields hence it must carry three tensor fields. What this means is that if the present attempt on a unified theory is correct, then, there must exist an-as-yet undiscovered force.

8 New Geodesic Law

Lastly, before entering into a general discussion of the entire body of work presented herein, let us address the problem raised in §(3) of the geodesic law namely that it is neither invariant nor covariant under a change of the system of coordinates and/or change in the reference system. In order to derive the equation of motion in the GTR, one needs to formulate this equation first from a Gaussian coordinate system and thereafter make a transformation to a coordinate system of their choice. As already said in that section, this geodesic equation is in contempt of the very principle upon which the GTR is founded – namely the Principle *of* Relativity, which requires that one should be free to formulate the geodesic equation of motion in any legitimate coordinate system of their choice without having to be preferentially constrained to start from a Gaussian coordinate system. The geodesic law equation (9) is derived (upon making proper algebraic operations) from the Lagrangian:

$$\mathcal{L} = \frac{1}{2} g_{\mu\nu} \frac{dx^\mu}{ds} \frac{dx^\nu}{ds}, \tag{102}$$

by inserting this into the Lagrangian equation of motion, namely:

$$\frac{d}{ds}\left(\frac{\partial \mathcal{L}}{\partial \dot{x}^\mu}\right) - \frac{\partial \mathcal{L}}{\partial x^\mu} = 0. \tag{103}$$

In the present, our geometry's metric has been replaced by $\rho\varphi g_{\mu\nu}$, thus we will have to effect this into the Langragian by $\mathcal{L} \longmapsto \rho\varphi\mathcal{L}$, *i.e.*:

$$\mathcal{L} = \frac{1}{2} \rho\varphi g^{(a)}_{\mu\nu} \frac{dx^\mu}{ds} \frac{dx^\nu}{ds}, \tag{104}$$

Using this Lagrangian in (103), one arrives at the geodesic equation:

$$\frac{d^2 x^\lambda}{ds^2} + \bar{\Gamma}^{(a)\lambda}_{\mu\nu} \frac{dx^\mu}{ds} \frac{dx^\nu}{ds} = 0, \tag{105}$$

and this geodesic equation unlike the GTR geodesic equation (9), it does not require us to formulate the equation of motion in the preferred Gaussian systems, but in any coordinate system one chooses or desires and this is because $\bar{\Gamma}^\lambda_{\mu\nu}$ is a tensor!

We show, as a way of justification the importance of this equation, *i.e.* its correspondence with experience, in that, in the low energy and curvature regime, this equation (105); for the case $\lambda = j = (1, 2, 3)$, it reduces to the Lorentz equation. To show this, first let us make the setting $\beta^\mu = dx^\mu/ds = v^\mu/c$ where v^μ is the four velocity and second, we make the approximation for low energies that: $\beta^0 \sim 1$ and $\beta^k \ll 1$ and this means we will have to drop the terms $\beta^i\beta^j$ because these terms will be small, thus to first order approximation, we will have $\bar{\Gamma}^j_{0\nu}\beta^0\beta^\nu$ reduce to: $\bar{\Gamma}^j_{0\nu}\beta^0\beta^\nu \simeq \beta^\nu F^{(aj)j}_\nu + (G^j + Q^j)$. In this approximation of low energy and curvature, $|A^{(a)}_\mu| \sim 1$, and the derivatives $|\partial_\mu A^{(a)}_\nu| \ggg 1$, are significant. From all this, it follows that:

$$\frac{d^2 \boldsymbol{x}}{d\tau^2} = \overbrace{-\boldsymbol{\nabla}\Phi}^{\text{Gravitational Force}} - \overbrace{v^\nu F^{(aj)j}_\nu \hat{\boldsymbol{e}}_j}^{\text{Electromagnetic Force}} \overbrace{-\boldsymbol{\nabla}\Psi}^{\text{Quantum Force}}, \tag{106}$$

where $\hat{\boldsymbol{e}}_j$ are the ordinary xyz-orthonormal basis vectors on the xyz-grid, *i.e* $\hat{\boldsymbol{\imath}}$, $\hat{\boldsymbol{\jmath}}$, $\hat{\boldsymbol{k}}$ and, $\Psi = c^2 \ln\rho$, $\Phi = c^2 \ln\varphi$ and $ds = cd\tau$. This is the Lorentz equation for a particle travelling inside an electromagnetic field under the forces $(-\nabla\Psi)$ and $(-\nabla\Phi)$. The quantum force $(-\nabla\Psi)$ is a new force. Its presence (*vis* observations and physical experience) can be justified on the basis that it gives particles the random quantum mechanical properties that are observed in quantum particles. The best comparison we can make is to think of this random quantum force as a Bohmian Quantum Mechanical type of force, akin and redolent to David Bohm's objective interpretation of the wavefunction (Bohm 1952*a*, *b*, 1980, 1990; Bohm & Hiley 1993). We are going to deal with this force in more detail in Monograph (I), but in passing, we have this to say about this force, $(-\nabla\Psi)$.

In Monograph (I), we are going to champion an interpretation of Ψ in which this object is a random variable. To give the reader an idea, we know that if we have a system of particles whose wavefunctions are Ψ_j, these particles will undergo superposition such that the resultant wavefunction will be given $\Psi = \sum c_j \Psi_j$, where c_j are superposition parameters such that $\sum_j c^\dagger_j c_j = 1$. According to current wisdom $c^\dagger_j c_j$ is interpreted as being the probability that the system Ψ will be found in the state Ψ_j. In the new interpretation being championed in Monograph (I), the c_j's are dynamical random variables and the wavefunctions Ψ_j are exact well defined functions. If c_j are dynamic random variables and Ψ_j are exact well defined functions, then $\Psi = \sum c_j \Psi_j$ is a dynamic random function, hence $-\nabla\Psi$ is a dynamic random force which is expected to endow the *Quantum World* with its bizarre properties of randomness and probability. This force can be thought of as being the gravitational force at the quantum level, hence it is the quantum gravitational force operating at the atomic scale and $-\nabla\Phi$ is the gravitational force acting on the astronomical and cosmological scale. If this proves correct, then quantum and classical gravitation are described by two different forces. This invariably leads us to the fact that the gravitational force is not a unitary force in the present unified theory.

Though not convincing, what we are going to now is to justify the inclusion of the scalar field ϕ. We introduced this field so that emergent force from it – *i.e.*: $-\nabla\Phi$; will act as the gravitational field on astronomical scales while the emergent force from ψ – *i.e.*: $-\nabla\Psi$; will be a force that will act on the nuclear scale. If we did not include ϕ, we where going to have other than the forces, $cv^{\nu} F_{\nu}^{(aj)j} \hat{\boldsymbol{e}}_j$; just one force to identify with the gravitational force and this would have been $-\nabla\Psi$. Since this force is associated with quantum randomness, it would have been difficult to justify this as the force gravitation that act on the astronomical scale since on this scale, we do not observe significant randomness as in quantum mechanics. Depending on what experience detects, the inclusion of the scalar ϕ can be revised.

Now, we are going to point to a very important problem and this is the problem of mass. We are not going to solve it here, but merely state that this problem will be spread across the two monographs *i.e.* Monograph (I & II). Equation (106) as it stands suggests that the acceleration of a massive charged particle inside a combined gravitational and electric field should be independent of its mass to charge ratio. How can this be? Suppose we have a particle of inertial mass m inside the gravitational of the Sun. Suppose the gravitational field of the Sun is exactly as that described by Newton's Law. Further suppose the Sun has an electronic charge Q and also that this particle of inertial mass m has an electronic charge q. It goes without saying that the total force acting on the this particle in the combined gravitational and electric field of the Sun is:

$$\boldsymbol{F} = m\boldsymbol{a} = -\frac{G\mathcal{M}m}{r^2}\hat{\boldsymbol{r}} + \frac{1}{4\pi\varepsilon_0}\frac{Qq}{r^2}\hat{\boldsymbol{r}} \tag{107}$$

where ε_0 is the permittivity of free space and $\mathcal{M}$ is the mass of the Sun. From (107), neglecting the quantum force and as-well the magnetic force, it follows that the acceleration of this massive charged particle in the combined gravitational and electrical field of the Sun is:

$$\boldsymbol{a} = -\frac{G\mathcal{M}}{r^2}\hat{\boldsymbol{r}} + \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\left(\frac{q}{m}\right)\hat{\boldsymbol{r}}. \tag{108}$$

If the particle where not electronically charged, its acceleration in this combined gravitational and electrical field of the Sun would be independent of the particle's inertial mass. This means, in such a field as the combined gravitational and electronic field, the weak equivalence principle does not hold as the acceleration of a particle is dependent on its composition. In (106), information about the other particle in this combined gravitational and electrical field in not present. This means the acceleration of any particle, whether charged or not, will be independent of the electronic charge of the particle. This goes against our current understanding of the nature of gravitational and electrical fields as outlined above. In order to obtain results in conformity with experience, we will need to develop the theory of interaction of particles. As the UFT stands in present, it only deals with a single particle. As aforestated, this problem will solved across the two monographs.

9 Discussion & Conclusions

We believe that we have herein shown that it is possible to describe all the known forces of *Nature via* a 4D geometric theory that needs not the addition of extra dimensions as is the case with string and string related theories which today stand on the beacon of unification as the most favoured/promising theories. Although a lot of work is still to be done as mentioned in the penultimate of the introduction section that two volumes each spanning about 200 pages are at an advanced stage of preparation, in our most humble and modest opinion, we believe that the present attempt is a significant contribution to the endeavour of the unification program of physics. We are of the strong opinion that the reader should be their own judge. We shall state a few main points that lead us to believe that the present attempt is a significant contribution to the endeavour of the unification program of physics.

First, we know that in Riemann geometry and as-well in the GTR, the metric tensor is described by 10 different potentials and in-turn these potentials describe the gravitational force. In the RHS, the metric tensor is described by just 4 different potentials and not 10 and these potentials describe not the gravitational but the nuclear forces. At the very least, this is a paradigm shift we leads to a much simple model of spacetime and matter! The RHS on which the present theory is founded, is different from that of employed in Riemann spacetime in that the unit vectors of the RHS are variable at all points on this continuum. This property of the RHS that it has variable unit vectors make it fall into the category of Weyl's brilliant but failed geometry, but the RHS is different because the affinities are tensors; the meaning of which is that vectors do not change their length and angles under parallel transport.

An important out-come which lead to the ideas laid down here is the revision carried out of what is a reference system and a system of coordinates. This revision has lead us to the idea that it is erroneous to treat time much the same as we do when dealing with reference systems. It has been concluded that the way in which we have treated time and space when it comes to coordinate transformations since Minkowski's 1908 pronouncement in his now famous lecture when he said:

> *"The views of space and time which I wish to lay before you have sprung from the soil of experimental physics, and therein lies their strength. They are radical. Henceforth space by itself, and time by itself, are doomed to fade away into mere shadows, and only a kind of union of the two will preserve an independent reality."*

is partly at fault because this has lead us to treat time between reference systems and coordinate systems in a manner that makes no physical distinction between the two. If this is the case, that space and time be treated on an equal footing irrespective of whether we are dealing with space and time coordinates or reference systems, it could mean that the labelling of points in spacetime has a dynamic physical meaning – it has been argued that this is clearly not right and this is what leads us to coordinates systems that give different physical description of the same physical phenomena like with the Schwarzchild metric which has a singularity and this singularity is transformed away by the Eddington-Finkelstein coordinates. The physics from these two coordinate systems is very different yet they are suppose to describe the same physical event.

We note that while the gravitational force has here been brought under the same roof with the nuclear forces, it is not unified with these forces in a manner Michael Faraday had hoped, *i.e.*, it being inter-convertible to other forces just as the electronic and magnetic forces are inter-convertible into one another. It is not even what Einstein had envisaged in his 1929 interview with the journal Nature when he said[6]:

> *"Now, but only now, we know that the force which moves Electrons in their ellipses about the nuclei of the atom is the same force which moves our Earth in its annual course about the Sun, and it is the same force which brings to us the rays of light and heat which make life possible upon this planet."*

Einstein said these words when he was describing his then proposed final theory on distant parallelism that latter proved to have no resemblance with physical reality. Here (in the present attempt), we see that the gravitational field acts *via* the scalar field ϕ, this force acts on the astronomic scale; on the atomic scale it [gravity should] act(s) *via* the spinor ψ. On the sub-nuclear level, we have the Abelien gauge field A_{μ}. All these forces are united though addition *via* the tensorial affine:

[6]"News and Views", 1929, *Nature*, Vol. 123, pp.174-175.

$$\bar{\Gamma}^{\lambda}_{\mu\nu} = \underbrace{\overbrace{\Gamma^{\lambda}_{\mu\nu}\left(A^{(a)}_{\mu}\right)}^{\textbf{Nuclear Physics}} + \overbrace{Q^{\lambda}_{\mu\nu}(\psi)}^{\textbf{Atomic Physics}} + \overbrace{G^{\lambda}_{\mu\nu}(\phi)}^{\textbf{Astrophysics}}}_{\textbf{Unified Forces}} . \tag{109}$$

The above may give one the impression that gravitation and electricity are not unified in a manner Faraday had hoped. This is not the case; we believe that this work sets the stage for a unification of gravitation, electricity and magnetism. We should make mention that work on such a scheme is underway in Monograph (II). From this work, we find that the magnetic field of stellar bodies such as the Sun, Earth, and the stars that populate the heavens may very well have a gravitational origin.

It strongly appear that we have been able to achieve one thing that Einstein sought (not that Einstein's opinion is a fact, but merely that him begin the inspiration to many in this field his thoughts on the subject are important) in a unified theory – *i.e.*, the material field ψ must be part and parcel of the fabric of spacetime. Einstein is quoted as having said the left handside of his equation is like marble and the right handside is like wood and that he found wood so ugly that his dream was to turn wood into marble. These feelings of Einstein against his own GTR are better summed up in his own words in a letter to Georges Lemaître (1894 − 1966) the Belgian Roman Catholic priest on September 26, 1947:

> *"I have found it very ugly that the field equation should be composed of two logically independent terms which are connected by addition. About justification of such feelings concerning logical simplicity is to difficult to argue. I can not help to feel and I am unable to believe that such an ugly thing should be realized in Nature."*

Einstein hoped that the final theory must be such that the ponderable material function (ψ) must emerge from the geometry of the theory – this has been achieved. The wavefunction is part and parcel of the fabric of the RHS – it is part of the metric that defines this spacetime.

The equations discovered here – and more importantly the geodesic equation of motion; are completely gauge invariant and covariant under a change of the coordinate system as well as under a change of the reference system. The geodesic equation of motion reproduces the Lorentz equation of motion. However, we note that this equation needs a deeper inspection *viz* its meaning to the relation between inertia and gravitational mass.

The gravitational field of GTR is described by the metric and in the low energy regime, Einstein's equation predict the existence of gravitational radiation. Given that in the present theory, the metric no longer describes the gravitational force, but the electromagnetic potential *etc*, it follows from the present UFT that Einstein's gravitational waves may not exist! There are currently at least four major experiments running the effort of which is to detect Gravitational waves. These experiment are Laser Interferometer Gravitational Wave Observatory (LIGO)[7] which is a joint project between scientists at MIT and Caltech in the USA; The Virgo detector which is an Italian project; Geo 600 is a gravitational wave detector located in Hannover, Germany; AIGO which is an Australian project, is another gravitational wave detector.

The gravitational waves which the above mentioned experiments are searching for are those understood from the GTR and these are caused by a periodic time varying curvature which is caused by the presence of say, a spinning mass. In the GTR paradigm, the more massive the object is, the greater the curvature it causes, and hence the more intense the gravity if this mass is spinning. When these massive objects move around in spacetime, the curvature will change in accordance to the motion of the object thus causing ripples in spacetime which then spread outward at the speed of light like ripples on the surface of a pond. These ripples are what is then called gravitational waves in the GTR. To date no direct evidence of their existence has yet come forth.

Lastly, it is expected of a UFT to say something about darkmatter and darkenergy (about darkmatter and darkenergy, see *e.g.* Zwisky 1933, 1937; Rubin & Ford 1971; Rubin *et al.* 1985). The present reading is

[7] See *e.g.* `http://www.ligo.caltech.edu/LIGO_home.html`

silent on the matter. This does not mean it does not have anything to do with this subject. Work on the inclusion of darkmatter and darkenergy began earnestly with the reading Nyambuya (2009) in which the dark field (which explains the darkmatter and darkenergy) have been introduced as a four cosmological vector field Λ_μ. In this reading (Nyambuya 2009), we introduced this field by making the transformation $\partial_\mu \longmapsto \partial_\mu + \Lambda_\mu$. We introduced this four cosmological vector field to explain the apparent asymmetry between matter and antimatter. We note that in the present theory, this vector can be introduced by the addition of a dark-potential we $\phi_D = \exp\left(\int \Lambda_\mu dx^\mu\right)$ to the unit vector (23), *i.e.*: $\boldsymbol{e}_\mu^{(a)} = \frac{1}{2} i\phi\phi_D A_\mu \gamma_\mu^{(a)} \psi$ where $\Lambda_\mu = \Lambda_\mu(x^k, t)$. This leads to a cosmological-affine connection:

$$\mathrm{C}^\lambda_{\mu\nu} = \frac{1}{2} g^{\lambda\alpha}\left\{g_{\alpha\mu}\Lambda_\nu + g_{\nu\alpha}\Lambda_\mu - g_{\mu\nu}\Lambda_\alpha\right\}, \tag{110}$$

thus the resultant affine connection would be:

$$\bar{\Gamma}^\lambda_{\mu\nu} = \underbrace{\overbrace{\Gamma^\lambda_{\mu\nu}\left(A_\mu^{(a)}\right)}^{\textbf{Nuclear Physics}} + \overbrace{\mathrm{Q}^\lambda_{\mu\nu}(\psi)}^{\textbf{Atomic Physics}} + \overbrace{\mathrm{G}^\lambda_{\mu\nu}(\phi)}^{\textbf{Astrophysics}} + \overbrace{\mathrm{C}^\lambda_{\mu\nu}\left(\Lambda_\mu\right)}^{\textbf{Cosmology}}}_{\textbf{TOE}} \tag{111}$$

The new definition of the unit vector, *i.e.*, $\boldsymbol{e}_\mu^{(a)} = \frac{1}{2} i\phi\phi_D A_\mu \gamma_\mu^{(a)} \psi$, leads to the curved spacetime Dirac equations transforming to:

$$i A_\mu \gamma_\mu^{(a)} \left[\partial^\mu + \Lambda^\mu\right] \psi = \left(\frac{m_0 c}{\hbar}\right) \psi. \tag{112}$$

What this cosmological field does is that it leads to two perfectly symmetrical worlds of ponderable and non-ponderable material particles; where in the ponderable world, we have the Electrons, Protons *etc*, and in the non-ponderable world we have the dark-Electrons, dark-Protons *etc*. The ponderable particles have positive mass and energy while the non-ponderable have negative mass and energy.

Also, the introduction of the cosmological field leads to the geodesic equation of motion, to become:

$$\frac{d^2\mathbf{x}}{d\tau^2} - v^\nu F_\nu{}^j \boldsymbol{e}_j + \boldsymbol{\nabla} Q = -\boldsymbol{\nabla}\Phi - \boldsymbol{\Lambda} c^2 \tag{113}$$

where $\boldsymbol{\Lambda} = \Lambda^j \boldsymbol{e}_j$ where $j = 1, 2, 3$.

In the succeeding paragraph and those that follow thereafter, we shall discuss the the results of our investigation in point form. Our discussion will be limited to what we have discovered here and we shall not try to make comparisons of the ideas here with the many proposals of UFTs (*e.g.* Garrett 2007) and the reason for this is to avoid a much as is possible any confusion.

In closing, we would like to say that, while further work is underway, if it turns out that this theory is a true description of natural reality or anything to go by as we believe it to be, then, it is without doubt that the train and ground for a grander understanding of the natural *World* from a unified perspective has been set forth. It seems to us, this theory is something worthwhile for one to spend their time on because the mathematical scheme discovered here does appear to be connected with physics as we know it, thus, as per Dirac's philosophy, one should concentrate on getting the interesting mathematics [out and into the World].

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