A Multiple Particle System Equation Underlying the Klein-Gordon-Dirac-Schrödinger Equations

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Abstract

The purpose of this paper is to illustrate a fundamental, multiple particle, system equation for which the Klein-Gordon-Dirac-Schrödinger equations are single particle special cases. In the same manner that eigenvalues of the Schrödinger equation represents energy levels of an interacting atomic system, eigenvalues represent particle energies in an interacting system of particles. An equation is proposed that has vector solutions defined in Dirac, or Clifford algebra, that treats a collection of particles as a single system..The proposed solution is a descriptor of a symmetric, light speed expanding group of interacting particles having real, as well as the familiar QM constituents.

INTRODUCTION

This paper presents a system equation, and solutions termed a "systemfunction", which defines an expanding system of point particles, and the interactions between those particles. Embedded within the equation and the "systemfunction", are the single particle Klein Gordon Dirac expressions with standard wavefunction solutions for the particles in the system.

The interrelations between the standard QM equations are well known, but it is only under special conditions, that being field free, that there is a degree of connectivity [1]. Their most notable omission is the inability to define more than just a single particle, and one of the most notable issues, is the lack of connection between these equations, after the inclusion of the vector potential via the correspondence relation.

Field free KG

$$\left(-\frac{\partial^2}{\partial x^2}-\frac{\partial^2}{\partial y^2}-\frac{\partial^2}{\partial z^2}+\frac{\partial^2}{\partial ct^2}\right)\psi=-\frac{1}{\lambda\!\!\!^{-2}}\psi \qquad \textbf{(1)}$$

Field free Dirac

$$\left(+\gamma_1\frac{\partial}{\partial x}+\gamma_2\frac{\partial}{\partial y}+\gamma_3\frac{\partial}{\partial z}+\gamma_4\frac{\partial}{\partial ct}\right)\psi=\pm\frac{1}{\lambda\!\!\!^{-}}\psi \qquad \textbf{(2)}$$

Field free Schrödinger

$$\left(-\frac{\partial^2}{\partial x^2}-\frac{\partial^2}{\partial y^2}-\frac{\partial^2}{\partial z^2}-\frac{2i}{\lambda\!\!\!^{-}}\frac{\partial}{\partial ct}\right)\psi=\ 0 \qquad \textbf{(3)}$$

Where $\lambda\!\!\!^{-}$ is the particle Compton radius $\lambda\!\!\!^{-}=\hbar/mc$. (For general conventions see appendix [II])

The general system equation, we are proposing, is a descriptor of a time expanding system of charged, half spin particles. The inclusion of particles other than half-spin should be straightforward, but the particle moment function Eq. (12) , for such particles have not been included. Normal Quantum Mechanics expressions, (Klein-Gordon-Dirac-Schrödinger), represent single particles systems, separated from the rest of the universe, interacting through a coupled potential. The standard QM, coupling is by the insertion of an approximate representation of the potential through the correspondence relations, rather than an interaction of quantum mechanical wavefunctions. (See Appendix I, for a heuristic view of the issue.)

I. THE EQUATION

The proposed general expression for an expanding system of charged particles will be.

$$\frac{\partial}{\partial\left(\Re^2\right)}\vec{\psi}_1=K^2\,\vec{\psi}_1 \qquad \textbf{(4)}$$

That is the derivative with respect to a virtual displacement of the square of the expansion of the universe, has constant eigenvalues. Where $\Re^2$ the square of

radius of the universe, and is expanding at ct ($\Re = cT$). (This should not be confused with the second derivative). Note also that:

$$\partial\Re^2 = 2\Re\partial\Re = 2\Re\ \partial(ct) \tag{5}$$

The "systemfunction" $\vec{\psi}$ being proposed, as a solution to the system equation, is a matrix representation defined by Dirac or Clifford matrix, not the normal scalar, or scalar vector component "wavefunction". i.e.:

$$\vec{\psi}_n = e^{f\left[\gamma_1 f(x) + \gamma_2 f(y) + \gamma_3 f(z) + \gamma_4 f(ct)\right]} \tag{6}$$

This function will have real as well as imaginary components that are separable, leading to well known energy and frequency relations. This is not a probability type function though it has similarity with particle wavefunction. Eigenvalues represent one particle and as such $\vec{\psi}^*\vec{\psi}$ it is normalizable to one. The probability interpretation for the real parts of the function, is not applicable, but the imaginary parts are reasonably consistent with standard QM interpretation. The "systemfunction" will have values throughout the system of particles up to the limit of the extent of the space, at the radius of the universe. Points in the "systemfunction" are generally zero accept at extrema, which are particle locations that have discrete values, and represent eigenvalues of the system. Due to the symmetric vector nature of the function, the value of the function of the entire volume of space is zero at all points. Similar to a group of particles with a net charge, however the function could have non-zero values for a volume containing isolated group of particles.

Preliminaries

Since the systemfunction has a composition of both real and imaginary functions, we will require that the real and imaginary functions be separable such that:

$$\vec{\psi} = \vec{\psi}_R\vec{\psi}_I \tag{7}$$

and thus:

$$\vec{\psi}_R \frac{\partial}{\partial \Re^2} \vec{\psi}_I + \vec{\psi}_I \frac{\partial}{\partial \Re^2} \vec{\psi}_R = \left(\frac{1}{\mathbf{r_0}^2} + \frac{i}{2\Re \mathbf{r_0}} \right) \vec{\psi}_R \vec{\psi}_I \qquad \textbf{(8)}$$

Which can be separated into real and imaginary parts:

$$\frac{\partial}{\partial \Re^2} \vec{\psi}_R = \frac{1}{\mathbf{r_0}^2} \vec{\psi}_R \quad \text{Real}, \qquad \textbf{(9)}$$

$$\frac{\partial}{\partial \Re^2} \vec{\psi}_I = \frac{i}{2\Re \mathbf{r_0}} \vec{\psi}_I \quad \text{Imaginary}, \qquad \textbf{(10)}$$

The real equation is somewhat new but the imaginary part is closely related, but not identical to the Dirac equation. A discussion of the connection of the system equation and solutions to standard QM and the Dirac equation is included in Appendix V.

Illustration

The systemfunction solution of Eq.(4) that will be proposed is a vector product of the sum of the angular momentum of the individual particles defined from the observation point. The function thus, has a value related to the sum of the angular momentum of all the particles. In addition the cross products have a value related to the angular momentum of the interaction between those particles. Heuristically this would be for a single separated particle:

$$\vec{\psi} = e^{\left[\sum M \Re \vec{V}\right]^2} = e^{\left[\sum \left(M_1 \Re \vec{V}_1\right)^2 + \sum M \Re \vec{V}_m \sum M \Re \vec{V}_n\right]} \qquad \textbf{(11)}$$

Where all the vectors of $\Re$ and $\vec{V}$ are both defined by Dirac, or Clifford matrix. (See Appendix II and appendix IV for specifics)

The proposed systemfunction for the general expression, will be seen to exist in a four dimensional space, and normally has a non-zero, finite, exponent, only at the locus of a particle. The extrema points in the space, define eigensolutions for individual particles, and include their interactions with the other particles.

A. Single Particle Moment Function

For the systemfunction, from any given point in the system, there is a unique representative function that denotes an extrema associated with a particle and represents a characteristic angular momentum. We propose that function for a spin ½ fermion to be:

$$\vec{P}_m = \pm \frac{i\Re M\vec{V}_m \pm \frac{1}{2}\hbar}{Mcr_m/\alpha - \hbar} \qquad \textbf{(12)}$$

Which, when $r = 0$, is the "maximum" relativistic total angular momentum for a particle residing within the universe, and when $r > 0$ this is the maximum angular momentum of the particle associated with its electric energy. See appendix III for a discussion of the function, see appendix IV for a more detailed version, and see appendix II for notation conventions.

$\Re$, is again the time dependent radius of the universe . For simplification we will present this radius as a scalar, but for complete particle interactions, must also be a Clifford-Dirac vector. (See Appendix IV for details.) The $\pm$ indicates the particle charge, and spin. The minus sign in the numerator causes like charges to repel, and opposites to attract.

Using the specific representation from (12) in (11) gives:

$$\vec{\psi}_n = \mathbf{exp}\left(\sum_I^N P_I \sum_J^N P_J \right) \qquad \textbf{(13)}$$

and for the function evaluated at the locus of the n^{th} particle this would be:

$$\vec{\psi}_n = \mathbf{exp}\left(P_n^2 + \left(\sum_m^N P_m \right) P_n + P_n \left(\sum_m^N P_m \right) + + + \right) \qquad \textbf{(14)}$$

which as noted earlier is an eigensolution of Eq. **(4)**. Notable here is that the first term P_n^2 is the square of the moment of the n^{th} particle, and the cross terms would be the Clifford dot products of the interacting moments of the other particles in the system.

If we let the observation point be at the location of the n^{th} particle then the four-moments for the n ($r_n = 0$), and m particles ($r_m >> \mathbf{r}_m$) become:

$$\vec{P}_n = \pm\left(i\frac{\Re}{\lambda_n}\frac{\vec{V}_n}{c} \pm \frac{1}{2}\right), \qquad \vec{P}_m = \pm\,\alpha\left(i\frac{\Re}{r_m}\frac{\vec{V}_m}{c} \pm \frac{1}{2}\frac{\lambda_m}{r_m}\right) \qquad \textbf{(15)}$$

B. Lagrangian Equation of Motion

Putting in the moment for the n^{th} particle evaluated at $r = 0$ for P_n^2 gives the function representation of the free particle. Thus:

$$\vec{\psi}_n = \mathbf{exp}\left[-\left(\frac{\Re}{\lambda_n}\frac{\vec{V}_n}{c}\right)^2 \pm i\frac{\Re}{\lambda_n}\frac{\vec{V}_n}{c} + \frac{1}{4}\right], \qquad \textbf{(16)}$$

or, for illustration, explicitly in terms of mass and velocities:

$$\vec{\psi}_n = \mathbf{exp}\left[-\left(\frac{\Re M_n c}{\hbar}\right)^2\left(1-\frac{v^2}{c}\right) \pm i\,\frac{\Re M_n}{\hbar}\vec{V} + \frac{1}{4}\right] \qquad \textbf{(17)}$$

Real Terms

From Eq. **(4)**, taking the derivative and equating .the real portion as shown in Eq. **(9)**, this is just the free particle Lagrangian.

$$\frac{\partial P_n P_n}{\partial \Re^2} = -\left(\frac{M_n c}{\hbar}\right)^2\left(1-\frac{v_n^2}{c^2}\right) = -\left(\frac{M_{0n} c}{\hbar}\right)^2 \qquad \textbf{(18)}$$

Which is the proper result for the free particle.

Including the electromagnetic interacting, or cross terms of Eq. **(14)**, which are:

$$\vec{P}_n\left(\sum_m^N \vec{P}_m\right) + \left(\sum_m^N \vec{P}_m\right)\vec{P}_n = \pm\frac{\Re\Re}{\lambda_n}\left[\frac{\vec{V}_n}{c}\left(\sum\frac{\alpha}{r_m}\frac{\vec{V}_m}{c}\right) + \left(\sum\frac{\alpha}{r_m}\frac{\vec{V}_m}{c}\right)\frac{\vec{V}_n}{c}\right] \qquad \textbf{(19)}$$

The derivative of the real portion of this function with respect to $\Re^2$ is then:

$$\frac{\partial \sum \vec{P}_n \vec{P}_m}{\partial \Re^2} = \pm 2 \frac{M_n c}{\hbar} \sum \frac{\alpha}{r_m} \frac{\vec{V}_n}{c} \frac{\vec{V}_m}{c} = \pm 2 \frac{M_n}{\hbar^2} \sum \frac{Q^2}{r_m} \left(\frac{\vec{V}_n}{c} \bullet \frac{\vec{V}_m}{c} \right) \qquad \textbf{(20)}$$

Which is the sum of the dot products of the moments, or the electromagnetic interaction of the n particle with the m particles in the system. . This is a remarkable result, in that the vector potential is arising from the interaction of the wavefunction of the solution particle, with the wavefunctions of the other particles. This without resorting to the insertion of a potential field through the correspondence relations.

Combining the real terms of Eq.**(18)** and Eq.**(20)** and inserting into the real portion of Eq. **(9)**.

$$-\frac{\left(M_{0n} c^2\right)^2}{\hbar^2 c^2} = -\frac{\left(M_n c^2\right)^2}{\hbar^2 c^2} \left[\left(1 - \frac{v_n^2}{c^2} \right) \pm \frac{2}{M_n c^2} \sum \frac{Q^2}{r_m} \left(\frac{\vec{V}_n}{c} \bullet \frac{\vec{V}_m}{c} \right) \right], \qquad \textbf{(21)}$$

Which is the square of the classic Lagrangian for a moving particle in an electric field. Thus:

$$M_{0n} c^2 = M_n c^2 \sqrt{\left(1 - \frac{v_n^2}{c^2} \right) \pm \frac{2}{M_n c^2} \sum \frac{Q^2}{r_m} \left(\frac{V_n}{c} \bullet \frac{V_m}{c} \right)}, \qquad \textbf{(22)}$$

or:

$$M_{0n} c^2 = M_n c^2 \left(1 - \frac{1}{2} \frac{v_n^2}{c^2} \right) - \sum V_n \qquad \textbf{(23)}$$

Which is just the Lagrangian for a particle is the electromagnetic field of a collection of charged particles. As noted earlier, the inclusion of the vector radius in the moment will make this a more detailed expression. (see Appendix IV).

Imaginary Terms

Now considering the imaginary portion of Eq. **(17)**, and since $\Re = \Re_0 + ct$ the wavelength of is:

$$\frac{1}{\lambda} = \frac{1}{2\pi} \left| \frac{M_{0n}}{\hbar} \vec{V}_n \right| = \frac{p}{h} \qquad \textbf{(24)}$$

Which is the correct free particle deBroglie wavelengths for the velocity and the total energy. The frequencies are.

$$\omega_d = \frac{M_{0n} c v}{\hbar} \quad \& , \quad \omega_C \quad \frac{M_{0n} c^2}{\hbar} \qquad \textbf{(25)}$$

Which are the relativistic deBroglie kinetic frequency and the Compton free particle frequency.

There are two imaginary interaction type terms in the products of **(14)**, that constitute the well known Electric and spin orbital interaction energy of standard QM. Combining and adding to the free term gives:

$$i\left[M_n c \vec{V}_n \quad \pm \frac{Q^2}{r_m}\left(\frac{M_n}{M_m}\frac{\vec{V}_n}{c} \pm \frac{\vec{V}_m}{c} \right) \right] \frac{t}{\hbar}, \qquad \textbf{(26)}$$

Noting that The first term is the linear relativistic momentum p, the geometry of which determines the primary quantum numbers in an atomic system. The second grouped terms are the reciprocal spin orbital terms of the central and orbiting particles. The first of which is the momentum associated with the electric spin-energy of the central particle and the second is the momentum associated with the spin-electric energy of the orbiting particle. In an orbiting pair, $M_n V_n = -M_m V_m$ These interactions and the quantization conditions determine the fine structure energy levels in an atomic system.

In addition to the previous noted terms, there is a small spin-spin term which has no derivatives with respect to $\Re^2$ and thus doesn't contribute to the total energy. A detail representation of this is presumed to illustrate its participate in the mechanism of the exclusion principle.

$$\pm \frac{1}{2M_m c^2}\left(\frac{Q^2}{2r_m} \right) \qquad \textbf{(27)}$$

CONCLUSION

A multiple particle system equation for quantum mechanics, and its connection to physical phenomena has been presented. Although some of the detail has been left out for simplicity, and a lot of the obvious aspects have not been explored, it is clear that it represents a new approach to fundamental physics.

Though it represents a different perspective it is yet to be determined as to whether it provides any new physics. Though one could add gravitational contributions into the system equation, at this point it doesn't seem make a useful contribution, but since it addresses the entire universe as a system, aspects may have cosmological implications.

References :

[1] E. Comay, "Difficulties with the Klein-Gordon Equation", Journal-ref: Apeiron 11, No. 3, 1 (2004) arXiv:quant-ph/0312206,v3,8, Sep,2004.

[2] John David Jackson, Classical Electrodynamics Second Edition pp 411

[3] P. A. M. Dirac, The Principles of Quantum Mechanics, 4th ed., Oxford University Press,London (1958).

Appedix I

The current base equations for QM are the Relativistic Schrödinger equation or the Klein-Gordon and the Dirac equation with the potential incorporated by use of the "correspondence relation" . This method asserts that the total momentum of a charged particle in an external field is modified as such that.

$$p \rightarrow p - \frac{q}{c}A \qquad \textbf{(28)}$$

$$\frac{\partial}{\partial x_\mu} \rightarrow \left(\frac{\partial}{\partial x_\mu} - \frac{q}{c}A_\mu \right) \qquad \textbf{(29)}$$

The Schrödinger equation with fields included is:

$$\left(-\frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial y^2} - \frac{\partial^2}{\partial z^2} - i\frac{2}{\hbar}\left[\frac{\partial}{\partial ct} - \frac{\alpha}{r} \right] \right)\psi = 0 \qquad \textbf{(30)}$$

And the Dirac expression becomes:

$$\left(+\gamma_1\left(\frac{\partial}{\partial x} - \frac{q}{c}A_x \right) + \gamma_2\left(\frac{\partial}{\partial y} - \frac{q}{c}A_y \right) + \gamma_3\left(\frac{\partial}{\partial z} - \frac{q}{c}A_z \right) + \gamma_4\left(\frac{\partial}{\partial ct} - \frac{q}{c}\varphi \right) \right)\psi = \pm\frac{1}{\hbar}\psi \qquad \textbf{(31)}$$

Before this modification, that is for the free field solution, the Klein Gordon, and the Dirac equations are identical, in that the Dirac equation is a

factorization of the KG equation using the Dirac matrix. Once the modification has been included via the correspondence substitution, the equations, though both are accurate, are not equivalent, not even in interpretation [1]. The KG describes electromagnetic spin one particles in a potential, and the Dirac is a probability distribution of half spin particles.

It is asserted that including the potential, via the correspondence relation is the fundamental error that plagues further development of QM, in explaining physical phenomena. This is also believed to be source of the problem with renormalization. The insertion of the potential should be considered an approximation and only accurate to the extent that the approximation of the inverse square force is an accurate representation of the effect of the rest of the system.

A better understanding can be illustrated by considering the field free Dirac expression:

$$\gamma^{\mu}\left(\partial_{\mu}\psi\right)+\frac{1}{\mathbf{r}}\psi=0 \qquad \textbf{(32)}$$

Where we replace the isolated particle wavefunction with:

$$\psi\rightarrow\psi U \qquad \textbf{(33)}$$

Where U is the Hartree-Fock product wavefunction representing the effect of the entirety of the system excluding ψ.

$$U=\psi_1\psi_2\psi_3\psi_4... \qquad \textbf{(34)}$$

Substituting this into Eq. (32) then gives:

$$\gamma^{\mu}\left(U\partial_{\mu}\psi+\psi\partial_{\mu}U\right)+\frac{1}{\mathbf{r}}\psi U=0\,\mathbf{,} \qquad \textbf{(35)}$$

or:

$$\gamma^{\mu}\left(\partial_{\mu}\psi+\psi\frac{\partial_{\mu}U}{U}\right)+m\psi=0 \qquad \textbf{(36)}$$

Comparing this with the Dirac expression Eq. (31) with the correspondence related potential:

$$\gamma^{\mu}\left(\partial_{\mu}\psi-\frac{q}{c}A\psi\right)+\frac{1}{\mathbf{r}}\psi=0 \qquad \textbf{(37)}$$

We find:

$$\frac{\partial_\mu U}{U} = -\frac{q}{c} A \qquad \textbf{(38)}$$

Thus, A, which is normally inserted through the correspondence relation actually is an approximation of the electromagnetic effects of the rest of the system. In the case of the electron inverse radial potential, this only works to the extent that the potential is an accurate representation of the entirety of the rest of the system. The most notable defect of course is that the integration of the electrons energy becomes infinite, requiring renormalization.

One can argue that the Hartree-Fock product wavefunction Eq. (34), cannot represent multi-fermionic anti-symmetric system. This would be true, unless the individual wavefunction are represented by matrix functions which satisfying the Slater determinant conditions. It is expected that the solutions to the proposed equation can do this.

Appendix II

Definitions and Conventions

The radius of the universe:

$$\Re = cT = \Re_0 + ct$$

The Dirac matrix convention used in this development is

$$\gamma_1 = \begin{bmatrix} & & & -1 \\ & & -1 & \\ & +1 & & \\ +1 & & & \end{bmatrix} \quad \gamma_2 = \begin{bmatrix} & & & i \\ & & -i & \\ & -i & & \\ i & & & \end{bmatrix} \quad \gamma_3 = \begin{bmatrix} & & -1 & \\ & & & 1 \\ 1 & & & \\ & -1 & & \end{bmatrix} \quad \gamma_4 = \begin{bmatrix} & & & 1 \\ & & 1 & \\ & 1 & & \\ 1 & & & \end{bmatrix},$$

and

$$\gamma_1^{\,2} = -1, \quad \gamma_2^{\,2} = -1, \quad \gamma_3^{\,2} = -1, \quad \gamma_4^{\,2} = +1.$$

The product of the space coordinates is termed the spin matrix:

$$\vec{S} = \gamma_1 \gamma_2 \gamma_3 \qquad -\vec{S} = \gamma_3 \gamma_2 \gamma_1 \qquad \tau = \vec{S}\gamma_4 \qquad S = \begin{bmatrix} & & i & \\ & & & i \\ -i & & & \\ & -i & & \end{bmatrix}$$

The square is:

$$\vec{S}^2 = 1 \qquad \tau^2 = -1$$

The product of S with the coordinate vectors:

$$\vec{S}\ \gamma_n = \gamma_2\gamma_3,\ \ \gamma_1\gamma_3,\ \ \gamma_1\gamma_2$$

Which are the elements of the spin vector:

$$\sigma_1 = \gamma_2\gamma_3 \qquad \sigma_2 = \gamma_3\gamma_1 \qquad \sigma_3 = \gamma_2\gamma_1$$

The vector four velocity:

$$\vec{V} = \gamma_1 v_x + \gamma_2 v_y + \gamma_3 v_z + \gamma_4 c$$

Commutation relation with V and S

$$\begin{aligned}
\vec{S}\vec{V} &= \vec{S}\left(\gamma_1 v_x + \gamma_2 v_y + \gamma_3 v_z + \gamma_4 c\right) \\
\vec{V}\vec{S} &= \vec{S}\left(\gamma_1 v_x + \gamma_2 v_y + \gamma_3 v_z + \gamma_4 c\right)\vec{S} \\
\vec{S}\vec{V} + \vec{V}\vec{S} &= 2\,\vec{S}\left(\gamma_1 v_x + \gamma_2 v_y + \gamma_3 v_z\right) \\
&= 2\left(\gamma_2\gamma_3 v_x + \gamma_1\gamma_3 v_y + \gamma_1\gamma_2 v_z\right) \\
&= 2\,\sigma \bullet V
\end{aligned}$$

:

The product of two four velocities:

$$\vec{V}_m\vec{V}_n = \left(\gamma_1 v_{xn} + \gamma_2 v_{yn} + \gamma_3 v_{zn} + \gamma_4 c\right)\left(\gamma_1 v_{xm} + \gamma_2 v_{ym} + \gamma_3 v_{zm} + \gamma_4 c\right)$$

or

$$\vec{V}_m\vec{V}_n = -\vec{V}_m \bullet \vec{V}_n + \sigma \bullet V_m \times V_n + \gamma_4 c\,\Delta V$$

and commutation relation, or dot product:

$$\vec{V}_m\vec{V}_n + \vec{V}_m\vec{V}_n = 2\,\vec{V}_m \bullet \vec{V}_n$$

Appendix III

Discussion of the Moment function.

$$\pm\frac{i\Re M\vec{V}_m \pm \frac{1}{4}\hbar}{Mcr_m/\alpha-\hbar}$$

If the velocity and r_m goes to zero then $\vec{V}_m \to c$ the function becomes:

$$\pm\frac{\Re Mc}{\hbar}$$

Which is the "maximum" relativistic angular momentum a particle existing in the universe can have, since $\Re$ is the radius of the universe. Note that at the Compton radius the function has an extrema. But since it is an observation point there is no physical significance.

Focusing on the denominator and presuming $Mcr_m > \hbar$ we have:

$$=\frac{i\Re Mc}{\hbar}\left(\frac{1}{Mc^2}\frac{Q^2}{r_m}\right)$$

Which makes it the "maximum" angular momentum of the electric energy potential of the particle in units of $\hbar$ defined from at the origin

The moment function thus represents the "Total" relativistic angular momentum when $r=0$ and the "Total" angular momentum of the electric energy potential when $r>0$

Appendix IV

Vector Scalar Representation

For the purposes of this presentation the radius representing the location of the particles are treated as scalars. This is useful and simplifying for illustration, of the principle, however the solution space, includes the locus of all particles, and the interaction of multiple particles cannot be represented by radial scalars, there have to be vector represented positions. In Eq.16:

$$\vec{P}_m=\pm\frac{i\Re M\vec{V}_m \pm \frac{1}{2}\hbar}{Mcr_m/\alpha-\hbar}$$

in the preceding paper the $\Re$ in the distance from the observation point to the initial event (or big bang event) has been treated as a scalar. Technically the proper representation for $\Re$ is $\vec{\Re}$ a four vector from the observation point through the represented particle to the initial event, evaluated at $t = 0$

$$\vec{\Re} = \Re\left(\gamma_1 x_m + \gamma_2 y_m + \gamma_3 z_m + \gamma_4 ct\right)_{ct=0}$$

and The radius of the m particle r_m is just the magnitude of the location vector of the particle.

$$r_m = \left|\gamma_1 x_m + \gamma_2 y_m + \gamma_3 z_m + \gamma_4 ct\right|_{ct=0}$$

So we can represent

$$\vec{\Re} = \Re\,\frac{\vec{R}_m}{r_m}$$

Where ct is zero at the observation point, and $\frac{\vec{R}_m}{r_m}$ is a unit vector.

The $\hbar$ term is properly:

$$\frac{1}{2}\hbar\,\vec{s}\bullet\frac{\vec{R}_m}{r_m} = \frac{1}{2}\hbar\left(\gamma_1+\gamma_2+\gamma_3\right)\bullet\frac{\vec{R}_m}{r_m} = \frac{1}{2}\hbar$$

The proper velocity representation for the velocity becomes:

$$\vec{V}_n \to \frac{\vec{R}_m}{r_m}\vec{V}_n\frac{\vec{R}_m}{r_m}$$

And a more detailed representation of the moment function of Eq.(12) becomes.

$$\vec{P}_m = \pm\frac{i\Re M\frac{\vec{R}_m}{r_m}\vec{V}_m\frac{\vec{R}_m}{r_m} \pm \frac{1}{2}\hbar\frac{\vec{R}_m}{r_m}\bullet\left(\gamma_1+\gamma_2+\gamma_3\right)}{Mcr_m/\alpha-\hbar}$$

This provides the proper representation of the moment, and carrying fourth the previous calculations, one can arrive at the vector Lagrangian for the two particle interaction in which the observation particle has a velocity V_n .which is

the more detailed version and the proper two particle Lagrangian represented in Eq. (18) [2]

$$L_{ini} = \frac{Q^2}{r}\left[1 - \frac{1}{2}\frac{V_n \bullet V_m}{c^2} - \frac{1}{2}\left(\frac{\vec{R}_m}{r_m}\bullet\frac{V_n}{c}\right)\left(\frac{\vec{R}_m}{r_m}\bullet\frac{V_m}{c}\right)\right]$$

The velocity dot terms are the B field contributions.

Appendix V

Equivalence of equations

We have asserted that there is correspondence between the applying the Dirac operator to the standard scalar quantum mechanical wavefunctions and applying the expansion displacement operator to the imaginary part of the matrix systemfunction. This will be an illustration of the concept, but not a proof.

A component representative scalar wavefunction for a free particle would be:

$$\psi_Q \sim e^{\frac{m}{\hbar}\left(v\sqrt{x^2+y^2+z^2}+cct\right)}$$

Applying the Dirac operator to this would give:

$$\gamma_\mu \frac{\partial}{\partial x_\mu}\psi = \frac{m}{\hbar}\left(\gamma_1 v_x + \gamma_2 v_y + \gamma_3 v_z + \gamma_4 c\right)\psi = i\frac{m_0 c}{\hbar}\psi$$

Now if we take the imaginary matrix systemfunction for the free particle to be:

$$\vec{\psi} = e^{\frac{m}{\hbar}\left(\gamma_1 v_x + \gamma_2 v_y + \gamma_3 v_z + \gamma_4 c\right)\Re}$$

and apply the virtual displacement operator we get:

$$\frac{\partial}{\partial \Re}\vec{\psi} = \frac{m}{\hbar}\left(\gamma_1 v_x + \gamma_2 v_y + \gamma_3 v_z + \gamma_4 c\right)\vec{\psi} = \frac{m_0 c}{\hbar}\vec{\psi}$$

Note that the application of the expansion displacement operator to the matrix systemfunction procures the same derivative as the application of the matrix Dirac operator to the corresponding scalar wavefunction.

$$\gamma_\mu \frac{\partial}{\partial x_\mu} \psi = \frac{i}{\hbar} \psi \qquad \frac{\partial}{\partial \Re} \vec{\psi} = \frac{i}{\hbar} \vec{\psi}$$

It would be concluded that these are corresponding representations of the same physical system.