Probabilistic Interpretation of Quantum Mechanics with Schrödinger Quantization Rule

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Abstract

We quantize the probabilistic interpretation of quantum mechanics using Schrödinger quantization rule. We describe the probability of getting a quantum object in configuration space as the eigenvalue (image) of quantum mechanical probability (operator) satisfying Schrödinger probability eigenvalue equation. The deduction is used to obtain quantum description of systems which would be used to quantize many classical and quantum (differential) problems.

keywords Schrödinger Quantization Rule • Probability • Unity or zero-order differential operator

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Probability Eigenvalue Formalism

Schrödinger wave is given by

$$\psi(s(q_\alpha, t)) := \exp(\frac{i}{\hbar}s(q_\alpha, t)) \tag{1}$$

Its exact derivative w.r.t. action is given by

$$\frac{d\psi}{ds} = \frac{i}{\hbar}\psi \tag{2}$$

We propose a unity (zero-order differential) operator which satisfy for any function

$$\mathcal{I}f = f \tag{3}$$

as well for Schrödinger wave

$$\mathcal{I}|\psi\rangle = |\psi\rangle \tag{4}$$

Following deduction (4) for (2), we obtain

$$\mathcal{I}|\psi\rangle = -i\hbar\frac{d}{ds}|\psi\rangle \tag{5}$$

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which quantizes unity eigenoperator $\mathcal{I}$ to

$$\widehat{\mathcal{I}} = -i\hbar\frac{d}{ds} \tag{6}$$

satisfying unity eigenoperator equation

$$\widehat{\mathcal{I}}|\psi\rangle = \mathcal{I}|\psi\rangle \tag{7}$$

Probability (w) of getting a quantum object in real configuration space is provided unity. However, in operator representation, we modify this analysis by transforming probability (w) to unity operator ($\mathcal{I}$). By putting $w := \mathcal{I}$ in (5), we obtain probability eigenvalue equation

$$w|\psi\rangle + i\hbar\frac{d}{ds}|\psi\rangle = 0 \tag{8}$$

with probability operator

$$\widehat{w} = -i\hbar\frac{d}{ds} \tag{9}$$

satisfying Schrödinger probability eigenvalue equation

$$\widehat{w}|\psi\rangle = w|\psi\rangle \tag{10}$$

This is probability theory with Schrödinger quantization rule. Now probability is observed as the eigenvalue (image) of probability operator. Previous description of quantum probability theory (Born, 1926) is now modified (quantized) with Schrödinger representation

$$w = \int \psi^*\psi d\tau = \psi^{-1}\widehat{w}\psi \tag{11}$$

with $\widehat{w} = -i\hbar\frac{d}{ds}$. Here probability is described in both differential and integral representations. Both descriptions would help us to quantize classical and quantum problems.

Let differentiate

$$\langle\widehat{\mathcal{A}}\rangle = \langle\psi|\widehat{\mathcal{A}}|\psi\rangle = \int \psi^*\widehat{\mathcal{A}}\psi d\tau \tag{12}$$

exactly w.r.t. action with differential-integral rule

$$\widehat{f}\int \mathcal{K}(\tau,\tau')d\tau = \int \widehat{f}\mathcal{K}(\tau,\tau')d\tau \tag{13}$$

We obtain

$$\frac{d}{ds}\langle\widehat{\mathcal{A}}\rangle = \langle\frac{d\psi}{ds}|\widehat{\mathcal{A}}|\psi\rangle + \langle\psi|\frac{d\widehat{\mathcal{A}}}{ds}|\psi\rangle + \langle\psi|\widehat{\mathcal{A}}|\frac{d\psi}{ds}\rangle \tag{14}$$

Consider probability eigenvalue equation

$$|\frac{d\psi}{ds}\rangle = \frac{i}{\hbar}|\widehat{w}\psi\rangle, \langle\frac{d\psi}{ds}| = -\frac{i}{\hbar}\langle\widehat{w}^\dagger\psi| \tag{15}$$

We obtain

$$\frac{d}{ds}\langle\widehat{\mathcal{A}}\rangle = \langle\frac{d\widehat{\mathcal{A}}}{ds}\rangle - \frac{i}{\hbar}[\langle\widehat{w}^\dagger\psi|\widehat{\mathcal{A}}|\psi\rangle - \langle\psi|\widehat{\mathcal{A}}\widehat{w}|\psi\rangle] \tag{16}$$

Since the probability is a real aspect of nature, i.e., in operator representation, it must be hermitian

$$\langle\widehat{w}^\dagger\psi|\widehat{\mathcal{A}}|\psi\rangle = \langle\psi|\widehat{w}\widehat{\mathcal{A}}|\psi\rangle \tag{17}$$

which yields

$$\frac{d}{ds}\langle\widehat{\mathcal{A}}\rangle = \langle\frac{d\widehat{\mathcal{A}}}{ds}\rangle - \frac{i}{\hbar}\langle[\widehat{w},\widehat{\mathcal{A}}]_-\rangle \tag{18}$$

This is first order quantum description of systems which would be used to quantize first order differential problems. Considering the analogy, we further obtain second and third order quantum description of systems

$$\frac{d^2}{ds^2}\langle\widehat{\mathcal{A}}\rangle = \langle\frac{d^2\widehat{\mathcal{A}}}{ds^2}\rangle - \frac{i}{\hbar}\langle[[\widehat{w},\frac{d\widehat{\mathcal{A}}}{ds}]_- + \frac{d}{ds}[\widehat{w},\widehat{\mathcal{A}}]_- - \frac{i}{\hbar}[\widehat{w},[\widehat{w},\widehat{\mathcal{A}}]_-]_-]\rangle \tag{19}$$

and

$$\begin{aligned}\frac{d^3}{ds^3}\langle\widehat{\mathcal{A}}\rangle = \langle\frac{d^3\widehat{\mathcal{A}}}{ds^3}\rangle - \langle[[\widehat{w},\frac{d^2\widehat{\mathcal{A}}}{ds^2}]_- + \frac{d}{ds}[\widehat{w},\frac{d\widehat{\mathcal{A}}}{ds}]_- + \frac{d^2}{ds^2}[\widehat{w},\widehat{\mathcal{A}}]_- \\ -\frac{i}{\hbar}[[\widehat{w},[\widehat{w},\frac{d\widehat{\mathcal{A}}}{ds}]_-]_- + [\widehat{w},\frac{d}{ds}[\widehat{w},\widehat{\mathcal{A}}]_-]_- + \frac{d}{ds}[\widehat{w},[\widehat{w},\widehat{\mathcal{A}}]_-]_- \\ -\frac{i}{\hbar}[\widehat{w},[\widehat{w},[\widehat{w},\widehat{\mathcal{A}}]_-]_-]_-]]\rangle\end{aligned} \tag{20}$$

These are second and third order quantum description of systems which would be used to quantize second and third order differential problems.

Appendix

Unity Operator

Unity operator (eigenoperator) is deduced as a zero-order (ordinary or partial) differential operator (irrespective of with respect to what) defined as

$$\mathcal{I} := \partial_x^0, (x = q, p, t, ...) \tag{21}$$

We have observed in Mathematical analysis that a zero-order differential operator does not change the function to which it is applied which leads to deduce it unity operator satisfying $\mathcal{I}f = f$. For example, in Ostrogradsky transformation, zero-order prime of generalized co-ordinate $q_{(n)}, (n = 0, 1, 2, 3, ...)$ for $n = 0$ is given by q. It may be extended to $q_{(n)} = \mathcal{I}q = q$ for $n = 0$ with $\mathcal{I} := d_t^0$. The deduction is less applicable in Mathematical analysis but is very important to treat quantum problems. Unity operator is quantized to $\widehat{\mathcal{I}} := -i\hbar\frac{d}{ds}$ satisfying unity eigenoperator equation $\widehat{\mathcal{I}}|\psi\rangle = \mathcal{I}|\psi\rangle$ while treating quantum problems. For example, a quantum transformation with $\psi_{(n)}, (n = 0, 1, 2, 3, ...)$ (being n-order exact or partial derivative of ψ w.r.t. any variable x) is extended for $n = 0$, $\psi_{(n)} = \mathcal{I}\psi = \psi$ with $\mathcal{I} := \partial_x^0$. This is a quantum problem and we quantize $\mathcal{I}$ to $\widehat{\mathcal{I}}$ which yields $\psi_{(n)} + i\hbar\frac{d\psi}{ds} = 0$ for $n = 0$.

Conclusion

The work has attempted to deduce probability theory using Schrödinger quantization rule. Quantum probability theory is extended with Schrödinger representation. Probability of getting a quantum object in (real) configuration space is described as the eigenvalue (image) of probability operator. The deduction is used to obtain quantum description of systems which would be used to quantize (many order) differential problems. An attempt to quantize integral problems by using integral description of probability theory may be developed by using integral mathematical (operator) techniques (such as unity operator being zero-order differential operator). The operator (eigenvalue) representation of probability would be used to modify quantum probability theory in differential manner of observation.

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