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On the distribution of prime numbers in the intervals defined by the Fibonacci numbers

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Abstract

The number of primes in the inclusive intervals defined by consecutive Fibonacci numbers exhibits interesting behavior between the Fibonacci numbers 55 and 196418. Specifically, starting with the interval [55, 89] through the interval [121393,196418] the ratio of the number of primes in successive intervals is a value that alternates high, low, high, low, etc.

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Accordingly, in the table below:

- the first column contains row number,
- the second and third columns contain a pair of consecutive Fibonacci numbers,
- the fourth column, the number of primes in the inclusive interval defined by these Fibonacci numbers, and
- the fifth column, the ratio of the number of primes in successive intervals. (So, for row number 9 divided by row number 10: 5 primes / 3 primes = 1.6666... .)

As noted above, in column five, starting at row 10 (Fibonacci numbers 55 and 89), these ratios zigzag high, low, high, low, etc. up to row 26 (Fibonacci numbers 121393 and 196418):

```
01, 000000001, 000000001, 000000000, 00.UNDEFINED
02, 000000001, 000000002, 000000001, 01,UNDEFINED
03, 000000002, 000000003, 000000002, 02.00000
04, 000000003, 000000005, 000000002, 01.00000
05, 000000005, 000000008, 000000002, 01.00000
06, 000000008, 000000013, 000000002, 01.00000
07, 000000013, 000000021, 000000003, 01.50000
```

```
08, 000000021, 000000034, 000000003, 01.00000
09, 000000034, 000000055, 000000005, 01.66667


10, 000000055, 000000089, 000000008, 01.60000 <- ZIGZAG START
11, 000000089, 000000144, 000000011, 01.37500

12, 000000144, 000000233, 000000017, 01.54545
13, 000000233, 000000377, 000000024, 01.41176

14, 000000377, 000000610, 000000037, 01.54167
15, 000000610, 000000987, 000000055, 01.48649

16, 000000987, 000001597, 000000085, 01.54545
17, 000001597, 000002584, 000000126, 01.48235

18, 000002584, 000004181, 000000198, 01.57143
19, 000004181, 000006765, 000000297, 01.50000

20, 000006765, 000010946, 000000458, 01.54209
21, 000010946, 000017711, 000000704, 01.53712

22, 000017711, 000028657, 000001088, 01.54545
23, 000028657, 000046368, 000001674, 01.53860

24, 000046368, 000075025, 000002602, 01.55436
25, 000075025, 000121393, 000004029, 01.54842

26, 000121393, 000196418, 000006263, 01.55448 <- ZIGZAG END
```

Moreover, if one takes successive intervals as defined by $[\ F_n, ceiling(F_{n+2}/2)]$, two zigzag patterns arise (in rows 3 to 13, and 14 to 28, below). Accordingly, in the table below, the second and third columns are now F_n and $ceiling(F_{n+2}/2)$, where the first zigzag begins at row 3 and ends at row 13, and the second zigzag begins at row 14 and ends at row 28.

```
01, 000000001, 000000001, 000000000, UNDEFINED
02, 000000001, 000000002, 000000001, UNDEFINED


03, 000000002, 000000003, 000000002, 02.00000 <- START OF 1ST ZIGZAG
04, 000000003, 000000004, 000000001, 00.50000

05, 000000005, 000000007, 000000002, 02.00000
06, 000000008, 000000011, 000000001, 00.50000

07, 000000013, 000000017, 000000002, 02.00000
```

```
08, 000000021, 000000028, 000000001, 00.50000

09, 000000034, 000000045, 000000003, 03.00000
10, 000000055, 000000072, 000000004, 01.33333

11, 000000089, 000000117, 000000007, 01.75000
12, 000000144, 000000189, 000000008, 01.14286

13, 000000233, 000000305, 000000012, 01.50000 <- END OF 1ST ZIGZAG


14, 000000377, 000000494, 000000020, 01.66667 <- START OF 2ND ZIGZAG
15, 000000610, 000000799, 000000028, 01.40000

16, 000000987, 000001292, 000000044, 01.57143
17, 000001597, 000002091, 000000066, 01.50000

18, 000002584, 000003383, 000000100, 01.51515
19, 000004181, 000005473, 000000148, 01.48000

20, 000006765, 000008856, 000000232, 01.56757
21, 000010946, 000014329, 000000352, 01.51724

22, 000017711, 000023184, 000000553, 01.57102
23, 000028657, 000037513, 000000851, 01.53888

24, 000046368, 000060697, 000001324, 01.55582
25, 000075025, 000098209, 000002035, 01.53701

26, 000121393, 000158906, 000003166, 01.55577
27, 000196418, 000257115, 000004917, 01.55306

28, 000317811, 000416020, 000007656, 01.55705 <- END OF 2ND ZIGZAG
```

The above zigzags fail to continue when the above tables are extended, but it is not clear whether such zigzags might not later reappear, or related interesting phenomena arise.