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ELECTROMAGNETIC STEALTH WITH PARALLEL ELECTRIC AND MAGNETIC FIELDS

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Electromagnetic Stealth with Parallel Electric and Magnetic Fields

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Abstract
We analyze a theoretical example of parallel electric and magnetic fields in a hypothetical anisotropic medium with varying susceptibility. We deduce the polarization characteristics and we discuss the conditions under which this could be utilized in electromagnetic invisibility.

1. Introduction

In a previous work [1] we have presented a general method for solving the Beltrami problem for a spherically symmetric vector potential. This class of fields has been previously discussed in the literature [2-7], with respect to their peculiar property of having both the electric and magnetic components in parallel. This situation caused an initial controversy which was settled down after the realization that such special cases constitute very special solutions of Maxwell equations that can only exist within cavities with prescribed boundary conditions [7]. In fact, an experimental realization of such a state in a laser cavity with opposite circularly polarized modes was shown in [5].
In the present work we present an example which proves that the application of such peculiar states is far from exhausted. In particular, in section 2 we consider a spherical region where a varying charge density followed by the associated polarization and current sources exists and we analyze the general form of solutions of Maxwell equations under the condition that the electric field is an eigen-rotation field [9-10] although the magnetic field is not.
This approach differs from previous studies where the magnetic field was supposed to satisfy a *Beltrami* condition which is a special class of eigen-rotation fields. In section 3, we present a special multipole solution for the eigen-rotation equation from which we derive the generalized susceptibility and permittivity of the enclosing space. In section 4, we discuss the possibility that such solutions could have potential applications with respect to electromagnetic invisibility (plasma sheath).

2. Parallel electric and magnetic fields

We start from a polarizable medium which contains a harmonically time varying radial charge distribution $\rho(r,t) = \rho(r,\theta,\phi)\exp(\mathbf{i}\omega t)$ where ω stands for a monochromatic frequency, inside a spherical region of maximum radius R, an associated current **J** and a polarization **P**. The need for the addition of a polarization term will become apparent in the next section. Our aim will be to define the spatial

dependence of the quantities ρ, **J** and **P** under the condition of parallelization of the **E** and **B** fields and find the characteristics of the medium.
Denoting with ε_0 and μ the respective permittivity and permeability of the vacuum, Maxwell equations in the SI system take the form

$$\begin{aligned} &\nabla \bullet \mathbf{D} = \rho, \quad \nabla \bullet \mathbf{B} = 0 \\ &\nabla \times \mathbf{E} = -\partial_t \mathbf{B} \\ &\nabla \times \mathbf{H} = \partial_t \mathbf{D} + \mathbf{J} \end{aligned} \tag{1a}$$

with the constitutive relations

$$\begin{aligned} &\mathbf{D} = \varepsilon_0 \mathbf{E} + \mathbf{P} \\ &\mathbf{H} = \frac{\mathbf{B}}{\mu} \end{aligned} \tag{1b}$$

The last of (1a) can be rewritten in terms of a polarization current source term taking into account the constitutive relations as

$$\begin{aligned} &\nabla \bullet \mathbf{D} = \rho, \quad \nabla \bullet \mathbf{B} = 0 \\ &\nabla \times \mathbf{E} = -\partial_t \mathbf{B} \\ &\nabla \times \mathbf{B} = \varepsilon_0 \mu \partial_t \mathbf{E} + \mu \partial_t \mathbf{P} + \mathbf{J} \end{aligned} \tag{2}$$

We assume a harmonic time-dependence of the form $\exp(\mathbf{i}\omega t)$ and a linear Ohm's law such that $\mathbf{J} = \sigma\mathbf{E}$ so that we can rewrite the last two as

$$\begin{aligned} &\nabla \times \mathbf{E} = -\mathbf{i}\omega \mathbf{B} \\ &\nabla \times \mathbf{B} = \mathbf{i}\varepsilon_0 \mu \omega \mathbf{E} + \mathbf{i}\mu\omega \mathbf{P} + \sigma \mathbf{E} \end{aligned} \tag{3}$$

The central hypothesis of this section is that we can find solutions for the sources such that ***E//B*.** We express this through the condition

$$\mathbf{B} = \Lambda(r,\omega)\mathbf{E} \tag{4}$$

where Λ is an arbitrary scalar function. Substituting this into the first of (3) we derive the eigen-rotation equation

$$\nabla \times \mathbf{E} = -\mathbf{i}\omega\Lambda\mathbf{E} \tag{5}$$

This is similar with the equation describing a Beltrami flow in hydrodynamics but without the additional condition for **E** to be a purely solenoidal field [9-10]. The term $\mathbf{i}\omega\Lambda$ is then the field's eigen-vorticity. Applying the divergence operator to the *lhs* leads naturally to the condition $\nabla(\Lambda\mathbf{E}) = 0$ which results in the relation

$$\nabla \mathbf{E} = -\left(\frac{\nabla\Lambda}{\Lambda}\right) \bullet \mathbf{E} \qquad (6)$$

This also automatically satisfies $\nabla \mathbf{B} = 0$. We then substitute (4) into the second of (3) which results in the equation

$$\nabla\Lambda \times \mathbf{E} + \Lambda(\nabla \times \mathbf{E}) = \mathbf{i}\varepsilon_0 \mu\omega \mathbf{E} + \mathbf{i}\mu\omega \mathbf{P} + \sigma \mathbf{E} \qquad (7)$$

From the above and using (5) we can find the polarization current in terms of the electric field as

$$\mathbf{P} = -\chi(r,\omega)\mathbf{E} - \frac{\mathbf{i}}{\mu\omega}\nabla\Lambda \times \mathbf{E} \qquad (8a)$$

$$\chi(r,\omega) = \varepsilon_0 + \sigma + \frac{\Lambda^2}{\mu} \qquad (8b)$$

We now see that in the absence of any polarization the current would have to obey a strongly non-linear form with unnatural characteristics like negative resistance. From (8a) we see that ***D*** can now be expressed as

$$\mathbf{D} = \varepsilon_0 \mathbf{E} + \mathbf{P} = -\frac{\Lambda^2}{\mu}\mathbf{E} - \frac{\mathbf{i}}{\mu\omega}\nabla\Lambda \times \mathbf{E} \qquad (9)$$

Accordingly, the charge distribution can now be found from the first of (2) as

$$\rho = \nabla \bullet \mathbf{D} = -\frac{\mathbf{i}}{\mu\omega}\nabla(\nabla\Lambda \times \mathbf{E}) - \frac{\Lambda}{\mu_0}\nabla\Lambda \bullet \mathbf{E} - \frac{\Lambda^2}{\mu_0}\nabla \mathbf{E} \qquad (10)$$

From standard vector identities we also have that

$$\nabla(\nabla\Lambda \times \mathbf{E}) = -\nabla\Lambda \bullet (\nabla \times \mathbf{E}) = \mathbf{i}\omega\Lambda\nabla\Lambda \bullet \mathbf{E} \qquad (11)$$

Using both (10) and (6) in (9) finally yields

$$\rho = \frac{\Lambda}{\mu_0}\nabla\Lambda \bullet \mathbf{E} \qquad (12)$$

3. Solutions of the eigen-rotation equation

Complete solution of the problem of sources for the previously prescribed electric and magnetic fields requires a solution of the eigen-rotation equation (5). We have provided a generic semi-analytical technique for solving such a problem elsewhere [1]. Here we will approach the problem from a different viewpoint that allows finding a special analytical solution that can be utilized for computations.

Specifically, we avoid the use of a vector potential and we recourse to a two step technique. We will have to introduce a set of vector spherical harmonics as $\vec{\mathrm{Y}}_{lm} = \mathrm{Y}_{lm}\hat{\mathbf{r}},\ \vec{\Psi}_{lm} = r\nabla \mathrm{Y}_{lm},\ \vec{\Phi}_{lm} = \vec{\mathbf{r}} \times \nabla \mathrm{Y}_{lm}$ where Y_{lm} are the usual scalar spherical harmonics. Next, we proceed in two steps. We seek for a set of dual fields with respect to the rotation operator that transform as

$$\begin{aligned} \nabla \times \mathbf{E}_1 &= \lambda_1 \mathbf{E}_2 \\ \nabla \times \mathbf{E}_2 &= \lambda_2 \mathbf{E}_1 \end{aligned} \tag{13}$$

A solution of the the eigen-rotation equation can be found from a linear combination of a set of special solutions of the above that allow a common factor. From standard identities of vector spherical harmonics as presented in [12] we have

$$\begin{aligned} \nabla \times f(r)\vec{\mathrm{Y}} &= -\frac{1}{r} f(r)\vec{\Phi} \\ \nabla \times f(r)\vec{\Phi} &= -\frac{l(l+1)}{r} f(r)\vec{\mathrm{Y}} - \left(\frac{df}{dr} + \frac{f}{r}\right)\vec{\Psi} \end{aligned} \tag{14}$$

We see that the second part of the *rhs* of (14) cancels out if we choose $f(r) = k/r$ which leads to the choice

$$\mathbf{E}_1 = \frac{k}{r}\vec{\mathrm{Y}}_{lm},\ \ \mathbf{E}_2 = \frac{k}{r}\vec{\Phi}_{lm} \tag{15}$$

for which (13) is satisfied with $\lambda_1 = -1/r,\ \lambda_2 = -\lambda_0/r,\ \lambda_0 = l(l+1)$. In order to find a symmetric linear combination of these fields we assume a set of coefficients that depend on λ_0 and write the total field in the form

$$\mathbf{E} = \lambda_0^{\mu}\mathbf{E}_1 + \lambda_0^{\nu}\mathbf{E}_2 \tag{16}$$

where μ and ν are unknown exponents. Then we observe that the action of the rotation operator is to interchange the coefficients of (16) in the form

$$\nabla \times \mathbf{E} = \lambda_1(\lambda_0^{\nu+1}\mathbf{E}_1 + \lambda_0^{\mu}\mathbf{E}_2) \tag{17}$$

In order for the final combination to be written in terms of the original we rewrite the above as

$$\nabla \times \mathbf{E} = \lambda_1(\lambda_0^{\nu-\mu+1}\lambda_0^{\mu}\mathbf{E}_1 + \lambda_0^{\mu-\nu}\lambda_0^{\nu}\mathbf{E}_2) \tag{18}$$

In order to get a common factor the following condition must be satisfied

$$\lambda_0^{\nu-\mu+1} = \lambda_0^{\mu-\nu} \iff \lambda_0^{2(\nu-\mu)+1} = 1 \tag{19}$$

This is equivalent to the equation $2(\nu-\mu)+1=0$ or $\mu=\nu+1/2, \nu=0,1,2,...$. Then from (17) we find the eigen-vorticity as $\lambda' = \lambda_1\lambda_0^{1/2} = -\frac{\sqrt{l(l+1)}}{r}$. We can thus write a family of special solutions of the eigen-rotation equation in the form

$$\mathbf{E}^{(\nu)} = \frac{k}{r}(L^{2\nu+1}\vec{Y}_{lm} + L^{2\nu}\vec{\Phi}_{lm}), l > 0 \tag{20}$$

where we introduced the abbreviation $L=\sqrt{l(l+1)}$. From (5) we now find that the Λ factor has the form $\Lambda(r,\omega) = -\mathbf{i}\frac{L}{r\omega}$ from which we deduce the susceptibility function as

$$\chi(r,\omega) = \varepsilon_0 + \sigma + \frac{1}{\mu}\left(\frac{L}{r\omega}\right)^2 \tag{21}$$

We may now separate the overall polarization into an isotropic and an anisotropic part as

$$\begin{aligned} \mathbf{P} &= -\chi\mathbf{E} + \mathbf{P}_A \\ \mathbf{P}_A &= -\frac{\mathbf{i}}{\mu\omega}\nabla\Lambda \times \mathbf{E} = \frac{L^{2\nu+1}}{\mu r^2\omega^2}\left(\hat{\mathbf{r}} \times \vec{\Phi}_{lm}\right) \end{aligned} \tag{22}$$

We notice that both parts diverge as $r \to 0$ or $\omega \to 0$. In fact, both terms diverge as $1/(r^2\omega^2)$ taking into account that $\mathrm{Y}_{lm}, \Phi_{lm}$ are functions of the spherical angles and $\hat{\mathbf{r}}$ is a unit vector. For the charge distribution we also have that

$$\rho = \frac{L^{2\nu+3}}{\mu r^3\omega^2}\mathrm{Y}_{lm} \tag{23}$$

Assuming that we can set up appropriate boundary conditions in an interior and exterior spherical surface $R_1 < r < R_2$, we may suppress the divergence to a scale less than R_1 by taking the product $R_1\omega \to 1$ in which case the divergence can be reduced to a very low scale by increasing the frequency. Thus, for a frequency of the order of GHz the divergence region for both parts can be reduced to a radius of less than 1 nm.

4. Conclusion

The previous sections are devoted to an abstract treatment of a hypothetical polarizable medium capable of sustaining parallel electric and magnetic fields. In section 3 an exact analytical form of the polarizability and the susceptibility of such a medium were derived. The important property of the prescribed solutions of Maxwell equations resumes into their ability to hold a large amount of electromagnetic energy due to radiation cancellation ($\mathbf{E} \times \mathbf{B} = 0$).

The last decade has seen an increasing effort towards optical invisibility cloaks using special metamaterials with negative refractive index which were first introduced with the pioneering work of J. Pendry and others [13 - 14]. In our treatment, a different possibility arises due to the radiation cancellation condition. While in ordinary invisibility the effort is towards “light bending” around an object similar to gravitational lensing, in our case it appears that in principle radiation could get trapped inside a spherical region. This in practice would represent an alternative method of shielding an area against any passive detection method.

In the present report we found that a solution with non-radiating parallel electric and magnetic fields is possible inside an anisotropic polarizable medium with a varying charge concentration and polarization current.

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