Seven Conjectures in Geometry and Number Theory

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Abstract:
In this short paper we propose four conjectures in synthetic geometry that generalize Erdos-Mordell Theorem, and three conjectures in number theory that generalize Fermat Numbers.

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1. **Four Geometrical Conjectures:**

a) Let M be an interior point in a $A_1A_2...A_n$ convex polygon and P_i the projection of M on A_iA_{i+1}, $i = 1, 2, 3, ..., n$.
Then

$$\sum_{i=1}^{n} \overline{MA_i} \geq c \sum_{i=1}^{n} \overline{MP_i}$$

where c is a constant to be found.

For n=3, it was conjectured by Erdös in 1935, and solved by Mordell in 1937 and Kazarinoff in 1945. In this case $c = 2$ and the result is called the Erdös-Mordell Theorem.

b) More generally: If the projections P_i are considered under a given oriented angle $\alpha \neq 90$ degrees, what happens with the above inequality?

c) In a 3-space, we make the same generalization for a convex polyhedron with n vertexes and m faces:

$$\sum_{i=1}^{n} \overline{MA_i} \geq c_1 \sum_{j=1}^{m} \overline{MP_j}$$

where P_j, $1 \leq j \leq m$, are projections of M on all faces of the polyhedron, and c_1 is a constant to be determined.

[Kazarinoff conjectured that for the tetrahedron

$$\sum_{i=1}^{4}\overline{MA_i} \geq 2\sqrt{2}\sum_{i=1}^{4}\overline{MP_i}$$

and this is the best possible].

d) Furthermore, does the below inequality hold?

$$\sum_{i=1}^{n}\overline{MA_i} \geq c_2\sum_{k=1}^{r}\overline{MT_k}$$

where T_k, $1 \leq k \leq r$, are projections of M on all sides of the polyhedron, and c_2 is a constant to be determined.

2. Three Number Theory Conjectures (Generalization of Fermat Numbers):

Let's consider $a,\ b$ integers ≥ 2 and c an integer such that $(a,\ c) = 1$.

One constructs the function $P(k) = a^{b^k} + c$, where $k \in \{0,1,2,...\}$.

Then:

a) For any given triplet $(a,\ b,\ c)$ there is at least a k_0 such that $P(k_0)$ is prime.

b) Does there exist a non-trivial triplet $(a,\ b,\ c)$ such that $P(k)$ is prime for all $k \geq 0$?

c) Is it possible to find a triplet $(a,\ b,\ c)$ such that $P(k)$ is prime for infinitely many k's?

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