A Multiple Particle System Equation Underlying the Klein-Gordon-Dirac-Schrödinger Equations

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Abstract

The purpose of this paper is to illustrate a fundamental, multiple particle, system equation for which the Klein-Gordon-Dirac-Schrödinger equations are single particle special cases. In the same manner that eigenvalues of the Schrödinger equation represents energy levels of an interacting atomic system, eigenvalues represent particle energies in an interacting system of particles. An equation and a solution is proposed that treats all of the particles in the universe as a single system. The proposed solution is a descriptor of a symmetric, light speed expanding group of interacting particles having familiar constituents.

INTRODUCTION

The success and the accuracy of Quantum Mechanics has been the most oustanding achievement in the history of science, bet it is limited in the scope of its application to macroscopic systems. All of the parameters associated with the microscopic systems, spin inetria, energy, physical constants, etc, are obviously induced by the universe at large. The values associated with the physical constants that apply to the microscopic systems have to be the result of the effect of the entire system. This paper is an attempt to define the entire system as one integrated function. That one function having energy levels that are the particles and their interactions with the other particles in the system.

The approach is somewhat unusual in that it postulates both a differential equation, and a solution to the universe of particles. From there, with as few assumptions as possible, the standard equations and rules of physical interactions are recovered, primarily illustrated are those of QM and E &M. Presented is a system equation, and solutions termed a "Systemfunction", Θ which in effect, can be considered a space, of eigensolutions for an expanding system of point particles.

The primary features of this development is the inclusion in the particle function not only the internal structure of the particle, but the external structure as well. Standard particle functions are separate from, and are acted on by external potentials and functions, whereas in this development all of the external interactions are an integral part of the function itself.

Reviewing the standard QM equations in the current notation.

Field free KG

$$\left(-\frac{\partial^2}{\partial x^2}-\frac{\partial^2}{\partial y^2}-\frac{\partial^2}{\partial z^2}+\frac{\partial^2}{\partial ct^2}\right)\psi=-\frac{1}{\tilde{r}_0^2}\psi \tag{1}$$

Field free Dirac

$$\left(+\gamma_1\frac{\partial}{\partial x}+\gamma_2\frac{\partial}{\partial y}+\gamma_3\frac{\partial}{\partial z}+\gamma_4\frac{\partial}{\partial ct}\right)\psi=i\frac{1}{\tilde{r}_0}\psi \tag{2}$$

Field free Schrödinger

$$\left(-\frac{\partial^2}{\partial x^2}-\frac{\partial^2}{\partial y^2}-\frac{\partial^2}{\partial z^2}-2i\frac{1}{\tilde{r}_0}\frac{\partial}{\partial ct}\right)\psi= 0 \tag{3}$$

Where $\tilde{r}_0$ is the particle Compton radius $\tilde{r}_0 = m_0 c/\hbar$. Of the rest mass of a particle (For general conventions, and notation, see appendix [II])

The system equation, being proposed, is a descriptor of a time expanding system of charged, half spin particles. The inclusion of particles other than half-spin will be left for later. Normal quantum mechanical expressions, (Klein-Gordon-Dirac-Schrödinger), represent single particles systems, separated from the rest of the universe, interacting through a coupling potential. The standard QM, coupling is by the insertion of a representation of the potential through the correspondence relations, and is not part of the wavefunction, but is action on the wavefunction. (See Appendix I, for a heuristic view of the issue.)

I. THE GENERAL EQUATION

The system equation being proposed for the Systemfunction will be.

$$\left(\frac{\partial^2}{\partial\left(X^2\right)^2}+\frac{\partial^2}{\partial\left(Y^2\right)^2}+\frac{\partial^2}{\partial\left(Z^2\right)^2}-\frac{\partial^2}{\partial\left(\Re^2\right)^2}\right)\tilde{\Theta}=0 \tag{4}$$

That is the derivative with respect to a virtual displacement of the square of the expansion of the universe. $\Re^2 = cT$ expanding at ct, and X, Y, & Z are the coordinates of the expanding sphere of the universe. A presumption is made that the time and spaces coordinates have s separation constant such that:

$$\left(\frac{\partial^2}{\partial\left(X^2\right)^2}+\frac{\partial^2}{\partial\left(Y^2\right)^2}+\frac{\partial^2}{\partial\left(Z^2\right)^2}\right)\tilde{\Theta} \;=\; \frac{\partial^2}{\partial\left(\Re^2\right)^2}\tilde{\Theta} \;=K\tilde{\Theta} \tag{5}$$

Thus:

$$\frac{\partial}{\partial\left(\Re^2\right)}\tilde{\Theta} \;=K\tilde{\Theta} \tag{6}$$

Not the same K. From this $\tilde{\Theta}$ is seen to be a real scalar function, but is presumed to be the product of a mixed real and imaginary function:

$$\tilde{\Theta}=\Theta^*\Theta\,. \tag{7}$$

We can then have:

$$\left(\gamma^\mu\frac{\partial}{\partial\left(X^2\right)_\mu}\right)\left(\gamma^\mu\frac{\partial}{\partial\left(X^2\right)_\mu}\right)\Theta^*\Theta=0 \tag{8}$$

Inserting a unitary matrix $A^*A=1$ into the function allows us to separate one of the bracketed terms into two column matrix such that:

$$\left(\gamma^\mu\frac{\partial}{\partial\left(X^2\right)_\mu}\right)\left(\Theta^*\Theta\right)\left(\gamma^\mu\frac{\partial}{\Theta^*\partial\left(X^2\right)_\mu}\;\Theta^*\mathbf{a}^*+\mathbf{a}\gamma^\mu\frac{\partial}{\Theta\partial\left(X^2\right)}\Theta_\mu\right)=0 \tag{9}$$

The left bracket of this expression is zero because of Eq. (4). Since the left and right are identical then the right side must be zero also. We will presume that the two terms can individually be independent and thus equal to a constant matrix. We then have.

$$\gamma^\mu\frac{\partial}{\partial\left(X^2\right)_\mu}\;\Theta^*\mathbf{a}^*=-iK_2\Theta^*\mathbf{a}^* \tag{10}$$

$$\gamma^\mu\frac{\partial}{\partial\left(X^2\right)_\mu}\;\Theta\,\mathbf{a}\quad=+iK_2\Theta\mathbf{a} \tag{11}$$

The left bracket of this expression has two terms that are matrix functions having both real and imaginary components. But from Eq.(4) the real portion is:

$$\mathrm{real}\left[\gamma^{\mu}\frac{\partial}{\partial\left(X^{2}\right)}\Theta_{\mu}\right]=0, \tag{12}$$

or:

$$\frac{\partial}{\partial\left(\Re^{2}\right)}\Theta=-K_{1}\Theta \tag{13}$$

Where K_2 is constant separation matrix between the space and time variables. Since the universe is quite large and the radius $(\Re)$ is slow changing we can change the expression Eq.(10) , and Eq.(11) , into local coordinate differential. Details are shown in appendix XX.

$$\gamma^{\mu}\frac{\partial}{2\Re\partial(x)_{\mu}}\ \Theta^{*}\mathbf{a}^{*}=-iK_{2}\Theta^{*}\mathbf{a}^{*} \tag{14}$$

$$\gamma^{\mu}\frac{\partial}{2\Re\partial(x)_{\mu}}\ \Theta\mathbf{a}\quad=+iK_{2}\Theta\mathbf{a} \tag{15}$$

For simplicity and a foreknowledge of the development we will set:

$$K_{1}=-\frac{1}{\tilde{r}_{0}^{2}}=\left(\frac{m_{0}c}{\hbar}\right)^{2} \tag{16}$$

and:

$$K_{2}=\pm\frac{1}{2\Re\tilde{r}_{0}}\ I \tag{17}$$

and thus we have for our imaginary expression Eq. (11), in local coordinates:

$$\gamma^{\mu}\frac{\partial}{\partial(x)_{\mu}}\Theta\mathbf{a}=\pm\frac{i}{\tilde{r}_{0}}\ \Theta\mathbf{a} \tag{18}$$

Eq. (18),has the appearance of the Dirac expression, but not quite. It is a column matrix equation, however and its connection to the Dirac equation will be explored in Appendix XX.

There is a presumption that the Systemfunction, Θ being proposed, can be represented by a sum of eigenfunctions.

$$\Theta = \sum c \, \vec{\Theta}_n \tag{19}$$

which we will designate as the systemmatrix and note that the real terms cancel:

$$\Theta = -\begin{bmatrix} (K_1 + iK_2)_1 1\vec{\Theta}_1 & & & \\ & (K_1 + iK_2)_2 \vec{\Theta}_2 & & \\ & & .. & \\ & & & (K_1 + iK_2)_n \vec{\Theta}_n \end{bmatrix} \tag{20}$$

II. THE GENERAL SOLUTION

The Systemfunction

Our purpose now, is to design a Systemfunction that is a solution to our general equation. We will assert it to be the space of eigensolutions, each of which is a solution for a particular particle Eq. (20). The linear sum of these eigensolutions being the complete systemfunction.

Single Particle Action

We will propose that the systemfunction is somehow a product of the actions of the individual particles in the system. The action for the m particle being the path integral over all possible paths from the initiation of the universe, until the current time as observed at a given point.

From standard QM the nominal wavefunction for a particle can be stated as:

$$\psi(x,t) = Ae^{i\int \frac{L}{\hbar}dt} = Ae^{i\frac{S}{\hbar}} \tag{21}$$

And for the wavefunction of a collection of independent particles this is:

$$\psi(x,t) = Ae^{i\left(\int \frac{L_1}{\hbar}dt + \int \frac{L_2}{\hbar}dt \ \cdots\right)} = Ae^{\frac{i}{\hbar}\sum_n S_n} \tag{22}$$

In an analogy with this will presume the systemfunction to be defined by:

$$\Theta_m = Ae^{\int_0^T \frac{L^2{}_m}{\hbar}dt} = Ae^{\sum_N S_n \sum_N S_n} \tag{23}$$

Where we have a square of the sum of the actions of the individual particles. In this case it is apparent that L^2 does not represent the sum of non-interacting particles But the squares and cross products of a collection of individual particle actions. Thus this is not a function representing an isolated particle, but the collection of particles. It is to be shown that the cross product of these properly defined actions represent the potentials and E & M interactions between the particles. For a given eigenvalue the eigenfunction will represent the entire particle, including its interactions with the rest of the system.

Particle Action, Half Spin particles

The action for a particle is taken to be the path integral of the Lagrangian from one point in time to a second point in time, over all possible paths. In this case we will require the first point to be the initiation point of the universe, and the final point, to the current time, as observed at a point in the center of the system. An observer of the value of the function at the center of the system, will observe this value in retarded time. Note that the evaluation point of this action, which is at the center of the coordinate system, is not the same as the observation point, which can be at a distance, and is observed in retarded time. (See appendix III for further discussion.)

For Eq. (23), the action of a one half spin particle, from the start of the universe to its present position, as evaluated at the coordinate origin, and observed at the a distance r from origin is proposed to be:

$$\vec{s}_m = \frac{\pm i\left|\Re - r\right| m_m c\vec{V}_m \pm \frac{1}{2}\hbar\vec{\eta}}{\left(m_m c r_m / \alpha\right) - \hbar} \tag{24}$$

$\Re$ is the radius of the universe at the origin of the coordinate system, r is the distance of an observation point from the origin. m_m is the relativistic mass of the m particle, $\vec{V}_m$ is the four-velocity, in Clifford matrix, and $\vec{\eta}$ is a null vector associated with the spin of the particle. r_m is the distance from the origin of the coordinate system to the m particle. The first $\pm$ is associated with the particle charge and the second is associated with the spin null vector.

Note that $|\Re - r|$ is time dependent, and makes the action retarded with respect to the observer and is dependent on the distance from the origin to the distance of the observation point. A variation in r does not represent a change in the value of the function, but the delay in the time the function is observed

Although the coordinates may be defined as linear, the general equation Eq. (4), defined above, is not linear in the coordinates, so the position of the coordinates system affects the value of the function. A change in the location of the origin with respect to the particle changes the value of the action due to the $1/r_m$ dependence.

The Systemfunction can be evaluated at any point in the collection of particle actions, but is always at the center of the system. Only at the locus of the action of a particle $r_n = 0$ Will it have an eigensolution.

Expanding the function With the first particle being the n^{th} particle this would be:

$$\vec{\Theta}_n = \exp\left(\vec{S}_n^2 + \left(\sum_m^N \vec{S}_m \right) \vec{S}_n + \vec{S}_n \left(\sum_m^N \vec{S}_m \right) + + + \right) \tag{25}$$

Notable here is that the first term $\vec{S}_n^2$ is the square of the action for the single particle, and will define the exponent of the free particle. The cross terms would be the Clifford products of the interacting actions of the other particles in the system.

Letting the coordinate system be located coincident with the n th particle $r_n = 0$, and $r_m >> 0$ The actions of Eq. (24) become for the n and m particle.

$$\vec{S}_n = \pm i \frac{|\Re - r|}{\tilde{r}_n} \vec{V}_n \pm \frac{1}{2} \vec{\eta}_n, \qquad \vec{S}_m = \pm \alpha \left(i \frac{|\Re - r|}{r_m} \vec{V}_m \pm \frac{1}{2} \frac{\tilde{r}_m}{r_m} \vec{\eta}_m \right) \tag{26}$$

The Free particle

The Systemfunction for the free particle is just:

$$\vec{\Theta}_n = e^{\vec{S}_n{}^2} \tag{27}$$

Putting in the moment for the n^{th} particle evaluated at $r = 0$ for $\vec{S}_n^2$ gives the Systemfunction representation of the free particle (See **Appendix IV** for details):

$$\vec{\Theta}_n = \exp\left[-\left(\frac{|\Re - r|}{\tilde{r}_n} \vec{V}_n \right)^2 + \underset{n}{\overset{C}{\pm}} \underset{n}{\overset{S}{\pm}} \; i \frac{|\Re - r|}{\tilde{r}_n} \left(\vec{V}_n \bullet \vec{\eta}_n \right) \right], \tag{28}$$

Since this the function for the free particle. we can illustrate it explicitly in terms of mass and velocities.

$$\vec{\Theta}_n = \exp\left[-\left(\frac{|\Re - r| m_n c}{\hbar} \right)^2 \left(1 - \frac{\vec{v}^2}{c^2} \right) + \underset{n}{\overset{C}{\pm}} \underset{n}{\overset{S}{\pm}} \, i \frac{|\Re - r| m_n c}{\hbar} \left(1 - \vec{v}_n \bullet \vec{\eta}_n \right) \right] \tag{29}$$

Note that this is the combined real and imaginary function for the free Particle defined at the particle center, and has an extremely small real value. For an electron $\sim \exp(-75)$,

The **real** function From Eq.(13) is:

$$\vec{\Theta}_{nR} = e^{-\left(\frac{|\Re - r| M_n c}{\hbar} \right)^2 \left(1 - \frac{\vec{v}^2}{c^2} \right)} \tag{30}$$

Which With Eq. (16), just returns the coordinate independent energy, momentum equation of motion for a free particle.

$$\frac{\partial}{\partial \Re^2} \vec{\Theta} = \frac{\partial}{\partial \Re^2} \exp\left(\vec{S}_n{}^2 \right) = -\frac{1}{\tilde{r}_0^2} \vec{\Theta} \tag{31}$$

or

$$\left(\frac{m_0 c}{\hbar} \right)^2 = \left(\frac{m_n c}{\hbar} \right)^2 \left(1 - \frac{\vec{v}^2}{c^2} \right) \tag{32}$$

Which is just the free particle relativistic equation of motion

For the **imaginary** function of the free particle Eq. (18), where we note that $\Re \rightarrow \Re_0 + ct$ and $i\Re_0$ is just a constant phase factor that can be left out is:

$$\vec{\Theta}_{nI} = e^{\overset{C}{\underset{n}{\pm}}\overset{S}{\underset{n}{\pm}} \frac{|\Re_0+ct-r|m_n c}{\hbar}(1-\vec{v}_n\bullet\vec{\eta}_n)} \rightarrow e^{\overset{C}{\underset{n}{\pm}}\overset{S}{\underset{n}{\pm}}(\vec{p}_n\bullet\vec{\eta}_n - m_n c)|ct-r|/\hbar} \tag{33}$$

or:

$$\vec{\Theta}_{nI} = e^{\overset{C}{\underset{n}{\pm}}\overset{S}{\underset{n}{\pm}}\, i\,(ct-r)\frac{M_n c}{\hbar}(1-\vec{v}_n\bullet\vec{\eta}_n)} \tag{34}$$

The sign of the exponent is the product of the sign of the charge and spin. This results in both the equation Eq.(18) , and the proposed solution, being similar in appearance to the Dirac function, and its common solution, The corresponding Dirac Wavefunction is:

$$\psi = u_{\vec{p}} e^{i(\vec{p}\bullet\vec{r}-Et)/\hbar} \tag{35}$$

See appendix xx for the relation between the free particle systemfunction and the Dirac free particle function.

Some of the features of Eq. (34), are easily determined. The velocity wavelength is:

$$\frac{1}{\lambda} = \frac{1}{2\pi}\left|\frac{M_{0n}}{\hbar}\vec{V}_n\right| = \frac{p}{h} \tag{36}$$

Which is the correct free particle deBroglie wavelengths for the velocity and the total energy. The frequencies are.

$$\omega_d = \frac{M_{0n} cv}{\hbar} \quad \&, \quad \omega_C \quad \frac{M_{0n} c^2}{\hbar} \tag{37}$$

Which are the relativistic deBroglie kinetic frequency and the Compton free particle frequency.

In **Appendix VII**, we show that the systemfunction for a free particle can be considered a composition particle consisting of a Dirac type particle, and a half spin, zero rest mass, light speed particle. The second particle having characteristics of a neutrino.

Total Particle Function

The foregoing was just the free particle expression. To illustrate the **real** function representing the particle is inclusive of the interactions, all the terms can be included in Eq. (16) . This gives the rest of the expression including the electromagnetic interaction. (See **Appendix IV** for details.)

$$\frac{\partial}{\partial\Re^2}\vec{\Theta} = \frac{\partial}{\partial\Re^2}\exp\left(\vec{S}_n{}^2 + \vec{S}_n\ \ \vec{S}_m + \vec{S}_m\vec{S}_n \bullet\bullet\bullet\right) = -\frac{1}{\tilde{r}_0^2}\vec{\Theta} \tag{38}$$

or

$$\left(\frac{m_{n0}c}{\hbar}\right)^2 \ = \ +\frac{m_n{}^2c^2}{\hbar^2}\left(1-\frac{\vec{v}_n{}^2}{c^2}\right) - \sum_n^{C}\!\pm\sum_m^{C}\!\pm 2\frac{m_n}{\hbar^2}\frac{Q^2}{r_m}\left(1-\frac{\vec{v}_m}{c}\bullet\frac{\vec{v}_n}{c}\right), \tag{39}$$

Which is the square of the Lagrangian for a moving charged particle in the presence of another moving charged particle. Taking the square root and summing over a collection of particles gives the familiar expression.

$$m_{n0}c^2 \approx +m_nc^2\left[\left(1-\frac{1}{2}\frac{\vec{v}_n{}^2}{c^2}\right) \pm \sum_m \frac{Q^2}{r_m}\left(1-\frac{\vec{v}_m}{c}\bullet\frac{\vec{v}_n}{c}\right)\right] \tag{40}$$

Which is just the Lagrangian for a particle in the presence of a collection of charged moving particles.

For the **imaginary** function that includes the other particles in the system is)See Appendix XX for details),we have, from Eq. (18),

$$\gamma^\mu\frac{\partial}{2\Re\partial(x)_\mu}e^{\overset{C}{\underset{n}{\pm}}\overset{S}{\underset{n}{\pm}}\ i\frac{\Re}{\tilde{r}_n}\vec{V}_n\bullet\vec{\eta}_n\ + i\alpha\frac{\Re}{r_m}\left(\overset{S}{\underset{n}{\pm}}\overset{C}{\underset{m}{\pm}}\vec{V}_m\bullet\vec{\eta}_n\ \ \overset{C}{\underset{n}{\pm}}\overset{S}{\underset{m}{\pm}}\ \vec{V}_n\bullet\vec{\eta}_m\frac{m_n}{m_m}\right)}\mathbf{a} = \pm\frac{i}{2\Re\tilde{r}_0}\ I\vec{\Theta}_n\mathbf{a} \tag{41}$$

Since we have already noted the free particle aspects, we will let the velocity of the n particle be zero and evaluate the energy aspects of the interactions.

This leaves only the time differential in Eq. (18).

$$\frac{1}{\tilde{r}_{0n}} = \frac{\partial}{\partial(ct)_\mu}\left(\overset{C}{\underset{n}{\pm}}\overset{S}{\underset{n}{\pm}}\; i\frac{\Re}{\tilde{r}_n}\vec{V}_n \bullet \vec{\eta}_n \;+ i\alpha\frac{\Re}{r_m}\left(\overset{S}{\underset{n}{\pm}}\overset{C}{\underset{m}{\pm}}\vec{V}_m \bullet \vec{\eta}_n \;\; \overset{C}{\underset{n}{\pm}}\overset{S}{\underset{m}{\pm}}\; \vec{V}_n \bullet \vec{\eta}_m \frac{m_n}{m_m}\right)\right) \quad \vec{V}_n \to 1, \qquad (42)$$

or:

$$\pm i\frac{m_{0n}c}{\hbar} = \overset{C}{\underset{n}{\pm}}\overset{S}{\underset{n}{\pm}}\left[i\;\frac{mc}{\hbar} \;+ i\,\frac{\alpha}{r_m}\left(\left(\vec{V}_m \bullet \vec{\eta}\right) + \frac{m_n}{m_m}\right)\right] \qquad (43)$$

or

$$\frac{m_{0n}c^2}{\hbar} = \frac{mc^2}{\hbar} \;+ \frac{Q^2}{\hbar r_m}\left(\left(\vec{V}_m \bullet \vec{\eta} + \frac{m_n}{m_m}\right)\right) \qquad (44)$$

since we know $Q\vec{V}_m / r_m$ is the potential of a moving charge,

$$Q^2\frac{\vec{V}_m \bullet \vec{\eta}}{r_m} \qquad (45)$$

Is the spin energy of a ½ spin particle with a 2 g spin factor in the field of a moving point charged particle. The m_n / m_m term is just the center of mass correction term to the potential.

This same potential energy is derived in the Dirac expression by adding the potential to the differential via the correspondence relations, and in that case is not a part of the wavefunction.

Similarities and contrasts with the Dirac functions:

$\vec{\Theta}_I$	ψ
Value of point function at the location of the particle. Observable in retarded time at distance $r = c\Delta t$	Location probability amplitude through time and space for point particle.
Phase velocity is c	Velocity not apparent, but has eigenvalue $\pm c$
Velocity wave is a plane wave in a direction of the spin axis, having deBroglie wavelength. Compton wave is spherical.	Velocity wave is a plane wave along he direction of travel, with wavelength equal deBroglie wavelength. No Compton space

	wavelength apparent.
Frequency: both deBroglie and Compton. ie Zitterbewegung	Frequency: Compton. Only. ie Zitterbewegung
Correct spin, particle interaction energy	Correct spin, particle interaction energy

CONCLUSION

A multiple particle system equation for the universe, and its connection to quantum mechanics has been demonstrated. If viable it is clear that it represents a new approach to particle dynamics, and perhaps opens a window into the relation between the particles, and the expanding universe. A point to note is, that by considering only the relativistic dynamics, and property of particles, interactions can be defined without the necessity of defining fields.

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Appedix I

The current base equations for QM are the Relativistic Schrödinger equation or the Klein-Gordon and the Dirac equation with the potential incorporated by use of the "correspondence relation" . This method asserts that the total momentum of a charged particle in an external field is modified as such that.

$$p \rightarrow p - \frac{q}{c}A \tag{1.1}$$

$$\frac{\partial}{\partial x_\mu} \rightarrow \left(\frac{\partial}{\partial x_\mu} - i\frac{q}{c}A_\mu \right) \tag{1.2}$$

The Schrödinger equation with fields included is:

$$\left(-\frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial y^2} - \frac{\partial^2}{\partial z^2} - \frac{2}{\mathbf{r}}\left[i\frac{\partial}{\partial ct} - \frac{\alpha}{r} \right] \right)\psi = 0 \tag{1.3}$$

And the field included Dirac expression becomes:

$$\left(+\gamma_1\left(\frac{\partial}{\partial x} - \frac{q}{c}A_x \right) + \gamma_2\left(\frac{\partial}{\partial y} - \frac{q}{c}A_y \right) + \gamma_3\left(\frac{\partial}{\partial z} - \frac{q}{c}A_z \right) + \gamma_4\left(\frac{\partial}{\partial ct} - \frac{q}{c}\varphi \right) \right)\psi = i\frac{1}{\tilde{r}}\psi \tag{1.4}$$

Before this modification, that is for the free field solution, the Klein-Gordon, and the Dirac equations are identical, in that the Dirac equation is a factorization of the KG equation using the Dirac matrix. Once the modification has been included via the correspondence substitution, the equations, are not equivalent, not even in interpretation [1]. The KG describes electromagnetic spin one particles in a potential, and the Dirac is a probability distribution of half spin particles.

It is asserted that including the potential, via the correspondence relation is the fundamental error plaguing QM, in explaining physical phenomena. The insertion of an infinite potential has to be considered an approximation and only accurate to the extent that the approximation of the inverse square potential is an accurate representation.

Appendix II

Definitions, Notation, and Conventions

The radius of the universe $\quad \Re = cT = \Re_0 + ct$

Four velocity of m particle	$\gamma^{\mu}\left(\frac{v}{c}\right)_{m\mu} = \vec{V}$
Three velocity of m particle	$\gamma^{k}\left(v\right)_{mk} = \vec{v}_m$
Null vector	$\left(\gamma^{k}\vec{r}_k + \gamma_4 \lvert r \rvert\right) / \lvert r \rvert$
Mass	m
Rest mass	m_0
Compton radius	$\tilde{r} = \frac{\hbar}{mc}$
Compton radius rest mass	$\tilde{r}_0 = \frac{\hbar}{m_0 c}$
Sign of m particle charge	$\overset{C}{\underset{m}{\pm}}$
Sign of m particle spin	$\overset{S}{\underset{m}{\pm}}$

The Dirac matrix convention used in this development is

$$\gamma_1 = \begin{bmatrix} & & & +1 \\ & & +1 & \\ & -1 & & \\ -1 & & & \end{bmatrix} \quad \gamma_2 = \begin{bmatrix} & & & -i \\ & & i & \\ & i & & \\ -i & & & \end{bmatrix} \quad \gamma_3 = \begin{bmatrix} & & 1 & \\ & & & -1 \\ -1 & & & \\ & 1 & & \end{bmatrix} \quad \gamma_0 = \begin{bmatrix} & & 1 & \\ & & & 1 \\ 1 & & & \\ & 1 & & \end{bmatrix} \qquad (2.1)$$

,

and

$$\gamma_1^{\ 2} = -1, \quad \gamma_2^{\ 2} = -1, \quad \gamma_3^{\ 2} = -1, \quad \gamma_4^{\ 2} = +1.$$

The product of the space coordinates is termed the spin matrix:

$$\bar{\sigma} = \gamma_1\gamma_2\gamma_3 \qquad -\bar{\sigma} = \gamma_3\gamma_2\gamma_1 \qquad \tau = \bar{\sigma}\gamma_4 \qquad \bar{\sigma} = \begin{bmatrix} & & -i & \\ & & & -i \\ i & & & \\ & i & & \end{bmatrix} \qquad (2.2)$$

The square is:

$$\bar{\sigma}^2 = 1 \qquad \tau^2 = -1$$

The product of $\bar{\sigma}^2$ with the coordinate vectors:

$$\bar{\sigma}\gamma_{1,2,3} = \bar{\sigma}\gamma_{1.2,3}$$
$$\bar{\sigma}\gamma_4 = -\gamma_4\bar{\sigma}$$

$$\bar{\sigma}\ \gamma_n = \ \gamma_2\gamma_3, \ \ \gamma_1\gamma_3, \ \ \gamma_1\gamma_2 \tag{2.3}$$

Which are the elements of the spin vector:

$$\sigma_1 = \gamma_2\gamma_3 \qquad \sigma_2 = \gamma_3\gamma_1 \qquad \sigma_3 = \gamma_2\gamma_1 \tag{2.4}$$

The vector four velocity:

$$\vec{V} = \left(\gamma_1 v_x + \gamma_2 v_y + \gamma_3 v_z + \gamma_4 c\right)/c \tag{2.5}$$

Commutation relation with V and S

$$\begin{aligned}
\bar{\sigma}\vec{V} &= \bar{\sigma}\left(\gamma_1 v_x + \gamma_2 v_y + \gamma_3 v_z + \gamma_4 c\right)/c \\
\vec{V}\bar{\sigma} &= \left(\gamma_1 v_x + \gamma_2 v_y + \gamma_3 v_z + \gamma_4 c\right)\bar{\sigma}/c \\
\bar{\sigma}\vec{V} + \vec{V}\bar{\sigma} &= 2\,\vec{S}\left(\gamma_1 v_x + \gamma_2 v_y + \gamma_3 v_z\right)/c \\
&= 2\left(\gamma_2\gamma_3 v_x + \gamma_1\gamma_3 v_y + \gamma_1\gamma_2 v_z\right)/c \\
&= 2\,\sigma \bullet V
\end{aligned} \tag{2.6}$$

$$\begin{aligned}
&\left(\sigma\bullet\vec{v}_n + \gamma_4 c\right)\left(\sigma\bullet\vec{v}_n + \gamma_4 c\right) = \bar{\sigma}\left(\vec{v}_n + \bar{\sigma}\gamma_4 c\right)\left(\vec{v}_n + \gamma_4\bar{\sigma}c\right)\bar{\sigma} \\
&\left(\sigma\bullet\vec{v}_n + \gamma_4 c\right)\left(\sigma\bullet\vec{v}_m + \gamma_4 c\right) = \bar{\sigma}\left(\vec{v}_n + \bar{\sigma}\gamma_4 c\right)\left(\vec{v}_m + \gamma_4\bar{\sigma}c\right)\bar{\sigma} \\
&= \vec{v}_m\vec{v}_n + c^2 \\
&\left(\sigma\bullet\vec{v}_n + \gamma_4 c\right)\left(\sigma\bullet\vec{v}_n + \gamma_4 c\right) + \left(\sigma\bullet\vec{v}_n + \gamma_4 c\right)\left(\sigma\bullet\vec{v}_n + \gamma_4 c\right) \\
&= 2\vec{v}_m\vec{v}_n + 2c^2 = 2\vec{V}_n\bullet\vec{V}_m
\end{aligned} \tag{2.7}$$

:

The product of two four velocities:

$$\vec{V}_n\vec{V}_m = \left(\gamma_1 v_{xn} + \gamma_2 v_{yn} + \gamma_3 v_{zn} + \gamma_4 c\right)\left(\gamma_1 v_{xm} + \gamma_2 v_{ym} + \gamma_3 v_{zm} + \gamma_4 c\right)/c^2 \tag{2.8}$$

or

$$\vec{V}_n\vec{V}_m = -\vec{V}_n \bullet \vec{V}_m + \left[\sigma \bullet \vec{v}_n \times \vec{v}_m + \gamma_4 c\left(\vec{v}_m - \vec{v}_n\right)\right]/c^2 \tag{2.9}$$

The inner product :

$$\vec{V}_n\vec{V}_m + \vec{V}_m\vec{V}_n = 2\vec{V}_n \bullet \vec{V}_m \tag{2.10}$$

The outer product:

$$\vec{V}_n\vec{V}_m - \vec{V}_m\vec{V}_n = \left[2\bar{\sigma} \bullet \vec{v}_n \times \vec{v}_m + 2\gamma_4 c\left(\vec{v}_m - \vec{v}_n\right)\right]/c^2 \tag{2.11}$$

Appendix III

Discussion of the action vector

The action for a particle is taken to be the path integral of the Lagrangian from one point in time to a second point in time, over all possible paths. In this case we will require the first point to be the initiation point of the universe, and the final point, to the current time, as observed at a point in the center of the system. An observer of the value of the function at the center of the system, will observe this value in retarded time. Note that the evaluation point of this action, which is at the center of the coordinate system, is not the same as the observation point, which can be at a distance, and is observed in retarded time.

For Eq. (23), the action of a one half spin particle, from the start of the universe to its present position, as evaluated at the coordinate origin, and observed at the a distance r from origin is proposed to be:

$$\vec{s}_m = \frac{\pm i\left|\Re - r\right| m_m c\vec{V}_m \pm \frac{1}{2}\hbar\vec{\eta}}{\left(m_m c r_m/\alpha\right) - \hbar} \tag{3.1}$$

$\Re$ is the radius of the universe at the origin of the coordinate system, r is the distance to an observation point from the origin. m_m is the relativistic mass of the m particle, $\vec{V}_m$ is the four-velocity, in Clifford matrix, and $\vec{\eta}$ is a null vector associated with the spin of the particle. r_m is the distance from the evaluation point at the origin of the coordinate system to the m particle. The first $\pm$ is associated with the particle charge and the second is associated with the spin null vector.

If the velocity and r_m are set to zero the function becomes:

$$\vec{S} \pm \gamma_4 \frac{\Re mc}{\hbar} \tag{3.2}$$

Since $\Re$ is the radius of the universe this is the "maximum" action a particle existing at that location in the universe, can have, Note that at the Compton radius $r_m = \tilde{r}$ the function becomes infinite. But this is an evaluation point, and has no physical significance.

Focusing on the denominator and presuming $Mcr_m > \hbar$ and the velocity is zero we have:

$$\vec{S} = \frac{i\Re m\vec{V}_m}{\hbar}\left(\frac{1}{mc^2}\frac{Q^2}{r_m}\right) \to \frac{i\Re}{\hbar}\left(\frac{1}{c}\frac{Q^2}{r_m}\right)\gamma_4 \tag{3.3}$$

Which makes it the action of the electric energy potential of that particle, as evaluated at the origin.

The action vector function for a particle has a maximum value when $r = 0$ and the minimum when $r \to \Re$. At the origin the action for a particle is diminished by $\Re / r$, making its contribution to the complete function proportional to its observed cosmic age.

Appendix IV

Action Product Details

This is the details of the products of the actions of the individual particles for Eq.(25).

Starting with the particle action:

$$\vec{S} = \frac{\pm i\left|\mathfrak{R}-r\right| Mc\vec{V}_m \pm \frac{1}{2}\hbar\vec{\eta}}{\left(Mcr/\alpha\right)-\hbar} \tag{4.1}$$

We evaluate the center of the coordinate system at the locus of the n particle and at a distance for all the other m particles.

$$\vec{S}_n = \left(\overset{C}{\underset{n}{\pm}} i\frac{\left|\mathfrak{R}-r\right|}{\tilde{r}_n}\vec{V}_n \overset{S}{\underset{n}{\pm}}\frac{1}{2}\vec{\eta}_n\right),\ r_n = 0 \qquad \vec{S}_m = \alpha\left(\overset{C}{\underset{m}{\pm}} i\frac{\left|\mathfrak{R}-r\right|}{r_m}\vec{V}_m \ \overset{S}{\underset{m}{\pm}}\frac{1}{2}\frac{\tilde{r}_m}{r_m}\vec{\eta}_m\right),\ \ \left(Mcr_m/\alpha\right) >> \hbar \tag{4.2}$$

The square of (n) particle action is:

$$\vec{S}_n{}^2 = \left(\overset{C}{\underset{n}{\pm}} i\frac{\left|\mathfrak{R}-r\right|}{\tilde{r}_n}\vec{V}_n \overset{S}{\underset{n}{\pm}}\frac{1}{2}\vec{\eta}_n\right)\left(\overset{C}{\underset{n}{\pm}} i\frac{\left|\mathfrak{R}-r\right|}{\tilde{r}_n}\vec{V}_n \overset{S}{\underset{n}{\pm}}\frac{1}{2}\vec{\eta}_n\right) \tag{4.3}$$

or:

$$\vec{S}_n{}^2 = \left(\overset{C}{\underset{n}{\pm}}\overset{C}{\underset{n}{\pm}} i\frac{\left|\mathfrak{R}-r\right|}{\tilde{r}_n}\vec{V}_n\, i\frac{\left|\mathfrak{R}-r\right|}{\tilde{r}_n}\vec{V}_n \quad \cancel{\overset{S}{\underset{n}{\pm}}\overset{S}{\underset{n}{\pm}}\frac{1}{2}\vec{\eta}_n\frac{1}{2}\vec{\eta}_n} \quad \overset{C}{\underset{n}{\pm}}\overset{S}{\underset{n}{\pm}}\ i\frac{\left|\mathfrak{R}-r\right|}{\tilde{r}_n}\vec{V}_n\frac{1}{2}\vec{\eta}_n \quad \overset{C}{\underset{n}{\pm}}\overset{S}{\underset{n}{\pm}}\ \frac{1}{2}\vec{\eta}_n\, i\frac{\left|\mathfrak{R}-r\right|}{\tilde{r}_n}\vec{V}_n\right) \tag{4.4}$$

Which is also the square of the action for the free particle:

$$\vec{S}_n{}^2 = \left(-\left[\frac{\left|\mathfrak{R}-r\right|}{\tilde{r}_n}\vec{V}_n\right]^2 \quad + \quad \overset{C}{\underset{n}{\pm}}\overset{S}{\underset{n}{\pm}} i\frac{1}{2}\frac{\left|\mathfrak{R}-r\right|}{\tilde{r}_n}\left[\vec{V}_n\vec{\eta}_n + \vec{\eta}_n\vec{V}_n\right]\right) \tag{4.5}$$

Note the sign of the imaginary term is a product of the charge, and spin signs. $\left(\overset{C}{\underset{n}{\pm}}\overset{S}{\underset{n}{\pm}}\right)$

$$\vec{S}_n{}^2 = \left(-\left[\frac{\left|\mathfrak{R}-r\right|}{\tilde{r}_n}\vec{V}_n\right]^2 + \overset{C}{\underset{n}{\pm}}\overset{S}{\underset{n}{\pm}}\quad i\frac{\left|\mathfrak{R}-r\right|}{\tilde{r}_n}\vec{V}_n\bullet\vec{\eta}_n\right) \tag{4.6}$$

Note that the sum of the product of the four velocity and the null vector is just the inner dot product, and is a scalar.

An eigenfunction for a free particle from Eq., and Eq. (23) would then be:

$$\vec{\Theta}_n = Ae^{\left(S_n{}^2\right)} = Ae^{\left[-\left(\frac{\left|\mathfrak{R}-r\right|}{\tilde{r}_n}\vec{V}_n\right)^2 \ + \ \overset{C}{\underset{n}{\pm}}\overset{S}{\underset{n}{\pm}} i\frac{\left|\mathfrak{R}-r\right|}{\tilde{r}_n}\vec{V}_n\bullet\vec{\eta}_n\right]} \tag{4.7}$$

The imaginary free particle function is then:

$$\Theta_{nI} = e^{\overset{C}{\underset{n}{\pm}}\overset{S}{\underset{n}{\pm}}\left[i\frac{|\Re - r|}{\tilde{r}_n}\left(\vec{V}_n \bullet \vec{\eta}_n\right)\right]} \tag{4.8}$$

And the real free particle function is:

$$\Theta_{nR} = Ae^{-\left(\frac{|\Re-r|}{\tilde{r}_n}\vec{V}_n\right)^2} \tag{4.9}$$

Now we take the products of the m and n particle

$$\vec{S}_n \ \vec{S}_m = \pm\left(\overset{C}{\underset{n}{\pm}} i\frac{\Re}{\tilde{r}_n}\vec{V}_n \overset{S}{\underset{n}{\pm}}\frac{1}{2}\vec{\eta}_n\right)\left[\alpha\left(\overset{C}{\underset{m}{\pm}} i\frac{\Re}{r_m}\vec{V}_m \ \overset{S}{\underset{m}{\pm}}\frac{1}{2}\frac{\tilde{r}_m}{r_m}\vec{\eta}_m\right)\right] \tag{4.10}$$

$$\vec{S}_n \ \vec{S}_m = \alpha\left(\overset{C}{\underset{n}{\pm}}\overset{C}{\underset{m}{\pm}} i\frac{\Re}{\tilde{r}_n}\vec{V}_n i\frac{\Re}{r_m}\vec{V}_m \ \overset{S}{\underset{n}{\pm}}\overset{C}{\underset{m}{\pm}}\frac{1}{2}\vec{\eta}_n i\frac{\Re}{r_m}\vec{V}_m \ \overset{C}{\underset{n}{\pm}}\overset{S}{\underset{m}{\pm}} i\frac{\Re}{\tilde{r}_n}\vec{V}_n\frac{1}{2}\frac{\tilde{r}_m}{r_m}\vec{\eta}_m \ \overset{S}{\underset{n}{\pm}}\overset{S}{\underset{m}{\pm}}\frac{1}{2}\vec{\eta}_n\frac{1}{2}\frac{\tilde{r}_m}{r_m}\vec{\eta}_m\right) \tag{4.11}$$

Recollecting terms.

$$\vec{S}_n \ \vec{S}_m = \alpha\left[-\overset{C}{\underset{n}{\pm}}\overset{C}{\underset{m}{\pm}}\left(\frac{\Re^2}{\tilde{r}_n r_m}\vec{V}_n\vec{V}_m\right) + \overset{S}{\underset{n}{\pm}}\overset{S}{\underset{m}{\pm}}\frac{1}{4}\frac{\tilde{r}_m}{r_m}\vec{\eta}_n\vec{\eta}_m + i\frac{1}{2}\frac{\Re}{r_m}\left(\overset{S}{\underset{n}{\pm}}\overset{C}{\underset{m}{\pm}}\vec{\eta}_n\vec{V}_m \ \overset{C}{\underset{n}{\pm}}\overset{S}{\underset{m}{\pm}} \ \vec{V}_n\vec{\eta}_m\frac{\tilde{r}_m}{\tilde{r}_n}\right)\right] \tag{4.12}$$

Now the reverse order:

$$\vec{S}_m\vec{S}_n = \alpha\left[-\overset{C}{\underset{n}{\pm}}\overset{C}{\underset{m}{\pm}}\left(\frac{\Re^2}{\tilde{r}_n r_m}\vec{V}_m\vec{V}_n\right) + \overset{S}{\underset{n}{\pm}}\overset{S}{\underset{m}{\pm}}\frac{1}{4}\frac{\tilde{r}_m}{r_m}\vec{\eta}_m\vec{\eta}_n + i\frac{1}{2}\frac{\Re}{r_m}\left(\overset{S}{\underset{n}{\pm}}\overset{C}{\underset{m}{\pm}}\vec{V}_m\vec{\eta}_n \ \overset{C}{\underset{n}{\pm}}\overset{S}{\underset{m}{\pm}} \ \vec{\eta}_m\vec{V}_n\frac{\tilde{r}_m}{\tilde{r}_n}\right)\right] \tag{4.13}$$

Thus the sum is:

$$\vec{S}_n\vec{S}_m + \vec{S}_m\vec{S}_n = \alpha\left[\begin{array}{l} -\overset{C}{\underset{n}{\pm}}\overset{C}{\underset{m}{\pm}}2\frac{\Re^2}{\tilde{r}_n \ r_m}\vec{V}_m \bullet \vec{V}_n + \overset{S}{\underset{n}{\pm}}\overset{S}{\underset{m}{\pm}}\frac{1}{2}\frac{\tilde{r}_m}{r_m}\vec{\eta}_m \bullet \vec{\eta}_n \\ \qquad + i\frac{\Re}{r_m}\left(\overset{S}{\underset{n}{\pm}}\overset{C}{\underset{m}{\pm}}\vec{V}_m \bullet \vec{\eta}_n + \overset{C}{\underset{n}{\pm}}\overset{S}{\underset{m}{\pm}} \ \vec{V}_n \bullet \vec{\eta}_m \frac{m_n}{m_m}\right)\end{array}\right] \tag{4.14}$$

Note the anti-symmetric terms cancel leaving the inner dot products, as in the n square terms.

The sum of all the terms is then for an interacting particle is then:

$$\vec{S}_n{}^2 + \vec{S}_n\vec{S}_m + \vec{S}_m\vec{S}_n = \left[\begin{matrix} -\left(\frac{\Re}{\tilde{r}_n}\vec{V}_n\right)^2 - \underset{n}{\overset{C}{\pm}}\underset{m}{\overset{C}{\pm}}\frac{2\alpha\Re^2}{\tilde{r}_n\ r_m}\vec{V}_m\bullet\vec{V}_n + \underset{n}{\overset{S}{\pm}}\underset{m}{\overset{S}{\pm}}\frac{\alpha}{2}\frac{\tilde{r}_m}{r_m}\vec{\eta}_m\bullet\vec{\eta}_n \\ + \underset{n}{\overset{C}{\pm}}\underset{n}{\overset{S}{\pm}} i\frac{\Re}{\tilde{r}_n}\vec{V}_n\bullet\vec{\eta}_n + i\alpha\frac{\Re}{r_m}\left(+\underset{n}{\overset{S}{\pm}}\underset{m}{\overset{C}{\pm}}\vec{V}_m\bullet\vec{\eta}_n + \underset{n}{\overset{C}{\pm}}\underset{m}{\overset{S}{\pm}}\vec{V}_n\bullet\vec{\eta}_m\frac{m_n}{m_m}\right) \end{matrix}\right] \quad (4.15)$$

From Eq. and Eq.(23) the real part of the function for the n particle is:

$$\vec{\Theta}_{nR} = Ae^{\left(\sum_N S_n \sum_N S_n\right)_R} = e^{\left[-\left(\frac{\Re}{\tilde{r}_n}\vec{V}_n\right)^2 - \underset{n}{\overset{C}{\pm}}\underset{m}{\overset{C}{\pm}}\frac{2\alpha\Re^2}{\tilde{r}_n\ r_m}\vec{V}_m\bullet\vec{V}_n + \underset{n}{\overset{S}{\pm}}\underset{m}{\overset{S}{\pm}}\frac{\alpha}{2}\frac{\tilde{r}_m}{r_m}\vec{\eta}_m\bullet\vec{\eta}_n\right]} \quad (4.16)$$

From Eq. and Eq.(23) the imaginary eigenfunction for the n particle

$$\vec{\Theta}_{nI} = Ae^{\left(\sum_N S_n \sum_N S_n\right)_I} = Ae^{\left[+\underset{n}{\overset{C}{\pm}}\underset{n}{\overset{S}{\pm}} i\frac{\Re}{\tilde{r}_n}\vec{V}_n\bullet\vec{\eta}_n + i\alpha\frac{\Re}{r_m}\left(+\underset{n}{\overset{S}{\pm}}\underset{m}{\overset{C}{\pm}}\vec{V}_m\bullet\vec{\eta}_n + \underset{n}{\overset{C}{\pm}}\underset{m}{\overset{S}{\pm}}\vec{V}_n\bullet\vec{\eta}_m\frac{m_n}{m_m}\right)\right]_I} \quad (4.17)$$

Taking the real part of this and putting into Eq. (6), and differentiating We can arrive at the actual square of the Lagrangian for a particle in the potential of a second particle.

$$-\frac{1}{\tilde{r}_{n0}^2} = -\left(\frac{1}{\tilde{r}_n}\vec{V}_n\right)^2 - \underset{n}{\overset{C}{\pm}}\underset{m}{\overset{C}{\pm}}\frac{2\alpha}{\tilde{r}_n\ r_m}\vec{V}_m\bullet\vec{V}_{nn} \quad (4.18)$$

$$-\left(\frac{m_{n0}c}{\hbar}\right)^2 = -\left(\frac{m_n c}{\hbar}\vec{V}_n\right)^2 - \underset{n}{\overset{C}{\pm}}\underset{m}{\overset{C}{\pm}} 2\frac{m_n}{\hbar^2}\frac{Q^2}{r_m}\vec{V}_m\bullet\vec{V}_n$$

$$\text{or:} \quad (4.19)$$

$$\left(\frac{m_{n0}c}{\hbar}\right)^2 = +\frac{m_n{}^2c^2}{\hbar^2}\left(1-\frac{\vec{v}_n{}^2}{c^2}\right) - \underset{n}{\overset{C}{\pm}}\underset{m}{\overset{C}{\pm}} 2\frac{m_n}{\hbar^2}\frac{Q^2}{r_m}\left(1-\frac{\vec{v}_m}{c}\bullet\frac{\vec{v}_n}{c}\right)$$

Taking the square root:

$$\frac{m_{n0}c}{\hbar}=+\sqrt{\left(\frac{m_n c}{\hbar}\right)^2\left(-\vec{V}_n^{\,2}-\mathop{\pm}_{n}^{C}\mathop{\pm}_{m}^{C}\frac{2}{m_n c^2}\frac{Q^2}{r_m}\vec{V}_m\bullet\vec{V}_n\right)}=+\frac{m_n c}{\hbar}\sqrt{\vec{V}_n^{\,2}-\mathop{\pm}_{n}^{C}\mathop{\pm}_{m}^{C}\frac{2}{m_n c^2}\frac{Q^2}{r_m}\vec{V}_m\bullet\vec{V}_n}$$

(4.20)

or:

$$\frac{m_{n0}c}{\hbar}=+\frac{m_n c}{\hbar}\sqrt{\left[1-\frac{\vec{v}_n^{\,2}}{c^2}\right]\mathop{\pm}_{n}^{C}\mathop{\pm}_{m}^{C}\,2\left(\frac{1}{M_n c^2}\frac{Q^2}{r_m}\left(1-\frac{\vec{v}_m}{c}\bullet\frac{\vec{v}_n}{c}\right)\right)} \tag{4.21}$$

or:

$$m_{n0}c^2\approx+m_n c^2\left[1-\frac{1}{2}\frac{\vec{v}_n^{\,2}}{c^2}\right]\pm\left(\frac{Q^2}{r_m}\left(1-\frac{\vec{v}_m}{c}\bullet\frac{\vec{v}_n}{c}\right)\right) \tag{4.22}$$

Thus we have arrived at the Lagrangian for the eigenvalue from its eigenfunction without appealing to an external potential.

Appendix VI

The Change to Local Co-ordinates

Noting that Eq.(8), is the change in the function with respect to a displacement of the expanding light cone at the edge of the universe, we can make an approximation, and convert to differentials of the function with respect to local coordinates. Since in a spherical universe, X,Y,Z are just the maximum value of the x, y, & z coordinates, and from the center, equal to the radius, $\Re$, and ΔX is the same as a change in a local frame $\partial X=\partial x$, we can have:

$$\begin{aligned}\partial\left(X^2\right)&=2X\partial x=2\Re\,\partial x\\ \partial\left(Y^2\right)&=2Y\partial y=2\Re\,\partial y\\ \partial\left(Z^2\right)&=2Z\partial z=2\Re\,\partial z\\ \partial\left(\Re^2\right)&=2\Re\partial x=2\Re\partial\left(ct\right)\end{aligned}\quad, \tag{6.1}$$

And thus arrive at a normal Clifford expression for the matrix function.

$$\left(\gamma_1 \frac{\partial}{2\Re\partial x} + \gamma_2 \frac{\partial}{2\Re\partial y} + \gamma_3 \frac{\partial}{2\Re\partial z} + \gamma_4 \frac{\partial}{2\Re\partial(ct)}\right)\Theta = K\Theta, \tag{6.2}$$

or:

$$\gamma^{\mu} \frac{\partial}{\partial\left(x^2\right)_{\mu}}\Theta = K\Theta \tag{6.3}$$

Appendix VII

A. Connection to Dirac Expression

The purpose of this section is to show the connection of the imaginary portion of the free particle systemfunction to the Dirac free particle.

Although the interpretations of the functions are different, there are structural similarities. The most notable structural difference is the phase velocity of the systemfunction $c(t - r/c)$, whereas the Dirac function has no constant phase velocity $Et - \vec{p}\bullet\vec{r}$.
In this section we intend to show that the function for the free particle systemfunction can be separated into a product of functions, one of which is equivalent to the Dirac particle. The other a massless particle, having velocity equal to c.

From Eq. 30. We have for the imaginary function from our systemfunction expression for a free particle :

$$\vec{\Theta}_{nI} = e^{\overset{C}{\underset{n}{\pm}}\overset{S}{\underset{n}{\pm}} i\,(ct-r)\frac{M_n c}{\hbar}(1-\vec{v}_n\bullet\vec{\eta}_n)} \tag{7.1}$$

And from Eq. 31 we have the Dirac function for the free particle to be.

$$\psi = u_{\vec{p}} e^{i\left(Et-\vec{p}\bullet\vec{r}\right)/\hbar} \tag{7.2}$$

Both expressions would appear to be a solution to the Klein Gordon expression, however the systemfunction does not appear be a solution to the Dirac expression.

Our imaginary portion of the systemfunction Eq. , is:

$$\gamma^{\mu} \frac{\partial}{\partial\left(x^{2}\right)} \Theta_{\mu} = +iK_{2}\Theta \tag{7.3}$$

And is a solution to the matrix equation:

$$\gamma^{\mu} \frac{\partial}{\partial(x)_{\mu}} \Theta = \pm \frac{i}{2\Re \tilde{r}_{0}} \; I \tag{7.4}$$

A judicious factoring of Θ can yield a product that one factor is functionally equivalent to the Dirac free particle function, and another term which is a time space compliment function, ie:

$$\vec{\Theta} = \Theta_{1}\Theta_{2} = e^{\pm\left(\frac{m_{n}c}{\hbar}ct - \frac{\vec{p}_{n}}{\hbar}\bullet\vec{\eta}_{n}r\right)} \times e^{\pm\left(+\frac{m_{n}c}{\hbar}r - \frac{\vec{p}_{n}}{\hbar}\bullet\vec{\eta}_{n}ct\right)} \tag{7.5}$$

By applying a transform consisting of a unit matrix $A^{*}A$ the function can be converted into a sum of equations one functionally equivalent to the Dirac equation and and another complimentary equation.

$$\vec{\Theta} = \Theta_{1}\mathbf{a}^{*}\mathbf{a}\Theta_{2} \tag{7.6}$$

Where:

$$\mathbf{a}^{*}\mathbf{a} = \begin{bmatrix} a_{1} \\ a_{2} \\ a_{3} \\ a_{4} \end{bmatrix} \begin{bmatrix} a_{1} & a_{2} & a_{3} & a_{4} \end{bmatrix}^{*} = I \tag{7.7}$$

If we separate the K_{2} matrix such that $K_{2} = \mathbf{k}_{a} + \mathbf{k}_{b} = f\left(\frac{m_{0}c}{\hbar}\right)$ we then have:

$$\gamma^{\mu}\frac{\partial}{\Theta_1\partial\left(x^2\right)}\vec{\Theta}_1\mathbf{a}^* + \gamma^{\mu}\frac{\partial}{\vec{\Theta}_2\partial\left(x^2\right)}\vec{\Theta}_2\mathbf{a} = +i\left(k_a + k_b\right) \tag{7.8}$$

and we set:

$$\gamma^{\mu}\frac{\partial}{\partial\left(x^2\right)_{\mu}}\vec{\Theta}_1\mathbf{a}^* = ik_a\Theta_1 \tag{7.9}$$

$$\gamma^{\mu}\frac{\partial}{\partial\left(x^2\right)_{\mu}}\vec{\Theta}_2\mathbf{a} = ik_b\vec{\Theta}_2 \tag{7.10}$$

This is in effect, dividing the function into two particles, each of which can be shown to be solution to the Dirac equation. The problem sets in when the evaluating k_a and k_b, which are functions of the rest mass. If they are set equal, or apportioned, the particle modes have unusual characteristics having unequal up and down values. The approach that seems to be most appropriate is to set $k_a = K_2 = m_0 c/\hbar$ and $k_b = 0$. This result in the first being identical to the dirac particle, and the other a massless light speed particle.

For the first Eq. ,Which is identical in form to the linear Dirac equation:

$$\gamma^{\mu}\frac{\partial}{\partial(x)_{\mu}}e^{i\left(\frac{m_n c}{\hbar}ct-\frac{\vec{p}_n}{\hbar}\bullet\vec{\eta}_n r\right)} = i\left(\gamma_0\frac{m_n c}{\hbar} - \gamma_1\vec{p}_x - \gamma_2\vec{p}_y - \gamma_3\vec{p}_z\right)\Theta_1 \tag{7.11}$$

We have:

$$\left(\gamma_0\frac{m_n c}{\hbar} - \gamma_1\vec{p}_x - \gamma_2\vec{p}_y - \gamma_3\vec{p}_z\right)A^* = k_a \tag{7.12}$$

Using a common set of Dirac matrix.

$$\gamma_1 = \begin{bmatrix} & & & +1 \\ & & +1 & \\ & -1 & & \\ -1 & & & \end{bmatrix},\quad \gamma_2 = \begin{bmatrix} & & & -i \\ & & i & \\ & i & & \\ -i & & & \end{bmatrix},\quad \gamma_3 = \begin{bmatrix} & & 1 & \\ & & & -1 \\ -1 & & & \\ & 1 & & \end{bmatrix},\quad \gamma_0 = \begin{bmatrix} & & 1 & \\ & & & 1 \\ 1 & & & \\ & 1 & & \end{bmatrix} \tag{7.13}$$

Setting $K_2 = k_a$ and $k_b = 0$ The 1st expression is just the normal dirac equation.

$$\begin{bmatrix} & & \frac{m_n c}{\hbar}-\vec{p}_z & -\vec{p}_x+i\vec{p}_y \\ & & -\vec{p}_x-i\vec{p}_y & \frac{m_n c}{\hbar}+\vec{p}_z \\ \frac{m_n c}{\hbar}+\vec{p}_z & \vec{p}_x-i\vec{p}_y & & \\ \vec{p}_x+i\vec{p}_y & \frac{m_n c}{\hbar}-\vec{p}_z & & \end{bmatrix} A^* = k_a \begin{bmatrix} \frac{m_0 c}{\hbar} & & & \\ & \frac{m_0 c}{\hbar} & & \\ & & \frac{m_0 c}{\hbar} & \\ & & & \frac{m_0 c}{\hbar} \end{bmatrix} \tag{7.14}$$

Now for simplicity we set the velocity to be along the z axis, and A^*To be a column matrix of constants,

$$\mathbf{a}^* = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{bmatrix} \tag{7.15}$$

we have the normal four simultaneous equations in.

$$\begin{bmatrix} -\frac{m_0 c}{\hbar}a_1 & & \left(\frac{m_n c}{\hbar}-\vec{p}_z\right)a_3 & \\ & -\frac{m_0 c}{\hbar}a_2 & & \left(\frac{m_n c}{\hbar}+\vec{p}_z\right)a_4 \\ \left(\frac{m_n c}{\hbar}+\vec{p}_z\right)a_1 & & -\frac{m_0 c}{\hbar}a_3 & \\ & \left(\frac{m_n c}{\hbar}-\vec{p}_z\right)a_2 & & -\frac{m_0 c}{\hbar}a_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{7.16}$$

Solving for the a's

$$\frac{m_0 c}{\hbar}a_1 = \left(\vec{p}_z - \frac{mc}{\hbar}\right)a_3 \quad , \text{ or } \quad \frac{m_0 c}{\hbar}a_3 = \left(\vec{p}_z + \frac{mc}{\hbar}\right)a_1 \tag{7.17}$$

We can then have:

$$\frac{a_1}{a_3}=\frac{\left(\vec{p}_z-\frac{mc}{\hbar}\right)}{\frac{m_0c}{\hbar}}\left[\frac{\sqrt{\left(\vec{p}_z+\frac{mc}{\hbar}\right)}}{\sqrt{\left(\vec{p}_z+\frac{mc}{\hbar}\right)}}\right]=\frac{\sqrt{\left(\vec{p}_z-\frac{mc}{\hbar}\right)}}{\sqrt{\left(\vec{p}_z+\frac{mc}{\hbar}\right)}}$$

$$\frac{a_1}{a_3}=\frac{\frac{m_0c}{\hbar}}{\left(\vec{p}_z+\frac{mc}{\hbar}\right)}\left[\frac{\sqrt{\left(\vec{p}_z-\frac{mc}{\hbar}\right)}}{\sqrt{\left(\vec{p}_z-\frac{mc}{\hbar}\right)}}\right]=\frac{\sqrt{\left(\vec{p}_z-\frac{mc}{\hbar}\right)}}{\sqrt{\left(\vec{p}_z+\frac{mc}{\hbar}\right)}} \tag{7.18}$$

Similarly:

$$\left(\frac{m_nc}{\hbar}-\vec{p}_z\right)a_2-\frac{1}{2}\frac{m_0c}{\hbar}a_4 \qquad \frac{a_2}{a_4}=\frac{\frac{m_0c}{\hbar}}{\left(\frac{m_nc}{\hbar}-\vec{p}_z\right)}\left[\frac{\sqrt{\left(\frac{m_nc}{\hbar}+\vec{p}_z\right)}}{\sqrt{\left(\frac{m_nc}{\hbar}+\vec{p}_z\right)}}\right]=\frac{\sqrt{\left(\frac{m_nc}{\hbar}+\vec{p}_z\right)}}{\sqrt{\left(\frac{m_nc}{\hbar}-\vec{p}_z\right)}}$$

$$\left(\frac{m_nc}{\hbar}+\vec{p}_z\right)a_4-\frac{1}{2}\frac{m_0c}{\hbar}a_2 \qquad \frac{a_2}{a_4}=\frac{\left(\frac{m_nc}{\hbar}+\vec{p}_z\right)}{\frac{m_0c}{\hbar}}\left[\frac{\sqrt{\left(\frac{m_nc}{\hbar}-\vec{p}_z\right)}}{\sqrt{\left(\frac{m_nc}{\hbar}-\vec{p}_z\right)}}\right]=\frac{\sqrt{\left(\frac{m_nc}{\hbar}+\vec{p}_z\right)}}{\sqrt{\left(\frac{m_nc}{\hbar}-\vec{p}_z\right)}} \tag{7.19}$$

If we set $k_a = k_b$ one can still arrive at a solution, but the magnitudes of the modes have inequivalent up and down components. The modes and options are:

$$\Theta_1 = \left\{ \begin{array}{l} \alpha_1 \begin{bmatrix} \sqrt{\left(\frac{m_n c}{\hbar} - \vec{p}_z\right)} \\ 0 \\ \sqrt{\left(\frac{m_n c}{\hbar} + \vec{p}_z\right)} \\ 0 \end{bmatrix} + \beta_1 \begin{bmatrix} 0 \\ \sqrt{\left(\frac{m_n c}{\hbar} + \vec{p}_z\right)} \\ 0 \\ 2\sqrt{\left(\frac{m_n c}{\hbar} - \vec{p}_z\right)} \end{bmatrix} \\ + \ \alpha_2 \begin{bmatrix} \sqrt{\left(\frac{m_n c}{\hbar} + \vec{p}_z\right)} \\ 0 \\ \sqrt{\left(\frac{m_n c}{\hbar} - \vec{p}_z\right)} \\ 0 \end{bmatrix} + \beta_2 \begin{bmatrix} 0 \\ \sqrt{\left(\frac{m_n c}{\hbar} - \vec{p}_z\right)} \\ 0 \\ \sqrt{\left(\frac{m_n c}{\hbar} + \vec{p}_z\right)} \end{bmatrix} \end{array} \right\} e^{\pm\left(\frac{m_n c}{\hbar} ct - \frac{\vec{p}_n}{\hbar} \bullet \vec{\eta}_n r\right)} \tag{7.20}$$

Where the α's and β's are arbitrary constants

The second expression Eq. , it does have similar solutions to the first Eq. ,

Now looking at the second equation, we have:

$$\begin{aligned} \gamma^\mu \frac{\partial}{\partial(x)_\mu} e^{i\left(\frac{m_n c}{\hbar} r - \frac{\vec{p}_n}{\hbar} \bullet \vec{\eta}_n ct\right)} &= i\left(\frac{m_n c}{\hbar}\left(\frac{\gamma_1 x + \gamma_2 y + \gamma_3 z}{r}\right) - \gamma_0 \frac{\vec{p}_n}{\hbar} \bullet \vec{\eta}_n\right)\Theta_2 \\ &= \left[i k_b \Theta_2 \vec{V} = \left(\gamma_1 v_x + \gamma_2 v_y + \gamma_3 v_z + \gamma_4 c\right) / c \vec{V}_n \vec{V}_m \right] \Theta_2 \\ &= \left[\left(\gamma_1 v_{xn} + \gamma_2 v_{yn} + \gamma_3 v_{zn} + \gamma_4 c\right)\left(\gamma_1 v_{xm} + \gamma_2 v_{ym} + \gamma_3 v_{zm} + \gamma_4 c\right) / c^2 \right] \Theta_2 \end{aligned} \tag{7.21}$$

or:

$$\left(\frac{m_n c}{\hbar}\left(\frac{\gamma_1 x + \gamma_2 y + \gamma_3 z}{r}\right) - \gamma_0 \frac{\vec{p}_n}{\hbar} \bullet \vec{\eta}_n\right) == k_b \tag{7.22}$$

Explicitly if $\kappa_2 = k_b$:

$$A\begin{bmatrix} & & \frac{m_n c}{\hbar}\frac{z}{r}-\frac{\vec{p}_n}{\hbar}\bullet\vec{\eta}_n & \frac{m_n c}{\hbar}\left(\frac{x}{r}-i\frac{y}{r}\right) \\ & & \frac{m_n c}{\hbar}\left(\frac{x}{r}+i\frac{y}{r}\right) & -\frac{m_n c}{\hbar}\frac{z}{r}-\frac{\vec{p}_n}{\hbar}\bullet\vec{\eta}_n \\ -\frac{m_n c}{\hbar}\frac{z}{r}-\frac{\vec{p}_n}{\hbar}\bullet\vec{\eta}_n & \frac{m_n c}{\hbar}\left(\frac{x}{r}+i\frac{y}{r}\right) & & \\ \frac{m_n c}{\hbar}\left(\frac{x}{r}-i\frac{y}{r}\right) & \frac{m_n c}{\hbar}\frac{z}{r}-\frac{\vec{p}_n}{\hbar}\bullet\vec{\eta}_n & & \end{bmatrix}=\begin{bmatrix}0 & 0 & 0 & 0\end{bmatrix} \qquad (7.23)$$

Where $A = a'_1, a'_2, a'_3, a'_4$,

Now if we take the system to be such that the spin is aligned with the momentum along the z axis and multiply the components of A, We have:

$$\begin{bmatrix} & & \left(\frac{m_n c}{\hbar}\frac{z}{r}-\frac{\vec{p}_n}{\hbar}\bullet\vec{\eta}_n\right)a'_1 & \\ & & & \left(-\frac{m_n c}{\hbar}\frac{z}{r}-\frac{\vec{p}_n}{\hbar}\bullet\vec{\eta}_n\right)a'_2 \\ \left(-\frac{m_n c}{\hbar}\frac{z}{r}-\frac{\vec{p}_n}{\hbar}\bullet\vec{\eta}_n\right)a'_3 & & & \\ & \left(\frac{m_n c}{\hbar}\frac{z}{r}-\frac{\vec{p}_n}{\hbar}\bullet\vec{\eta}_n\right)a'_4 & & \end{bmatrix}=\begin{bmatrix}0 & 0 & 0 & 0\end{bmatrix} \qquad (7.24)$$

Solving for the a's

$$a'_1\left(\frac{m_n c}{\hbar}-\frac{\vec{p}_z}{\hbar}\right)=0, \quad a'_2\left(\frac{m_n c}{\hbar}+\frac{\vec{p}_z}{\hbar}\right)=0\ , \quad a'_3\left(\frac{m_n c}{\hbar}+\frac{\vec{p}_z}{\hbar}\right)=0\ , \quad a'_4\left(\frac{m_n c}{\hbar}-\frac{\vec{p}}{\hbar}\right)=0 \qquad (7.25)$$

Which are satisfied if:

$$a'_1=\left(\frac{m_n c}{\hbar}+\frac{\vec{p}_z}{\hbar}\right), \quad a'_2=\left(\frac{m_n c}{\hbar}-\frac{\vec{p}_z}{\hbar}\right), \quad a'_3=\left(\frac{m_n c}{\hbar}-\frac{\vec{p}_z}{\hbar}\right), \quad a'_4=\left(\frac{m_n c}{\hbar}+\frac{\vec{p}}{\hbar}\right)=0 \qquad (7.26)$$

Since the a's are not zero because $\mathbf{a}^*\mathbf{a}=1$ the particles momentum times c is the total energy. Since the modes have to be the compliment to the Dirac particle, the spin is one half, and the rest mass is zero. The particle thus has the characteristics of a neutrino.

The modes are somewhat more restrictive than in the case of Eq, (7.20),

$$\Theta_2 = \left\{ \alpha'_1 \begin{bmatrix} \sqrt{\left(\frac{m_n c}{\hbar} + \vec{p}_z\right)} \\ \sqrt{\left(\frac{m_n c}{\hbar} - \vec{p}_z\right)} \\ \sqrt{\left(\frac{m_n c}{\hbar} - \vec{p}_z\right)} \\ \sqrt{\left(\frac{m_n c}{\hbar} + \vec{p}_z\right)} \end{bmatrix} \right\} e^{\pm\left(+\frac{m_n c}{\hbar} r \; - \frac{\vec{p}_n}{\hbar} \bullet \vec{\eta}_n c t\right)} \tag{7.27}$$

From the modes of the two functions if we require $\mathbf{a}^*\mathbf{a}$ to be 1 thus putting the rest of the constants and matching modes, we have:

$$\mathbf{a}^* = \left\{ \begin{matrix} \frac{1}{\sqrt{\frac{m_0 c}{\hbar}}} \begin{bmatrix} \sqrt{\left(\frac{m_n c}{\hbar} + \vec{p}_z\right)} \\ 0 \\ \sqrt{\left(\frac{m_n c}{\hbar} - \vec{p}_z\right)} \\ 0 \end{bmatrix} + \frac{1}{\sqrt{\frac{m_0 c}{\hbar}}} \begin{bmatrix} 0 \\ \sqrt{\left(\frac{m_n c}{\hbar} - \vec{p}_z\right)} \\ 0 \\ \sqrt{\left(\frac{m_n c}{\hbar} + \vec{p}_z\right)} \end{bmatrix} \\ + \frac{1}{\sqrt{\frac{m_0 c}{\hbar}}} \begin{bmatrix} \sqrt{\left(\frac{m_n c}{\hbar} - \vec{p}_z\right)} \\ 0 \\ \sqrt{\left(\frac{m_n c}{\hbar} + \vec{p}_z\right)} \\ 0 \end{bmatrix} + \frac{1}{\sqrt{\frac{m_0 c}{\hbar}}} \begin{bmatrix} 0 \\ \sqrt{\left(\frac{m_n c}{\hbar} + \vec{p}_z\right)} \\ 0 \\ \sqrt{\left(\frac{m_n c}{\hbar} - \vec{p}_z\right)} \end{bmatrix} \end{matrix} \right\} \mathbf{a} \left\{ \frac{1}{\sqrt{\frac{m_0 c}{\hbar}}} \begin{bmatrix} \sqrt{\left(\frac{m_n c}{\hbar} + \vec{p}_z\right)} \\ \sqrt{\left(\frac{m_n c}{\hbar} - \vec{p}_z\right)} \\ \sqrt{\left(\frac{m_n c}{\hbar} - \vec{p}_z\right)} \\ \sqrt{\left(\frac{m_n c}{\hbar} + \vec{p}_z\right)} \end{bmatrix} \right\} \tag{7.28}$$

Picking the matching for A^* modes, we have for the A^*A:

$$A^*A = \frac{1}{\sqrt{\frac{m_0 c}{\hbar}}} \left[\begin{bmatrix} \sqrt{\left(\frac{m_n c}{\hbar} - \vec{p}_z\right)} \\ \sqrt{\left(\frac{m_n c}{\hbar} + \vec{p}_z\right)} \\ \sqrt{\left(\frac{m_n c}{\hbar} + \vec{p}_z\right)} \\ \sqrt{\left(\frac{m_n c}{\hbar} - \vec{p}_z\right)} \end{bmatrix} \sqrt{\left(\frac{m_n c}{\hbar} + \vec{p}_z\right)} \;, \sqrt{\left(\frac{m_n c}{\hbar} - \vec{p}_z\right)} \;, \sqrt{\left(\frac{m_n c}{\hbar} - \vec{p}_z\right)} \;, \sqrt{\left(\frac{m_n c}{\hbar} + \vec{p}_z\right)} \right] \frac{1}{\sqrt{\frac{m_0 c}{\hbar}}} = 1 \tag{7.29}$$

The second particle, has zero rest mass, a velocity of c, and a spin of ½ since it is a solution to the homogeneous Dirac equation. These are the characteristics of the neutrino, and thus one could suggest that the systemfunction free particle is a composition particle consisting of a Dirac type particle and a neutrino.