Infinite Smarandache Groupoids

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Abstract

It is proved that there are infinitely many infinite Smarandache Groupoids.

Key words: Binary operation, Groupoid, Semigroup, Prime number, Function.

1. Introduction

The study of groupoids is very rare and meager; according to W.B.Vasantha Kandasamy, the only reason to attribute to this is that it may be due to the fact that there is no natural way by which groupoids can be constructed.

The study of Smarandache Algebraic Structures was initiated in the year 1998 by Raul Padilla following a paper written by Florentin Smarandache called "Special Algebraic Structures". In his research Padilla treated the Smarandache Algebraic Structures mainly with associative binary operation. Since then the subject has been pursued by a growing number of researchers. In [11], a systematic development of the basic non-associative algebraic structures *viz* Smarandache Groupoids, was given by W.B.Vasantha Kandasamy. Smarandache Groupoids exhibit simultaneously the properties of a semigroup and a groupoid.

In [11], most of the examples of Smarandache Groupoids, given by W.B. Vasantha Kandasamy, are finite. Further, it is said that finding Samarndache Groupoids of infinite order, seems to be a very difficult task and left as an open problem. In this paper we give infinitely many infinite Smarandache Groupoids by proving a theorem: "There are infinitely many infinite Smarandache Groupoids."

In section 2 we recall some definitions and examples pertaining to groupoids, Smarandache Groupoids and integers. For basic definitions please refer [11].

2. Preliminaries

Definition 2.1([11]) Given an arbitrary set P a mapping of P x P into P is called a binary operation of P. Given such a mapping σ **:**P x P$\rightarrow$P , we use it to define a product $*$ on P by declaring a $*$ b = c, if σ (a,b) = c.

Definition 2.2([11]) A nonempty set of elements G is said to form a groupoid if, in G, there is defined a binary operation called product denoted by $*$ such that a $*$ b $\in$ G, for all a , b $\in$ G.

It is important to mention here that the binary operation $*$ defined on the set G need not be associative, that is, (a $*$ b) $*$ c $\neq$ a $*$ (b $*$ c), in general for a, b, c $\in$ G. So, we can say the groupoid (G , $*$) is a set on which there is defined a non-associative binary operation which is closed on G. Examples of groupoids can, naturally, be found in the literature.

We call the order of the groupoid G to be the number of distinct elements in it. If the number of elements in G is finite we say the groupoid is of finite order or finite groupoid otherwise we say G is an infinite groupoid.

Definition 2.3([4]). A semigroup is a nonempty set S, in which for every ordered pair of elements a, b $\in$ S there is defined a binary operation $*$ called their product a $*$ b such that a $*$ b $\in$ G and we have (a $*$ b) $*$ c = a $*$ (b $*$ c.) for all a , b, c $\in$ S.

Definition 2.4([5]). An integer n is called prime if n > 1 and if the only positive divisors of n are 1 and n.

Theorem 2.5(Euclid).. There are infinitely many prime numbers.

Definition 2.6([11]). A Smarandache Groupoid (G, $*$) is a groupoid which has a proper subset S, S $\subset$ G, such that S under the operations of G is a semigroup.

Example 2.7. Let (G , ∗) be a groupoid given by the following table.

∗	0	1	2	3	4	5
0	0	3	0	3	0	3
1	1	4	1	4	1	4
2	2	5	2	5	2	5
3	3	0	3	0	3	0
4	4	1	4	1	4	1
5	5	2	5	2	5	2

Clearly, $S_1 = \{ 0 , 3 \}$, $S_2 = \{ 1 , 4 \}$ and $S_3 = \{ 2 , 5 \}$ are proper subsets of G which are semigroups of G. So, (G , ∗) is a Smarandache Groupoid.

Definition 2.8: Let G be a Smarandache Groupoid, if the number of elements in G is finite we say G is a finite Smarandache Groupoid otherwise, infinite Smarandache Groupoid.

Definition 2.9: Let G be a Smarandache Groupoid, G is said to be a Smarandache commutative groupoid if there is a proper subset, which is a semigroup, is a commutative semigroup.

3. Proof of the Theorem.

In this section we prove our main theorem that "There are infinitely many infinite Smarandache Groupoids. Here, for any real number x, [x] means the greatest integer less than or equal to x.

Theorem 3.1: There are infinitely many infinite Smarandache Groupoids.
Proof: We construct infinitely many infinite Smarandache Groupoids in two ways.

(3.1.1). Define the operation $*$ on the set Z of integers by $x * y = \left[\frac{x}{p}\right] + \left[\frac{y}{p}\right]$,

where p is a prime number , for all x, y $\in$ Z. It is immediate that $*$ is a binary operation on Z as $\left[\frac{x}{p}\right]$, $\left[\frac{y}{p}\right]$ are always integers.

Next, we observe that the operation $*$ is not associative. For, take p = 2.

$(2 * 3) * 4 = \left(\left[\frac{2}{2}\right] + \left[\frac{3}{2}\right] \right) * 4 = 2 * 4 = \left[\frac{2}{2}\right] + \left[\frac{4}{2}\right] = 3.$

On the other hand, $2 * (3 * 4) = 2 * \left(\left[\frac{3}{2}\right] + \left[\frac{4}{2}\right] \right) = 2 * 3 = \left[\frac{2}{2}\right] + \left[\frac{3}{2}\right] = 2.$

So, the operation $*$ is not associative. Hence, the structure (Z , $*$) is an infinite groupoid.

Now, we show that the groupoid (Z , $*$) is a Smarandache Groupoid. For a given prime number p , consider the proper subset S = { 0 , 1 , 2, 3 , …. , $p - 1$ } of the set Z. We can easily see that (S , $*$) is a semigroup as $a * b = 0$ for all a , b $\in$ S. Further, for a given prime p , there are many proper subsets of Z, which are always semigroups under $*$. Hence, (Z , $*$) is an infinite Smarandache Groupoid.

In view of the definition of the operation $*$ and *Theorem [2.5]* it is evident that there are infinitely many ways to define the operation $*$ as for each prime number there is such a binary operation on Z . Hence, our assertion is established.

(3.1.2). Let Z^+ be the set of positive integers and p be a prime number. For a given $a \in Z^+$, define a function $f_a : pz^+ \longrightarrow z^+$ by $f_a(x) = \left[\frac{x}{pa}\right] + 1$, for all $x \in Z^+$.

Now, we define the operation $*$ on Z^+ as $x * y = f_x(y)$, for all $x , y \in Z^+$.

It is obvious that $*$ is a binary operation on Z^+ as $x * y = f_x(y) = \left[\frac{y}{px}\right] + 1$ is always a positive integer, for all $x , y \in Z^+$.

Next, we observe that $*$ is not associative . For, take $p = 3$.

$$(4 * 5) * 6 = (f_4(5)) * 6 = \left(\left[\frac{5}{12} \right] + 1 \right) * 6 = 1 * 6 = f_1(6) = \left[\frac{6}{3} \right] + 1 = 3 .$$

On the otherhand,

$$4 * (5 * 6) = 4 * (f_5(6)) = 4 * \left(\left[\frac{6}{15} \right] + 1 \right) = 4 * 1 = f_4(1) = \left[\frac{1}{12} \right] + 1 = 1$$

So, the operation $*$ is not associative. Hence, (Z^+, $*$) is an infinite groupoid.

Now, we show that (Z^+, $*$) is a Smarandache Groupoid. For a given prime number p, consider a proper subset S = { 1 , 2 , . . . , $p - 1$ } of the set Z^+. We can easily see that (S , $*$) is a semigroup as $a * b = f_a(b) = \left[\frac{b}{pa} \right] + 1 = 1$, for all a , b $\in$ S.

Further, for a given prime p there are many proper subsets of Z^+ which are semigroups. Hence, (Z^+, $*$) is an infinite Smarandache Groupoid. inview of the argument provided in (3.1.1), our assertion is established immediately

Corollary 3.2. There are infinitely many infinite Smarandache commutative groupoids.

Proof *:* Obvious, as the binary operation $*$ defined on the set Z of integers in (3.1.1) is commutative.

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